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ON UNEXPLAINED PRICE DIFFERENCES

by

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ON UNEXPLAINED PRICE DIFFERENCES

1 INTRODUCTION

Real life phenomena are not always very well explained by economic theory. Large differences in the price of apparently homogenous goods in the same location or in different locations which cannot be attributed to differences in production costs, tariffs and the like, is an example. Still, considerable price dispersion is a fact of life, observed in many studies.[1] Milton Friedman, after having played tennis one morning in Stockholm in 1977, asked the question:"How come that this tennis racket costs twice as much in Sweden as in England, although it is produced in the U.S and there are no tariff differences?"

In this paper I show that there can be a great difference in the price of identical goods in different locations, even though the stores have identical "production costs", i.e., face the same gross purchase price and wages. The reason is that a market with incompletely informed consumers, searching in an optimal way, can have two equilibria. One is at the monopoly price. The other is a price dispersion equilibrium with firms charging prices from marginal cost and up with most of the prices just above marginal cost.[2] Which one of these two equilibria that will prevail depends mainly on the distribution of income. Thus, in one location the

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[1] See for instance Pratt, Wise and Zeckhauser (1979).

[2] For a statement of the general problem, see Rothschild(73). For an analysis of price dispersion in search market with alternative consumer search strategies, see Burdett and Judd (79).

monopoly price could be the equilibrium price, while in another location (city, country) with a different income distribution there could be price disperson equilibrium.

2 THE MODEL[1]

2.1 Consumer behavior

In each period k consumers enter the market. They will make one purchase after having found an acceptable price. Then they leave the market and are replaced by new consumers. They are all confronting a price distribution, represented by the density function f(p). They know the shape of this.[2] However, they don't know which firm is charging which price. They can, however, themselves, investigate the market. At a cost c they can contact a store and be informed of its selling price.

It is well known that an optimal searching consumer will follow a so called reservation price rule[3] which means that there exists a price, reservation price, such that further search is worthwhile if the price found is above the reservation price, while any price below the reservation price is acceptable.

This could be shown in the following way. If the searching consumer at a stage in his search process has found p_m as the lowest price offer, the expected benefit from further search is:

$$\Psi(p_m) = \int_0^{p_m} (p_m - p)\ f(p)\ dp \tag{1}$$

Integrating by parts gives:

$$\Psi(p_m) = \int_0^{p_m} F(p)\ dp$$

where

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[1] The model is identical with the one used in Axell (76) and Axell (77).

[2] For analysis of the case when the consumers do not know the distribution, see Axell (74).

[3] Se for instance Lippman, McCall (1976).

$$F(p) = \int_0^{p_m} f(p)\, dp \tag{2}$$

From this follows that a risk-neutral consumer should follow the following rules:

$$\Psi(p_m) > c \rightarrow \text{keep searching} \tag{3a}$$

$$\Psi(p_m) \leq c \rightarrow \text{stop and accept } p_m \tag{3b}$$

From this we can see that there is a certain price, R, called the reservation price, such that the consumer will accept any offer below or equal to R, and reject any price above R. Clearly, R is the solution to the equation:

$$\Psi(R) = c \tag{4}$$

Obviously, the function $R(c)$ is monotonically increasing. In particular, it is strictly monotonically increasing if $F(p)$ is also strictly monotonically increasing. Then, in this case, there is a unique reservation price for each search cost.

If now the consumers have different search costs, i.e., there is a distribution of search cost, represented by a density function $\gamma(c)$, what is then the density function of reservation prices?

The reservation price is the solution to

$$\int_0^R F(p)\, dp = c \tag{5}$$

If we denote $\int_0^p F(s)\, ds$ by $\tilde{F}(p)$, then $p = \tilde{F}^{-1}(c)$ is the inverse relation, which is possible to use if $\tilde{F}$ is strictly monotonic.

The probability that a given consumer has a search cost less than or equal to $\bar{c}$ is:

$$pr(c \leq \bar{c}) = \int_0^{\bar{c}} \gamma(c)dc. \tag{6}$$

Hence,

$$pr(R \leq \bar{R}) = \int_{\tilde{F}^{-1}(c) \leq \bar{R}} \gamma(c)dc. \tag{7}$$

Now define $\bar{c}$ by

$$\tilde{F}(\bar{R}) = \bar{c}. \tag{8}$$

$R \leq \bar{R}$ if and only if $c \leq \bar{c}$.
Then

$$pr(R \leq \bar{R}) = \int_0^{\bar{c}} \gamma(c)dc \tag{9}$$

and (differentiating (9))

$$g(\bar{R}) = \gamma(\bar{c}) \frac{d\bar{c}}{d\bar{R}}, \tag{10}$$

where $g(\bar{R})$ is the density function of reservation prices.
We also have

$$\frac{d\bar{c}}{d\bar{R}} = \tilde{F}'(\bar{R}) = F(\bar{R}) = \int_0^{\bar{R}} f(p)dp. \tag{11}$$

Then

$$g(\bar{R}) = \gamma[\tilde{F}(\bar{R})]\frac{d\bar{c}}{d\bar{R}} = \gamma[\tilde{F}(\bar{R})]F(\bar{R}) \tag{12}$$

which is the density function for reservation prices when the density function for search costs is $\gamma(\cdot)$.

Let us summing up what we have done until now. If the p.d.f[1] for the distribution of prices is $f(p)$ and the p.d.f for the distribution of search costs is $\gamma(c)$, then the p.d.f. for the distribution of reservation prices is

$$g(R) = \gamma[\tilde{F}(R)] \cdot F(R) \tag{13}$$

where

$$F(p) = \int_0^p f(s)ds \tag{14}$$

and

$$\tilde{F}(p) = \int_0^p F(s)ds\ . \tag{15}$$

However, we are interested to derive a p.d.f. describing frequencies of consumers at prices where they actually buy. Let us call the p.d.f of consumers at prices where they actually buy the stopping price distribution and denote it $\omega(p)$ and the corresponding cumulative distribution function $\Omega(p)$.

