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Social Networks and Crime Decisions: The Role of Social Structure in Facilitating Delinquent Behavior

by Antoni Calvó-Armengol and Yves Zenou

IUI, The Research Institute of Industrial Economics
P.O. Box 5501
SE-114 85 Stockholm
Sweden

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Antoni Calvó-Armengol† Yves Zenou‡

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Abstract

We develop a model in which delinquents compete with each other in criminal activities but may benefit from being friends with other criminals by learning and acquiring proper know-how on the crime business. By taking the social network connecting agents as given, we study the subgame perfect Nash equilibrium of this game in which individuals decide first to work or to become a criminal and then the crime effort provided if criminals. We show that this game always has a pure strategy subgame perfect Nash equilibrium that we characterize. Ex ante identical individuals connected through a network can end up with very different equilibrium outcomes: either employed, or isolated criminal or criminals in networks. We also show that multiple equilibria with different number of active criminals and levels of involvement in crime activities may coexist and are only driven by the geometry of the pattern of links connecting criminals. Using the equilibrium concept of pairwise-stable networks, we then show that the multiplicity of equilibrium outcomes holds even when we allow for endogenous network formation.

Keywords: strategic interactions, multiple equilibria, pairwise-stable networks.

JEL Classification: C72, K42, Z13

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†Department of Economics, Universitat Autònoma de Barcelona, Edifici B, 08193 Bellaterra (Barcelona), Spain. Email: antoni.calvo@uab.es. http://selene.uab.es/acalvo

‡Corresponding author: IUI, University of Southampton and GAINS. Address of correspondence: IUI, The Research Institute of Industrial Economics, Box 5501, 114 85 Stockholm, Sweden. E-mail: yvesz@iui.se

1 Introduction

Stark differences in crime rates are commonly observed across different social groups and/or locations displaying otherwise identical economic fundamentals. One of the main explanations put forward to account for this puzzle is the presence of social multiplier effects on individual crime decisions. Social multiplier effects arise when, as the result of social interactions, individual decisions feed each other and, altogether, generate a premium for the observed aggregate outcome.[1]

In the criminology/sociology literature, the influence of friends on criminal behavior has been acknowledged for a long time (see, e.g., Shaw and McKay 1942, Sutherland 1947, Sarnecki 2001 and War 2002).[2] In economics, the empirical evidence collected so far suggests that peer effects are, indeed, very strong in criminal decisions. For instance, Case and Katz (1991), using data from the 1989 NBER survey of youths living in low-income Boston neighborhoods, find that a 10 percent increase in the neighborhood juvenile crime rate increases the individual probability to become a delinquent by 2.3 percent. Also, using data from the Moving to Opportunity experiment, Ludwig *et al.* (2001) estimate that relocating families from high- to low-poverty neighborhoods reduces juvenile arrests for violent offences by 30 to 50 percent of the arrest rate for control groups.[3]

Even though the empirical literature emphasizes the role of social interactions in crime behavior, theoretical economic models accounting for this fact are a bit scarcer. The aim of this paper is precisely to study the role of social networks and social structure in facilitating criminal behavior.

For this purpose, we develop a model of crime decision where delinquents exert both a positive and a negative externality on each other. More precisely, we assume that delinquents compete with each other in criminal activities, but benefit from being friends with other criminals by learning and acquiring proper know-how on the crime business. The negative externality induced by the competition for the booty is global, and affects all criminals in the crime pool. On the contrary, the positive externality of know-how sharing is of local nature, and is tailored to the details of the local geometry of friendship links. Throughout, the network of social contacts is modelled by a graph where nodes stem for the players and links represent direct friendship relationships. Following the standard crime model (Becker, 1968), each individual has to make a choice between becoming a criminal and participating in the labor market (these two activities being mutually exclusive) by implementing a cost-benefit analysis. The geometry of the network links and the local positive externalities they induce are a key determinant of individual crime decisions.

In the first part of the paper, the social network connecting agents is exogenously given. We analyze the subgame perfect Nash equilibrium of this game in which individuals decide first to work or to become a criminal and then the crime effort provided if criminals. We consider two cases: general networks, that have

[1] With respect to what would be observed if individual outcomes were merely juxtaposed to each other, without cross influences of any sort.

[2] For instance, in their seminal study, Shaw and McKay (1942) show that delinquent boys in certain areas of US cities have contact not only with other delinquents who are their contemporaries but also with older offenders, who in turn had contact with delinquents preceding them, and so on ... This contact means that the traditions of delinquency can be and are transmitted down through successive generations of boys and across members of the same generation, in much the same way that language and other social forms are transmitted.

[3] Two more recent papers by Chen and Shapiro (2003) and Bayer, Pintoff and Pozen (2003) find also strong peer effects in crime by investigating the influence individuals serving time in the same facility have on the subsequent criminal behavior of offenders. The former shows that worsening prison conditions significantly increases post-release crime. The latter provides strong evidence that, for many types of crimes, learning is significantly enhanced by access to peers with experience with that crime.

arbitrary geometry, and regular networks, in which each criminal has exactly the same number of links.[4] For general networks, we establish conditions under which this game always has a pure strategy subgame perfect Nash equilibrium that we fully characterize. One of our main result is to show that different equilibria with different number of active criminals and differing levels of involvement in crime activities may coexist. Thus *ex ante identical individuals connected through a network can end up with very different equilibrium outcomes: either employed, or isolated criminal or criminals in networks.* This ex post heterogeneity is amenable to the presence of geometric asymmetries in the locations held by individuals in the original network (some being initially very well-connected, some not), and to the corresponding graph-theoretical restrictions inherited by the geometry of the sub-graphs of the original graph.[5] With regular networks, we obtain a closed-form expression for equilibrium crime effort levels and conclude that the social setting and its tightness have a multiplier effect on aggregate observed crime levels, which we refer to as the *network effect* on crime. The social network is thus a channel both for equilibrium multiplicity and for multiplier effects.[6]

In the second part of the paper, we explicitly deal with the issue of endogenous network formation. Using the equilibrium concept of pairwise-stable networks (i.e. a network is pairwise-stable if no player gains by cutting an existing link, and no two players gain by forming a new link), we show that all complete networks as well as all induced networks that are complete are pairwise stable. We also show that, both for general and regular networks, the multiplicity of equilibrium outcomes holds even when we allow for endogenous network formation. In this case, the multiplicity of equilibria is not directly driven by the geometry of the network connecting players. Rather, it is indirectly related to the network modelling approach and reflects the structural coordination problem that plagues decentralized procedures of network formation.

To link social interactions with crime is not new as it has been done in partial equilibrium setups (Sah, 1991, and Glaeser, Sacerdote and Scheinkman, 1996, 2002), in dynamic general equilibrium models (İmrohoroğlu, Merlo and Rupert, 2000, and Lochner, 2002) and in search-theoretic frameworks (Burdett, Lagos and Wright, 2003, 2004, and Huang, Laing and Wang, 2003). What is new, however, is to study the impact of the *network structure* and its geometric details on individual and aggregate criminal behavior, and to show that the social setting is a new plausible channel for equilibrium multiplicity.

