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**SEARCH MARKET MODELS: A SURVEY**

by

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Comments Welcome  
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This work was undertaken while visiting IUI, Stockholm, in the Spring of 1989. I would like to thank them for making my stay so enjoyable.

## Search Market Models: A Survey

During the last twenty years a large literature has developed on the analysis of markets where firms set their own prices and consumers search. Models of this type are termed here search market models. Although a goal of this study is to review this literature, the focus of attention will not be on individual contributions. Instead, I will attempt to outline the logic behind the models used in the literature and demonstrate the importance of particular restrictions. Particular attention will be paid to those restrictions that generate a dispersed price equilibrium as such equilibria have clearly been the center of much attention.

An important element of search market models is that consumers search for low prices and this search process is costly. Luckily, there is a vast store of knowledge readily available; the search literature (see Lippman and McCall (1976) for a more than useful survey). The consumer search literature (and the even larger one on worker search) analyzes the behavior of consumers (workers) looking for a low price (high wage) in a market in which there is a non-degenerate distribution of prices (wages). Such models are "partial partial-equilibrium" in they only consider one side of a single market as they take the behavior of firms as exogenous. Search market models can be usefully viewed as an attempt to embed reasonable firm behavior into consumer search models and then to characterize the resulting equilibria. Of course, different search models will generate different conclusions.

Contributions to the consumer search literature can be usefully partitioned by the method of search assumed to be used by the consumers.

Four methods of search will be analyzed in this survey: sequential, non-sequential, noisy, and repeated. They encompass most, if not all, methods discussed in the literature to date. Although sequential search has dominated the consumer search literature, it will be shown there exists environments where each of these methods of search dominate the others in the sense that it minimizes a consumer's expected cost of purchasing a unit of a homogeneous good. It should be stressed that in the market models to be considered the product being sold is homogeneous. Thus, quality search is ignored.

The methods of search to be analyzed here can be briefly described as follows. With sequential search a consumer pays a cost to observe a price. The consumer then chooses either to purchase at the offered price, or search again. Thus, when analyzing sequential search behavior the results from the optimal stopping literature can be utilized. Non-sequential search, on the other hand, assumes a consumer chooses the number of price observations to make. After observing these prices, the consumer purchases at the lowest price observed. Obviously, the literature on optimal sample size in Statistics is relevant in this case. Noisy search is a straightforward generalization of sequential search. As before, a consumer pays a cost to observe a price. In this case, however, there is a positive probability more than one offer is received. Thus, at a given cost a consumer knows the probability  $n$  prices will be observed,  $n \geq 1$ . The last method of search considered, repeated search, is significantly different than the others as it assumes that consumers repeatedly purchase. A consumer in this case is attached to a firm and repeatedly purchases from it. From time to time, however, the consumer learns about the price offered by another firm and changes

attachment if the price is lower than that previously faced.

Given any of these methods of search (or any other well specified one for that matter it is possible to characterize the strategy that minimizes the expected total cost of purchasing a unit of the good conditional on (a) the distribution of prices charged,  $F$ , and (b) the cost of search faced by the consumer. Such a task is performed in Section 2. Allowing consumers to differ in the cost of search they face, the distribution of strategies used by consumers conditional on  $F$  is readily obtained.

Suppose firms are essentially the same in that they face the same constant marginal cost of production. The results stated above about the distribution of strategies can be used to construct an expected profit function conditional on  $F$  which specifies the expected profit of any firm charging any price  $p$ . These conditional expected profit functions are, of course, dependant on the method of search used by consumers as it is this that determines the distribution of 'optimal' strategies.

Such conditional expected profit functions are then used as parameters in a price-posting game played by firms. Each firm, taking the distribution of prices charged by other firms,  $F$ , as given (as well as the search strategies of consumers), selects a price that maximizes its expected profit. An equilibrium distribution of prices,  $F^*$ , is such that the associated expected profit function is a constant on all prices on the support of  $F^*$ , and less elsewhere. Thus, a firm believing the prices charged by other firms is given by  $F^*$ , maximizes its expected profit at all prices on the support of  $F^*$ .

Suppose all firms are essentially the same and consumers only differ in the search costs they face (all search costs being bounded away from

zero). For any given method of search the following questions appear natural. (1) Will a market equilibrium exist? (2) Given an equilibrium exists, is the equilibrium distribution of prices degenerate or dispersed? (3) What are the consequences of allowing firms and/or consumers to differ in some well defined way? Although some attempt is made to answer (3), the focus of attention is on answering (1) and (2) for each of the four methods of search outlined above.

An early and important result in search market models was presented by Diamond (1971). He showed within a sequential search environment, there exists a market equilibrium where all firms charge the monopoly price. This result became the irritant that stimulated much work. The desire to obtain a dispersed price equilibrium was clearly the motivation of great amount of work, although the monopoly price equilibrium appears, in retrospect, remarkably robust.

When all firms face the same constant marginal cost and all consumers face search costs bounded away from zero the equilibria obtained in the literature to date are of two types; those with a monopoly price equilibrium and those with a dispersed price equilibrium. Of course, some models have both. The results obtained can be summarized as follows.

TABLE 1

	Monopoly Price Equilibrium	Dispersed Price Equilibrium
Sequential Search	Yes	No
Non-sequential Search	Yes	Yes
Noisy Search	No	Yes
Repeated Search	No	Yes

As can be seen in Table 1, a dispersed price equilibrium cannot exist in the sequential search environment, whereas in a noisy search environment the only search market equilibrium implies a non-degenerate distribution of prices.

Unfortunately, price dynamics will not be discussed. Although many contributors to this area were clearly motivated by the adage "if all firms are price-takers who changes price", few studies focus on price change behavior. The large majority concentrate on what prices firms offer in equilibrium; where each firm chooses its price based on its belief about the action of others. How firms acquired their "equilibrium" beliefs is usually not questioned. Most who have considered price change behavior have assumed firms follow some ad hoc rule (early examples are provided by Fisher (1971) and (1973), and Axell (1974)). The study by Rothschild (1974) provides an interesting (although largely ignored) attempt to model optimal price change behavior.

No attempt will be made to cover some important aspects of this literature. First, and most importantly, firms throughout this survey are assumed to be price-posters who are not willing to bargain with customers about price. The fascinating and important contributions made in the sequential-bargaining literature will not be reviewed (see Binmore and Rubinstein (1989) for a recent survey of this topic). Second, general equilibrium considerations will be ignored. Although this is clearly an important topic it has been somewhat neglected (Albrecht, Axell, and Lang (1985), Diamond (1985), and Wernerfeldt (1987) present such models). Finally, empirical work is not discussed. It is worth noting, however, that in such work it has been shown that significant

price dispersion characterizes many markets even those where the product traded appears homogeneous (Stigler (1962), Pratt, Wise, and Zeckhauser (1979), and Dahlby and West (1983) provide good examples of this literature).

After outlining the basic framework to be used in Section 1, the four methods of search to be considered are analyzed in Section 2. In each case the objective is to characterize that strategy that minimizes the expected cost of purchasing a unit of the good. These results are embedded in a market model in Section 3. The resulting equilibria are derived and described.