The p.d.f $\omega(p)$ for a consumer with reservation price $\bar{R}$ is:

$$\omega(p) \begin{cases} \dfrac{f(p)}{\int_0^{\bar{R}} f(p)dp} & \text{if } p \leq \bar{R} \\ \\ 0 & \text{if } p > \bar{R} \end{cases} \tag{16}$$

which is illustrated in figure 1.

However, the search costs differ between consumers and is described by a p.d.f $\gamma(c)$. In this case

[1] P.d.f is short for probability density function.

Figure 1.

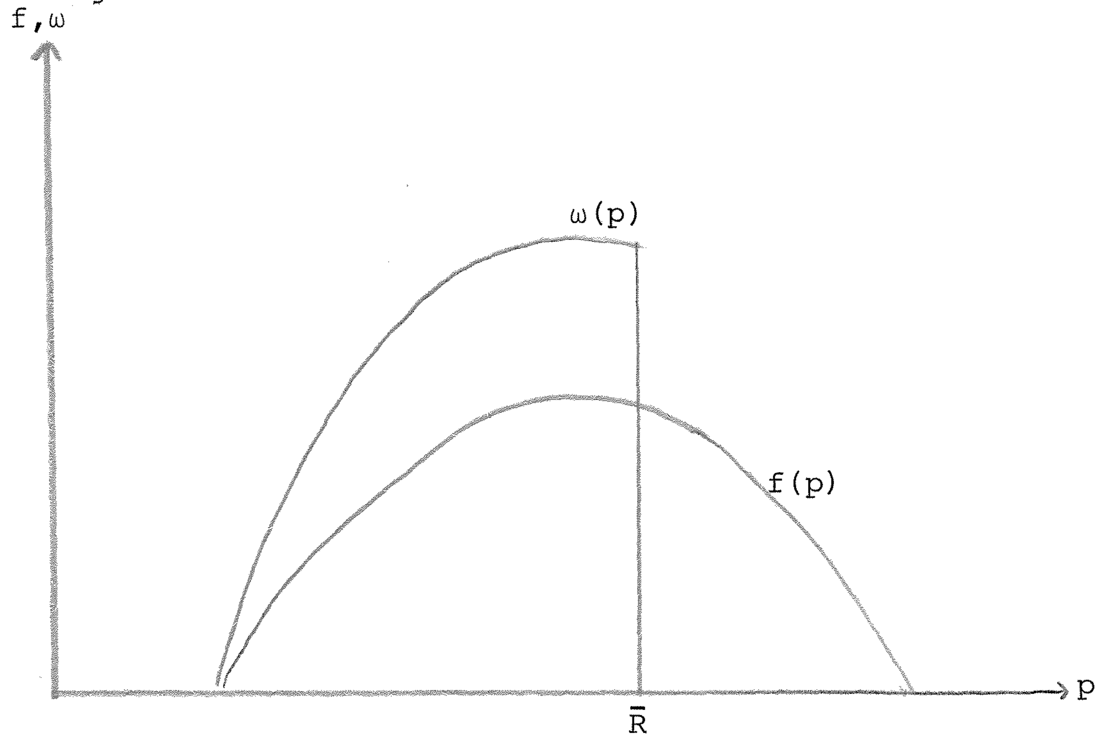
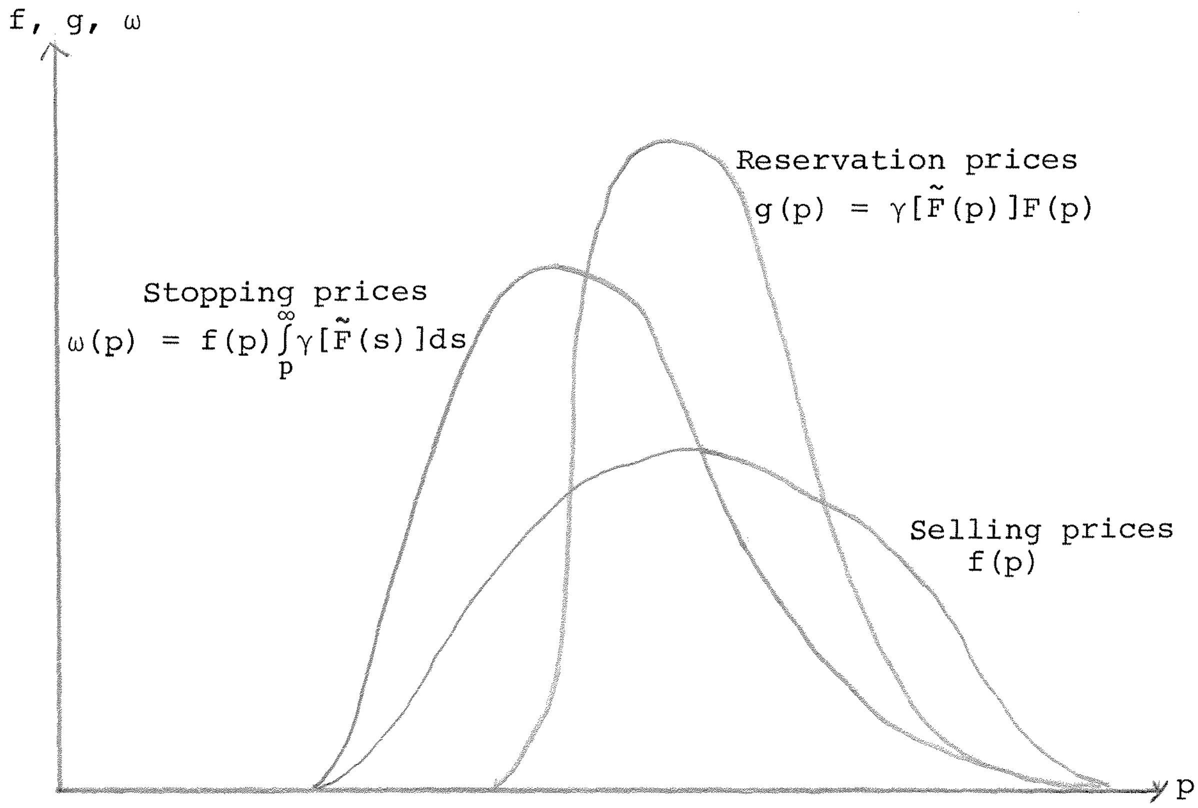


