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EXPLAINING PARALLEL MOBILE TELEPHONE NETWORKS: A THEORETICAL MODEL

by

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Explaining Parallel Mobile Telephone Networks: A Theoretical Model*

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1. Introduction

The production of telecommunication services is sometimes suggested as an example of a natural monopoly. One major reason for this is the presence of economics of scale. The existing technology requires large physical investments while the marginal cost of production is low.[1] [2] We believe that if the conventional telephone cable-network may be a natural monopoly, so may the mobile telephone network for the same reasons.

The markets for mobile telecommunication services have generally been deregulated earlier and more extensively than other telecommunication markets. If the

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[1]However, economic theory shows that economics of scale do not suffice for an industry to be a natural monopoly. It is even shown that economics of scale *and economics of scope*, together, is not a sufficient condition for an industry to be a natural monopoly. (Panzar[17], pp. 26.)

[2]Empirically, the large number of studies investigating whether the telecommunication industry is a natural monopoly or not have been inconclusive in the sense of reaching different conclusions. For some examples, see Evans and Heckman [5], Fuss[10], Röller[18], and Shin and Ying[20].

production of mobile telephone services is a natural monopoly and if competition is not mitigated then only one firm should be able to make non-negative profits, i.e. in a deregulated market we should observe one single producer of mobile telecommunication services. Yet, even in a small country like Sweden there are three parallel digital networks using the same technological standard and two analog networks.[3] There are several reasons to why the forces of a free market may not have stabilized the market structure. For example, the rapid technological progress may let new firms enter the market and successfully compete with the incumbent/incumbents. Competition may also be lessened by the small number of firms and the presence of switching costs.

This study has two purposes: (a) To study the strategic interaction between a small number of firms on a market characterized by the features of the mobile-telephone market in order to explain the co-existence of two firms with increasing returns to scale technology. (b) To study how and when prices and subsidies are used to deter entry.

The study consists of two parts, each dealing with one of the two topics. In the first part (chapter 2) we examine the strategic relation between two firms that repeatedly interacts. The main result is that competition is mitigated because an aggressive behavior by one firm today may be punished tomorrow by the other firm. The long arm of the future makes competition softer and allows two firms to make non-negative profits in the long run even though the existing technology exhibits increasing returns to scale. The second part (chapter 3) studies an entry deterrence game in which an incumbent can adopt alternative strategies to deter entry or force an already established firm to leave the market. The behavior of the incumbent is governed by how heavily future profits are discounted. The choice of strategy can be viewed as a choice of how to distribute payoffs over time. A patient incumbent deters entry early while an impatient incumbent waits.

At the end of each part there is a numerical example. All proofs are provided in an appendix at the end of the paper.

Even though the two models are quite different there are similarities. This is because of some distinctive features of the telecommunication market and we will briefly discuss them before proceeding with the models. First, production and consumption of the communication service is instantaneous. Hence, competition must be in prices because a firm can not produce a quantity of the service and then

[3]The three digital GSM-networks are owned by Comvik, Europolitan, and Telia Mobitel. GSM is a European digital standard. The analog networks are NMT-450 and NMT-900. They are owned by Telia Mobitel.

bring it to the market where it is sold at the market price. Second, the presence of economies of scale due to large investments and a low marginal cost. Third, it is (to some extent) costly for consumers to switch supplier. Switching costs can generally be divided into three categories: (i) transaction costs esteeming from the switching itself, (ii) learning costs due to differences in the service, and (iii) artificial switching costs such as coupon systems and product differentiation.[4] Here the service is treated as a homogeneous good which rules out the second category of switching costs. For the cause of simplicity the artificial switching cost is assumed to be exogenously given and the different categories are merged. An example of switching costs in the mobile telephone market is the fixed subscription fee which can not be regained if the subscription is canceled. A transaction cost must not necessarily be monetary, it may be viewed as the monetary equivalent of required effort to change supplier. Examples of effort is time spent to search for alternative suppliers and information about their service, price, and terms of payment. Furthermore, the consumer can not keep her old mobile telephone number when switching supplier. She must devote both time and effort to inform all those who have her old number about her new number. Fourth, there is a spill over of demand between the producers. A price-cut by firm A increases A's demand, but also firm B's demand. This is the call-back effect. A fraction of the increased number of calls made in firm A is made to subscribers of firm B. Some of these are induced to make a call as a response to the incoming call. Thus, B's demand increases as well.[5] (The call-back effect is not modelled in the second model but this does not change the qualitative results. Nor is entry regulated.)

2. A General Two Firm Model

The model presented below is a simple dynamic model consisting of a infinitely repeated game. A repeated game does not allow past play to influence the action space of the current period, or feasible payoffs. The game can, nevertheless, provide some understanding of long-term relations between firms that repeatedly interact. The horizon for investments in the telecommunication industry is long why long-term relations in terms of market structure and competition are of vital interest. A second argument for using a repeated game is that entry is regulated and that the number of repeatedly interacting firms is small. The infinite time

[4] See Nilssen[16].

[5] Appelbe et al[1] estimate the call back effect to be as large as 0.5, i.e. one new call was made as a response to every two incoming calls.

horizon can be interpreted as a situation where the players in each period think the game continues one more period with high probability.[6]

Producers of mobile telecommunication equipment are in general not suppliers of the telecommunication service. The suppliers are therefore likely to have the same access to new technology. For the sake of simplicity we restrict ourselves to only consider the case of two firms. The model is not intended to explain the pricing strategy used by an entering firm. It presumes that the two competing firms already are established. For convenience we let the two firms serve the same number of consumers.

There are two competing firms, 1 and 2. In each repetition of the stage game the two firms simultaneously undertakes one action each, denoted p_i, and the action is to set a price. The demand is normalized and equal to zero for all prices larger than 1. $[0,1]$ is therefore the action space of firm i and $\pi_i : [0,1]^2 \rightarrow \Re$ is the per period profit of firm i as a function of its own price and the price set by the other firm.

Let $\mathbf{p}^t = (p_1^t, p_2^t)$ denote the prices played in period t and let $\pi^t(\mathbf{p}^t) = (\pi_1^t(\mathbf{p}^t), \pi_2^t(\mathbf{p}^t))$ denote the payoff/profit vector in t. $0 \leq \beta \leq 1$ is the common discount factor with which both firms discount future profits. The history of the game in period t is the sequence of realized prices in all periods before t. The repeated game starts in period $t = 1$ and has no history in that period. A firm observes the other firm's price and the history is common knowledge. A pure strategy of firm i in the repeated game is a sequence of decision rules, one for each period, that map possible period$-t$ histories to prices. Denote i's decision rule in period t σ_i^t. The pure strategy of firm i, $\sigma_i = (\sigma_i^1, ..., \sigma_i^t, ...)$ must specify i's choice of action in all contingencies, even those that i does not expect to occur.

[6]Other classes of dynamic games are used to study firms pricing strategies over time. These games are often played over a finite number of periods, usually two or three periods. This is more of interest when a new firm enters the market, as in chapter 3. One standard result in dynamic models is that prices are lower in the first period when firms try to tie consumers in, and that it increases over time, see Nilssen[16]. Beggs and Klemperer[2] show that profits are higher in the presence of switching costs and that entry is more attractive than in the absence of switching costs, even though the entrant must overcome the disadvantage that a fraction of the market is already captured by the incumbent. If firms have equal costs then the market structure converges to equal market shares.

2.1. The Stage Game

To simplify notation we omit superscript t that denotes in which period the stage game is played. The call back effect is modeled in the following way. The demand of firm j as a function of its own price is $\frac{1}{2}(1-p_j)$. The increase in number of calls made from j due to a price-cut from p_j^0 to p_j^1 is $\frac{1}{2}(p_j^0 - p_j^1)$. Assume that the probability of a j-consumer to make a call to another j-consumer is equal to that of making a call to a i-consumer. Then, the increase in number of calls made to other firms is $\frac{1}{2}(p_j^0 - p_j^1)$. Furthermore, assume that one call is made from firm i for every second incoming call. The demand increase of i is found by dividing $\frac{1}{2}(p_j^0 - p_j^1)$ by $\frac{1}{2}$ which gives the increase in incoming calls to firm i divided by two. The same reasoning holds of course for j's total demand. s is the switching cost consumers face when changing supplier. The switching cost need not to be monetary, but the effort of switching can be translated into a monetary cost, $s \geq 0$. The switching cost is assumed to be equally large for all consumers and if a firm's price is understuck by $s+\varepsilon$ then its demand is zero. The firm who underbids meets the demand $\frac{5}{4}(1-p_i))$. If no firm underbids the other by $s+\varepsilon$ then they share the market and serve the same number of subscribers. Thus, the per-period demand of firm i is:

$$D_i(\mathbf{p}) = \begin{cases} 0 & if \quad p_i - p_j > s \\ \frac{1}{2}\left(1 - p_i + \frac{1}{2}(1-p_j)\right) & if \quad |p_i - p_j| \leq s \\ \frac{5}{4}(1-p_i)) & if \quad p_i - p_j < -s \end{cases} \tag{2.1}$$

where $\mathbf{p} = (p_1, p_2)$. The demand facing firm 1 as a function of its own price given the price of firm 2 and s is shown in figure 1 (below).