Before outlining the general framework it will be useful to specify some terminology. A distribution function  $F$  is right-continuous such that

$$F(x) = \lim_{e \rightarrow 0} F(x-e) + v(p)$$

where  $v(p)$  is the mass at  $p$ , if such a mass exists. The support of  $F$  is defined as the points of increase of  $F$ .  $F$  is said to be degenerate if its support is a real number; otherwise  $F$  is said to be dispersed. The infimum of the support of  $F$ , will be called the minimum, whereas the supremum is called the maximum. Although this last piece of terminology is loose it much simplifies the exposition and should lead to no confusion.

## 1. The General Framework

The objective is to specify a general market model in which stores set their own prices and consumers search. To focus on the more unusual aspects of the framework developed the simplest possible assumptions will be made about the standard elements.



Suppose a large fixed number of both consumers and stores participate in a market in which the stores sell a homogeneous product. Without any real loss of generality, let the number one indicate the measure ("number") of both consumers and stores. Assuming stores face the same production costs and consumers have the same demand function much simplifies the analysis and acts as a reference point when more complicated assumptions are imposed. In particular, the following two restrictions are used initially.

(A1) Each store faces a constant marginal cost that, without any further loss of generality, is set equal to zero.

(A2) Given a consumer has decided to purchase from a store offering price  $p$ , one unit is bought as long as  $p \leq z$ , otherwise, zero is bought. (For obvious reasons  $z$  will be termed the monopoly price).

As will be shown later, relaxing (A1) and/or (A2) constitutes the basis of many studies in this area.

At the start of the period under consideration each store chooses the price it will charge for the period and then waits for customers. No revision of prices is allowed. This implies the prices charged by stores at any time during the period can be represented by the same distribution function  $F$ , which may or may not be degenerate.

When defining equilibrium practically all studies have used the next restriction (to my knowledge, Rothschild (1974) is alone in taking an alternative approach).

(A3) Consumers and stores act as if they know the distribution function  $F$ .

Of course, (A3) is usually couched in different terms. Typically, it is assumed that consumers and stores have (point) expectations about  $F$

that are correct (at least in equilibrium). Such complications will be ignored here as the formal development of this type of arguments is well known.

Although consumers act as if they know  $F$ , they don't know which store is offering what price. To learn the particular price charged by a store a consumer has to contact (search) that store. Each search, however, is costly to the consumer. Specifically, a search cost,  $c$ , has to be paid for each contact. This cost may differ for different consumers. The search process of a consumer is completed when he or she chooses to purchase (according to (A2)) from a store he or she has searched.

Let  $\mathcal{F}$  denote the space of distribution functions whose supports are contained in the interval  $[0, z]$ . By virtue of (A1) and (A2), a store will sell to no one if it charges a price greater than  $z$ , and it guarantees itself a loss if it offers a negative price. Hence, assume without any loss of generality any distribution of prices,  $F$ , is an element of  $\mathcal{F}$ .

A search rule is defined to be a complete specification of how to search and when to purchase for a given  $F$ . Such a rule can take many forms; purchase from the first store encountered whose price is less than  $z$ ; observe six prices and then purchase from the lowest price observed; purchase from the first store contacted, etc.. In the next section four classes of search rules are considered; sequential search rules, non-sequential search rules, noisy search rules, and repeated search rules. These four classes of search rules encompass the vast majority of search rules considered in the literature to date. With each of the search rules considered it is assumed that contacting stores is a random phenomena. Specifically, the following restriction is used.

(A4) The price observed at any search is envisaged as the realization of a random draw from  $F$ .

Thus, (A4) implies  $F(p)$  denotes the probability any store contacted is offering a price no greater than  $p$ . Very few studies have used alternatives to (A4). Two exceptions are provided by Salop (1976) and Burdett and Vishwanath (1989). Salop assumed the consumer associates with each of the  $n$  stores in the market a distribution of prices,  $F_i(p)$ ,  $i = 1, 2, \dots, n$ . Hence,  $F_i(p)$  denotes the probability store  $i$  is charging a price no greater than  $p$ . In this case, the consumer must choose a search order as well as decide on what prices to accept if offered. Burdett and Vishwanath, on the other hand, assume consumers are more likely to contact large stores rather than smaller ones. Thus, if  $m_i$  denotes the number of customers purchasing from store  $i$ , and total number of consumers is normalized to one, then  $m_i$  is assumed to indicate the probability a consumer will contact store  $i$  given a contact is made. Both these alternative restrictions lead to significantly different conclusions than those to be obtained here.

Let  $s$  denote a particular class of search rules. For a given class of search rules, a consumer will choose the particular rule that minimizes his or her expected total cost of purchasing. As will be shown later, with each of the class of search rules considered, the particular rule that minimizes a consumer's expected total cost of purchasing can be fully described by a real number. With sequential, noisy, and repeated search rules this number will be the highest price acceptable to a consumer, whereas with non-sequential search the number is interpreted as the number of stores contacted before the purchase decision is made. Of course, such a number will depend in general on the

search costs faced by consumers. Let  $G(c)$  denote the proportion of consumers who face a search cost no greater than  $c$ .

For a given distribution of search costs among consumers,  $G(\cdot)$ , and a distribution of prices offered,  $F(\cdot)$ , let  $H_s(\cdot|G,F)$  denote the distribution function that describes the rules that minimize the expected total cost of purchasing among search rules of class  $s$ , i.e.,  $H_s(x|G,F)$  denotes the proportion of consumers whose cost minimizing search rule can be described by a number no greater than  $x$ .

For a distribution of search rules,  $H_s$ , the stores are envisaged to play a price-posting game. In particular, each store chooses a price that maximizes its expected profits subject to the search behavior of consumers,  $H_s$ , and the distribution of prices charged by other firms,  $F$ . An equilibrium to this price-posting game is defined by an  $(F_s, v_s)$  such that

$$(R1) \quad \Pi_s(p|F,H) = v_s, \text{ if } p \text{ is on support of } F_s, \text{ and}$$

$$(R2) \quad \Pi_s(p|F,H) \leq v_s, \text{ otherwise.}$$

Thus, at such an equilibrium each firm maximizes its expected profit at any price on the support of  $F$ . Within this context, a search market equilibrium is a solution to the price-posting game with the added condition that  $H$  is generated by each consumer minimizing his or her expected cost of purchasing. Formally, a search market equilibrium when the environment is indicated by  $s$  is defined by  $(F_s, v_s, H_s)$  is such that  $(F_s, v_s)$  is a solution to the price posting game given  $H_s$ , i.e., (R1) and (R2) are satisfied by  $(F_s, v_s)$  given  $H_s$ , and

$$(R3) \quad H_s \text{ is generated by each consumer minimizing his or her expected total cost purchasing given } F_s \text{ and } G.$$

Before considering search rules in detail, claims are made about

solutions to the price-posting game played by firms in two special cases. The first states that if the rules followed by consumers guarantees that each consumer observes at least two prices before purchasing, then the unique equilibrium to the price-posting game is where each store offers the competitive price.

Claim 1.1

If the distribution of search rules,  $H$ , is such that all consumers will observe at least two prices with probability one for any  $F \in \mathcal{F}$ , there exists a unique solution to the price-posting game. At this equilibrium all firms will charge the competitive price, 0.

