Inflation, Taxation and Capital Cost

Villy Bergström and Jan Södersten

INTRODUCTION

The world inflation of the 1970's has called for a growing literature on the causes as well as the effects of the inflation surge. The literature on the effects of inflation has been partly normative by dealing with indexing the economy to avoid distortions added by inflation--to already existing ones--through the tax system.

A large part of the recent literature on the distorting effects of inflation deals with profit taxation and the cost of capital. Another part deals with inflation and taxation of income in the household sector.

In this paper we deal both with the profit taxation of the business sector and the income taxation of the household sector. The central concept of our analysis is the cost of capital and our intention is to make a detailed analysis of how taxation influences capital cost in times of inflation.

When there is inflation there are distortions produced by the tax system because not all real costs are deductible for taxation and because not all real income is included in taxable profits. Also costs of debt and equity become distorted.

With the tax regimes existing in most countries there are four different distorting factors that operate in times of inflation. Two of these are due to the construction of the system of corporate profit taxation (points 1 and 2 below) and two due to income taxation of households (points 3 and 4 below).

1) When depreciation allowances are based on historical costs under corporate tax laws inflation undermines their real significance. Therefore, part of capital consumption may be included in the tax base (or accelerated depreciations are diminished in real terms). Hereby capital cost increases.

2) When the nominal interest on debt is deductible against corporate profits a real amortization is in fact deductible, when market rates of interest on debt are adjusted to the rate of inflation. Therefore, capital cost is reduced.

3) When nominal capital gains on household holdings of corporate stocks are taxed, capital cost is increased.

4) When the nominal rate of return on the household's alternative financial investments is taxed, capital cost is reduced.

The result of our analysis indicates that for most reasonable assumptions the net outcome of these effects is to lower capital cost, when both profit tax and personal taxes on dividends and capital gains are taken into account.

When the effects of inflation on capital accumulation of private firms are analyzed in the literature, the analysis is often limited to the system of profit taxation.[1]

However, an interesting line of development of the analysis of inflationary effects through the tax system is represented by Feldstein and different co-authors.[2] These authors include also income taxation in the household sector and they use a general equilibrium framework, (as compared to the authors mentioned in note 1 below whose models are more partial) to study how inflation influences i.e. costs of equity and debt and the debt-to-equity ratio. But with the general equilibrium framework the corporate tax system is stylized and does not allow a detailed analysis of how capital cost is influenced by tax laws in times of inflation. For instance, accelerated depreciation is disregarded, which restricts the results.

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[1] See e.g. the paper by Tideman and Tucker (1976, especially appendix A). The authors claim that inflation increases capital cost for all kinds of investment. Their numerical analysis rests upon a model that is not fully presented in their paper. It seems, though, that the objective of their model firm is not to maximize stockholders' required rate of return -- the cost of equity -- but by the average cost of equity and debt (less the rate of inflation). (Cf Nelson, 1976.) Another example is Sumner (1973). Contrary to Tideman and Tucker, Sumner holds (p. 30) that the net result of points 1 and 2 above is inconclusive. At low inflation rates an increased rate of inflation would tend to increase capital cost, wheras capital cost would be decreased at high rates of inflation by further increases.

[2] See Feldstein (1976) and Feldstein, Green and Sheshinski (1978).

Another (implicit) assumption is that one dollar of retained earnings creates a capital gain of one dollar. This would not be the case--due to differential taxation of dividends and capital gains--on an optimal growth path.[1]

When the distortionary effects of inflation on capital cost via the tax system are analyzed, different norms can be used. The inflationary situation can be compared to resource allocation in a world without inflation and free of tax distortions.[2] The other way is to compare capital cost with the inflationary distortions introduced in times of inflation by the construction of the tax system to capital cost with those distortions present that are due to the tax system at zero rate of inflation.[3]

If the tax system represents a deliberate choice on the part of the government to intervene in the allocation of resources but the tax system was constructed without regard to inflation, this

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[1] Feldstein and Summers (1978) in a recent paper discuss the effects of inflation on the maximum nominal interest rate a firm can afford to pay on a "standard" investment. Their analysis is similar to ours in that the complexities of the actual tax system are taken into account. They differ, however, by basing their analysis of capital gains taxation on the ad hoc assumption that a dollar of retained earnings will produce a dollar's worth of capital gains. For a criticism of this assumption, see e.g. Bergström and Södersten (III:5 in this volume) and Auerbach (1979).

[2] This norm is used by Sandmo (1974) in his short comments on inflation.

[3] This norm is inherent in the numerical analysis of Tideman and Tucker (1976).

second norm should be used. The idea that depreciation rules for tax purposes should reflect a real economic loss of value has a very limited scope in Sweden as well as in several other countries. By way of accelerating depreciation allowances governments make effective tax rates lower than statutory tax rates, not primarily to compensate for historical cost depreciation in times of inflation.[1]

Therefore, when we discuss effects of inflation on capital cost our main norm of comparison is capital cost with those distortions present that are due to taxation of profits and household income at zero rate of inflation. We also discuss briefly the over all norm of capital cost with no tax distortions (and a zero rate of inflation).

The model used for this paper and which is presented in the next section is in the Jorgenson[2] tradition of a firm aiming at maximizing the value of its shares in the portfolios of stockholders. The gross cost of capital of this firm, financed by equity and debt in a given proportion, is derived. The cost of equity and debt are then taken at their nominal values as the firm is assumed to observe them on the capital market.

We then analyze the net real cost of capital, where market rates of return are adjusted for in-

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[1] See Bergström (1977) and Södersten (1978).