Both firms have access to an increasing returns-to-scale technology with a fixed marginal cost. The marginal cost may differ between the firms and for the moment we exclude the fixed cost because it will not affect the marginal incentives of any of the firms. It will be introduced later on. The stage-game profit of firm i exclusive of the fixed cost is:

$$\pi_i(\mathbf{p}) = D_i(\mathbf{p})(p_i - c_i). \tag{2.2}$$

For convenience, let superscript m indicate a monopoly position of firm i, i.e. $\pi_i^m(p_i)$ is the monopoly profit of i (exclusive of the fixed cost). We now turn to the special case where the switching cost is infinite, .i.e. $s = +\infty$, and consumers are completely locked in. Each firm i acts as a monopolist and maximizes the stage game profit:

$$\max_{p_i} \pi_i. \tag{2.3}$$

The two firms' first order conditions constitute a system of two linear equations expressing i's profit maximizing price as a function of the own marginal cost and j's price (and vice versa), i.e. $\widetilde{p}_i = \widetilde{p}_i(p_j, c_i)$. Let $\widetilde{\mathbf{p}}^* = (\widetilde{p}_1^*, \widetilde{p}_2^*)$ denote the unique solution to the system.

There exists a smallest value of s, denoted $\widetilde{s}_i$, such that for any smaller value of s firm i can not charge $\widetilde{p}_i$ without making it a best response for j to cut its price by $s + \varepsilon$ and become a monopolist. Thus,

Proposition 1 *(i) If $s < \widetilde{s} = \max[\widetilde{s}_1, \widetilde{s}_2]$ then there exists no Nash equilibrium in pure strategies to the stage game.*
(ii) If $s \geq \widetilde{s} = \max[\widetilde{s}_1, \widetilde{s}_2]$ then is $\widetilde{\mathbf{p}}^*$ is the unique Nash equilibrium to the stage game.

Proposition 1 (ii) says that if $s \geq \widetilde{s}$ then is playing $p_i^t = \widetilde{p}^*$ for all t and all $i = 1, 2$. also a Nash equilibrium to the repeated game. (i) says that if the stage game is played only once and $s < \widetilde{s}$ then the only Nash equilibrium to the stage game is in mixed strategies. The reason to the non-existence result for low enough switching costs is quite simple. The game may be viewed as a matching-pennies game in which the basic actions are "undercut" and "price above". If firms 1 chooses to charge $p_1' > p_1$ where p_1 is such that firm 2 is indifferent between undercut by $s + \varepsilon$ and price above by s or charge $\widetilde{p}_2$, then 2 will undercut with probability 1. p_1' can not be 1's best reply. On the other hand, if 1 charges $p_1'' < p_1$ then 2 will charge a price above p_1'' with probability 1. Again, p_1'' can not be a best reply. Nor is charging exactly p_1 a best reply by the same reasons. Clearly, any equilibrium strategy in the stage game must involve randomizing.[7]

Depending on the size of the switching cost there are three different best-reply patterns in the stage game. They are illustrated in figure 2 - 4 where $s = 0.1$, $c_1 = 0$, and $c_2 = 0.1$. In figure 2 and 3 there exists no Nash equilibrium in pure strategies which easily is seen because the best-reply correspondence of firm 1, $R_1(p_2; s, c_1)$, never intersects firm 2's best-reply correspondence, $R_2(p_1; s, c_2)$. In figure 4 is the switching cost sufficiently large for the reaction curves to intersect and there exists a unique Nash equilibrium, namely $\widetilde{\mathbf{p}}^*$.

[7]Firm 1 randomizes between $\widetilde{p}_2^* - s - \varepsilon$ and $\widetilde{p}_1^*$ and firm 2 between $\widetilde{p}_1^* - s - \varepsilon$ and $\widetilde{p}_2^*$.

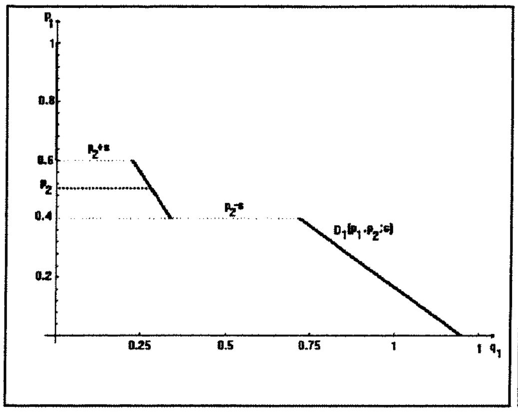

Figure 1: The demand curve.

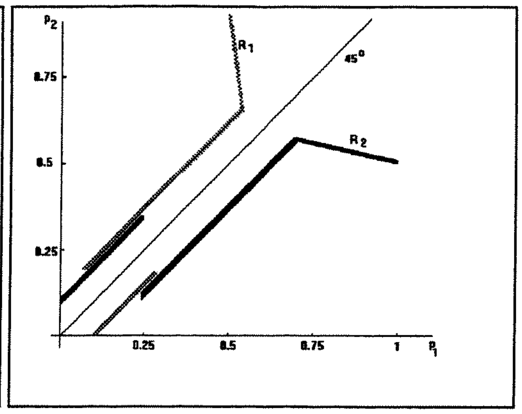

Figure 2: $s < \tilde{s}_1 < \tilde{s}_2$.

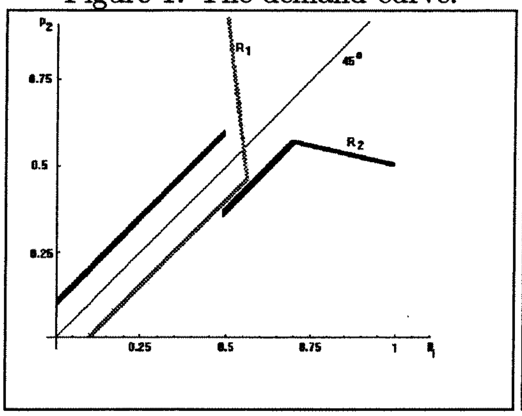

Figure 3: $\tilde{s}_1 < s < \tilde{s}_2$.

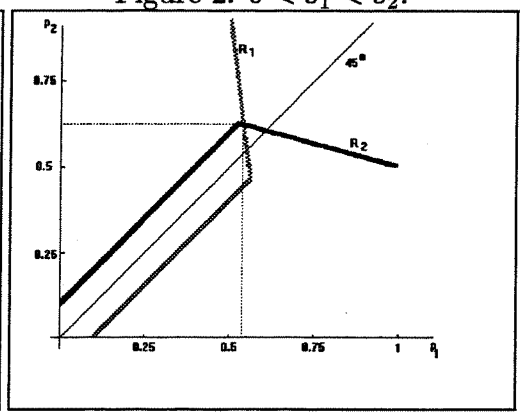

Figure 4: $\tilde{s}_1 < \tilde{s}_2 < s$.

2.2. The Repeated Game

As mentioned earlier repetition of the stage game allows the firms to condition the play in period t on the history of the game. This idea is captured in the well known concept of *trigger strategies.* A firm using a trigger strategy rewards "co-operative" play by its opponent and punishes deviations. The punishment phase may last for a large number of periods and after a fulfilled punishment the firm returns to the co-operative play. When the stage game is repeated infinitely we can use a well established result presented in theorem 1. The lowest profit a firm can guarantee itself for any p_j and s is $\underline{\pi}_i = \pi_i\left(R_i(p_j), p_j\right)$ where $p_j \geq c_j$ is chosen to minimize $\underline{\pi}_i$. We assume that when j chooses price to minimize i's profit the price must induce the own profit to be non-negative, i.e. $p_j \geq c_j$. Any lower price would not be a credible threat.

Definition 1 *The set of feasible payoff vectors to the repeated game is* $V = \textit{convex hull}\left\{\pi : \pi(\mathbf{p}) \text{ for all } \mathbf{p} \in [0,1]^2\right\}$.

Theorem 1 *(folk theorem) For every feasible payoff vector* π *with* $\pi_i > \underline{\pi}_i$ *for all players* i*, there exists a* $\underline{\beta} < 1$ *such that for all* $\beta \in \left(\underline{\beta}, 1\right)$ *there is a Nash equilibrium of the repeated game with payoffs* π*.*[8]

The folk theorem lets us state the first corollary:

Corollary 1 *If* $s \geq 0$ *and* β *is sufficiently high then* $\underline{\pi}_i < \pi_i\left(\tilde{\mathbf{p}}^*\right)$ *and there exists infinitely many Nash equilibria to the repeated game.*

Corollary 1 states that there are infinitely many equilibria for all values of s given that β is sufficiently large. In order to be able to discriminate among these equilibria we add three requirements that an equilibrium must satisfy.