Proof

For given  $F$ , let  $p^*$  denote the highest price in the market and assume  $p^* > 0$ . First, suppose  $p^*$  is not a mass point of  $F$ . In this case a store charging price  $p^*$  will sell to no one. Second, suppose  $F$  has a mass point at  $p^*$ . A store offering price  $p^*$  will significantly increase its expected number of purchasing consumers by offering  $p^* - \epsilon$  ( $\epsilon > 0$ ), no matter how small  $\epsilon$  is, whereas by making  $\epsilon$  small enough the reduction in the expected profit per customer can be made as small it likes. Hence,  $\Pi_s(p^* - \epsilon | F, H) > \Pi_s(p^* | F, H)$ . Thus, given the maximum price  $p^* > 0$ ,  $F$  cannot satisfy (R1) and (R2). As all consumers search twice by assumption it is immediate that all firms charging price 0 satisfy (R1) and (R2). This completes the proof.

Consider for the moment a Bertrand oligopoly model within the above framework. In this case a continuum of store set their own prices and then wait for customers. Bertrand assumed price information was costless. With such a restriction consumers will learn the price offered by all stores no matter what they think is the distribution of prices offered.

Thus, the hypothesis of claim 1.1 is satisfied in the Bertrand case. Consumers will only purchase from the stores offering the lowest price in the market and the only equilibrium is where all stores offer price 0. Claim 1.1 strengthens somewhat the Bertrand result in that even if consumers only observe two prices (independently of the distribution of prices offered) before making the decision to purchase, the only equilibrium to the price-posting game is where all stores offer the competitive price. However, if all firms offer the same price and price information is costly why should a consumer observe more than one price before purchasing? The next claim deals with this situation.

#### Claim 1.2

If the distribution of search rules,  $H$ , is such that all consumers make only one price observation with probability one for all  $F \in \mathcal{F}$ , there exists a unique solution to the price posting game. At this equilibrium all stores offer the monopoly price.

#### Proof.

If all consumers make one price observation, the expected profit of a firm is independent of the price charged by other stores. Each store has a local monopoly in this case and will maximize its expected profits by charging  $z$ , the monopoly price. This completes the proof.

## 2 Methods of Search

Within the context of the framework specified above, a search strategy defines for each  $F \in \mathcal{F}$  the (possibly different) search rules to be used. Four classes of search strategies are analyzed here. Two of them, sequential search strategies and non-sequential search strategies, are directly comparable in that both utilize assume a consumer faces

essentially the same environment. The other two, noisy and repeated search strategies, are not comparable as they both assume consumers face different environments.

The following example may help illustrate the difference between sequential and non-sequential search. A couple of years ago I decided to purchase a short-wave radio and knew type I wanted. Such radios are sold mainly by specialty dealers who are distributed all over the country. Although I had some idea of prices charged for this good, i.e., I thought I knew the distribution of prices in the market, I was unaware of which dealer was charging what price. Three options appeared promising:

(i) Phone a dealer (at average cost (say) \$2), enquire about the price offered, and then decide whether to purchase or phone another dealer.

(ii) Write to a dealer (at cost (say) \$0.25), wait for its reply (say 10 days on average), and then decide either to purchase, or write another letter.

(iii) Write to a selection of dealers all at once, wait until they all reply, and then either purchase from the dealer offering the lowest price, or not purchase at all.

With each of these options, the particular search rule that minimizes the expected cost of purchasing given the distribution of prices faced can (at least in theory) be calculated. The three expected cost minimizing search rules can then be compared to determine the rule that minimizes the expected cost of purchasing among the options considered.

Given there is no discounting, the above problem has a known solution. Obviously, option (ii) obviously dominates (i) in this case. Further, given the cost minimizing search rule is used with both options

it can be shown (ii) must yield a lower expected total cost of purchasing than (iii) (DeGroot (1971) presents the details). Of course, the actual search rule to be used with option (ii) must be still be calculated.

The problem is not so simple, however, when the consumer discounts the future. Given a sufficiently high discount rate, options (i) and (iii) can be shown to dominate (ii). Whether, option (i) dominates (iii) depends on the discount rate used as well as the difference in the price of a phone call relative to the cost of writing a letter. Both options can yield the smallest expected total cost of purchasing, depending on the value of these parameters.

To simplify, suppose a break-down in the telephone system rules out option (i). Further, assume the consumer must have the radio in two weeks, or not at all. As only one price can be observed with option (ii), it is obvious that option (iii) will be preferred option (ii).

Options (i) and (ii) above are examples of sequential search rules, where a decision to purchase or not is made after each price offer is received, whereas option (iii) is a form of non-sequential search rule, where a decision to purchase or not is only made after a number of prices have been observed. With sequential search the problem is to determine which prices, if observed, should be accepted, whereas with non-sequential search the problem is to determine how many letters should be written.

Although it is often claimed that sequential search dominates non-sequential search, the above example illustrates this need not be the case. What is true is that sequential search dominates non-sequential search when there is no discounting.

The two other classes of search rules considered are not directly



comparable to sequential and non-sequential search as different assumptions are made about the environment faced. Both sequential and non-sequential search are deterministic in the sense that a consumer always receives the number of price quotes he or she pays for. This restriction can of course be weakened in several ways. Suppose, for example, when a consumer writes off for  $n$  price quotes there is a probability,  $b$ , any store will not reply. Hence,  $b^n$  denotes the probability no offers will be received. Such a modification to the non-sequential search model will obviously lead to a different strategy minimizing the expected cost of purchasing. In the present study, similar complications are embedded in the sequential search framework. In particular, it is assumed that when a consumer pays for a price quote there is a probability more than one will be received. Returning to the example at the start of this section, suppose on phoning a dealer to ask about the price offered, you are informed they do not have the item you require in stock but they know the prices offered by (say) two other dealers in the market. Searching within such an environment will be termed noisy search. As will be shown later, this small difference between noisy search and sequential search leads to dramatically different equilibria when the actions of stores are taken into account.

The final method of search describes a more passive method of search than those described above. In this case a consumer is assumed to be attached to a store and repeatedly purchase from it. Every now and then, however, the consumer learns the price offered by another store. If this price is lower than that currently faced, the consumer changes attachment, otherwise it is ignored. Such behavior is not uncommon in markets for goods that are repeatedly purchased such as milk, bread,

gasoline, etc.

## 2(a) Sequential Search

Well over 80 percent of the contributions to the search literature have assumed consumers use sequential search (see Lippman and McCall (1976) for an excellent survey). In this case, at cost  $c$  ( $c > 0$ ) a consumer receives a price offer from a store. The consumer then either purchases from that store or pays  $c$  again for another price observation. Such modeling is a reasonable formalization of consumer behavior when (say) he or she phones stores for price quotes, or in any situation where a consumer visits stores sequentially enquiring about the prices they offer.

It is well known that the strategy that minimizes a consumer's expected cost of purchasing in a sequential search environment is a reservation price strategy. If reservation price  $x$  is used, a consumer purchases from the first store whose price offer is observed to be no greater than  $x$ . The expected cost of purchasing when reservation price  $x$  is used can be written as

$$(2.1) \quad V(x) = c + \Pr\{p \leq x\}E\{p|p \leq x\} + \Pr\{p > x\}V(x)$$

The particular reservation price that minimizes the expected cost of purchasing,  $R$ , satisfies  $R = V(R)$ . This claim is easily established if  $F$  has a density as then (2.1) is differentiable with respect to  $x$ . Indeed, the proof is only a little more complicated if  $F$  is not differentiable. Utilizing this fact and integrating (2.1) by parts yields

$$(2.2) \quad c = \int_0^R F(p) dp$$

Of course, those consumers whose search costs are great enough will find  $c$  is greater than the right-hand side of (2.2) for any  $R$  on the support of  $F$ . Such consumers will purchase a unit from the first store encountered as long as the price offered is no greater than  $z$ . Hence, without loss of generality, let  $R^*$  denote the effective reservation price of a consumer, where

$$(2.3) \quad R^* = \min(R, z)$$

The claims of interest here on sequential search can now be stated.