Figure 2.



the (cumulative) distribution function $\Omega(p)$ is:[1]

$$\Omega(p) = \int_0^p \gamma[\tilde{F}(s)] \cdot F(s)ds + F(p) \int_p^\infty \gamma[\tilde{F}(s)]ds. \tag{17}$$

We can derive the p.d.f $\omega(p)$ by differentiating $\Omega(p)$, and doing so we get:

$$\omega(p) = \Omega'(p) = f(p)\int_p^\infty \gamma[\tilde{F}(s)]ds. \tag{18}$$

Figure 2 helps us summing up the result until now. We started with a given distribution of firms across prices, desribed by the p.d.f $f(p)$.

With a given distribution of consumers across search costs, represented by the p.d.f $\gamma(c)$, and assuming that each consumer search in accordance with an optimal sequential stopping rule (3a and 3b), the p.d.f of reservation prices could be derived as

$$g(p) = \gamma[\tilde{F}(p)]\ F(p) \quad (19$$

The distribution of consumers across prices showing where they actually buy, the stopping price distribution, represented by the p.d.f $\omega(p)$ was then derived as

$$\omega(p) = f(p)\int_p^\infty \gamma[\tilde{F}(s)]ds. \tag{20}$$

We are now in position to describe the firm environment.

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1 For a derivation, consult Appendix. A1.

2.2 The firm side

2.2.1 The demand curve

Now let us derive the demand curve facing a firm. We ask the question: what is the demand a firm charging the prise p_i compared to a firm charging p_j where $p_i \neq p_j$?

Let us consult figure 3 for clearifying the question. The frequency of firms charging prices in the interval $(p_i - \Delta p,\ p_i + \Delta p)$ represents by the staple $f(p_i)$ in figure 3, while the frequency of consumers buying in this interval is described by $\omega(p_i)$. Then $\frac{\omega(p_i)}{f(p_i)}$ is a measure of the relative frequency of buying consumers per firm at p_i. At p_j the corresponding staples are $f(p_j)$ and $\omega(p_j)$, and the relative frequency of consumer per firm is $\omega(p_j)/f(p_j)$. Then it is completely clear that the firms in this market faces a negatively sloped demand curve.

The demand curve could be derived in the following way. Assume that k consumers enter the market per period and, after having found a store charging a price below their reservation price, buy one unit, leave the market and are replaced by k new consumers. In a price interval $(p, p+\Delta p)$ the quantity sold per period is:

$$k[\Omega(p+\Delta p) - \Omega(p)] \tag{20}$$

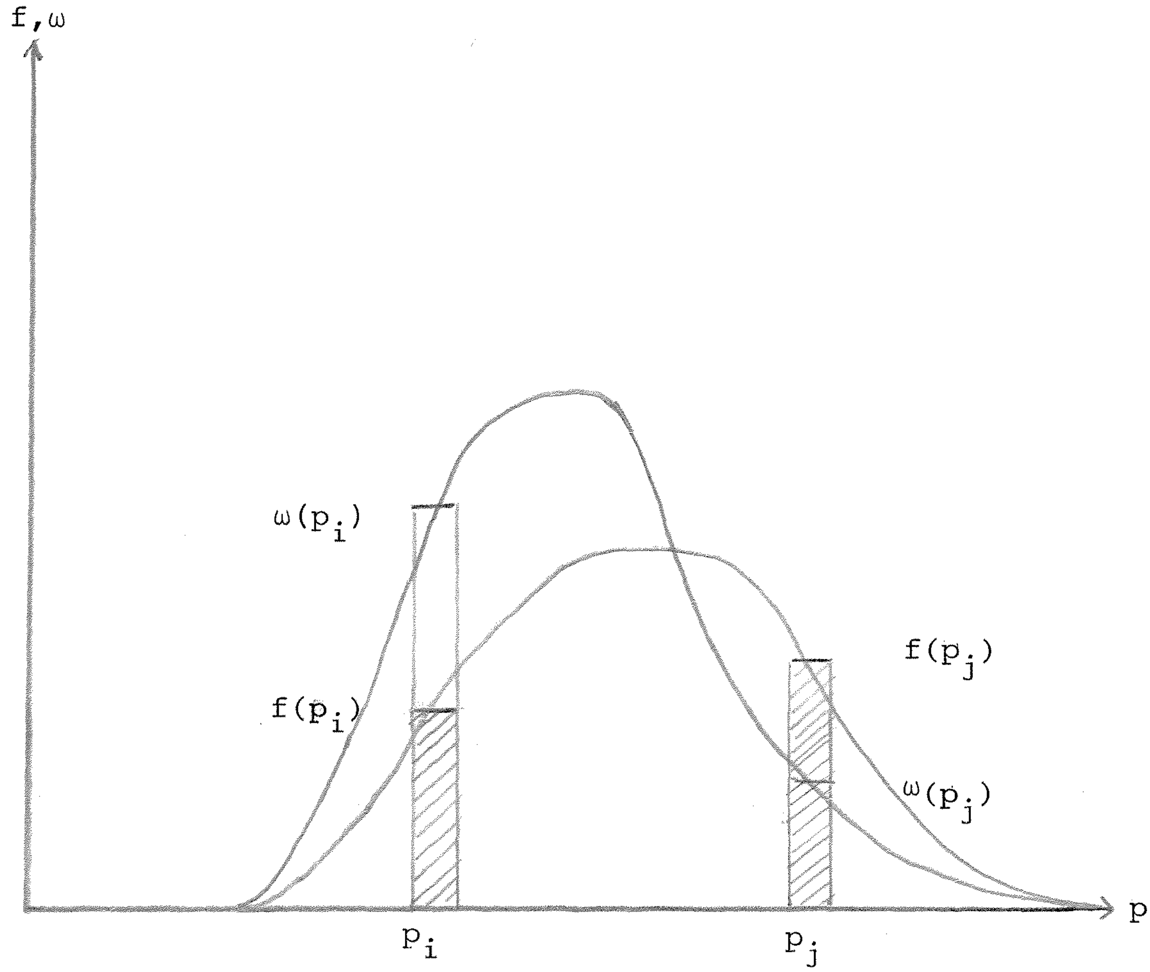
The number of firms in that interval is

$$m\,[F(p+\Delta p) - F(p)] \tag{21}$$

where m is the total number of firms in the market.

The quantity sold <u>per</u> firm is then

Figure 3.



$$\frac{k[\Omega(p+\Delta p) - \Omega(p)]}{m[F(p+\Delta p) - F(p)]} \tag{22}$$

Letting Δp approach zero we then get:

$$q(p) = \frac{k}{m} \cdot \frac{\omega(p)}{f(p)} \tag{23}$$

which is expected demand for a firm charging p. It is also, regarded as a function of p, the demand curve a firm is facing.

Using (20) we arrive at

$$q(p) = \frac{k}{m} \int_p^\infty \gamma[\tilde{F}(s)]ds \tag{23}$$

which is the "firm's demand curve" if each consume buys only one unit.

The elasticity of demand (η) is then

$$= \frac{\gamma[\tilde{F}(p)]p}{\int_p^\infty \gamma[\tilde{F}(s)]ds} \tag{24}$$

Now, instead, we assume that the consumers are sensitive to the price and buy an amount (or size) dependent on the price found. Let us assume that all consumers have the same individual demand curve $d(p)$. Then the demand curve will be:

$$q(p) = \frac{k}{m}\, d(p) \int_p^\infty \gamma[\tilde{F}(s)]ds \tag{25}$$

In this case the elasticity is $\eta+e$, when e is the elasticity of $d(p)$

3 MARKET EQUILIBRIUM

We are now looking for an equilibrium distribution of firms in this market.

First something about the equilibrium concept. We can find the equilibrium configuration of a model in either of two ways. One way is to specify the reaction pattern by the agents of the model (market) from period to period giving raise to differen/ce/tial equations describing how the endogenous variables change over time. The equilibrium is then found when these differential equations equals zero, i.e. the model (economy or market) repeat itself period after period.

Another way to find equilibrium is to applicate for instance the Nash conditions to a static model. The Nash conditions means that an equilibrium must have the property that all agents of the model (market, economy) have to be in an optimum with respect to variables that they can influence. In an ordinary market then this imply that the consumers choose consumption of different commodities and labor supply (if they are allowed) in such a way that their utility is maximized. The firms, on the same time, choose supply and production technique so that their profit is maximized.

These two equilibrium concepts are of course not isolated from each others. A dynamic formulated model would not change between periods if agents get maximum utility or profit, and the dynamic of a static model would be characterized by change if the agent could improve their situation by means of changes in their behavior.

Here we will apply the Nash condition for noncooperative equilibrium. The consumers are in the model always in optimum, i.e., they are always undertaking optimal search effort.

Then it remains, in finding the Nash equilibrium configuration of the market, to have the firms in optimum, i.e. profit maximum.

3.1 THE FIRM

Now let us derive the profit function for the firm.

The demand is according to (23)

$$q(p) = \frac{k}{m} d(p) \int_p^\infty \gamma(\tilde{F}(s))\, ds \tag{26}$$

The profit π as a function of p is then:

$$\pi(p) = p \frac{k}{m} d(p) \int_p^\infty \gamma(\tilde{F}(s))\, ds - C(q(p)) \tag{27}$$

where C(q) is the firm's cost function as a function of quantity sold, and q(p) must be in accordance with (26).

Let us assume that we have constant marginal cost and thereby a cost function of the type

$$C(q) = C_1 + mc \cdot q \tag{28}$$

Then (27) becomes

$$\pi(p) = (p-mc)\ d(p) \frac{k}{m} \int_p^\infty \gamma[\tilde{F}(s)]ds - C_1 \tag{29}$$

The condition for the market to be in equilibrium is then that all firms are charging profit maximum prices, i.e. $\frac{d\pi}{dp} = 0$.

Differentiating (29) with respect to price gives

$$\frac{d\pi}{dp} = \frac{k}{m}\left\{\int_p^\infty \gamma(\tilde{F}(s))\, ds \cdot \left[d(p) + \frac{dd}{dp}(p-mc)\right] + \right.$$
$$\left. + (mc-p)d(p)\ \gamma(\tilde{F}(p))\right\} \tag{30}$$

We are now in position to derive the configuration of the equilibrium price distribution. The condition is straightforward, $\frac{d\pi}{dp}$ must equal zero. The solution then could be that this condition is fulfilled at a unigue price or at a price interval (or for all prices).