Requirement 1 *Only equilibria in stationary pure strategies are considered.*

We require the price to be constant over time because the action space and set of feasible payoff vectors to the stage game is unaffected by previous play. That is, the continuation game is the same in each period. We also argue that it is not likely that a firm randomizes over well defined price alternatives.

Requirement 2 *The equilibrium price vector must be such that the best downward price deviation by any firm does not dominate the best upward price deviation in the actual period.*

The second restriction is quite strong but the motivation is simple. The restriction is meant to capture the idea that lies behind the concept of *conjectural variations* which is used to introduce some dynamic thoughts into static oligopoly models. Each firm takes its conjecture of the other firms response into consideration when making its price decision. A stronger version (consistent variations) require the firm to conjecture the other firms to play their best reply to the own action in the neighborhood of an equilibrium. Loosely speaking, the idea is to set the prices so that it is no longer an unique one-period best reply for the individual

[8]The theorem is picked from Fudenberg and Tirole[9] (page 152) but some of the notation has been changed. Fudenberg and Tirole[9] and Fudenberg and Maskin[8] are recommended for readers interested in repeated games and the folk theorem.

firm to undercut the other firm's price and become monopolist. No firm will then have a strict, even short-term, incentive to deviate downward and thereby trigger an aggressive response from the other firm.[9]

Requirement 3 *Given requirement 1 and 2 the equilibrium price vector is chosen so that the individual profit of firm 1 and 2 is maximized.*

If i charges p_i then the best downward deviation by j is to charge $p_i - s - \varepsilon$ and the best upward deviation is to charge $p_i + s$ or, if $p_i + s \geq \widetilde{p}_j$, to charge $\widetilde{p}_j$. Requirement 2 says that p_i should be such that:

$$\pi_j^m (p_j = p_i - s) \leq \max \left[\pi_j (p_j = p_i + s, p_i), \pi_i (\widetilde{p}_j, p_i) \right]. \tag{2.4}$$

Denote the upper bound of an equilibrium price vector satisfying (2.4) $\widehat{\mathbf{p}} = (\widehat{p}_1, \widehat{p}_2)$. For any $p_i \leq \widehat{p}_i$ j's best reply is $p_j = \min [\widehat{p}_i + s, \widetilde{p}_i]$ and for any $p_i \geq \widehat{p}_i$ the best reply is $p_j = p_i - s - \varepsilon$. If $p_i = \widehat{p}_i$ then j is indifferent between the best downward and the best upward deviation.

Proposition 2 *Requirements 1-3 make* $\mathbf{p}^* = (p_1^*, p_2^*)$ *where* $p_i^* = \min [\widehat{p}_i, \widetilde{p}_i]$ *the unique equilibrium price vector in the infinitely repeated game.*

Corollary 2 *If* $c_i < c_j$ *and* $s < \widetilde{s}$ *then* $p_i^* > p_j^*$.

Corollary 2 tells us that the low-cost firm also is the high-price firm. The reason is quite intuitive. The marginal cost of j directly sets the upper bound of i's equilibrium price if $s < \widetilde{s}_i$. This is easily seen in equation (2.4). It is j's own marginal cost that determines the temptation for j to undercut i's price by $s + \varepsilon$. A lower marginal cost increases the temptation.

Let $\overline{s}_i$ be the smallest s such that j charges $\widetilde{p}_j \leq \widehat{p}_i + s$ and recall that $\widetilde{s}_i$ is the smallest value of s such that i can charge $\widetilde{p}_i$ without risking having its price undercut by $s + \varepsilon$, i.e. satisfying equation (2.4) when charging $\widetilde{p}_i$.

Proposition 3 *(i)* $p_i^* = c_i$ *if* $s = 0$.
(ii) $\frac{\partial p_i^*}{\partial s} > 1$ *for all* $0 < s < \frac{5(1+19\sqrt{2})(1-c_j)}{721} < \overline{s}_i$.
(iii) $0 < \frac{\partial p_i^*}{\partial s} < 1$ *if* $\frac{5(1+19\sqrt{2})(1-c_j)}{721} \leq s < \widetilde{s}_i$.
(iv) $\frac{\partial p_i^*}{\partial s} < 0$ if $\widetilde{s}_j > s \geq \widetilde{s}_i$ and $\frac{\partial p_i^*}{\partial s} = 0$ if $s \geq \max [\widetilde{s}_j, \widetilde{s}_i]$.

[9] For a short introduction to conjectural variations, see Shapiro[19], and for consistent variation, see Bresnahan[3].

That is, the equilibrium market price can not exceed the marginal cost if it is costless to switch supplier. If the production requires investments (fixed costs) then marginal-cost pricing can not be a sustainable equilibrium (sufficient conditions will be discussed below). This is to say that if there are increasing returns to scale and no switching cost, then only one firm can survive in the long run. As we increase s, the upper bound of the equilibrium market price increases more than the switching cost. This is because a firm can not price discriminate. If it lowers its price in order to increase its market-share, it must charge the same price from all its customers. Thus, a firm that cuts its price incurs a loss proportional to the number of old customers and this makes a price-cut less attractive. Consequently, the sustainable market price exceeds the marginal cost with a larger amount than the switching cost. A further increase in s makes the equilibrium price increase by less than the increase in s. This is hardly surprising. The profit function is quadratic in the own price. Hence, the marginal profit falls with the own price why the incentives to increase the own price falls as well. But so do the marginal incentive to cut prices, but at a slower rate. Therefore, the speed with which $\widehat{p}_i$ increases falls with s. Alternatively, π_i is quadratic in p_i making $\widehat{p}_i$ contain a root-term of s. Clearly, $\widehat{p}_i$ is a strictly concave function of s. Finally, if the switching cost allows i to charge $\widetilde{p}_i$ but not j to charge $\widetilde{p}_j$, then an increase in s lets j raise its price which in turn lowers i's demand and thus the price and profit of i (proposition $3iv$ and corollary $3i$).

Proposition 4 *(i)* $0 < \frac{\partial p_i^*}{\partial c_j} < 1$ *if* $s < \widetilde{s}_i$.
(ii) $\frac{\partial p_i^*}{\partial c_i} = 0$ *if* $s < \widetilde{s}_i$.

Corollary 3 *(i)* $\frac{\partial \pi_i}{\partial s} < 0$ *for all* $\widetilde{s}_i \leq s < \widetilde{s}_j$.
(ii) $\frac{\partial \pi_i}{\partial s} = 0$ *for all* $s \geq \max\left[\widetilde{s}_1, \widetilde{s}_2\right]$.
(iii) $\frac{\partial \pi_i}{\partial c_i} < 0$ *for all* $s > 0$.

One additional condition for the equilibrium price vector to be sustainable is that the fixed cost of firm i is covered. Let F be the investment cost and τ the number of periods the investment lasts. We have seen that for any s is:

$$\pi_i\left(\mathbf{p}^*\right) \geq 0 \text{ for } i = 1, 2. \tag{2.5}$$

This is not sufficient to guarantee the existence of two firms in the long run. For a firm to reinvest or a firm to enter the market the following condition must hold:

$$\sum_{t=0}^{\tau-1} \beta^t \pi_i\left(\mathbf{p}^*\right) \geq F. \tag{2.6}$$

Otherwise, at least one of the firms can not cover its investment costs. If the market is large enough, or F small enough, so that an oligopoly can survive then there exists a $s > 0$ such that condition (2.6) is satisfied. If, on the other hand, only a monopoly can survive then one firm must leave the market, perhaps after a war of attrition.

2.3. Technological Progress - A Two-Firm Example

We have created an economic framework in which the impact of technological change is easy to analyze. As an example, a process innovation lowering i's the marginal cost will not allow i to charge a higher price but will force j to lower its price. π_i increases while π_j decreases and maybe j is forced to leave the market in the long run. The technological progress treated covers only process innovations, not product innovations. There is, however, one exception. If a product innovation only lowers the switching cost then it can be analyzed in the model.