#### Claim 2.1

Given sequential search and a particular cost of search  $c$  ( $c > 0$ ):

- (a) the strategy that minimizes the expected cost of purchasing is described by an effective reservation price,  $R^*$ , defined by (2.2) and (2.3);
- (b) the effective reservation price is strictly greater than the lowest price offered (the infimum of the support of  $F$ ) when the lowest price offered is strictly less than  $z$ .

#### Proof

The basic reasoning required to establish claim (a) has been indicated above. (a detailed proof is given in Lippman and McCall (1976)). To establish claim (b) suppose a consumer who faces search cost  $c$  receives price offer  $p_0 + e$ , where  $p_0$  denotes the lowest in the market and

$$(2.4) \quad 0 < e < \min\{c, z - p_0\}$$

Clearly, the consumer will prefer to purchase at  $p_0 + e$  rather than search again. Hence,  $R > p_0$  for any strictly positive search cost and

this completes the proof.

## 2(b) Non-sequential Search

This was the first type of search behavior to be analyzed in detail (see Stigler (1962)) although it has been ignored somewhat in recent years. This lack of attention is somewhat difficult to explain as it describes a method of search we all use from time to time and it leads to significantly different predictions than those that flow from sequential search. Nevertheless, Hey (1974), Wilde and Schwartz (1979), Braverman (1980), Chan and Leland (1982), Burdett and Judd (1983), and Wilde (1977,1987) have all considered variations within this framework.

Non-sequential search is perhaps best understood by thinking of a consumer writing off for price quotes. Suppose a consumer writes letters to  $n$  different firms enquiring about price. Let  $c$  ( $c > 0$ ) in this case denote the cost per letter written. After mailing the letters the consumer waits for the firms to reply. Each of the  $n$  price offers received is assumed to be the realization of an independent random draw from  $F$ . The consumer then purchases from the lowest price offer received as long as that price is no greater than  $z$ . The objective is to choose the number of letters to write that minimizes the expected cost of purchasing a unit of the good when search costs are taken into account.

The expected total cost of purchasing when  $n$  letters are written can be expressed as

$$(2.5) \quad V(n) = cn + n \int p[1-F(p)]^{n-1} dF(p)$$

Letting  $n$  be a real valued variable it is straightforward to check that  $V(\cdot)$  is a convex function of  $n$  with a unique minimum. This guarantees

there is at least one solution when only integer solutions are considered. Obviously, a consumer will choose an integer  $n^*$  such that

$$(2.6) \quad V(n^*) \leq V(n)$$

for all integers  $n$ .

#### Claim 2.2

Given non-sequential search with search cost  $c > 0$ :

(a) There exists either a unique integer  $n^*$  that minimizes the expected cost of purchasing, or there are two consecutive integers  $n^*$  and  $n^*+1$  that both minimize the expected cost of purchasing.

(b) If  $F$  is degenerate, then  $n^* = 1$ .

#### Proof

The proof of (a) follows from the analysis presented above. As there is obviously no expected reduction in the expected price paid, (b) follows immediately and this completes the proof.

#### 2(c) Noisy Search

Noisy search is a simple extension of sequential search. At cost  $c$  ( $c > 0$ ) a consumer receives a price quote; as with sequential search. In this case, however, more than one price quote may be received per search cost paid. Suppose, for example,  $c$  is the return bus fare to visit any store in the market. On the journey the individual may meet a friend who tells him or her of the price offered by a different store than that to be visited. Formally, let  $\alpha$  ( $0 < \alpha \leq 1$ ) denote the probability one price offer is received per search. Keeping things as simple as possible, let  $(1-\alpha)$  be the probability 2 offers are received per search.

Of course, a positive probability can be assigned to receiving 3, or more, price observations, but little is gained. After receiving one or two price offers from a search the consumer purchases from the lower priced firm, or pays  $c$  and searches again. Similar situations has been analyzed by Burdett and Judd (1983) and Fershtman and Fishman (1989).

It is straightforward to show that the strategy which minimizes the expected cost of purchasing a unit of the good can be characterized by a reservation price,  $Q$ . At this reservation price the expected cost of purchasing at the reservation price equals the expected cost of continuing to search,  $W(p)$ , and

$$(2.7) W(Q) = c + \alpha \int_0^Q p dF(p) + (1-\alpha) \int_0^Q p dG(p) + \alpha F(Q)W(Q) + (1-\alpha)G(Q)W(Q)$$

where  $G(p) = [1 - (1-F(p))^2]$  is the probability the lowest price observed when two offers are received is no greater than  $p$ . Substituting into (2.7) and integrating by parts yields

$$(2.8) c = \int_0^Q F(p) dp + (1-\alpha) \left[ \int_0^Q F(p) dp - \int_0^Q F(p)^2 dp \right]$$

As with sequential search, an individual's search cost may be so great that (2.8) is not satisfied for any  $Q \leq z$ . Note that if  $\alpha = 1$ , (2.8) becomes the same as (2.2) in the standard sequential search model. Again, there is no loss of generality if an individual is assumed to use an effective reservation price  $Q^*$ , where

$$(2.9) \quad Q^* = \min\{Q, z\}.$$

The following claim summarizes the above analysis and thus no formal

proof is presented.

### Claim 2.3

Given noisy search and a particular search cost  $c > 0$ , the strategy that minimizes the expected cost of purchasing a unit is characterized by an effective reservation price,  $Q^*$ , described by (2.8) and (2.9).

### 2(d) Repeated Search

In the search methods considered so far the consumers only purchase once. In many markets, such as the market for gasoline, consumers repeatedly purchase the product. The search strategy envisaged here involves no complicated strategy as a consumers merely moves to a lower priced store when one is encountered. The consequence of such search behavior has been considered recently by Mortensen (1986), Wernerfeldt (1988) and Burdett and Mortensen (1989). Indeed, the repeated search framework is in many respects the same as that analyzed in the sequential bargaining literature. Of course, here firms post prices and refuse to bargain.

The following story may help explain the situation envisaged. Suppose an individual moves to a new location and wishes to consume a loaf of bread a day. To simplify, suppose there are a large number of bakers in this town who are all equally competent at baking a loaf. On entering the town the individual chooses a bakery and starts to purchase a loaf a day (as long as the price is not too great). As time passes, however, the individual learns the prices offered by other bakeries. Every time he or she learns of one offering a lower price than that currently faced, the individual changes attachment. In the above story history plays a role in that the consumer remembers which bakery he or

she is currently attached, and only moves when a lower priced one is encountered.

To formalize, assume on entering the market a consumer is allocated to a store such that  $F(p)$  denotes the probability he or she is allocated to a firm offering a price no more than  $p$ . The consumer then repeatedly purchases a unit per instant from the store as long as the price offered is no greater than  $z$ . As time passes, however, the consumer learns about the prices offered by other stores. Let  $\lambda$  denote the arrival rate of another price offer. If such a price offer received is less than that currently faced the consumer changes attachment, otherwise the offer is ignored.