4.1 SINGLE PRICE EQUILIBRIUM

Let us first put the question: If there exists a single price equilibrium, i.e. an equilibrium with degenerated price distribution, at which price is then this equilibrium?

It is easy to show that if all consumers have search costs greater than zero and all firms charge the same price, p_i, then:

$$\int_{p_i}^{\infty} \gamma(\tilde{F}(s))\ ds = 1 \tag{31}$$

and

$$\gamma(\tilde{F}(p_i)) = 0 \tag{32}$$

(31) says that integrating from the common price p_i to infinity, we will include all firms (which is obvious because all firms charge p_i). (32) follows from the fact that if all firms charge p_i, all consumers have reservation prices greater than p_i.

Using (31) och (32) in examining $\frac{d\pi}{dp} = 0$ in (30), we arrive at:

$$d(p_i) + \frac{dd}{dp_i}(p_i - mc) = 0 \tag{33}$$

If the elasticity of the individual demand curve is denoted e as before, the solution to (33) is

$$p_i = mc \frac{e}{e+1} \tag{34}$$

Thus, there exists a price that will be an equilibrium price for a degenerated price distribution. This is the same price as a monopolist that controls the whole market would charge.[1] The similarity to the solution of monopolistic competiting is appealing. There is, however, an important difference. In monopolistic competition the elasticity in the "mark-up factor" between marginal cost and price is determined from both the individual elasticity and the flow of customers among firms with similar products. Thus the elasticity in monopolistic competition is fairly large and the deviation between price and marginal cost is thereby small. The single price equilibrium in a search market is instead equal to the price a monopolist would charge, which means that the difference between price and marginal cost is much greater than in monopolistic competition.

4.2 PRICE DISPERSION EQUILIBRIUM

Now, let us investigate whether or not there could be an equilibrium with price dispersion in a search market. Assuming that the consumers buy one unit each (i.e. $d(p) = 1$), we have (from differentiating (29))

$$\frac{d\pi}{dp} = \frac{k}{m}\left[\int_p^\infty \gamma(\tilde{F}(s))ds + (mc-p) \cdot \gamma(\tilde{F}(p))\right] \tag{35}$$

We then have to solve the differential equation

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[1] This result was found by Peter Diamond and published in Diamond (1971).

$$\int_{p}^{\infty} \gamma(\tilde{F}(s))\ ds + (mc-p)\ \gamma(\tilde{F}(p)) = 0 \qquad (36)$$

The solution to this is[1]:

$$\int_{p}^{\infty} \gamma(\tilde{F}(s))\ ds = \frac{B}{p-mc} \qquad p > mc \qquad (37)$$

where B is a positive constant.

4.3 NECESSARY AND SUFFICIENT CONDITIONS FOR A PRICE DISPERSION EQUILIBRIUM

Now we are in position to examine the conditions for the existence of an alternative equilibrium to the single price equilibrium at monopoly price earlier showed.

The problem is the following: Given a density function $\gamma(\cdot)$ and two positive constants B and mc, search a continuous probability distribution, with a continuous density function f on (mc, ∞) such that

$$\int_{p}^{\infty} \gamma(\tilde{F}(s))\ ds = \frac{B}{p-mc} \qquad p > mc \qquad (38)$$

where $\tilde{F}$ is given by

$$F(p) = \int_{0}^{p} f(s)\ ds \qquad (39)$$

$$\tilde{F}(p) = \int_{0}^{p} F(s)\ ds \qquad (40)$$

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[1] Proof: See appendix.A2.

The question is then. For which search cost distributions $\gamma(\cdot)$ is this solvable?

Proposition: Necessary and sufficient conditions on $\gamma(\cdot)$ for the problem above to have a solution are the following:

i) γ is defined on $(0,\infty)$

$\gamma \in C^2$, i.e., γ is twice differentiable

$$\gamma' < 0$$

$$\gamma'' > 0$$

$$\gamma(c) \to 0 \text{ when } c \to \infty$$

$$\gamma(c) \to \infty \text{ when } c \to 0+$$

ii)

$$\frac{\gamma(c)^{3/2}}{\gamma'(c)} \quad \text{is decreasing}$$

iii)

$$\lim_{c \to \infty} \frac{\gamma(c)^{3/2}}{\gamma'(c)} = -\frac{\sqrt{B}}{2}$$

iV

$$\lim_{c \to 0+} \frac{\gamma(c)^{3/2}}{\gamma'(c)} = 0$$

Proof for the necessary conditions:

$$\int_p^\infty \gamma(\tilde{F}(s))\, ds = \frac{B}{p-mc} \qquad p > mc$$

$$\Rightarrow$$

$$\gamma(\tilde{F}(p)) = \frac{B}{(p-mc)^2} \qquad p > mc$$

$\tilde{F}(p)$ is increasing and $\tilde{F}((mc, \infty)) = (0, \infty)$ so that γ *has to be defined on* $(0, \infty)$. Further γ has to be strictly decreasing because $\frac{B}{(p-mc)^2}$ is. By letting $p \to mc+$ and $p \to \infty$ respectively, we see that