Consider the case of two firms with marginal costs $c_1 = 0$ and $c_2 = 0.1$. If consumers can not switch supplier then maximizes i π_i with respect to the own price taking p_j as given. From the first order condition the profit maximizing p_i can be written as a function of c_i and p_j:

$$\tilde{p}_i(p_j; c_i) = \frac{5 + 4c_i - p_j}{8}. \tag{2.7}$$

If consumers are able to switch supplier but faces a switching cost then it is possible for one firm to underbid the other and become monopolist. Using requirement 1-3 we know from proposition 2 that $p_i^* = \min[\widehat{p}_i, \tilde{p}_i]$ is the only equilibrium price for firm i in the infinitely repeated game. The equilibrium pair of prices must be such that it does not pay for i to charge $p_j - (s + \varepsilon)$ and become monopolist relative to the best upward deviation $p_j + s$. Thus, we require:

$$\begin{array}{c} \frac{5}{4}\left(1 - p_j - (s+\varepsilon)\right)\left(p_j - (s+\varepsilon) - c_i\right) \leq \\ \frac{1}{2}\frac{5}{4}\left(1 - p_j - s\right)\left(p_j + s - c_i\right) \;\; \textit{for } i,j = 1,2 \textit{ and } i \neq j \end{array} \tag{2.8}$$

Letting (2.8) hold with equality p_j can be written as a function of c_i, $\widehat{p}_j(c_i; s)$. Since (2.8) is quadratic in p_j we get two alternative expressions. The two expressions are the lower and upper bound of an interval of values of p_j violating the inequality. Only the lower bound is of relevance and any equilibrium price of firm j must not exceed this lower bound:

$$\widehat{p}_j \leq \frac{1 + c_i}{2} + \frac{29}{10}s - \sqrt{\left(\frac{1 - c_i}{2}\right)^2 - s\frac{1 - c_i}{10} + \frac{721}{100}s^2}. \tag{2.9}$$

For some higher values of s is $\widetilde{p}_i(p_j) \leq p_j + s$ and the best upward response by firm i is $\widetilde{p}_i(p_j)$. Then the following inequality must hold:

$$\widehat{p}_j \leq \frac{76(1+c_i)+9+160s}{161} - \sqrt{\frac{20(1-c_i)^2+9s(1-c_i)-s^2}{161^2}} \quad (2.10)$$

For the selected values of c_1, c_2, and s is (2.8) binding and $(\widehat{p}_1, \widehat{p}_2) = (0.325, 0.231)$. As we increase the switching cost $\widehat{p}_i$ approaches $\widetilde{p}_i$ and at $s = \widetilde{s}_i$ they are equal. In the example is $(\widetilde{s}_1, \widetilde{s}_2) = (0.288, 0.41)$. For any larger value of s than $\max[\widetilde{s}_1, \widetilde{s}_2]$ is the equilibrium pair of prices given by given by the unique solution to

$$\begin{aligned} \widetilde{p}_1(\widetilde{p}_2^*; c_1) &= \widetilde{p}_1^* \\ \widetilde{p}_2(\widetilde{p}_1^*; c_2) &= \widetilde{p}_2^* \end{aligned}$$

which is $(\widetilde{p}_1^*, \widetilde{p}_2^*) = (0.54, 0.597)$. However, the selected value of s is far too low and the two equilibrium prices are given by the binding equation (2.9), i.e. $(p_1^*, p_2^*) = (\widehat{p}_1, \widehat{p}_2) = (0.325, 0.231)$ which is the unique equilibrium pair of prices to the repeated game. The equilibrium prices for some other combinations of marginal and switching costs are given in table 1:

	$c_1 = 0$	$c_1 = 0.1$	$c_1 = 0.2$
(p_1^*, p_2^*)	$(0.325, 0.231)$	$(0.325, 0.325)$	$(0.325, 0.417)$
$(\overline{s}_1, \overline{s}_2)$	$(0.189, 0.21)$	$(0.189, 0.189)$	$(0.189, 0.168)$
$(\widetilde{s}_1, \widetilde{s}_2)$	$(0.288, 0.41)$	$(0.331, 0, 331)$	$(0.373, 0.251)$

Table 1: The values are computed for $c_2 = 0.1$ and $s = 0.1$.

3. Prices and Subsidies as Signals of Strength

Limit pricing and predatory pricing are (usually costly) pricing strategies used by one firm to signal strength when there is uncertainty over that firm's costs. The intention is to discourage entry (limit pricing) of other firms or to encourage established firms to leave the market (predatory pricing). Following LeBlanc[13] we merge the two literatures of limit pricing and predatory pricing. In this second part of the paper we study a simple and informal two-firm model with price competition and switching costs. The aim is to investigate under what circumstances the incumbent chooses limit pricing and under what circumstances predatory pricing is chosen.

The entry-deterrence game is played over 3 periods. There are two firms, one incumbent and one potential entrant, and a market for a (homogenous) telecommunication service. The game begins with nature assigning one of two types to the incumbent. The incumbent can be either of a low-cost type or of a high-cost type. The entrant is commonly known to be of the high-cost type. In the first period the incumbent has a monopoly position and the demand is determined by the charged price. Having observed the incumbent's price in the first period the entrant makes his entry decision. If he decides to enter, then he must invest F, and if he decides not to enter then the incumbent remains monopolist in period 2 and 3. The active firms, the incumbent and eventually the entrant, fight over market shares in the second period. They do this by subsidizing consumers subscription fees and the incumbent acts as first mover. Consumers switch from the incumbent to the entrant if the entrant's subsidy exceeds the incumbent's by more than their switching cost. The entrant has the opportunity to leave the market after receiving the period-2 payoff. If the entrant decides to leave the market a fraction of the investment cost is recovered. In the third period the firms supply their respective share of the market with the service.

In the duopoly case the entrant's profit is higher if the incumbent is of the high-cost type than of the low-cost type. Thus, it is in the interest of the incumbent to look strong to discourage entry. If he is of the low-cost type he may engage in a costly signal in the first period, i.e. to charge a price lower than he otherwise would have done. This price is such that only a low-cost incumbent has an incentive to set such a price, given that it with certainty convey his true type to the entrant who stays out. For the sake of simplicity, we will restrict our attention to fully separating equilibria in pure strategies. That is, equilibria in which the incumbent, if being of the high-cost type, sets a low subsidy that allows the entrant to conquer a strictly positive share of the market. The low-cost incumbent sets a predatory subsidy such that the entrant is indifferent between staying and leaving the market. Being second mover the entrant observes the incumbent's subsidy before setting the own subsidy.

A patient low-cost incumbent chooses to deter entry early because he values future profits highly. He takes a cost in the first period in order to increase future profits. Being less patient he uses a predatory subsidy in the second period instead. This allows him to profit maximize in the first period at the expense of future profits. A very impatient incumbent will not even use the predatory subsidy.

There is also a possibility of pooling. Both the high-cost and the low-cost type charges the low-cost type's one-period preferred (monopoly) price in the first period and makes it impossible for the potential entrant to distinguish between them. The entrant chooses not to enter. Pooling occurs when the high-cost incumbent is sufficiently patient and the probability of the incumbent being of the low-cost type is high.

3.1. The Entry Deterrence Game

We use the same notation as in section 2 but with some redefinition. The reader should be aware of this. There are two firms and a market for a homogeneous service, firm 1 is the incumbent and firm 2 the potential entrant.[10] Let $C = \{\underline{c}, \overline{c}\}$ denote the set of possible types where $\underline{c}$ is the low-cost type and $\overline{c}$ the high-cost type. $T = \{1, 2, 3\}$ is the set of periods over which the game is played. The incumbent's type is private information but the distribution from which the type is drawn is common knowledge. Let λ denote the probability of the incumbent being of the low-cost type and let λ_2^t denote the entrant's belief at time t. $p_i^t \in \Re_+$ is the non-negative price set by firm i in period $t \in T$. Observe that this notation is made for convenience. The prices set in the second period, p_1^2 and p_2^2, are *subsidies* (transfers) from respective firm to the consumers and are not be confused with ordinary prices. For $t = 2$ let a zero subsidy by the entrant, $p_2^2 = 0$, denote his decision to leave the market. The entrant, if leaving the market, receives αF in the third period where $\alpha \in [0, 1]$ is the pre-determined share of the

[10] For a more complex analysis with three types, Cournot competition, and no switching costs, see LeBlanc[13].

investment that is recovered. The switching cost is uniformly distributed over the unit interval and the number of consumers is normalized to one. Consumers are assumed to act myopically, i.e. they switch in the second period if the difference in subsidies is greater than their switching cost without taking the third period into consideration.[11] $g \geq 0$ is the demand curve's intercept on the quantity axis. The demand functions are:

$$t=1 \quad D_1^1(p_1^1) = \begin{cases} (g-p_1^1) & if \quad 0 \leq p_1^1 \leq g \\ 0 & otherwise \end{cases}$$

$$t=2 \quad \begin{array}{l} D_1^2(p_1^2,p_2^2) = \begin{cases} 1 & if \quad p_2^2 - p_1^2 < 0 \\ (1-(p_2^2-p_1^2)) & if \quad 0 \leq p_2^2 - p_1^2 \leq 1 \\ 0 & if \quad p_2^2 - p_1^2 > 1 \end{cases} \\ \\ D_2^2(p_1^2,p_2^2) = \begin{cases} 0 & if \quad p_2^2 - p_1^2 < 0 \\ (p_2^2-p_1^2) & if \quad 0 \leq p_2^2 - p_1^2 \leq 1 \\ 1 & if \quad p_2^2 - p_1^2 > 1 \end{cases} \end{array}$$

$$t=3 \quad \begin{array}{l} D_1^3(p_1^3,p_2^3) = \begin{cases} D_1^2(p_1^2,p_2^2)\,(g-p_1^3) & if \quad 0 \leq p_1^3 \leq g \\ 0 & otherwise \end{cases} \\ \\ D_2^3(p_1^3,p_2^3) = \begin{cases} D_2^2(p_1^2,p_2^2)\,(g-p_2^3) & if \quad 0 \leq p_2^3 \leq g \\ 0 & otherwise \end{cases} \end{array}$$

Let $\pi_i^m(p_i;c_i)$ denote the monopoly profit of i being of type $c_i \in C$ and let $p_i^m(c_i)$ be the profit maximizing monopoly price. $\pi_i^3(p_i^3;p_1^2,p_2^2,c_i)$ is i's duopoly profit in the third period as a function of its price, p_i^3, and given the market share determined in $t=2$. Finally, if $p_2^2 = 0$ then is $\pi_2^3(p_2^3;p_1^2,p_2^2,c_i) = \alpha F$ for all p_2^3, i.e. a leaving decision is irrevocable. The firms have the common discount factor $\beta \in [0,1]$. Formally, the incumbent's price in period t is a function of the history of chosen actions, i.e. prices, subsidies, and firm 2's entry decision, up to that period. The belief, λ_2^t, is updated according to Baye's rule and as a function of the history.