Two further elements of the model outlined above need to be added to complete the story. First, assume  $\delta$  is the turnover rate of consumers in that  $\delta h$  denotes the probability any consumer leaves the market for good in small time period  $h$ , where  $\delta$  is the parameter of a Poisson process. Second, assume any consumer who leaves the market is replaced by a new one who is randomly allocated to a firm.

### 3 Search Market Equilibrium

The object here is to consider firm behavior and the resulting market equilibrium when stores take the search behavior of consumers as given. Each of the four methods of search analyzed above will be considered in turn.

#### 3(a) Sequential Search

Although all stores are assumed to be essentially the same, consumers are allowed to differ in the search costs they face. Let  $G(c)$



denote the probability a randomly selected consumer faces a search cost less no greater than  $c$ . By virtue of (2.2) and (2.3), for given  $G(\cdot)$  and  $F$  it is possible to calculate the distribution of effective reservation prices,  $H(\cdot)$ . A store will sell to a consumer who contacts it if and only if its price is no greater than the consumer's reservation price. Thus, the distribution of reservation prices,  $H$ , and the distribution of prices charged by other stores,  $F$ , a store can calculate the expected profit function,  $\Pi_1(\cdot|F,H)$ . Obviously, the function  $\Pi_1(\cdot|F,H)$  can be quite complicated. Luckily, few facts about this function need to be known to establish the essential results.

### Claim 3.1

(a) If the support of  $G(\cdot)$  is on the strictly positive reals then there exists an a search market equilibrium with sequential search,  $(F_1, v_1, H_1)$ , where all stores offer the monopoly price  $z$ .

(b) If the support of  $G(\cdot)$  is on the strictly positive reals and bounded away from zero, the unique search market equilibrium with sequential search,  $(F_1, v_1, H_1)$ , is where all stores offer the same price  $z$ .

### Proof

Given Claim 1.2 the existence of an equilibrium where all stores charge the monopoly price,  $z$ , follows immediately. Note, if  $F$  were degenerate at some  $p < z$ , then Claim 2.1(b) establishes a store offering a slightly greater price than  $p$  obtains a greater expected profit than at  $p$ , and thus such distributions can't be part of an equilibrium.

To establish Claim 3.1(b) suppose  $F$  is the non-degenerate distribution of prices offered in the market. Claim 2.1(b) and the assumed distribution of search costs imply there exists a  $p^1$  strictly greater than the lowest price offered in the market such that  $R \leq p^1$  for

all  $c$  on the support of  $G(\cdot)$ . Thus, any store offering a price no greater than  $p^1$  expects the same number of customers. As profits per customer are strictly increasing with price, stores charging a price less than  $p^1$  cannot be maximizing profits. This completes the proof as  $F$  was an arbitrary non-degenerate distribution.

Thus, what is termed the monopoly price equilibrium exists if all consumers have strictly positive search costs. Further, if search costs are bounded away from zero, i.e., there is not a significant measure of consumers with arbitrarily small search costs, the monopoly price equilibrium is unique.

It should be noted that allowing market entry does not disturb the basic result. Above the measure of stores and consumers was assumed to be fixed. Relaxing this assumption a little, let  $(F_{1k}, v_{1k}, H_{1k})$  denote the unique search market equilibrium when  $k$  denotes the "number" of consumers per store. For simplicity assume all consumers face the same search costs. In this case, for given  $k$ , the unique equilibrium is where all stores charge  $z$  and thus earn  $kz$  in profits. Such a profit may induce market entry of stores. Nevertheless, the equilibrium at any given smaller  $k$  ( $k > 0$ ) all stores still charge  $z$ ; only the equilibrium profit falls. As the number of consumers becomes small relative to the number of stores the expected profit approaches zero although each store still charges the monopoly price.

The monopoly price result presented above is unsatisfactory to many economists. As far as I can determine, the reasoning behind this dissatisfaction is as follows. If the equilibrium search model outlined above and the monopoly price equilibrium are accepted as a reasonable description of how a market works, then implicit in this view is the

belief the competitive model is not a good one in the sense it does not approximate market equilibrium behavior as frictions become small. To illustrate, let  $(F_{1c}, v_{1c}, H_{1c})$  denote the unique search market equilibrium when all consumers face search cost  $c$  ( $c > 0$ ). By virtue of Claim 2.1(b), if the given  $c$  is strictly positive, all stores will offer the monopoly price in equilibrium (only when the common search cost is zero does the competitive equilibrium hold). Hence, even when search cost become small the competitive equilibrium is not a good approximation of the search market one. The competitive market model is of course the most useful element of an economist's tool-kit, and many feel it would be too great a sacrifice to give it up because of this one result.

On the other hand, the monopoly price equilibrium appears to be remarkably robust to changes in assumptions within the sequential search framework. In the search market literature utilizing sequential search a wide variety of alternative restrictions have been used within the basic framework. The monopoly price equilibrium typically turns out to be one of the possible equilibria. This conclusion is all the more remarkable as many studies have been conscious attempts to obtain a price dispersion in equilibrium. For reasons that are not obvious, the monopoly price equilibrium is usually ignored and attention placed on the other equilibria. Perhaps, an indifference principle, where each possible equilibria generated by a model is given equal weight, would be more appropriate.

Price dispersion in equilibrium can be obtained within the sequential search market framework but some form of heterogeneity is required. Heterogeneity among consumers is considered first. An important contribution within this context was made by Axell (1977).

Specifically, within the context of a sequential search environment with strictly positive search costs, Axell demonstrates a non-degenerate price distribution exists as an equilibrium (as well as the monopoly price equilibrium) if the support of given distribution of search costs among consumers is not bounded away from zero and the slope of this distribution satisfied some inequality constraints. von zur Muehlen (1980) and Rob (1985) establish similar results. Unfortunately, although it would be interesting to investigate why the given distribution of search costs should satisfy the stated constraints, this has not yet been taken up in detail.

In a similar vein, Salop and Stiglitz (1979) show a non-degenerate equilibrium distribution exists if a proportion of consumers have essentially zero search; the remainder of consumers having positive search costs. More recently, Diamond (1987) has shown a dispersed price equilibrium can exist if all consumers face the same strictly positive search costs but differ in the maximum price they are willing to pay. Specifically, Diamond assumes a given proportion of consumers are willing to pay at most  $z_1$  for unit of the good, whereas the others will pay at most  $z_2$  ( $z_2 > z_1$ ). In this case a two price equilibrium may exist where some store offer  $z_1$  and others  $z_2$ . Albrcht and Axell (1984) establish a similar result within the context of a labor market.

So far it has been assumed that stores are essentially the same. Reinganum (1979) demonstrated that if stores have different production costs and consumers have downward sloping demand curves then a non-degenerate equilibrium distribution of prices offered can exist. Similar results are also obtained by MacMinn (1980) and Carlson and McAfee (1983). Such a result is of great interest as it can be interpreted as

showing inefficient stores can exist in the long run within such a framework.

### 3(b) Non-Sequential Search

As before, the distribution of search costs among consumers is indicated by  $G(\cdot)$ . For any  $F \in \mathcal{F}$  and  $G$  it is straightforward to derive the distribution of number of price observations among consumers via (2.5) and (2.6). From such a construct it is easy to show that monopoly price equilibrium is both alive and well in the non-sequential search environment.