In Sah (1991), the social setting only affects the individual perception of the costs of crime, and is thus conducive to a higher or a lower *sense of impunity*. In our model, the social network leads to a higher or a lower *real impunity* or vulnerability to punishment as it affects the available know-how or crime technology. As in Glaeser, Sacerdote and Scheinkman (1996), criminal interconnections act as a social multiplier on aggregate crime. Contrary to this paper, though, our model does not posit any ex ante heterogeneity across agents, neither it is restricted to the particular case of regular lattices. Rather, the interaction structure is modelled as a general graph that connects ex ante homogenous individuals. Our results then relate the details of the geometry of the network links to individual and aggregate outcomes. In particular, we show

[4] Consider a population of n agents. Then both the *complete graph* (in which each agent is in direct relationship with every other agents so that each has $n-1$ direct contacts) and the *circle* (in which each agent has two direct contacts) are examples of regular networks where all agents have the same number of direct contacts and, thus, hold symmetric positions in the network. By contrast, in a *star-shaped network* (where one central agent is in direct contact with all the other peripheral agents who, in turn, are only connected to this central agentin the star), agents hold very asymmetric positions, as one agent has $n-1$ direct links while all the other agents only have 1 direct link. General networks with arbitrary geometry encompass all these cases.

[5] That may arise even when the original graph is regular, as in the example with $n = 4$ agents on a circle.

[6] Networks of contacts also play a central role in labor markets by disseminating information among workers and by matching job-seekers with vacancies. Calvó-Armengol (2003) and Calvó-Armengol and Jackson (2003a, 2003b) propose respectively a static and a dynamic model of the labor market that accounts for the social network through which agents hear about jobs, and displaying multiplicity of equilibria and social multiplier effects.

that the social multiplier effect on crime increases with the density of the pattern of connections.[7]

İmrohoroğlu, Merlo and Rupert (2000) and Lochner (2002) both propose dynamic general equilibrium models in which social interactions are introduced via political economy (majority voting) and education respectively. Our approach is quite different since the model is static and the social setting is modeled by a network of relationships in which friends give information on how to reduce the probability to be caught.

Finally, recent studies use search-matching models to analyze crime behavior (Burdett, Lagos and Wright, 2003, 2004, and Huang, Laing and Wang, 2003). All these models generate multiple equilibria, as in our framework. However, the mechanisms at work are quite different. In the search approach, the strategic interaction in wage setting can be the source of multiple equilibria (Burdett, Lagos and Wright, 2003, 2004),[8] while in our model, it is mainly the structure of the network that is responsible for it.

The plan of the paper is as follows. In the next section, we present the model. Section 3 deals with exogenous networks while endogenous networks are studied in section 4. Finally, section 5 concludes. All proofs are relegated to the appendix.

2 The model

The network $N = \{1, \ldots, n\}$ is a finite set of agents. Agents are connected by a network of social connections. We represent social connections by a graph g, where $g_{ij} = 1$ if i knows j and $g_{ij} = 0$ otherwise. Links are taken to be reciprocal, so that $g_{ij} = g_{ji}$. By convention, $g_{ii} = 0$.

Individual actions Agents may either be criminals or participate in the labor force. Agents in the labor force earn a wage w, while those involved in criminal activities receive an expected payoff equal to:

$$d_i = p_i (y_i - f) + (1 - p_i) y_i = y_i - p_i f,$$

where y_i is i's booty, p_i his probability to be caught and f the corresponding fine.

Agents choose between entering the labor market or becoming a criminal based on the payoffs attached to those activities, respectively w and d_i.[9] Criminals also decide how much effort $e_i > 0$ they devote to delinquent behavior. By convention, $e_i = 0$ means that agent i enters the labor market.

Denote by $e = (e_1, \ldots, e_n)$ a population crime effort profile and by e_{-i} the crime effort profile of all agents but i. We assume that the booty y_i of some criminal i depends on e the following way. First, $\partial y_i (e) / \partial e_i \geq 0$,

[7] Another related contribution is Verdier and Zenou (2004) where the focus is on the geographical space. If everybody believes that blacks are more criminal than whites (even if there is no basis for this), they show that ex ante identical agents who only differ by the color of their skin (blacks versus whites) may have very different outcomes in terms of crime behavior because of amplifying effects. It is because the authors consider the complete interactions of three markets (crime, labor and housing) and because workers rationally anticipate these interactions that they obtain these magnification results.

[8] In Huang, Laing and Wang (2003) multiple equilibria stem from three effects: the interdiction effect (higher crime rate reduces the likelihood that any given criminal is interdicted by the authorities, which in turn lowers the expected costs of committing crime), the appropriation effect (higher crime rate lowers the expected returns to formal employment) and the human capital effect (higher crime rate reduces human capital accumulation, which discourages firms from entering the community, which raises unemployment and thus crime).

[9] Because our model is quite complex, the labor market is kept as simple as possible. However, friends provide information not only on crime but also on job opportunities. The interaction between these two networks (crime and labor market) is certainly an important feature of crime decisions since delinquents have in general few employed friends who can provide information about jobs. The issue is investigated in Calvó-Armengol, Verdier and Zenou (2003) in a context of much simplier networks in which individuals only belong to mutually exclusive two-person groups (dyads).

that is, the individual booty increase with one's involvement in crime $e_i > 0$. Second, $\partial y_i(e)/\partial e_{-i} \leq 0$,[10] which reflects rivalry or competition in individual crime gross payoffs.

Network payoffs Besides from being competitors in the crime market, criminals may also benefit from having criminal mates. This benefit may take the form of know-how sharing about delinquent behavior between individuals that know each other. We assume that the higher the criminal connections to a criminal and/or the higher the involvement in criminal activities of these connections, the lower his individual probability to be caught. The idea is as follows. There is no formal way of learning to become a criminal, no proper "school" providing an organized transmission of the objective skills needed to undertake successful criminal activities. Given this lack of formal institutional arrangement, we believe that the most natural and efficient way to learn to become a criminal is through the interaction with other criminals.[11] Delinquents learn from other criminals belonging to the same network how to commit crime in a more efficient way by sharing the know-how about the "technology" of crime. In our model, we capture this local nature of the mechanism through which skills are acquired by relating the individual probability to be caught to the crime level involvement of one's direct mates, and by assuming that this probability decreases with the corresponding local aggregate level of crime. In other words, in our model, individuals *learn* illegal conduct from others but *practice* it alone.

Formally, given a network g and a crime profile e, $\partial p_i(e)/\partial e_j \leq 0$ whenever $g_{ij} = 1$. We also assume that $\partial p_i(e)/\partial e_i \geq 0$, that is, the probability to be caught increases with one's exposure to crime.

As a result, because of our two assumptions, $\partial y_i(e)/\partial e_{-i} \leq 0$ and $\partial p_i(e)/\partial e_j \leq 0$ whenever $g_{ij} = 1$. Criminal friends (social connections) create both positive and negative externalities between each other. The competitive effect acts as a negative externality exerted by every individual criminal to every other individual in the delinquent pool. By contrast, the positive externality in the form of know-how is only exerted at the local level, among those directly connected individuals. The span and intensity of this latter externality is determined by the shape of the social network and varies across individuals (and their graph locations) with the geometry of the network of links. In particular, ex ante identical individuals holding asymmetric network locations face different externality intensities. Because these externalities are at the heart of the cost-benefit analysis undertaken at the individual level, the equilibrium pattern of network decisions is likely to reflect ex post heterogeneity among individuals. In solving the tension between the two forces, the geometry of the network of connections plays a major role, that we explore.