One major problem with signaling games is the multiplicity of equilibria due to unrestricted out of equilibrium beliefs. Following large parts of the existing static literature on signaling games we restrict ourselves to reasonable out of equilibrium beliefs.[12]

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[11] Letting consumers act non-myopically will not change the qualitative results of the model. It will only affect the required difference in subsidies, not the pattern of consumer behavior.

[12] For example, see Cho and Kreps[4].

Assumption 1. *Separating equilibrium strategies are efficient.*

That is, in a separating equilibrium the low-cost incumbent distinguish herself by in the second period choose a predatory subsidy, p^{α}, such that the entrant is *indifferent* between playing her profit maximizing response/subsidy and leaving the market.

Assumption 2. *An eventual preentry pooling price is the low-cost type's single-period profit maximizing price.*

Assume instead the preentry pooling price to be the high-cost incumbents monopoly price. Assumption 2 then says that an entrant observing a lower, and out of equilibrium, price must believe the deviator to be of the low-cost type with at least some certain probability. This is because only a low cost incumbent can possibly gain from such a deviation. A high-cost deviator would lose for sure. Thus, the high-cost type would never deviate and that is why the entrant should assign the high-cost deviator sufficiently low probability. Then, it is not possible to sustain a pooling equilibrium where both types of incumbent plays the high-cost incumbents single-period preferred price.

Assumption 3. *The low-cost incumbent chooses her most preferred equilibrium given assumptions 1 - 2.*

The game is most easily solved by studying profit maximizing firms and using backward induction. Backward induction is to solve the game backwards from the last period to the first. The market shares are pre-determined in the last period and there is no reputation to invest in since there is no period after this. The only equilibrium behavior in the last period is short-run profit maximizing. Firm i solves:

$$\max_{p_i^3} D_i^2(p_1^2, p_2^2)\left(g - p_i^3\right)\left(p_i^3 - c_i\right).$$

The profit maximizing price in $t = 3$ is $p_i^{3*} = p_i^m(c_i) = \frac{g+c_i}{2}$. Well aware of this, i's maximization problem in $t = 2$ boils down to:

$$\max_{p_i^2} D_i^2(p_1^2, p_2^2)\left(\beta\pi_i^m\left(p_i^{3*}; c_i\right) - p_i^2\right) \tag{3.1}$$

The monopoly profit received in the last period increases for each consumer that i attracts, but a subsidy must be paid in the second period. This is why

p_i^2 enters with a negative sign in the right-hand bracket. The entrant, acting as second mover and taking the incumbent's subsidy as given, subsidizes subscription until the marginal revenue of attracting one more subscriber equals the marginal cost of subsidizing. Rewriting the first order condition to (3.1) for i=2 and solving for p_2^2 lets us define $p_2^2(p_1^2)$:

$$p_2^2\left(p_1^2\right) = \frac{p_1^2 + \beta\pi_2^m\left(p_2^{3*};\overline{c}\right)}{2}.$$

Of course, playing $p_2^2(p_1^2)$ is only profit maximizing if it yields a higher profit than leaving the market:

$$D_2^2\left(p_1^2, p_2^2\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^2\left(p_1^2\right)\right) \geq \alpha\beta F. \tag{3.2}$$

Letting the inequality bind and solving (3.2) for p_1^2 gives $p^\alpha = p^\alpha(\overline{c}, \beta, \alpha, F, g)$. When the incumbent's subsidy is greater than p^α it is better for the entrant to leave the market, i.e. to play $p_2^2 = 0$, than to stay and subsidize each subscription with $p_2(p^\alpha)$. The entrant is indifferent between leaving and staying if $p_1^2 = p^\alpha$ but for simplicity we assume him to leave the market. Let $R_2^2(p_1^2)$ be the entrant's best-reply correspondence in the second period:

$$R_2^2\left(p_1^2\right) = \begin{cases} p_2^2\left(p_1^2\right) & if \;\; p_1^2 < p^\alpha \\ 0 & if \;\; p_1^2 \geq p^\alpha \end{cases}.$$

Acting as first mover in the second period the incumbent may either subsidize with p^α and make the entrant leave the market or subsidize with a smaller amount and share the market.

Proposition 5: *(i)* $\frac{\partial R_2^2}{\partial g} \geq 0$, *(ii)* $\frac{\partial R_2^2}{\partial \beta} \geq 0$, *(iii)* $\frac{\partial R_2^2}{\partial c_2} \leq 0$,
(iv) $\frac{\partial p\alpha}{\partial g} \geq 0$, *(v)* $\frac{\partial p\alpha}{\partial a} \leq 0$, *(vi)* $\frac{\partial p\alpha}{\partial F} \leq 0$,
(vii) $\frac{\partial p\alpha}{\partial c_2} \leq 0$, *and (viii)* $\frac{\partial p\alpha}{\partial \beta} \geq 0$ *if* $\beta \geq \frac{\alpha}{\left(\frac{g-\overline{c}}{2}\right)^2}$.

Proposition 5 tells us that the entrant's willingness to subsidize subscription increases with the size of the market and discount factor. It decreases with the own marginal cost. The third-period profit decreases with the own marginal cost and thereby the willingness to subsidize in the second period. An increase in demand increases the $t = 3$-profit and the entrant's willingness to subsidize in the second. Analogously, a higher discount factor increases the willingness to subsidize because the discounted value of a future profit increases. The incumbent's subsidy at which the entrant is indifferent between staying and leaving the market increases with the demand and discount factor and decreases with the entrant's marginal cost by the same reasons. p^{α} decreases with the size of the investment and the recovered share. This increases the payoff from leaving the market and increases the entrant's required $t = 3$-profit from staying. Consequently, the entrants maximal subsidy decreases and so do the incumbent's predatory subsidy. The incumbent chooses to force the entrant to leave if:

$$\beta\pi_1^m\left(p_1^{3*};c_1\right) - p^{\alpha} \geq D_1^2\left(p_1^2, p_2^2\left(p_1^2\right)\right)\left(\beta\pi_1^m\left(p_1^{3*};c_1\right) - p_1^2\right) \tag{3.3}$$

for some $p_1 \in [0, p^{\alpha})$. If we let the inequality bind we can write the minimal discount factor as a function of the two firms marginal costs, the share of the investment that is recovered, the size of the investment, and the size of the market. Denote this critical value $\overline{\beta}^2 = \overline{\beta}^2\left(c_1, \overline{c}, \alpha, g, F\right)$ where superscript denotes period. If the discount factor is smaller than $\overline{\beta}^2$ then the future $t = 3$-profits of playing p^{α} is to heavily discounted to make it worth playing p^{α} in the second period.

The entrant will enter the market if the expected present value of entering is greater than the present value of required investments. Recall that λ_2^t denotes the entrant's belief at time t. Assume that $\overline{\beta}^2\left(\underline{c}, \overline{c}, \alpha, g, F\right) \leq \beta \leq \overline{\beta}^2\left(\overline{c}, \overline{c}, \alpha, g, F\right)$. Then there is a $\overline{\lambda}_2^1$ such that for any $\lambda_2^1 < \overline{\lambda}_2^1$ the entrant decides to enter the market and to stay out if $\lambda_2^1 \geq \overline{\lambda}_2^1$. That is, at $\overline{\lambda}_2^1$ the expected profit from entering is equal to the required investment. $\overline{\lambda}_2^1$ is found by letting the inequality bind and solving:

$$\beta\left(\lambda_2^1 D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) + (1-\lambda_2^1)\beta\alpha F\right) \geq F \tag{3.4}$$

with respect to λ_2^1. Let $\overline{\lambda}_2^1\left(\underline{c}, \overline{c}, \beta, g, \alpha, F\right)$ denote the solution. $p_1^{2*} < p^{\alpha}$ is the high-cost incumbent's optimal subsidy in period 2 and $p_2^{2*} > 0$ the entrant's best reply.

Proposition 6: *(i)* $\frac{\partial \overline{\lambda}_2^1}{\partial F} \leq 0$, *(ii)* $\frac{\partial \overline{\lambda}_2^1}{\partial \alpha} \geq 0$, *and (iii)* $\frac{\partial \overline{\lambda}_2^1}{\partial \beta} \geq 0$.