#### Claim 3.2

Given non-sequential search with the distribution of search costs,  $G(\cdot)$ , having its support on the strictly positive reals, there exists a search market equilibrium,  $(F_2, v_2, H_2)$ , where all firms offer the monopoly price.

#### Proof

This claim follows immediately from Claim 1.2 and 2.2 (b).

As will be shown, an equilibrium with a non-degenerate distribution of prices may also exist with non-sequential search. To establish that a dispersed price equilibrium can exist it is assumed that all consumers face the same cost of search,  $c$ .

Let  $\Pi_2(\cdot | F, c)$  denote the expected profit function in this special case. By virtue of Claim 2.2(a), at an equilibrium (if one exists) all consumers will choose to observe the same number of prices, or be indifferent between observing  $n$  and  $n+1$  prices. This fact and Claims 1.1 and 1.2 guarantee that at any dispersed price equilibrium some consumers must observe one price, the remainder observing two prices.

Assume for the moment that consumers are indifferent between making one price or two price observations and assume  $q$  ( $0 < q < 1$ ) consumers search once and  $(1-q)$  search twice. In this case, the expected profit of any store can be written as

$$(3.1) \Pi_2(p|F,c) = p[q + (1-q)[(1-F(p)) + v(p)/2]]$$

for all  $p$  on the support of  $F$ , where  $v(p)$  denotes mass of stores charging exactly price  $p$ . Note that a consumer who observes two prices the same is assumed to use a fair coin to determine from which store to purchase.

From (3.1) it is straightforward to establish the following three facts (Burdett and Judd (1983) provide the detailed arguments):

- (A) A store cannot be maximizing its profits by offering a price  $p > 0$  where  $p$  is a mass point of  $F$ ;
- (B) A store charging the highest price maximizes its expected profits by charging the monopoly price;
- (C) If  $F(p) = F(p+e)$ ,  $e > 0$ , where  $1 > F(p) > 0$ , then a store offering  $p$  cannot be maximizing its expected profit.

From (A)-(C) it follows that if an equilibrium distribution exists then it must be continuous on its connected support and  $z$  denotes the highest price. Hence, (3.1) and (A)-(C) imply (R1) and (R2) are satisfied by  $(F(.|q),v)$ , given a proportion  $q$  of consumers search once ( $0 < q < 1$ ), if and only if the distribution,  $F(.|q)$ , can be written as

$$(3.2) F(p|q) = \begin{cases} 0, & \text{if } p < t(q) \\ 1 - [(z/p) - 1][q/(2(1-q))], & \text{if } t(q) \leq p < z \\ 1, & \text{if } p > z \end{cases}$$

and  $v = qz$ , where  $t(q) = zq/(2-q)$ .

To establish the existence of a dispersed price search market equilibrium in the case under consideration we need only to check there exists at least one  $F(.|q)$  that implies consumers will be indifferent between searching once and twice. By virtue of (2.5) and (2.6), the expected reduction in the price paid by observing two prices instead of one price, indicated by  $S(2|q)$ , can be written as

$$(3.3) \quad S(2|q) = \int_{t(q)}^z F(p|q) dp - \int_{t(q)}^z F(p|q)^2 dp$$

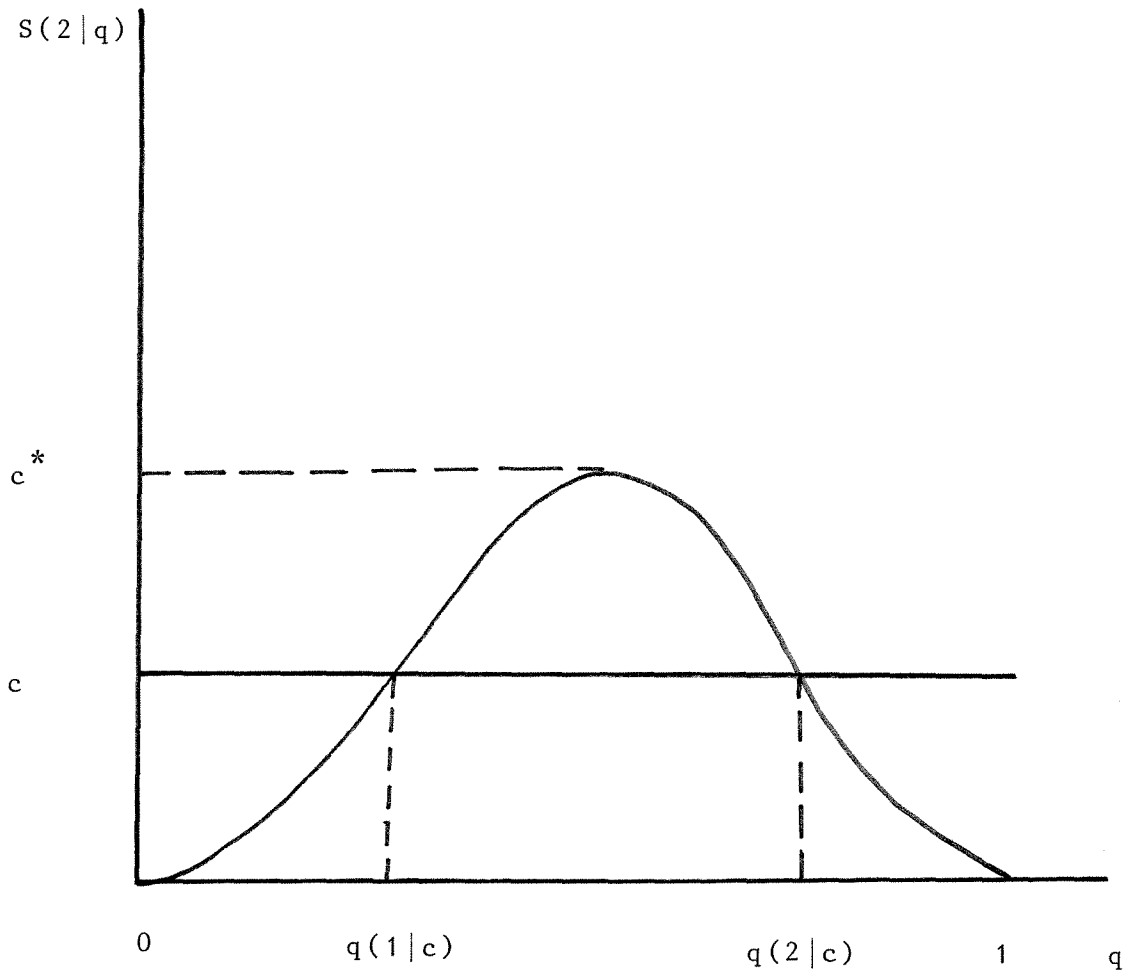
Substituting (3.2) into (3.3) and manipulating a great deal yields

$$(3.6) \quad S(2|q) = z \left[ \frac{q}{2(1-q)^2} \ln \left[ \frac{2-q}{q} \right] - \frac{q}{1-q} \right]$$

Taking the derivatives of  $S(2|q)$  with respects to  $q$  establishes  $S(2|.)$  is quasi-concave with a maximum at  $q^*$  ( $0 < q^* < 1$ ) and  $S(2|0) = S(2|1) = 0$  (Burdett and Judd (1983) present the details of such an exercise). Such an  $S(2|q)$  is illustrated in Figure 1.