Given a fine f, the individual expected gains from becoming a criminal are thus $d_i(e, f, g) = y_i(e) - p_i(e, g) f$. For sake of tractability, we set:

$$\begin{cases} y_i(e) = e_i(1 - \sum_{j \in N} e_j) \\ p_i(e, g) = p_0 e_i(1 - \sum_{j \in N} g_{ij} e_j) \end{cases}$$

Letting $\phi = p_0 f$ denote the marginal expected punishment cost for an isolated agent, individual expected payoffs are thus:

$$\pi_i(e, w, \phi, g) = \begin{cases} e_i(1 - \sum_{j \in N} e_j) - \phi e_i(1 - \sum_{j \in N} g_{ij} e_j), \text{ if } e_i > 0 \\ w, \text{ if } e_i = 0 \end{cases} \quad (1)$$

[10] For the component-wise ordering for e_{-i}.

[11] Sutherland (1947) and Akers (1998) expressly argue that criminal behavior is learned from others in the same way that *all* human behavior is learned. Indeed, young people may be influenced by their peers in all categories of behavior - music, speech, dress, sports, and *delinquency*.

The crime decision game Consider some given network g connecting individuals, a wage $w > 0$, and a marginal expected punishment cost for an isolated individual ϕ. Consider the following two-stage decision game.

In the first stage, players in N decide to enter the labor market or to become criminals. This is a simple binary decision.

In the second stage, all players who have decided to become criminals choose non-cooperatively the crime effort level $e_i > 0$ they provide.

The payoffs corresponding to this game are given in (1). To simplify notations, we do not explicitly describe the strategies of this sequential game. The decision to enter the labor market (resp. become a criminal) at the first stage is implicitly captured by setting $e_i = 0$ (resp. $e_i > 0$) at the second stage.

Observe that the payoff $\pi_i(e_i, e_{-i})$ for player i as a function of e_i is discontinuous at 0. Observe also that payoffs depend on the underlying network connecting players. To be more precise, only those network links that connect active criminals among themselves influence such payoffs. Hence, the actual network topology that indeed affects payoffs is shaped by the strategies adopted by the players. The impact of individual decisions on the criminals' network and on payoffs thus goes either ways. This interplay between individual strategies and interaction structure is at the heart of our analysis.

We denote the crime decision game by $\Gamma(w, \phi, g)$.

3 Exogenous networks

In this section, we first characterize the subgame perfect Nash equilibria in pure strategies of the crime decision game for *any* network geometry connecting players. We also provide an algorithm to compute such equilibria for *general* networks. Then, for the particular class of networks with a *regular* geometry, that is, such that all players in the network have exactly the same number of direct links, we analyze some properties of the corresponding equilibria for which we obtain closed-form expressions.

3.1 General networks

Agent i's gains from crime are:

$$d_i(e, \phi, g) = e_i(1 - \sum_{j \in N} e_j) - \phi e_i(1 - \sum_{j \in N} g_{ij} e_j),$$

with cross-derivatives $\partial^2 d_i / \partial e_i \partial e_j = -1 + \phi g_{ij}$. When $\phi < 1$, crime effort decisions are strategic substitutes, while when $\phi \geq 1$, crime effort decisions are (local) strategic complements, that is, $g_{ij} = 1$ implies $\partial^2 d_i / \partial e_i \partial e_j \geq 0$.[12] In this latter case, the equilibria of the crime decision game are simple to characterize.

Lemma 1 *When $\phi \geq 1$, at the unique subgame perfect Nash equilibrium of $\Gamma(w, \phi, g)$, all agents enter the labor force.*

From now on we restrict to the case $\phi < 1$.

In what follows, we characterize all the subgame perfect Nash equilibria in pure strategies of $\Gamma(w, \phi, g)$. We proceed backwards. To obtain the Nash equilibria of the second-stage game, we introduce an auxiliary game

[12] Strictly speaking, crime efforts are strategic substitutes or strategic complements insofar we ignore the outside opportunity to enter the labor force or, equivalently, we set $w = -\infty$.

The auxiliary game Consider the following game played on the network g. Players in N select an effort level $e_i \in \mathbb{R}$, and player i's payoffs are $d_i(e, \phi, g)$. Denote this game by $\Gamma^*(\phi, g)$.

Lemma 2 *There exists a finite set $\mathcal{Z} \in \mathbb{R}$ such that, for all $\phi \in (0,1) \backslash \mathcal{Z}$ and all networks g, the set of Nash equilibria in pure strategies of $\Gamma^*(\phi, g)$ exists and is unique.*

We refer to the situations in which $\phi \notin \mathcal{Z}$ as generic situations. Since $\mathcal{Z}$ is a finite set, the whole set of nongeneric situations has Lebesgue measure of zero. For the equilibrium analysis, we restrict from now on to generic situations.

For all ϕ and g, denote by $e^*(\phi, g) = (e_1^*(\phi, g), \ldots, e_n^*(\phi, g)) \in \mathbb{R}^n$ the unique Nash equilibrium in pure strategies of $\Gamma^*(\phi, g)$. This equilibrium is easy to obtain. It is the unique solution to the following linear system of n equations with n unknowns $e_1, ..., e_n$:

$$2e_i + \sum_{j \neq i}(1 - \phi g_{ij})e_j = 1 - \phi, i \in N. \tag{2}$$

The following result states that for small enough values of ϕ, active criminals exert a non-negative crime effort at the unique Nash equilibrium of the auxiliary game.

Lemma 3 *There exists a unique $0 < \overline{\phi} \leq 1$ such that, for all $\phi < \overline{\phi}$, the unique Nash equilibrium of $\Gamma^*(\phi, g)$ is such that $e_i^* \geq 0$, for all $i \in N$.*

From now on, we restrict to the case where $\phi < \overline{\phi}$.

Induced subnetworks Consider some network g on N.

Definition 1 *For all subset $S \subset N$ of players in N, let $g(S)$ be such that $g_{ij}(S) = 1$ if and only if both $g_{ij} = 1$ and $i, j \in S$. We say that $g(S)$ is the network induced by g on S.*

The network $g(S)$ is obtained the following way. Originally, all agents in N are connected according to the network g. Suppose now that we eliminate some player i. By eliminating player i, we also eliminate all the direct links in the original network g stemming from i. We denote by $g(N\backslash\{i\})$ the resulting network on the remaining population $N\backslash\{i\}$, that we term *the network induced by g on $N\backslash\{i\}$*. More generally, the network $g(S)$ induced by g on some given subset S of agents is obtained by eliminating in g every node corresponding to an agent in $N\backslash S$ and every direct link stemming from those agents.

In general, the geometry of the network $g(S)$ induced by g on S depends not only on the total number $|S|$ of players being removed from the original network, but also on the precise identity of the players in the set S.[13] This is best illustrated with a simple example.