If the size of the investment increases then the expected profit must increase as well to make it profitable (in expectation) for the entrant to enter the market. This requires a decrease in the critical probability of the incumbent to be of the low-cost type. A larger recovered share of the investment lowers the expected cost of entering which allows for a higher critical probability. A larger β makes the net present value of entering larger because future profits is less discounted. The critical probability increases with β.

Assume that the entrant believes the probability of the incumbent being of the low-cost type to be sufficiently low and enters if he does not receive any new information from the first period prices, i.e. $\lambda_2^1 < \overline{\lambda}_2^1 (\underline{c}, \overline{c}, \beta, g, \alpha, F)$. Then, the low cost incumbent has an incentive to signal his true type and deter entry. The signal p^l, or the limit price, must satisfy two restrictions:

$$\pi_1^m (p_i^m (\underline{c}) ; \underline{c}) + \beta \left(\beta \pi_1^m \left(p_1^{3*}; \underline{c}\right) - p^\alpha\right) \leq \pi_1^m \left(p^l; \underline{c}\right) + \beta^2 \pi_1^m \left(p_1^{3*}; \underline{c}\right) \tag{3.5}$$

$$\begin{aligned} & \pi_1^m (p_i^m (\overline{c}) ; \overline{c}) + \beta D_1^2 \left(p_1^{2*}, p_2^{2*}\right) \left(\beta \pi_1^m \left(p_1^{3*}; \overline{c}\right) - p_1^{2*}\right) \\ \geq \;& \pi_1^m \left(p^l; \overline{c}\right) + \beta^2 \pi_1^m \left(p_1^{3*}; \overline{c}\right). \end{aligned} \tag{3.6}$$

(3.5) says that the low-cost incumbent must receive a higher profit if setting the limit price in period 1 than if setting the monopoly price in the first period and enforce a leaving decision in the second by playing the predatory subsidy. The second restriction (3.6) says that a high-cost incumbent must not find it more profitable to deter entry in the first period than to set the monopoly price and receive the duopoly profits the second and third periods. Let p_1^l be such that (3.5) binds and p_2^l such that (3.6) binds. $p_1^l \leq p_2^l$ for a limit price to exist.

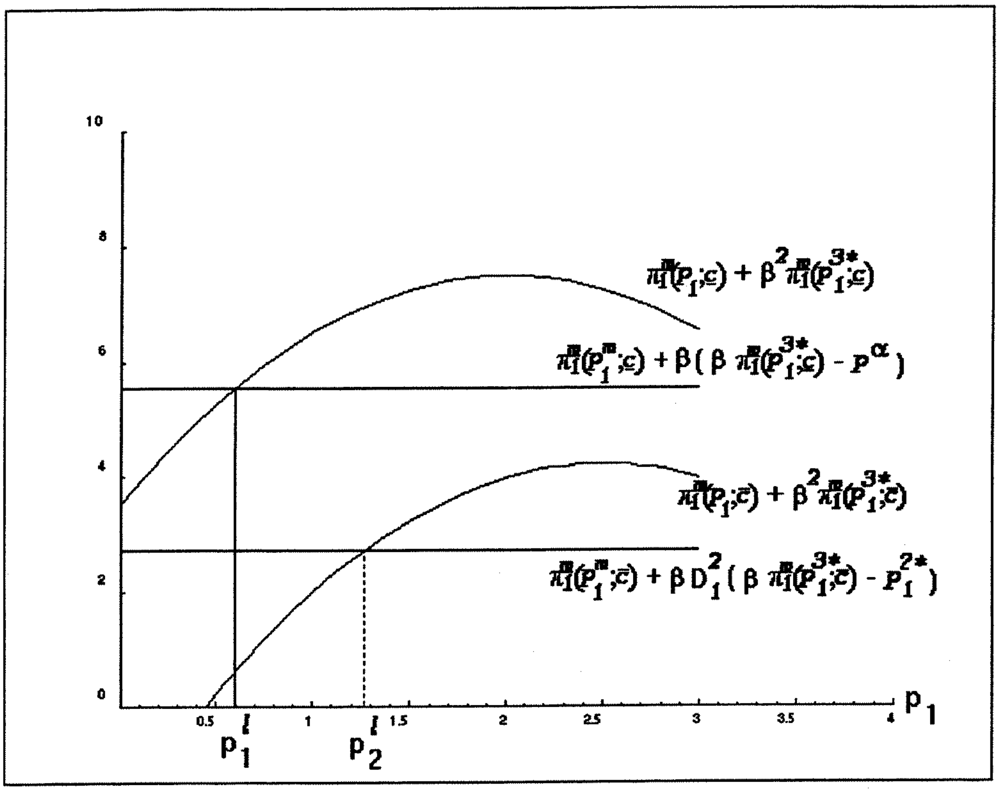

Figure 5: The limit price p_2^l.

The situation is illustrated in figure 5. At p_1^l is the low-cost incumbent indifferent between playing his monopoly price and a limit price in the first period. Analogously, at p_2^l is the high-cost incumbent indifferent between playing a limit price in the first period and with certainty be taken for the low-cost type and playing the monopoly price. If the monopoly price is played then the entrant enters and the incumbent receives the duopoly profit. Any price $p_1^l \leq p^l \leq p_2^l$ lets the low-cost incumbent signal his true type while it is profit maximizing for the high-cost type to play the monopoly price. The profit maximizing limit price for the low cost incumbent is $p^l = p_2^l$. The entrant enters only if $p_1^1 > p^l$. Given $\alpha, F, g, \underline{c}$, and $\overline{c}$ the equality $p_1^l = p_2^l$ implicitly defines a lower bound, $\overline{\beta}^1(\alpha, F, g, \underline{c}, \overline{c})$, for β. For any $\beta \geq \overline{\beta}^1$ the low-cost incumbent finds it worth while to deter entry in the first period. There is, of course a possibility of $\beta \leq \overline{\beta}^2 \leq \overline{\beta}^1$. Then the low-cost incumbent will not limit competition in any period by playing either the limit price or the predatory subsidy.

When $\lambda \geq \overline{\lambda}_1^2$ pooling may occur. Pooling is when both types of incumbent charge the same price in the first period. Since both types set the same price no new information is received by the entrant and $\lambda_1^2 = \lambda \geq \overline{\lambda}_1^2$. The entrant decides not to enter. The pooling price is by assumption 2 the low-cost incumbent's monopoly price. The high-cost incumbent chooses to pool if:

$$\begin{aligned} & \pi_1^m\left(p_i^m\left(\underline{c}\right);\overline{c}\right)+\beta^2\pi_1^m\left(p_1^{3*};\overline{c}\right) \\ \geq\ & \pi_1^m\left(p_i^m\left(\overline{c}\right);\overline{c}\right)+\beta D_1^2\left(p_1^{2*},p_2^{2*}\right)\left(\beta\pi_1^m\left(p_1^{3*};\overline{c}\right)-p_1^{2*}\right). \end{aligned} \tag{3.7}$$

(3.7) defines a lower bound which β must be greater than for pooling to occur. Denote the solution $\underline{\beta}^1\left(\underline{c},\overline{c},g\right)$. For an illustration see figure (6).

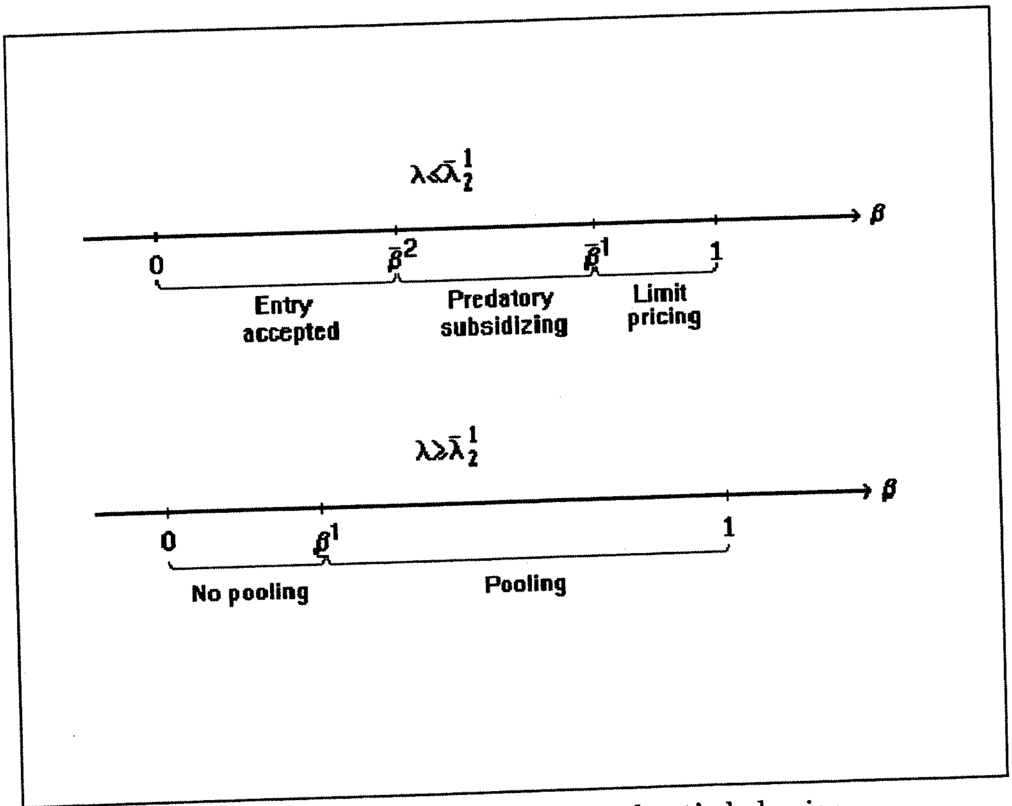

Figure 6: β determines the incumbent's behavior.