Consumers will be indifferent between making two and one price observation if and only if  $S(2|q) = c$ . Hence, a dispersed price equilibrium exists with non-sequential search (when stores face the same marginal cost and consumers face the same search cost) where (3.2) describes the distribution of prices offered and  $S(2|q) = c$ . Such an equilibrium can be guaranteed as long as  $c$  is low enough ( $c > 0$ ). Further, as  $S(2|q)$  is quasi-concave, if one dispersed price equilibrium

Figure 1





exists, then there will always be exactly two dispersed equilibria. The following claim summarizes the conclusions that can be reached from the above arguments.

### Claim 3.3

Suppose all consumers use non-sequential search and face the same search cost  $c$  ( $c > 0$ ). There exists a  $c^*$  such that

(a) if  $c > c^*$ , the unique search market equilibrium,  $(F_2, v_2, H_2)$ , where all stores offer the monopoly price and consumers search once;

(b) if  $c < c^*$ , there are three search market equilibria; one where all stores offer price  $z$ , and two others where distribution of price offers is dispersed. At any dispersed price equilibrium, (i) a fraction  $q$  of the consumers search once and  $(1-q)$  search twice, and at such a  $q$ ,  $S(2|q) = c$ , (ii) (3.4) denotes the distribution of prices paid, and (iii) the equilibrium profit is  $zq$ .

### Proof

Due to Claim 3.2, we only need concentrate on the dispersed price equilibria. Given the above analysis it is straightforward to check  $R(1)$ - $R(3)$  are satisfied in this case if and only if the distribution of prices offered,  $F(\cdot|q)$ , is given by (3.4) and  $S(2|q) = c$ . As  $S(2|\cdot)$  is unimodal with a finite maximum, the proof of the claims made follow from inspection of Figure 1. This completes the proof.

Note that at any dispersed price equilibrium presented above consumers are indifferent to making one or two price observations. Thus, for a particular equilibrium proportion,  $q$ , to search only once must be either a happy coincidence, or the accomplishment of an external agency. This weakness can obviously be overcome in a model where consumers have different search costs.

A dispersed price equilibrium in this case is simple to describe. At such an equilibrium the density of the distribution of prices has a negative slope on its support  $[z(q/(1-q)), z]$ , where  $q$  is the (equilibrium) proportion making only one price observation. Indeed, the equilibrium density function is convex. The expected profit to each store is  $zq$ .

What happens to the non-sequential search case as search costs become small? The monopoly price equilibrium obviously exists for any  $c > 0$ , and thus such an equilibrium is not disturbed by a reduction in search costs. It is, however, a different story with the two dispersed price equilibria. Let  $q(i,c)$  denote the equilibrium proportion who search only once when the search cost is  $c$  ( $0 < c < c^*$ ),  $i = 1, 2$ , where  $q(1,c) < q(2,c)$  as illustrated in Figure 1.

Consider for the moment only dispersed price equilibria. As the given  $c$  faced by consumers becomes small the equilibrium distribution associated with  $q(1,c)$  ( $q(2|q)$ ) becomes more (less) spread in that the smallest price in the market decreases (increases) whereas the highest price remains at  $z$ . Indeed, the distribution associated with  $q(2|q)$  converges to a mass point at the monopoly price as  $c$  goes to zero.

### 3(c) Noisy Search

Focussing on essentials, it will be assumed throughout this section that consumers face the same search cost  $c > 0$ . As before, let  $\alpha$  denote the consumer observes one price per search and  $(1-\alpha)$  the probability two prices are observed. To prevent repetition, assume  $0 < \alpha < 1$ . An immediate implication of this restriction is that at any equilibrium all will have the same effective reservation price,  $Q^*$ , that satisfies (2.8)

and (2.9). This, of course, implies no store will knowingly charge price greater than  $Q^*$  at an equilibrium. Hence, at any equilibrium (if one exists) the expected profit of a store charging price  $p$ , indicated in this case by  $\Pi_3(p|F,c)$ , can be written as

$$(3.5) \quad \Pi_3(p|F,c) = p[\alpha + (1-\alpha)[(1-F(p))+\zeta(p)/2], \quad \text{if } p \leq Q^*$$

$$= 0, \quad \text{if } p > Q^*$$

where  $Q^*$  satisfies (2.8) and (2.9), and  $\zeta(p)$  denotes the size of the mass at  $p$  given the distribution  $F$ . As before, a consumer is assumed to toss a fair coin to break ties in prices observed. Such mass points, however, can be quickly ruled out as an equilibrium phenomena. Suppose  $F$  has a mass point at  $p$  ( $p > 0$ ). A store offering price  $p$  can increase its expected profit by lowering its price a small amount as it will gain a significant number of customers relative the loss in its profit per customer. Further, as a store can obtain expected profit  $p\alpha > 0$ , if  $0 < p < Q^*$ , a mass point at  $p = 0$  can be ruled out at an equilibrium. Similar arguments establish that an equilibrium distribution of prices must have a connected support as well as being continuous.

Suppose for the moment all stores expect consumers to use effective reservation price  $Q^e$ . In this case (R1) and (R2) are satisfied by  $(F(.|Q^e), v(Q^e))$ , where

$$(3.6) \quad F(p|Q^e) = \begin{cases} 0, & \text{if } p < s(Q^e) \\ 1 - [(Q^e/p) - 1][\alpha/(2(1-\alpha))], & \text{if } s(Q^e) \leq p < Q^e \\ 1, & \text{if } p > Q^e \end{cases}$$

and  $v(Q^e) = Q^e\alpha$ , where  $s(Q^e) = [\alpha/(2-\alpha)]Q^e$ .

Suppose now that consumers face a distribution of prices given by

(3.6). As they all face the same search costs all will use the same reservation price, indicated by  $Q(Q^e)$ , and the same effective reservation price  $Q^*(Q^e) = \min\{z, Q(Q^e)\}$ . However, it is straightforward to establish that  $Q^*(\cdot)$  is continuous such that  $Q^*(0) > 0$  and  $Q^*(z) = z$ . Thus, there exists at least one search market equilibrium where  $Q^*(Q^e) = Q^e$ . It is now shown there exists only one search market equilibrium.

Note that the distribution of prices that satisfies (R1) and (R2) in this case is the same as (3.2) with  $z$  being replaced by  $Q^e$ . Further, (3.4) is the same as the term in bracket in (2.8). These facts can be exploited when substituting (3.6) into (2.8) where  $Q(Q^e) = Q^e$ .

$$c = \int_0^{Q^e} F(p) dp + Q^e(1-\alpha) \left[ \frac{\alpha}{2(1-\alpha)^2} \ln\left[\frac{2-\alpha}{\alpha}\right] - \frac{\alpha}{1-\alpha} \right]$$

$$(3.7) \quad c = (1-\alpha)Q^e$$

Obviously, if  $Q^e > z$ , then  $Q^* = z$ . The following claim summarizes the above analysis and thus no proof is given.

#### Claim 3.4

Suppose all consumers face the same cost of search  $c > 0$ . For given  $\alpha$  there exists a unique market equilibrium. At this equilibrium the dispersed distribution of prices offered,  $F^*$ , can be written as

$$(3.8) \quad F^*(p) = \begin{cases} 0, & \text{if } p < t(\alpha) \\ 1 - [\alpha/2(1-\alpha)][(Q^*/p)-1], & \text{if } t(\alpha) < p \leq Q^* \\ 1, & \text{if } p > Q^* \end{cases}$$

From (3.7) and (3.8) it is readily established that as the common search cost becomes small the equilibrium distribution of prices converges to a mass point at the competitive price. Thus, within the noisy search environment as frictions become small the market equilibrium begins to approximate the competitive equilibrium.