Suppose, for instance, that g is a star centered on i. Formally, $g \neq \emptyset$, and $jk \in g$ implies that either $j = i$ or $k = i$. Consider some subset $S \subset N$. Then, if $i \in S$, the induced network $g(S)$ is also a star centered on i, while if $i \notin S$, $g(S) = \emptyset$.

The outcome of the first-stage game is a partition of the population in a set S of criminals and a set $N\backslash S$ of workers, together with the network $g(S)$ induced by g on the pool S of criminals. By the remark

[13] Formally, the set $\{g(S) : S \subset N\}$ of induced networks generated by g embedded with the inclusion ordering on networks is a lattice, namely, it is a partially ordered set for this network inclusion ordering, and it both contains the union and the intersection of each pair of its elements.

above, the geometry of $g(S)$ generally depends on the precise label of the players in the criminal pool. As we will see below, this implies that the set of subgame perfect Nash equilibria of the crime decision game is generally not unique.

The subgame perfect Nash equilibria of the crime decision game The subgame perfect Nash equilibria of $\Gamma(w, \phi, g)$ are obtained from the Nash equilibria of the auxiliary game $\Gamma^*(\phi, g(S))$ played by subsets of players $S \subset N$ and connected by the corresponding induced network $g(S)$.

The way we compute the subgame perfect Nash equilibrium is as follows. Given n agents in the economy, we start with a given graph of relationships that could have any shape. We then compute the Nash equilibria in crime-effort levels of the auxiliary game for all possible subsets of criminals S, i.e., we determine the crime efforts $e_i^*(\phi, g(S))$, $i \in S$ for such players. Note that the network of know-how sharing among a given subset $S \subset N$ of criminals is given by the network $g(S)$ induced on S by the initial graph.[14] We then look at all the induced graphs of criminals that are indeed sustainable as a subgame perfect Nash equilibrium, where agents compare crime benefits with the outside option provided by the labor market (i.e. no unilateral deviation shall be profitable for neither type of agents, nor criminal, nor worker). Subgame perfect Nash equilibria are thus characterized by the subset S of active criminals, the network $g(S)$ connecting them, and the corresponding crime effort levels. All remaining players in $N \backslash S$ are workers. The network $g(S)$ induced by the initial graph on S and connecting active criminals is unambiguously defined.

To be more precise, for all $S \subset N$, denote by $e^*(\phi, g(S))$ the unique Nash equilibrium in pure strategies of $\Gamma^*(\phi, g(S))$. Define:

$$\begin{cases} \overline{m}(\phi, S) = \min\{e_i^*(\phi, g(S)) \mid i \in S\}, \text{ for all } S \subset N \\ \underline{m}(\phi, S) = \max\{e_j^*(\phi, g(S \cup \{j\})) \mid j \notin S\}, \text{ for all } S \subset N, S \neq N \\ \underline{m}(\phi, N) = 0 \end{cases}$$

We have the following result:

Proposition 1 *Let $S \subset N$. If $\overline{m}(\phi, S) > \sqrt{w} > \underline{m}(\phi, S) \geq 0$, then the crime decision profile $e = (e_1, ..., e_n)$ where $e_i = e_i^*(\phi, g(S))$ for all $i \in S$, and $e_i = 0$ otherwise is a subgame perfect Nash equilibrium of $\Gamma(w, \phi, g)$. Moreover, all the subgame perfect Nash equilibria of $\Gamma(w, \phi, g)$ are of this sort.*

In equilibrium, individual decisions are consistent across players and no unilateral deviation is profitable. Here, the implications are twofold. First, no active criminal in S gains by exerting a different effort level. This is true because the vector of crime efforts is an equilibrium of the auxiliary game $\Gamma^*(\phi, g(S))$ played by such criminals.

Second, no worker gains by becoming a criminal, neither a criminal gains by becoming a worker, which is equivalent to $\overline{m}(\phi, S) > \sqrt{w} > \underline{m}(\phi, S)$. Indeed, the payoff to a criminal exerting an effort $e_i^*(\phi, g(S))$ is equal to $e_i^*(\phi, g(S))^2$.[15] When $\overline{m}(\phi, S)^2 > w$, no criminal in S gains by entering the labor force. Symmetrically, when $w > \underline{m}(\phi, S)^2$, no worker in $N \backslash S$ gains by entering the crime business.

[14] Links in $g \backslash g(S)$ connect workers to workers or workers to criminals. Therefore, no actual know-how is being shared between the end nodes of such connections, and links in $g \backslash g(S)$ are redundant at every equilibrium where only agents in S are criminals.

[15] Recall that $e^*(\phi, g(S))$ is a Nash equilibrium of $\Gamma^*(\phi, g(S))$ or, equivalently, the unique solution to the system of equations (2). Therefore,

$$e_i^* = (1-\phi) - \sum_{j \in N} (1 - \phi g_{ij}) e_j^*.$$

By definition, and after some manipulation, $d_i(e^*) = e_i^*[(1-\phi) - \sum_{j \in N}(1-\phi g_{ij})e_j^*]$, implying that $d_i(e^*) = e_i^{*2}$.

Proposition 1 does not establish uniqueness of the subgame perfect Nash equilibria of $\Gamma(w, \phi, g)$. In fact, different crime patterns can generally be sustained at equilibrium for the same parameter values. It suffices that the condition

$$\overline{m}(\phi, S) > \sqrt{w} > \underline{m}(\phi, S) \tag{3}$$

holds for two different subsets S and S' of players. Note that $\overline{m}(\phi, S)$ and $\underline{m}(\phi, S)$ depend both on S and on the geometry of the network $g(S)$ induced by g on S, as can be seen from equations (2) from which $e_i^*(\phi, g(S))$, $i \in S$ are calculated. Given the lattice structure of the set of induced networks, it may well be the case that two subsets S and S' connected through two induced networks $g(S)$ and $g(S')$ of different geometry both satisfy (3). This may be true with two subsets S and S' of either identical or different size.

If S and S' have different sizes, then the number of active criminals differs across the two corresponding equilibria of $\Gamma(w, \phi, g)$. If S and S' have identical size, there is exactly the same number of criminals in both equilibria. Yet, because $g(S)$ and $g(S')$ may differ, both the individual and the aggregate crime effort level are generally different across the two equilibria.

Thus, one of our main result is to demonstrate that different equilibria with different number of active criminals and differing levels of involvement in crime activities may coexist. Note that *this multiplicity of equilibria is obtained with ex ante identical agents, and is only driven by the geometry of the pattern of links connecting them.* Note also that multiplicity of equilibria, as obtained in our model, is consistent with the high (unexplained) variance of crime levels across geographic locations (see, e.g., Glaeser, Sacerdote and Scheinkman, 1996).

The existence of subgame perfect Nash equilibria in pure strategies of $\Gamma(w, \phi, g)$ is a side-product of the previous characterization result.

Corollary 1 *For all w, ϕ and g, the crime decision game $\Gamma(w, \phi, g)$ has a pure strategy subgame perfect Nash equilibrium.*

Example Consider $n = 4$ players arranged on a circle g where each player has exactly two direct contacts. Fix $\phi \in (0, 1)$. Then, any of the crime pattern depicted in Figure 1 can be sustained as a subgame perfect Nash equilibrium for a suitable wage range.[16] Criminals (resp. workers) are depicted with full (resp. empty) nodes, and dashed lines represent the links that are deleted in each induced network.