The expressions $\overline{\beta}^2\left(\cdot\right)$, $\overline{\beta}^1\left(\cdot\right)$, and $\underline{\beta}^1\left(\cdot\right)$ easily get messy and the sign of various derivatives ambiguous. We therefore proceed with a numerical example.

3.2. A Numerical Example

We let the low-cost incumbent have zero marginal cost and the high cost-type a marginal cost equal to one, i.e. $C = \{0,1\}$. Let $g = 4$ and $\beta = 0.9$. The two types' of incumbent monopoly prices and profits are:

$$\begin{aligned}
p_1^m(\underline{c}) &= p_1^{3*} = \frac{g+\underline{c}}{2} = 2, \\
p_i^m(\overline{c}) &= p_2^{3*} = \frac{g+\overline{c}}{2} = 2.5 \; for \; i = 1,2, \\
\pi_1^m(p_1^m(\underline{c});\underline{c}) &= \left(\frac{g-\underline{c}}{2}\right)^2 = 4, \; and \\
\pi_i^m(p_i^m(\overline{c});\overline{c}) &= \left(\frac{g-\overline{c}}{2}\right)^2 = 2.25 \; for \; i = 1,2.
\end{aligned}$$

The monopoly prices are played in the third period and sometimes in the first period, depending on β. In the second period the high-cost incumbent and entrant play the equilibrium subsidies p_1^{2*} and p_2^{2*}. p_1^{2*} is given by:

$$\begin{aligned}
p_1^{2*} &= \arg\max_{p_1^2} D_1^2\left(p_1^2, R_2^2\left(p_1^2\right)\right)\left(\beta\pi_1^m\left(p_1^{3*};\overline{c}\right) - p_1^2\right) \\
&= \frac{b(g-\overline{c})^2}{4} - 1 = 1.025
\end{aligned}$$

and p_2^{2*} by:

$$p_2^{2*} = R_2^2\left(p_1^{2*}\right) = \frac{b(g-\overline{c})^2 - 2}{4} = 1.525.$$

That is, if both the incumbent and the entrant is of the high-cost type then the incumbent subsidizes each subscriber with 1.025 and the entrant with 1.525. The reason to why the entrant's equilibrium subsidy is higher than the incumbent's is that the entrant first must subsidize as much as the incumbent and then increase the subsidy to compensate for consumers' switching cost when changing supplier. Recall that the switching cost is uniformly distributed over the unit interval. The entrant's market share is given by:

$$D_2^2\left(p_1^{2*}, p_2^{2*}\right) = p_2^{2*} - p_1^{2*} = 0.5$$

which also is equal the high cost incumbent's share:

$$D_1^2\left(p_1^{2*}, p_2^{2*}\right) = 1 - \left(p_2^{2*} - p_1^{2*}\right) = 0.5.$$

For simplicity we let 10 percent of the entrant's investment be recovered if he leaves the market, $\alpha = .1$. Rearrange equation (3.4) that gives the critical value of λ:

$$\overline{\lambda}_2^1 = \frac{\beta D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) - F}{\beta\left(D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) - \beta\alpha F\right)}.$$

A necessary condition for the entrant to enter the market is:

$$\beta D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) \geq F.$$

which boils down to $F \leq \frac{\beta}{4} = 0.233333$. Let $F = 0.2$. Then is $\overline{\lambda}_2^1 = 0.119732$. With the specified values of α and F equation (3.3) gives:

$$p_1^{2\ \max} = 2.00242.$$

This is the highest subsidy that a low-cost incumbent is prepared to pay in order to make the entrant leave the market. The corresponding subsidy for the-high cost type is 1.525. For the entrant to stay equation (3.2) must be satisfied:

$$D_2^2\left(p_1^2, R_2^2\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - R_2^2\left(p_1^2\right)\right) \geq \alpha\beta F.$$

Solving for p_1^2 yields:

$$p_1^2 \leq p^\alpha = 1.75667.$$

Hence, a low-cost incumbent will find it worth while to force the entrant to leave the market while a high-cost incumbent will not. Some computations give that:

$$\begin{aligned}
\overline{\beta}^2\left(\underline{c}, \overline{c}, \alpha, g, F\right) &= 0.441994 \\
\overline{\beta}^2\left(\overline{c}, \overline{c}, \alpha, g, F\right) &= 3.125.
\end{aligned}$$

The discount factor $\beta = 0.9$ is far to low for the high-cost incumbent to find it profitable to make the entrant leave the market. It is, however, sufficiently high to induce a predatory behavior from the low-cost type of incumbent. Now, if $\lambda \leq \overline{\lambda}_2^1 = 0.119732$ and the entrant receives no further information about the incumbent's type through the observed prices. He will then enter the market. Knowing this, the low-cost incumbent will send a price signal if it is not to costly. This price signal is the limit price, p^l. For the given values of β, α, F, and g the lowest price in $t = 1$ that the high-cost type can accept is:

$$p_2^l = 1.23608$$

and for the low-cost type:

$$p_1^l = 0.818191.$$

Thus, it is profit maximizing for the low-cost incumbent to signal his type by setting a price equal to p_2^l in the first period. For any $\beta \geq \underline{\beta}^1(\underline{c}, \overline{c}, g) = 0.560446$ is $p_2^l \geq p_1^l$ and there exists a limit price. Furthermore, if $\lambda \geq \overline{\lambda}_2^1$ and $\beta \geq 0.44105$ then the high-cost incumbent pools in the first period.

3.3. Summary

We have seen that it is the impatience that governs the behavior of the low-cost incumbent. The choice of pricing strategy determines the distribution of costs and revenues over the periods. The patient incumbent will choose to play the limit price because this will maximize total profits but the signalling costs occurs in the first period. If the incumbent is less patient and chooses to play the monopoly price in $t = 1$ then the cost is postponed to the second period while profits occurs mainly in the first period. Total profits will however bee lower. The very impatient incumbent will not deter entry or force an eventual entrant to leave the market.

The entrant's behavior is determined by the incumbent's observed behavior in the first period and the entrant's prior of the incumbent being of the low-cost type. Entry will occur if the entrant beliefs the probability of the low-cost type to be sufficiently low.

4. Proofs

Proposition 1. *Let $R_i : [0.1] \twoheadrightarrow [0,1]$ be firm i's best-reply correspondence in the stage game and let it be defined by*

$$R_i(p_j) = \begin{cases} \min[p_j + s, \widetilde{p}_i] & if \;\; 0 \leq p_j \leq \widehat{p}_j \\ \min[p_j - s - \varepsilon, \widetilde{p}_i] & if \;\; \widehat{p}_j < p_j \leq 1 \end{cases}.$$

(i) Let $s > 0$ be such that $\widehat{p}_i < \widetilde{p}_i$ for some $i \in N$. Two observations are easily made:
(a) No pure strategy $p_i > \widehat{p}_i$ can be a part in a stage-game Nash equilibrium.
(b) No pure strategy $p_i < \widehat{p}_i$ can be a part in a stage-game equilibrium.
Hence, $p_i = \widehat{p}_i$ is the only possibility. We know that $p_j = \min[p_i + s, \widetilde{p}_j]$ and if $p_j = p_i + s$ then is $\pi_i(p_i + \varepsilon, p_j) > \pi_i(p_i, p_j)$ making p_i a non-best reply of

firm i. Analogously is $\pi_i(p_i+\varepsilon,p_j) > \pi_i(p_i,p_j)$ *if* $p_j = \widetilde{p}_j$ *and* $|p_i - \widetilde{p}_j| < s$. *If* $p_i - \widetilde{p}_j = s$ *then will* $\pi_j(p_i, p_j - \varepsilon) > \pi_j(p_i,p_j)$.
(ii) Let $s > 0$ *be such that* $\widehat{p}_i > \widetilde{p}_i$ *for all* $i \in N$. *Charging* $p_i = \widetilde{p}_i$ *is trivially the unique Nash equilibrium in pure strategies to the stage game.* 2