### 3(d) Repeated Search

The flow of consumers between stores as well as in and out of the market can be specified from the description given of a representative consumer's market history in Section 2(d). Let  $G(p,t)$  indicate the number of consumers at time  $t$  who are currently attached to a store offering a price no greater than  $p$ . As  $\delta$  consumers flow out of the market for good at each instant,  $\delta G(p,t)$  will leave those stores offering a price no greater than  $p$ . On the other hand, as those who leave the market are instantly replaced by new consumers who are randomly allocated to stores,  $\delta F(p)$  denotes the flow of new consumers to stores charging a price no greater than  $p$ . Finally, of the  $[1-G(p,t)]$  consumers attached to stores offering a price greater than  $p$ ,  $\lambda F(p)[1-G(p,t)]$  will flow to stores charging a price no greater than  $p$ . Hence, the time derivative of  $G(.,t)$  can be written as

$$(3.9) \quad dG(p,t)/dt = \delta F(p) + \lambda F(p)[1-G(p,t)] - \delta G(p,t)$$

As  $t$  becomes large  $G(.,t)$  will settle down to its steady-state level. Let  $G(p) = \lim G(p,t)$  for all  $p$  as  $t$  goes to infinity. It follows from (3.9) the unique steady-state distribution of prices paid by consumers can be written as

$$(3.10) \quad G(p) = [1+k]F(p)/[1+kF(p)]$$

for all  $p$  on the support of  $F$ , where the parameter  $k = \lambda/\delta$  can be interpreted as the average number of price observations received during a consumer's life; it also reflects how efficiently price information is disseminated in the market. By virtue of (3.10), it follows that the steady-state distribution of prices paid,  $G$ , is uniquely determined by the distribution of prices offered,  $F$ . The distribution  $F$  and  $G$  have the same support and  $F$  stochastically dominates  $G$ .

In a steady-state the number of consumers attached to stores offering a price in the range  $[p-e, p]$  is given by  $[G(p-e)-G(p)]$ . Further, there are  $[F(p-e)-F(p)]$  stores offering such prices. Letting  $e$  go to zero the number of consumers per store offering price  $p$ ,  $D_4(p, F)$ , is well defined for all  $p$  on the support of  $F$  and can be written as

$$(3.11) \quad D_4(p|F) = \lim_{e \rightarrow 0} [G(p-e)-G(p)]/[F(p-e)-F(p)] \\ = [(1+k)]/[1+kF(p)][1+kF(p^-)]$$

given  $F(p) = F(p^-) + v(p)$ , where  $v$  denotes the fraction (or mass) of stores offering price  $p$ , if there is such a mass. It follows immediately that  $D_4(., F)$  is strictly decreasing on the support of  $F$ , and discontinuous where  $F$  has a mass point.

Although (3.11) cannot be used to construct  $D_4(., F)$  for any  $p$  off the support of  $F$ , it is straightforward to show it is a constant on any connected interval off the support. Let  $p_0$  and  $p_1$  denote the lowest (infimum) and highest (supremum) price on the support of  $F$  respectively. A store charging a price less than  $p_0$  will attract all consumers who make contract with it and will only loose customers when they leave the market for good. On the other hand, a store charging a price greater than

$p_1$  will lose each of its customers as soon as they contact an other store. Given the consumer flows, this implies

$$(3.12a) \quad D_4(p|F) = (1+k), \text{ for all } p < p_0$$

$$(3.12b) \quad D_4(p|F) = 1/(1+k), \text{ for all } p > p_1$$

The above results specify the steady-state number of customers per store for any given distribution of prices offered. In particular,

(a)  $D_4(.,F)$  decreases as  $p$  increase and is strictly decreasing on the support of  $F$ ,

(b)  $D_4(.,F)$  is continuous at any  $p$  where  $F$  is continuous but is discontinuous at any mass point of  $F$ , and

(c)  $D_4(.,F)$  satisfies the equations of (3.12).

It is these conditional (steady-state) demand functions that are assumed to be the ones used by stores in the price-setting game they play.

A consumer attached to a store charging price  $p$  is assumed to purchase one unit per instant as long as the price offered is no greater than  $z$ . As the constant marginal cost of production faced by each store is zero, the expected steady-state profit flow faced by a store charging price  $p$  is  $\Pi_4(p|F) = pD_4(p|F)$ . As consumer behavior is trivial in this case, a steady-state equilibrium is defined in this case as a solution to the price-posting game when  $\Pi_4(.,F)$  is used by stores.

Before establishing the existence of a unique equilibrium it is shown our attention need only be focussed on continuous offer distributions. Suppose there exists a mass point at  $p'$ . As noted previously, this implies there is a discontinuity in  $D_4(.,F)$  at  $p'$ . Thus, by lowering its price a small amount less than  $p'$ , given  $p' > c$ , a store will increase its profit flow as it increases its number of customers significantly but

reduces its profit flow per customer hardly at all as  $\pi D_4(\cdot, F)$  is continuous by assumption. If there is a mass point at  $p' = c$ , then stores offering such a price make zero profit. Any store offering a price greater than  $c$  such that  $\pi > 0$  (and this is guaranteed by (3.12a)) will make strictly positive profits. Hence, when looking for possible equilibria, only those with continuous distributions will be considered. Given this result, any possible equilibrium  $D_4(\cdot, F)$  considered can be written as

$$(3.13) \quad D_4(p, F) = (1+k)/[1+kF(p)]^2$$

#### Claim 3.5

There exists a unique market equilibrium  $(F_4, v_4)$ . The equilibrium profit rate  $v_4$  satisfies

$$(3.14) \quad v_4 = z/(1+k) = p_0(1+k)$$

the equilibrium distribution of price offers,  $F_4$  can be written as

$$(3.15) \quad F_4(p) = [1/k][(1+k)[p/z]^{1/2} - 1]$$

for all  $p$  on its support.

#### Proof

First, note the highest priced store in the market will always offer charge the monopoly price,  $z$ , at an equilibrium. As stated previously, given the only candidates for an equilibrium offer distribution is continuous, the equations of (3.12a) imply the highest priced store in the market will always have  $m/(1+k)$  customers in an equilibrium.

By similar reasoning, the lowest priced store will have  $m(1+k)$



customers. Thus, the unique equilibrium rate of profit,  $v_4$ , must be as presented in (3.13). Manipulating (3.13) and (3.14) demonstrates the equilibrium offer distribution,  $F_4$ , must satisfy (3.15) on its support. This completes the proof.

As with the previous search methods considered it is of some interest to see what happens to equilibrium as "frictions" become small. In this case the parameter  $k$  describes how quickly price information is transmitted in the market. Considering a sequence of market equilibria as  $k$  becomes large represents a reduction in market "frictions". From (3.15) it follows that the equilibrium distribution of prices offered changes in the following way as larger and larger  $k$  are considered. Although the highest priced store always charges the monopoly price, the lowest price store charges a lower price the greater  $k$ . Furthermore, as  $k$  becomes large a greater proportion of stores offer lower prices.

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