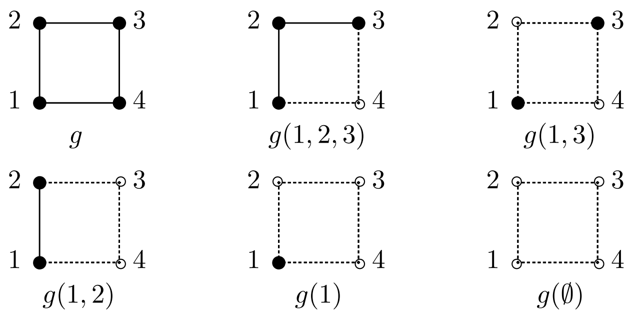


Figure 1. Crime decision equilibrium patterns for $n = 4$ players on a circle.

[16] In fact, $g(1, 3)$ can only emerge at equilibrium under some conditions both on w and on ϕ. See the appendix for more details.

For some wage range, both the set of players $\{1,2\}$ and the set of players $\{1,3\}$ can constitute the pool of active criminals at equilibrium.[17] Note that, in both cases, there are exactly two criminals at equilibrium. Yet, the aggregate crime rate is higher when criminals are in $\{1,2\}$ than in $\{1,3\}$. Indeed, criminals in $\{1,2\}$ connected by $g(1,2)$ are direct network mates, while criminals in $\{1,3\}$ connected by $g(1,3)$ do not know each other. As such, with $g(1,2)$, criminals possess a higher know-how on the crime business than with $g(1,3)$ and exert a higher individual crime effort level. This is in accordance with the empirical findings of Thornberry *et al.* (2003) who show that networks of criminals or gangs amplify delinquent behaviors

When the set of criminals at equilibrium is $\{1,2,3\}$, active criminals hold asymmetric positions in the corresponding know-how sharing network $g(1,2,3)$. Indeed, player 2 has two direct links with players 1 and 3, while the latter are only connected to player 2. At equilibrium, criminal 2 exerts a higher crime effort than criminals 1 and 3, reflecting the comparative knowledge advantage accruing from his central position in the network.

Finally, individual crime efforts decrease with the mass of active criminals across the different equilibria, which corresponds to a crowding out effect that results from competition for the booty.

3.2 Regular networks

In order to obtain closed-form solutions, we now focus on regular networks.

In a regular network, all agents have the same number of direct contacts k, that is,

$$\sum_{j\in N} g_{ij} = k, \text{ for all } i \in N.$$

We refer to k as the degree of g. For instance, the network connecting $n = 4$ players in the previous example is a regular network of degree 2, where each player has exactly two links.

The following result provides a general characterization of the subgame perfect Nash equilibria of the crime decision game where criminals, at equilibrium, are connected through a regular network.

Proposition 2 *At every subgame perfect Nash equilibrium of* $\Gamma(w,\phi,g)$ *with a set of criminals* S*, and where* $g(S)$ *is a regular network of degree* k*, the equilibrium crime effort levels are:*

$$e_i^*(\phi, g(S)) = \frac{1-\phi}{|S|+1-k\phi}, \textit{ for all } i \in S.$$

According to the previous expression, an increase in the per individual number of links k raises criminal effort. The aggregate level of crime $\sum_{i\in S} e_i^*$ also depends positively on the degree of the criminals' network. Criminal interconnections thus generate a crime level premium[18] that increases with the density of interconnections among criminals. The social setting and its tightness have a multiplier effect on aggregate observed crime levels which we refer to as the *network effect* on crime.

Also, both individual and aggregate crime decrease with the number $|S|$ of active criminals, which corresponds to the *crowding-out effect* on the booty induced by competition among criminals. Finally, the negative dependence of crime outcomes on the level of punishment ϕ corresponds to a standard *deterrence effect*.

[17] It suffices that $\frac{1-\phi}{3} > \sqrt{w} > \frac{(1+2\phi)(1-\phi)}{4+4\phi-2\phi^2}$. This wage range condition is non-empty whenever $\phi < \widetilde{\phi}$, for some uniquely defined $\widetilde{\phi} \in (0,1)$. See the appendix for more details.

[18] With respect to the case where social interactions among criminals are absent.

The previous characterization result does not guarantee existence of symmetric equilibria of the crime decision game on regular networks. Here, the equilibrium existence problem is, essentially, a *graph-theoretical existence problem*. It amounts to identifying topological conditions on the original network g such that $g(S)$ be regular of degree k. We ignore this problem by focusing on a particular type of networks where these topological conditions are always met and existence is trivially guaranteed.

Suppose that the initial network connecting players is the complete graph g^N on N, where every player is directly linked to every other player, that is, $g_{ij}^N = 1$, for all $i \neq j$. In g^N, each player has exactly $n-1$ connections.

Let $S \subset N$. Then, in the network $g^N(S)$ induced by g^N on S, every player in S is directly linked to every other player in S. In words, the network induced by g^N on any subset S of N is the complete graph on S. Formally, $g^N(S) = g^S$. With g^S, agents in S are fully interconnected and have exactly $|S|-1$ links each.

Define:

$$s^*(w, \phi) = \frac{1}{\sqrt{w}} - \frac{1+\phi}{1-\phi}$$

For simplicity, assume that $n \geq s^*(w, \phi)$.

Corollary 2 *If $\sqrt{w} \leq (1-\phi)/(1+\phi)$, there exists a subgame perfect Nash equilibrium of $\Gamma\left(w, \phi, g^N\right)$ with exactly $s^*(w, \phi)$ criminals fully interconnected and $n - s^*(w, \phi)$ workers. The criminal effort levels are:*

$$e^* = \frac{1-\phi}{1+\phi+s^*(w, \phi)(1-\phi)}.$$

With fully interconnected networks, aggregate crime increases with the total number of active criminals but at a decreasing rate.[19]

More precisely, with complete networks, the criminal size pool has both a positive and a negative effect on aggregate crime. First, crowding-out implies that aggregate crime decreases. Second, in a complete network, the individual number of links increases with the total number of criminals and, by the network effect, aggregate crime increases.

The network effect on crime also implies that the elasticity of aggregate crime with respect to the deterrence effort ϕ is a decreasing and concave function of ϕ.

4 Endogenous networks

In our model, the payoffs accruing to criminals depend both on their involvement in crime and on the underlying network connecting them. In the previous section, the network was exogenous and criminals select the crime effort level they wish to exert. In this section, networks become a manipulable device and criminals decide also with whom to establish links.

Pairwise-stable networks Consider some payoff function $u(g) = (u_1(g), \ldots, u_n(g))$ that assigns a payoff to every agent in N as a function of the underlying network g connecting them. The concept of pairwise-stability introduced by Jackson and Wolinsky (1996) provides a network equilibrium notion that delivers sharp results for our analysis.

We denote by $g+ij$ (resp. $g-ij$) the network obtained by adding (resp. deleting) ij to (resp. from) g.

[19] Formally, $\partial(s^*e^*)/\partial s^* > 0$ and $\partial^2(s^*e^*)/\partial s^{*2} < 0$.