Corollary 1. *Follows immediately from the folk theorem.* 2

Proposition 2. *Requirement 2 excludes all pure stationary strategies prescribing* $p_i^* > \widehat{p}_i$. *Requirement 3 requires firm i to maximize profits given requirement 1 and 2. If* $\widehat{p}_i < \widetilde{p}_i$ *then maximizes* $p_i^* = \widehat{p}_i$ *firm i's profit and if* $\widehat{p}_i > \widetilde{p}_i$ *then maximizes* $p_i^* = \widetilde{p}_i$ *the profit. Hence,* $p_i^* = \min[\widehat{p}_i, \widetilde{p}_i]$ *is the only* p_i *satisfying requirement 2 and 3. Requirement 1 only adds that* p_i^* *must be charged by firm i in every period.* 2

Corollary 2. *Follows directly from proposition and from studying the expression for* $\widehat{p}_i$, *see proposition 3.*

Proposition 3. *For all* $0 \le s < \widetilde{s}_i$ *is* $p_i^* = \widehat{p}_i$.
(i) If $0 \le s < \overline{s}_i$ *then is:*

$$\widehat{p}_i = \frac{1+c_j}{2} + \frac{29}{10}s \pm \sqrt{\frac{(1-c_j)^2}{4} - \frac{s(1-c_j)}{10} + \frac{721s^2}{100}}. \tag{0.1}$$

Setting $s = 0$ *yields* $\widehat{p}_i = c_j$ *and* $\widehat{p}_i = 1$. *That is,* p_i *must be either smaller than* c_j *or greater than 1. The latter solution is not allowed.*
(ii) Differentiating (0.1) with respect to s yields:

$$\frac{\partial \widehat{p}_i}{\partial s} = \frac{29}{10}s + \frac{5(1-c_j) - 721s}{10\sqrt{5^2(1-c_j)^2 - 10s(1+c_j) + 721s^2}}.$$

Requiring the derivative to be strictly greater than 1 and simplifying yields the following condition:

$$s' < \frac{5\left(1+19\sqrt{2}\right)(1-c_j)}{721}.$$

Letting $p_i = \widehat{p}_i$ *and solving the equality:*

$$\pi_j(p_j = p_i - s - \varepsilon, p_i) = \max[\pi_j(p_j = p_i + s, p_i), \pi_i(\widetilde{p}_j, p_i)]$$

with respect to s gives us:

$$\overline{s}_i = \frac{\left(65 + 45\sqrt{145}\right)(1-c_j)}{2894}.$$

Comparing s' and $\overline{s}_i$ we see that the latter is strictly greater for all $c_j < 1$.
(iii) For any $s' < s < \overline{s}_i$ it follows from (ii) that $\frac{\partial \widehat{p}_i}{\partial s} \in (0,1)$. For any $\overline{s}_i \leq s < \widetilde{s}_i$ is $\widehat{p}_i$ defined by:

$$\widehat{p}_i = \frac{76(1+c_j)+9+160s}{161} - \sqrt{\frac{160(20(1-c_j)^2+9s(1-c_j)-s^2}{161^2}} \tag{0.2}$$

Requiring (0.2) to be greater or equal to one and simplifying yields $s \geq 4(1-c_j)$ or $s \leq 5(1-c_j)$. Since $c_j \leq \frac{1}{2}$ by assumption this requires s to be strictly greater than 1 which is ruled out by assumption.
(iv) For any $\widetilde{s}_i \leq s \leq 1$ is $p_i^* = \widetilde{p}_i$. $\widetilde{p}_i$ is given by firm i's first order condition to maximization problem (2.3) in section 2.1 and is defined:

$$\widetilde{p}_i\left(p_j\right) = \frac{5+4c_i-p_j}{8}.$$

Differentiating with respect to s yields:

$$\frac{\partial \widetilde{p}_i}{\partial s} = -\frac{\frac{\partial p_j}{\partial s}}{8}.$$

Using that $\widetilde{s}_i \leq s < \widetilde{s}_j$ gives us that $\frac{\partial p_j}{\partial s} > 0$ and that $\frac{\partial \widetilde{p}_i}{\partial s} < 0$. If $s \geq \max\left[\widetilde{s}_1, \widetilde{s}_2\right]$ then is the solution to the system:

$$\begin{aligned} \widetilde{p}_1\left(p_2\right) &= p_1 \\ \widetilde{p}_2\left(p_1\right) &= p_2 \end{aligned}$$

not a function of s and $\frac{\partial p_i}{\partial s} = 0$. 2

Proposition 4. *For all $0 \leq s < \widetilde{s}_i$ is $p_i^* = \widehat{p}_i$.*
(i) Differentiate (0.1) with respect to c_j yields:

$$\frac{\partial \widetilde{p}_i}{\partial c_j} = \frac{1}{2} + \frac{5\left(1-c_j\right)-s}{2\sqrt{5^2\left(1-c_j\right)^2 - 10s\left(1+c_j\right) + 721s^2}}$$

The derivative is strictly greater than zero but strictly smaller than 1 if:

$$5\left(1-c_j\right) - s < \sqrt{\left(5\left(1-c_j\right)-s\right)^2 + 720s^2}.$$

The inequality is satisfied for all $s > 0$. We must also differentiate (0.2) with respect to c_j:

$$\frac{\partial \widetilde{p}_i}{\partial c_j} = \frac{76}{161} + \frac{2\sqrt{10}\left(40\left(1-c_j\right)+9s\right)}{161\sqrt{20\left(1-c_j\right)^2 + 9s\left(1-c_j\right) - s^2}}$$

The expression is strictly smaller than one for all $c_j < 1-0.2s$ *and strictly positive for all* $c_j < 1+0.7s$. *Hence,* $\frac{\partial p_i^*}{\partial c_j} \in (0,1)$ *for all* $c_j \in \left(0, \frac{1}{2}\right)$ *and* $s \in (0, \tilde{s}_i)$.
(ii) For all $s \in (0, \tilde{s}_i)$ *is* p_i^* *not a function of* c_i.
(iii) For any $\tilde{s}_i \leq s \leq 1$ *is* $p_i^* = \tilde{p}_i$. $\tilde{p}_i$ *is given by firm i's first order condition to maximization problem (2.3) in section 2.1 and is defined:*

$$\tilde{p}_i(p_j) = \frac{5+4c_i-p_j}{8}.$$

Differentiating with respect to c_i *gives* $\frac{\partial p_j}{\partial c_j} = \frac{1}{2}$. *Differentiating with respect to s yields:*

$$\frac{\partial \tilde{p}_i}{\partial c_j} = -\frac{\frac{\partial p_j}{\partial c_j}}{8}.$$

Using that $\tilde{s}_i \leq s < \tilde{s}_j$ *gives us that* $\frac{\partial p_j}{\partial c_j} = 0$ *and that* $\frac{\partial \tilde{p}_i}{\partial c_j} = 0$. *If* $s \geq \max[\tilde{s}_1, \tilde{s}_2]$ *then is the solution to the system:*

$$\begin{aligned} \tilde{p}_1(p_2) &= p_1 \\ \tilde{p}_2(p_1) &= p_2 \end{aligned}$$

not a function of s but of c_1 *and* c_2. $\frac{\partial p_j}{\partial c_j} > 0$ *which makes* $\frac{\partial \tilde{p}_i}{\partial c_j} < 0$. 2

Corollary 3. *(i) and (ii) follows from proposition 3 (iv) and the first order condition to equation(2.3). (iii) follows from proposition 4.* 2

Proposition 5. *(i)-(iii) follow trivially from definition of* $p_2^2(p_1^2)$ *and* R_2^2. *Solving (3.2) yields:*

$$p^{\alpha} = \frac{\beta(g-\overline{c})^2}{4} - 2\sqrt{\alpha\beta F}.$$

Requiring:

$$\beta D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) \geq F$$

gives that $F \leq \frac{\beta}{4}$. *From this follow (iv)-(viii) automatically.* 2

Proposition 6. *Solving:*

$$\overline{\lambda}_2^1 = \frac{\beta D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) - F}{\beta\left(D_2^2\left(p_1^{2*}, p_2^{2*}\left(p_1^2\right)\right)\left(\beta\pi_2^m\left(p_2^{3*};\overline{c}\right) - p_2^{2*}\right) - \alpha\beta F\right)}$$

yields:

$$\overline{\lambda}_2^1 = \frac{\beta - 4F}{\beta - 4\alpha\beta F}.$$

(ii) is trivial. (i):

$$\frac{\partial \overline{\lambda}_2^1}{\partial F} = -1 + \alpha\beta \frac{\beta - 4F}{\beta - 4\alpha\beta F} = -1 + \alpha\beta\overline{\lambda}_2^1 \leq 0$$

if $\overline{\lambda}_2^1 \in [0, 1]$. *(iii):*

$$\frac{\partial \overline{\lambda}_2^1}{\partial \beta} = \frac{1}{\beta - 4\alpha\beta F} - \frac{(\beta - 4F)(1 - 4\alpha F)}{(\beta - 4\alpha\beta F)^2} = 1 - \frac{\beta - 4F}{\beta} \geq 0. \ 2$$

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