$\gamma(c) \to 0$ when $c \to \infty$

and

$\gamma(c) \to \infty$ when $\to 0+$

Put $\Gamma = \gamma^{-1}$. Then we have

$$\tilde{F}(p) = \Gamma\left(\frac{B}{(p-mc)^2}\right).$$

or

$$\Gamma(c) = \tilde{F}\left(\sqrt{\frac{B}{c}} + mc\right)$$

from which we see that $\Gamma \in C^2$, $\Gamma' < 0$ and $\Gamma'' > 0$. From this we get $\underline{\gamma \in c^2,\ \gamma' < 0,\ \gamma'' > 0}$ and besides:

$$\Gamma'(c) = \frac{-\sqrt{B}\ F\left(\sqrt{\frac{B}{c}} + mc\right)}{2\ c^{3/2}}$$

or

$$c^{3/2}\ \Gamma'(c) = \frac{-\sqrt{B}}{2}\ F\left(\sqrt{\frac{B}{c}} + mc\right)$$

or

$$\frac{\gamma(c)}{\gamma'(c)}^{3/2} = \frac{-\sqrt{B}}{2}\ F\left(\sqrt{\frac{B}{\gamma(c)}} + mc\right)$$

Now we know that $\gamma(c)$ is strictly decreasing from ∞ to 0 when c goes from $0+$ to ∞, and since $F(p)$ increases from 0 to 1 when p goes from mc to ∞, we obtain from this the properties ii), iii) and iV).

Thus the proof of the necessary conditions is complete.

<u>Proof for the sufficient conditions</u>.

Because of the presumptions in i) there exists $\Gamma \stackrel{\text{def.}}{=\!=} \gamma^{-1} \in C^2$, and Γ is defined on $(0, \infty)$.

Put

$$F(p) \overset{\text{def.}}{=\!=} \begin{cases} \Gamma'\left(\dfrac{B}{(p-mc)^2}\right) \dfrac{-2B}{(p-mc)^3} & p \geq mc \\ \\ 0 & p < mc \end{cases}$$

Then it is obvious that $F \in C^1$ for $p > mc$. A reformulation gives:

$$F\left(\sqrt{\frac{B}{\gamma(c)}} + mc\right) = \frac{2\gamma(c)^{3/2}}{\sqrt{B}\gamma'(c)} \qquad c > 0$$

The presumptions on γ then imply that F is increasing, $F(mc+) = 0$ and $\lim_{p\to\infty} F(p) = 1$.

Consequently $f = F'$ solves the problem, for we have

$$F(p) = \frac{d}{dp}\,\Gamma\left(\frac{B}{(p-mc)^2}\right) \qquad p > mc$$

i.e.

$$\tilde{F}(p) = \Gamma\left(\frac{B}{(p-mc)^2}\right) \qquad p > mc$$

(note that $\Gamma(s) \to 0$ when $s \to \infty$)
i.e.

$$\gamma(\tilde{F}(p)) = \frac{B}{(p-mc)^2} \qquad p > mc$$

which is equivalent to

$$\int_p^\infty \gamma(\tilde{F}(s))\, ds = \frac{B}{p-mc} \qquad p > mc$$

(note that $\gamma(\tilde{F}(p)) \to 0$ when $p \to \infty$).

Thus the proof is complete.

<u>An example</u>

In order to illustrate the equilibrium, we now show an example.

The function

$$\gamma(c) = \frac{B}{\left(\frac{c}{2} + \sqrt{c + \frac{c^2}{4}}\right)^2}$$

satisfies the conditions i)-iV). The equilibrium density function is thus:

$$f(p) = \frac{2}{(p-mc+1)^3} \qquad p\varepsilon(mc, \infty)$$

because:

$$F(p) = 1 - \frac{1}{(p-mc+1)^2} \qquad p > mc$$

$$\tilde{F}(p) = p - mc - 1 + \frac{1}{p-mc+1} \qquad p > mc$$

$$\tilde{F}^{-1}(c) = mc + \frac{c}{2} + \sqrt{c + \frac{c^2}{4}} \qquad c > 0$$

Then

$$\gamma(c) = \frac{B}{(\tilde{F}^{-1}(c)-mc)^2} \qquad c > 0$$

and consequently

$$\gamma(\tilde{F}(p)) = \frac{B}{(p-mc)^2} \qquad p > mc$$

i.e.

$$\int_p^\infty \gamma(\tilde{F}(s))\, ds = \frac{B}{p-mc} \qquad p > mc$$

The derivation of necessary and sufficient conditions in this section tells us that for a search market to have another equilibrium than the degenerated monopoly price equilibrium, there must be some restrictions on the search cost distribution.

In sum, the requirement are that the search cost distribution must not be bounded away from zero, its corresponding density function must be negatively sloped and convex, and the "degree of convexity" must fulfill certain conditions.

Let us try to see what is going on here.

First, let us look at the single price equilibrium at the monopoly price. How is it that the monopoly price could be an equilibrium price in a market that in most respects fulfills the requirement for a perfectly competitive market? There are many sellers and buyers, a so called atomistic market. The commodity is homogeneous and sold in a single place. Firms and individuals are optimizing. Still the equilibrium price is the monopoly price, not the marginal cost price as in perfect competition.

We can understand this result easier if it is put in the following way. The demand for an individual firm can be regarded as the product of three factors: 1) the number of consumers, denoted by μ, who come into contact with the firm; 2) the share (λ) of those who are willing to buy at the offered price, and 3) the individual demand (d), i.e.

$$q(p_i) = \mu(p_i) \cdot \lambda(p_i) \cdot d(p_i)$$

where q is the demand for a firm charging p_i. q is a function of price as are μ, γ and d.

It is hard to believe that μ is dependent on the firm's offering price. Rather we can assume a given probability that a consumer will come into contact with a certain firm at each search step, a probability related to non-price factors.

λ, by contrast, obviously depends on the price. The lower the price, the greater the probability that it is below a certain consumer's reservation price. Lowering price, λ will rise until the lowest reservation price among the consumers is passed. Then λ becomes unity. Below this price the only price sensitivy comes from d(p), the individual demand curve.