Definition 2 *A network g is pairwise-stable for the payoff function u if and only if for all $ij \in g$, $u_i(g) \geq u_i(g - ij)$, and for all $ij \notin g$, $u_k(g) > u_k(g + ij)$ for some $k \in \{i, j\}$.*

In words, a network is pairwise-stable if no player gains by cutting an existing link, and no two players gain by forming a new link. Pairwise-stability thus only checks for one-link deviations.[20] It requires that any mutually beneficial link be formed at equilibrium but does not allow for multi-link severance.

Pairwise-stable networks can be interpreted as the limiting graphs of a dynamic procedure of network formation. Suppose, indeed, that players myopically add or sever links to improve their current status, and that only one link is added or removed at a time. When this process converges, the networks ultimately reached are pairwise-stable.[21]

Pairwise-stability in the crime decision game Consider some crime decision game $\Gamma(w, \phi, g)$.

Let S be the set of active criminals at some subgame perfect Nash equilibrium of $\Gamma(w, \phi, g)$. Criminals in S are connected by the induced network $g(S)$. Their payoffs are given by the unique Nash equilibrium outcome of the crime effort game $\Gamma^*(\phi, g(S))$. These payoffs depend on the geometry of the network connecting them. Some criminals may thus have incentives to manipulate the network at their advantage. Therefore, the network $g(S)$ need not necessarily be pairwise-stable for the Nash equilibrium payoffs of the crime effort game.

4.1 Regular networks

We first analyze regular networks. Because of the graph-theoretical existence problem of regular networks mentioned in section 3.2, we focus on a particular class of induced sub-networks: complete ones. The following result establishes that the criminal network is always pairwise-stable in any equilibrium where the set of criminals is fully interconnected, that is, when the graph connecting criminals is the complete network.

Proposition 3 *Consider some subgame perfect Nash equilibrium of $\Gamma(w, \phi, g)$ where the set of criminals is S and where $g(S) = g^S$. Then, g^S is pairwise-stable for the subgame perfect Nash equilibrium payoffs of the crime effort game played by criminals in S.*

When the pool of criminals is fully interconnected (complete network), they all exert the same crime effort. For any of such criminals, deleting one existing link amounts to receiving relatively less know-how on the crime business than the rest of criminals, and thus weakening ones' position in the competition for the booty, hitherto receiving a lower payoff. The fully interconnected network is thus pairwise-stable.

Observe that this proposition does not require that the initial graph be complete. It says that, if the induced sub-network is complete, at the subgame perfect Nash equilibrium, this sub-network is also pairwise stable.

4.2 General networks

At equilibrium, all pairwise-stable criminal networks, though, are not necessarily fully interconnected. We illustrate this fact with an example.

[20] This weak equilibrium concept is often interpreted as a necessary conditions for stronger stability concepts.

[21] Note that network transitions only concern one link at at time. The dynamics may thus lock in (the set of limiting graphs need not be unique) or cycle. See Jackson and Watts (2002) for more details.

Consider a population of $n = 4$ players. The possible equilibria of the crime decision game, satisfying the added requirement that the network connecting criminals be pairwise-stable, are depicted in Figure 2 as a function of w and ϕ. Full (resp. empty) nodes represent criminals (resp. workers).[22]

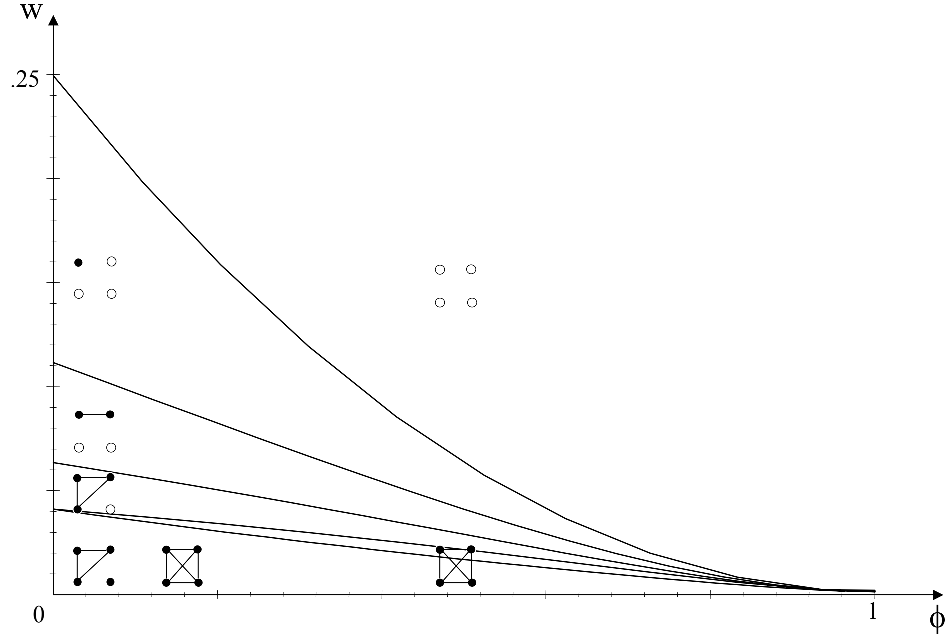


Figure 2. Pairwise-stable networks for the crime decision game as a function of w and ϕ for $n = 4$.

First, as established in Proposition 3, all active criminals being fully interconnected is a pairwise-stable network for every possible size of the criminal pool.

Second, with four active criminals, pairwise-stability is also obtained with a highly asymmetric network where three criminals are fully interconnected while one criminal is completely isolated. Moreover, for a certain region of the parameters' space, this asymmetric network and the complete network are both pairwise stable.

In this region of the parameters' space, when a group of three players is fully interconnected and forms a triad, the isolated player has not incentives to set a link with any of the players in the triad. Indeed, setting a new link has both a positive and a negative effect. First, know-how increases for the newly linked players, which is beneficial to both of them. Second, the newly linked partners exert a higher competitive pressure to

[22] Alternatively, consider the following three-stage game of network formation and crime decision. In the first stage, agents decide non-cooperatively with whom they wish to form links to share their know-how with. We assume that links are formed only when the mutual consent of the two concerned players is obtained. At the end of this first stage, a network of links is obtained. The second and the third stage then correspond to the crime decision game played on the network outcome of stage one. The potential multiplicity of the subgame perfect Nash equilibrium outcomes for the second and third stage crime decision game challenges a full-fledged equilibrium analysis of this three-stage game of network formation and crime decision. In the particular case where $n = 4$, though, the set of subgame perfect equilibria that satisfy the added requirement that any mutually beneficial link be formed at equilibrium coincide with the set of pairwise-stable networks in Figure 2.

the rest of players, which is detrimental to everybody, including themselves. Here, due to the asymmetry of links across players in the original graph, setting a link has an overall negative effect for the isolated player. Absent mutual consent, this link is not formed, and the original asymmetric network is pairwise-stable.[23]

5 Conclusion

The aim of this paper is to provide a theoretical framework that explains the observed differences in crime rates across locations displaying otherwise identical economic fundamentals. For that, we have developed a model in which delinquents compete with each other in criminal activities but may benefit from being friends with other criminals by learning and acquiring proper know-how on the crime business. By taking the social network connecting agents as given, we have studied the subgame perfect Nash equilibria of this game in which individuals decide first to work or to become a criminal and then the crime effort provided if criminals. We have showed that this game always has a pure strategy subgame perfect Nash equilibrium that we characterize. Ex ante identical individuals connected through a network can end up with very different equilibrium outcomes: either employed, or isolated criminal or criminals in networks.

We have also showed that multiple equilibria with different number of active criminals and levels of involvement in crime activities may coexist. This multiplicity of equilibria is driven by the geometry of the pattern of links connecting criminals. Therefore, according to our analysis, different locations with the same economic fundamentals need not experience the same crime level when the social arrangements differ across these areas. Our approach is general and allows for as many interaction structures as different graph geometries.[24]

Using the equilibrium concept of pairwise stability, we then show that the multiplicity of equilibrium outcomes holds even when we allow for endogenous network formation. In this case, the multiplicity of equilibria is not directly driven by the geometry of the network connecting players. Rather, it is indirectly related to the network modelling approach and reflects the structural coordination problem that plagues decentralized procedures of network formation. Indeed, because network relationships are intransitive by nature,[25] robustness checks to a single player link deviations yield, in general, to a broad array of stable geometric arrangements of bilateral links. Therefore, acknowledging the role of the social setting in crime behavior with a model where the details of the network structure matter (as we do here) is bound to produce a multiplicity of equilibrium outcomes.

[23] Note, though, that creating this link would trigger a dynamic process of link addition converging to the complete network, which Pareto dominates the original network in terms of criminal payoffs.

[24] Note also the central role played in our analysis by the subgraphs induced on subsets of agents by the original graph, and the (potential) multiplicity of network architecture that such subgraphs may display.

[25] That is, when player i is directly linked to player j, and player j is directly linked to player k, it does not necessarily imply that player i is directly linked with player k.

Appendix

1. Exogenous networks

Proof of Lemma 1: First, recall that $\pi_i(0, e_{-i}) = w$, for all e_{-i}. Suppose that $\pi_i(\widetilde{e}) > w$ for some $\widetilde{e}$. Necessarily, $\widetilde{e}_i > 0$. Then, $\pi_i(\widetilde{e}) = d_i(\widetilde{e}, \phi, g) = (1-\phi)\widetilde{e}_i(1 - \sum_{j\in N} \widetilde{e}_j) - \phi\widetilde{e}_i \sum_{j\in N}(1 - g_{ij})\widetilde{e}_j > w$. Given that both $(1-\phi)\widetilde{e}_i \leq 0$ and $-\phi\widetilde{e}_i \sum_{j\in N}(1-g_{ij})\widetilde{e}_j \leq 0$, necessarily $1 - \sum_{j\in N}\widetilde{e}_j < 0$, which implies that $y_i(\widetilde{e}) < 0$. ∎

Proof of Lemma 2: Taking first order conditions leads to the following best-response functions:

$$BR_i(e_{-i}) = \frac{1}{2}\left[1 - \sum_{j\neq i} e_j - \phi \sum_{j\neq i}(1-g_{ij})e_j\right], i \in N.$$

The equilibrium profiles e^* of $\Gamma^*(\phi, g)$ are the solutions to $BR_i\left(e^*_{-i}\right) = e^*_i$, $i \in N$. We obtain a linear system of n equations with n unknowns:

$$2e_i + \sum_{j\neq i}(1 - \phi g_{ij})e_j = 1 - \phi, i \in N \tag{4}$$

that we can write in matrix form $M(\phi, g)\cdot e = (1-\phi)\mathbf{1}_n$, where $m_{ii} = 2$, $m_{ij} = 1 - \phi g_{ij}$ for all $i \neq j$ and $\mathbf{1}'_n = (1, \ldots, 1)$. Denote by $\det(M(\phi, g))$ the determinant of $M(\phi, g)$. We show that there exists some finite set $\mathcal{Z} \in \mathbb{R}$ such that $\det(M(\phi, g)) \neq 0$, for all $\phi \notin \mathcal{Z}$ and for all g on N.

Consider some network g. It is readily checked that $\det(M(\phi, g))$ is a polynomial in ϕ of degree smaller than n.[26] Therefore, $\det(M(\phi, g))$ has at most n different roots $\left\{\widetilde{\phi}_1(g), \ldots, \widetilde{\phi}_m(g)\right\}$, $m \leq n$, such that $\det\left(M\left(\widetilde{\phi}_i(g), g\right)\right) = 0$ for all $1 \leq i \leq m$. Given that there are exactly $2^{\frac{n(n-1)}{2}}$ different networks g on N, the set of values $\mathcal{Z}$ of ϕ such that $\det(M(\phi, g)) = 0$ for some g on N is finite, with $|\mathcal{Z}| \leq n2^{\frac{n(n-1)}{2}}$. ∎

Proof of Lemma 3: Consider some network g. When $\phi = 0$, the unique solution to (2) is such that $e^*_i > 0$. By continuity, there exists $0 < \varepsilon(g) \leq 1$ such that the solutions to this system of equations are non-negative for all $\phi \in (0, \varepsilon(g))$. Let g^N such that $g^N_{ij} = 1$, for all $i \neq j$. Let $\overline{\phi} = \min\left\{\varepsilon(g) \mid g \subseteq g^N\right\}$. By construction, $0 < \overline{\phi} \leq 1$ and the solutions to (2) are all non-negative for all g on N whenever $\phi \in (0, \overline{\phi})$. ∎

Proof of Corollary 1: If there exists some $S \subset N$ such that $\overline{m}(\phi, S) > \underline{m}(\phi, S) \geq 0$, then Proposition 1 applies. Otherwise, all agents working is a subgame perfect Nash equilibrium. ∎

Example with $n = 4$ agents located on a circle: Let $N = \{1, 2, 3, 4\}$. For all subset $S \subset N$ of agents, let $e^*(g(S))$ denote the subgame perfect Nash equilibrium profile where only agents in S are criminals, while agents in $N\backslash S$ are workers. The network connecting criminals is then $g(S)$, that is, the network induced by g on S. Specializing and solving the equilibrium equations (2) of $\Gamma^*(\phi, g')$ for all induced network $g' \subset g$,

[26] Indeed, $\det(M(\phi, g))$ is a polynomial of highest degree in ϕ when $g_{ij} = 1$ for all $i \neq j$, in which case $\det(M(\phi, g)) = (1+\phi)^n + n(1-\phi)(1+\phi)^{n-1}$, which is a polynomial in ϕ of degree exactly n.

we obtain the following equilibrium effort levels:

Network g'	g	$g(1,2,3)$	$g(1,3)$	$g(1,2)$	$g(1)$	$g(\emptyset)$
$e_1^*(\phi, g')$	$\frac{1-\phi}{5-2\phi}$	$\frac{1-\phi^2}{4+4\phi-2\phi^2}$	$\frac{1-\phi}{3}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$	0
$e_2^*(\phi, g')$	$\frac{1-\phi}{5-2\phi}$	$\frac{(1+2\phi)(1-\phi)}{4+4\phi-2\phi^2}$	0	$\frac{1-\phi}{3-\phi}$	0	0
$e_3^*(\phi, g')$	$\frac{1-\phi}{5-2\phi}$	$\frac{1-\phi^2}{4+4\phi-2\phi^2}$	$\frac{1-\phi}{3}$	0	0	0
$e_4^*(\phi, g')$	$\frac{1-\phi}{5-2\phi}$	0	0	0	0	0

Simple algebra shows that $e_1^*(g(1)) > e_1^*(g(1,2)) > e_1^*(g(1,3)) > e_1^*(g(1,2,3)) > e_1^*(g)$. Also, $e_2^*(g(1,2,3)) > e_1^*(g(1,3))$ is equivalent to $2\phi^2 + 2\phi - 1 > 0$. This polynomial of order two in ϕ has exactly one root $\widetilde{\phi} = (\sqrt{3}-1)/2$ contained in $(0,1)$, and the inequality holds if and only if $\phi > \widetilde{\phi}$.