Imagine now that all firms charge the same price. What will happen to the profit for one particular firm if it increases or decreases its price? If it decreases its price a little, ε, it will experience a demand increase equal to $[d(p-\varepsilon) - d(p)]$. It will still get its "fair" share of the buyers, because no one will still search more than once, provided no buyer has zero search cost. The same is true if the firm tries to increase its price. It will experience a demand decrease equal to $[d(p) - d(p+\varepsilon)]$. If ε is small enough the firm will also in this case get its "fair" share of the buyers.

From this follows that the common price must be equal to $mc \frac{e}{e+1}$, the monopoly price, where e is the elasticity of d(p). The reason for this is that if the price is below the monopoly price, any firm could increase its profit by increasing its price. If the price is above, any firm could increase its profit by decreasing its price.

In the price dispersion equilibrium, the lower price of a low-price firm is exactly compensated by a higher demand, because buyers have different search costs. A low-price firm experiences a greater demand because its price is below more reservation prices than the price of a high-price firm. It is, however, important to notice that differences in search costs is not enough to give rise to a price dispersion equilibrium. The search cost distribution must not be bounded away from zero.

5. THE TWO PRICE EQUILIBRIUM OR THE UNEXPLAINED PRICE DIFFERENCES

Now let us return to the question put at the outset. Given that there are great differences in "the" price of an identical commodity in two locations, and that this price difference can not be explained by differences in costs, how can this difference be explained?

The foregoing analysis demonstrates that differences in income distribution could be an explanation. It shows

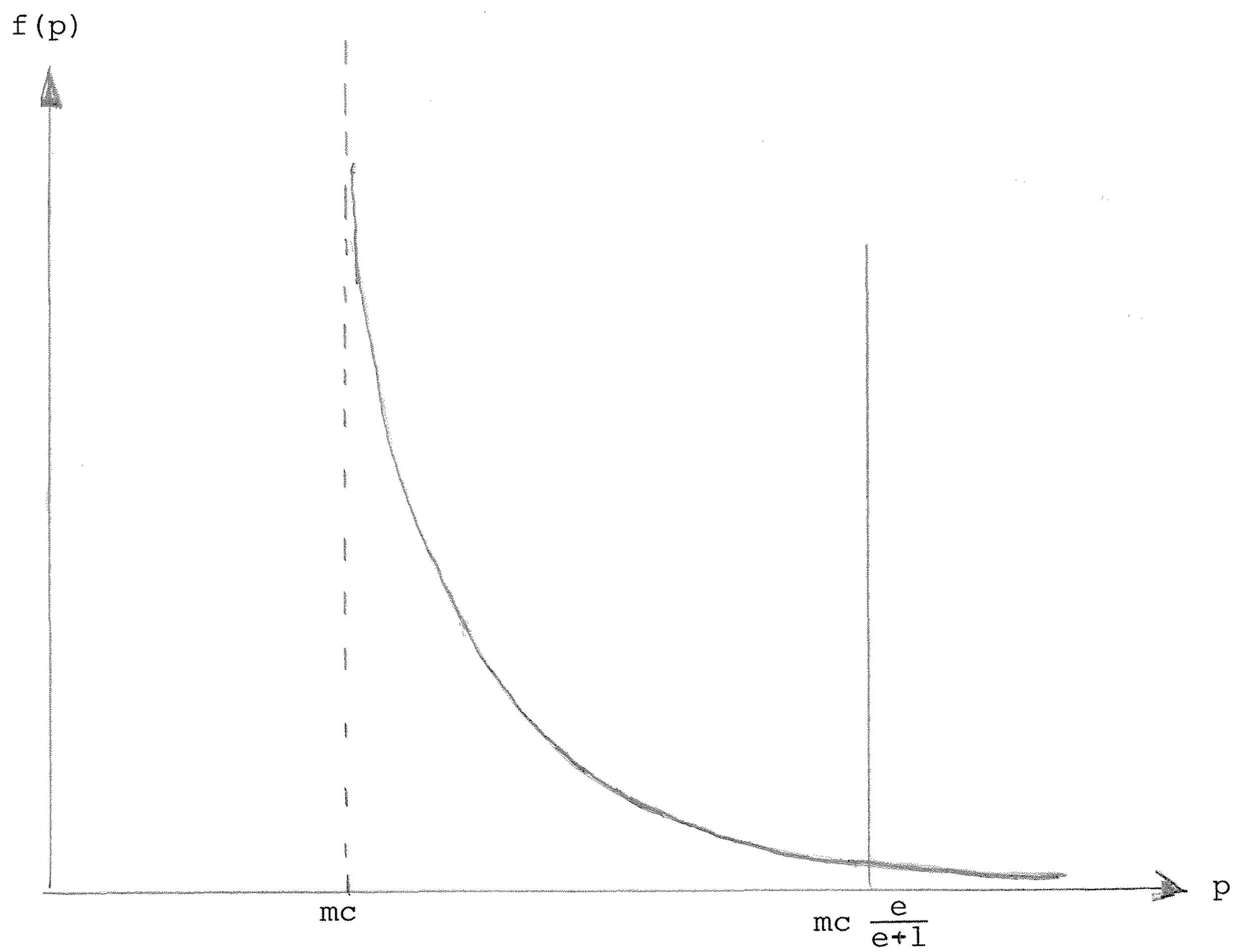

Figure 4.

that in a search market there can be either of two equilibria: (1) A degenerated equilibrium at the monopoly price, i.e., all firms charge a price equal to mc $\frac{e}{e+1}$. (2) A price dispersion equilibrium, where the density function starts at the marginal cost price and is negatively sloped and convex. The two equilibrium distributions are illustrated in Figure 4.

Now, what determines which one of the two equilibria will be at hand? The answer is that the "marginal cost equilibrium" case could only appear if the conditions derived in section 4.3 are fulfilled. The monopoly price equilibrium will be the case if anyone of the conditions for the price dispersed "marginal cost" equilibrium is not fulfilled.

The conditions concern the search cost distribution. The most important determinant of the search cost distribution is the income distribution. The reason for this is that the search cost is primarily the time cost. The time cost is for an optimizing individual equal to the wage. Therefore the income distribution could be regarded as a measurement of the search cost distribution. Thus, a reason for "unexplainable" differences in price between two locations could be different shape of the income distribution.

How reasonable are the conditions for the "marginal cost" case? The conditions that the density function should be downward sloping and convex are completely in line with what has been found in studies of income distributions. For instance the Pareto distribution, which has these properties, is commonly used as an income distribution.

More questionable is the condition that the search cost distribution must not be bounded away from zero. If no consumers have zero search costs, the monopoly price is the only possible equilibrium price. Some consumers can have zero search costs if (1) they experience positive utility from search *per se*, or (2) the rate of unemployment is high. The rationale for the latter would be that the unemployed have a very low opportunity cost in search.

The necessary requirement for the "marginal cost" equilibrium is that a sufficiently large proportion of the consumers have sufficiently low search costs. There is a provocative implication of this for the question of the costs and benefits of female labor force participation. Thus, a low labor force participation rate among women might imply a sufficiently large number of potential low search cost consumerss, namely the house wifes, to ensure a "marginal cost" equilibrium. If so, an increase in the labor force participation rate might reduce this group to such a degree that the conditions for the "marginal cost" solution are no longer fulfilled. Then the equilibrium configuration will change from the "marginal cost" price to the monopoly price.

The welfare implication of this is interesting. We know that the equilibrium in the traditional competitive model is, under some conditions, pareto optimal. The "marginal cost" solution in the present model could be regarded as fairly close to the competitive equilibrium. A switch of the market solution to the monopoly price because of an increase of the female participation rate, would then reduce the welfare of the economy.

APPENDIX

A1. $$\Omega(p_0) = pr(p \leq p_0) = \int_0^\infty pr(p \leq p_0|\bar{R})pr(R = \bar{R})d\bar{R} =$$

$$= \int_0^{p_0} pr(p \leq p_0|\bar{R})\ pr\ (R=\bar{R})d\bar{R} + \int_{p_0}^\infty pr(p \leq p_0|\bar{R})pr(R = \bar{R})d\bar{R} =$$

$$= \int_0^{p_0} pr(R = \bar{R})d\bar{R} + \int_{p_0}^\infty \frac{F(p_0)}{F(\bar{R})} g(\bar{R})d\bar{R} = \int_0^{p_0} g(\bar{R})d\bar{R} +$$

$$+ \int_{p_0}^\infty \frac{F(p_0)}{F(\bar{R})} \gamma[\tilde{F}(\bar{R})]\ F(\bar{R})d\bar{R} = \int_0^{p_0} \gamma[\tilde{F}(\bar{R})]F(\bar{R})d\bar{R} +$$

$$+ F(p_0) \int_{p_0}^\infty \gamma[\tilde{F}(\bar{R})]d\bar{R} \qquad \text{(A1)}$$

A2. The condition for equilibrium is:

$$\int_p^\infty \gamma[\tilde{F}(s)]ds + \gamma[\tilde{F}(P)](mc-p) = 0 \qquad \text{(A2)}$$

Let $\int_p^\infty \gamma[\tilde{F}(s)]ds$ be denoted by $q(p)$ and $\gamma[\tilde{F}(p)]$ by $-\frac{dq}{dp}$. Then, we have

$$q(p) - \frac{dq}{dp}(mc-p) = 0. \qquad \text{(A3)}$$

Rearranging terms and solving for the elasticity of demand, e, we get:

$$e = \frac{dq}{dp}\frac{p}{q} = \frac{p}{mc-p} \qquad \text{(A4)}$$

which shows how the elasticity must change as a function of p if the constant profit condition should be fulfilled.

Examining (A4) we see that

$$\lim_{p \to mc+} e = -\infty \qquad \text{(A5)}$$

$$\lim_{p\to\infty} e = -1 \tag{A6}$$

and

$$\frac{de}{dp} = \frac{mc}{(mc-p)^2} > 0 \text{ for all } p \neq mc. \tag{A7}$$

If $p < mc$ the elasticity must be positive except for $p < 0$. Then it follows that only $p > mc$ is economically meaningful.

Now, let us solve the differential equation (A3). It can be written

$$dq(p-mc) + qdp = 0 \tag{A8}$$

Dividing by $q(p-mc)$ we get:

$$\frac{dq}{q} + \frac{dp}{p-mc} = 0. \tag{A9}$$

The solution to (A9) is:

$$\int\frac{1}{q}\, dq + \int\frac{1}{p-mc}\, dp = A \tag{A10}$$

i.e.

$$\log q = A - \int\frac{1}{p-mc}\, dp \tag{A11}$$

Then, we have

$$q(p) = \exp\left[A - \int\frac{1}{p-mc}\, dp\right] \tag{A12}$$

Putting $e^A \equiv B$ we get:

$$q(p) = B\exp\left[-\int\frac{1}{p-mc}\, dp\right] \tag{A13}$$

and so

$$q(p) = B\exp\left[-\log(p-mc)\right] = \frac{B}{p-mc}. \tag{A14}$$

which is the shape of the demand curve, fulfilling the condition that the profit derivative will be zero at all prices, when marginal cost is constant.

Reinserting $\int_{p}^{\infty}\gamma[\tilde{F}(s)]ds$ for $q(p)$ we get:

$$\int_{p}^{\infty}\gamma[\tilde{F}(s)]ds = \frac{B}{p-mc} . \tag{A15}$$

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