We thus have the following inequalities:

$$\begin{cases} e_1^*(g(1)) > e_1^*(g(1,2)) > e_1^*(g(1,3)) > e_2^*(g(1,2,3)) > e_1^*(g(1,2,3)) > e_1^*(g), \text{ if } 0 < \phi < \widetilde{\phi} \\ e_1^*(g(1)) > e_1^*(g(1,2)) > e_2^*(g(1,2,3)) > e_1^*(g(1,3)) > e_1^*(g(1,2,3)) > e_1^*(g), \text{ if } \widetilde{\phi} < \phi < 1 \end{cases}$$

for some uniquely defined $\widetilde{\phi} \in (0,1)$. From these inequalities, we deduce the following:

Equilibrium network	g	$g(1,2,3)$	$g(1,3)$	$g(1,2)$	$g(1)$	$g(\emptyset)$
Upper bound for $\sqrt{w}$	$\frac{1-\phi}{5-2\phi}$	$\frac{1-\phi^2}{4+4\phi-2\phi^2}$	$\frac{1-\phi}{3}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$	$+\infty$
Lower bound for $\sqrt{w}$	0	$\frac{1-\phi}{5-2\phi}$	$\frac{(1+2\phi)(1-\phi)}{4+4\phi-2\phi^2}$	$\frac{1-\phi^2}{4+4\phi-2\phi^2}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$

∎

2. Endogenous networks

Proof of Proposition 3: Let S be the set of active criminals. Suppose that $g(S) = g^S$. Then, the individual crime effort level at equilibrium is:

$$\pi_i^*(g^S) = \frac{1-\phi}{2+(|S|-1)(1-\phi)}, \text{ for all } i \in S.$$

Suppose now that player i cuts some of the existing links ij in g^S. The resulting network is $g^S - ij$. Solving equations (2) yields the following equilibrium payoffs for criminal i in the pool S connected by $g^S - ij$:

$$\pi_i^*(g^S - ij) = \frac{(1-\phi)(1+\phi)}{6+3(|S|-3)(1-\phi)-2(|S|-2)(1-\phi)^2}.$$

Then, $\pi_i^*(g^S) \geq \pi_i^*(g^S - ij)$ is equivalent to

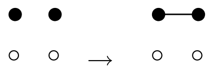
$$f(\phi) = (3|S|-5)(1-\phi)^2 - (3|S|-11)(1-\phi) - 6 \leq 0.$$

Given that $f''(\phi) > 0$, $f(0) = 0$ and $f(1) < 0$, this is true ∎

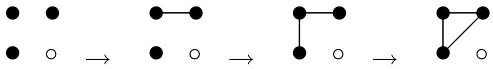
Endogenous networks with $n = 4$ agents: For each network configuration on $N = \{1,2,3,4\}$, we specialize and solve for the equilibrium equations (4). We analyze separately three cases with, respectively, two, three and four criminals. For each case, we establish the sequence of improving networks $g \to g'$, where

g' improves upon g if either (a) two agents i and j in g such that $ij \notin g$ mutually gain by linking each other, and $g' = g \cup \{ij\}$, or (b) one player in g unilaterally gains by seceding some existing links $ij_1, \dots ij_m \in g$, and $g' = g \backslash \{ij_1, \dots, ij_m\}$.

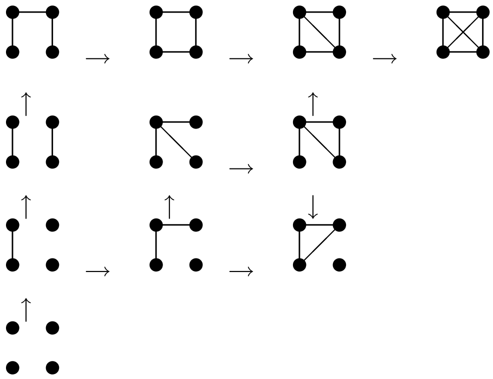
Two criminals: For all $\phi \in (0,1)$, we have:



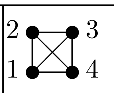
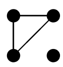
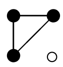
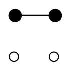
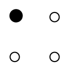
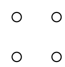
Three criminals: For all $\phi \in (0,1)$, we have:



Four criminals: For all $\phi \in (0,1)$, we have:



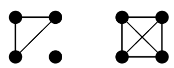
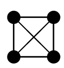
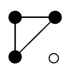
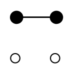
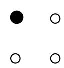
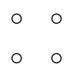
Given the previous sequences of improving paths, we can restrict to the following network configurations for the equilibrium analysis of the two-stage game, for which we compute the equilibrium crime effort levels $(e_1^*, \dots, e_4^*)$:

						
e_1^*	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{5-4\phi}$	$\frac{1-\phi}{4-2\phi}$	0	0	0
e_2^*	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{5-4\phi}$	$\frac{1-\phi}{4-2\phi}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$	0
e_3^*	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{5-4\phi}$	$\frac{1-\phi}{4-2\phi}$	$\frac{1-\phi}{3-\phi}$	0	0
e_4^*	$\frac{1-\phi}{5-3\phi}$	$\frac{(1-\phi)(2-\phi)}{10-8\phi}$	0	0	0	0

Pairwise-comparisons of payoffs leads to the following inequalities:

$$\frac{1-\phi}{2} > \frac{1-\phi}{3-\phi} > \frac{1-\phi}{4-2\phi} > \frac{1-\phi}{5-4\phi} > \frac{1-\phi}{5-3\phi} > \frac{(1-\phi)(2-\phi)}{10-8\phi}, \text{ for all } \phi \in (0,1)$$

from which we deduce the pairwise-subgame perfect Nash equilibrium networks when $\overline{m} > \sqrt{w} \geq \underline{m}$, for the following upper and lower bounds on $\sqrt{w}$:

						
$\overline{m}$	$\frac{(1-\phi)(2-\phi)}{10-8\phi}$	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{4-2\phi}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$	$+\infty$
$\underline{m}$	0	$\frac{(1-\phi)(2-\phi)}{10-8\phi}$	$\frac{1-\phi}{5-3\phi}$	$\frac{1-\phi}{4-2\phi}$	$\frac{1-\phi}{3-\phi}$	$\frac{1-\phi}{2}$

$\blacksquare$

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