[2] Jorgenson himself early introduced inflation into his model, but because he used depreciations for tax purposes on replacement values and did not have explicit debt financing the essence of the problem with inflation was concealed. See Jorgenson (1965) and (1968).

flation. This allows us to determine the net effects of inflation on capital cost. The analysis is first performed for corporate taxation only. Thereafter personal taxes are introduced. In the concluding section, different ways of indexing taxation to insulate the cost of capital from inflationary distortions are discussed.

1. BUSINESS TAXES ONLY

1.1 The Model

To analyze how inflation affects capital cost we will use a model similar to that presented in Bergström and Södersten (III:5 in this volume) with some special assumptions added.[1] First, we will assume that there is an expected rate of inflation of 100•p% on the price of capital goods, $P_K(s)$. Therefore we have $P_K(s)=P_K(v)e^{p(s-v)}$. Second, we assume that the firm keeps a constant debt ratio.

This last policy is introduced by assuming that the book value of outstanding debt, S(s), related to the current value of the capital stock, $P_K(s)K(s)$, is a constant:

$$\frac{S(s)}{P_K(s)K(s)} = h.$$

We also assume that the firm finances its gross investments by debt in the same relation, h, so

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[1] Note that different symbols are used in this paper. Cf. also Södersten (1975) and Bergström (1976).

that gross borrowing is $hP_K K(s)I(s)$, where $I(s)$ is gross real investment.

It is assumed that the stock of capital, $K(s)$, depreciates at the exponential decay rate, δ, and as capital gains per unit of capital through price inflation is p, the rate of amortization, to keep the debt ratio constant, is $(\delta-p)$.[1] [2]

It will be assumed that the firm can deduct a fraction γ of the book value of capital, $D(s)$, from profits for tax purposes and that profits so defined are taxed at the rate τ. The book value of capital is made up of investments at historical costs.

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[1] Without any amortization the stock of debt at point in time, s, would amount to

$$\int_{-\infty}^{s} hP_K(v)I(v)dv.$$

The current value of the firm's debt, when the rate of amortization is the rate of capacity depreciation less the rate of inflation $(\delta-p)$, is a fraction h of the current value of the capital stock:

$$S(s) = \int_{-\infty}^{s} P_K(s)e^{-p(s-v)}hI(v)e^{-(\delta-p)(s-v)}dv$$

$$= P_K(s) \int_{-\infty}^{s} hI(v)e^{-\delta(s-v)}dv$$

$$= hP_K(s)K(s).$$

[2] Failure to adjust the rate of amortization to the rate of capital gains through inflation would obviously result in changes in the average debt ratio. For the implication of this, see page 243 note 3.

Note also that the rate of amortization can be negative--$(\delta-p)<0$--meaning that the firm borrows on its appreciated capital stock (in excess of the gross borrowing to finance gross investment).

The management is assumed to maximize the value of the firm in the portfolios of the stockholders and to observe a rate of return, k, demanded by stockholders for investment in common stocks.

With product price P(s), wage rate w(s), labor input L(s), and interest rate i(s), the objective is to maximize the present value of all future cash flows.[1]

$$J = \int_{s=t}^{\infty} e^{-k(s-t)}\Big[(1-\tau(s))\{P(s)F[K(s),L(s)] - w(s)L(s) - i(s)hP_K(s)K(s)\} - (\delta-p)hP_K(s)K(s) - (1-h)P_K(s)I(s) + \gamma\tau(s)D(s)\Big], \qquad (1:1)$$

where F[K(s),L(s)] is a decreasing return to scale production function.

This maximization may not violate the two equations of motion:

$$\dot{K}(s) = I(s) - \delta K(s)$$

$$\dot{D}(s) = P_K(s)I(s) - \gamma D(s).$$

This is a control problem with control variables labor input, L(s) and gross investment, I(s) and the hamiltonian, H:

[1] Parameters assumed constant are written without time indices.

$$H = e^{-k(s-t)}\Big[\big(1-\tau(s)\big)\big(P(s)F\{K(s),L(s)\}-w(s)L(s)- i(s)hP_K(s)K(s)\big)-(\delta-p)hP_K(s)K(s)-(1-h)P_K(s)I(s)+ \gamma\tau(s)D(s)+\lambda_1(s)\{I(s) - \delta K(s)\}+ \lambda_2(s)\{P_K(s)I(s) - \lambda D(s)\}\Big] \quad (1:2)$$

We assume that this (properly defined) control problem has a solution which calls for decreasing returns to scale in production. We disregard, inter alia, that there would be instantaneous adjustments to the optimal path with infinitely large investment or disinvestment.

The necessary conditions used for (1:2) give:[1]

$$\frac{\partial H}{\partial I} = e^{-k(s-t)}\big[-(1-h)P_K+ \lambda_1+ \lambda_2 P_K\big] = 0 \quad (1:3)$$

and

$$\dot{\lambda}_1 + \big(1-\tau(t)\big)\big(PF_K-hiP_K\big) - (\delta-p)hP_K= \lambda_1(k+\delta) \quad (1:4a)$$

$$\dot{\lambda}_2 + \tau(t)\gamma = \lambda_2(k+\gamma) \quad (1:4b)$$

By solving the differential equations (1:4) we get for k, δ and γ constant (but $\tau(t)$ still a function of time):

$$\lambda_1= \int_{s=t}^{\infty} \big[\big(1-\tau(s)\big)\big(PF_K-hiP_K\big)-(\delta-p)hP_K\big]e^{-(k+\delta)(s-t)}ds \quad (1:5a)$$

$$\lambda_2 = \int_{s=t}^{\infty} \tau(s)\gamma e^{-(k+\gamma)(s-t)}ds \quad (1:5b)$$

[1] Time indices are skipped in most cases to save space. The optimal condition concerning labor input is not needed for our purposes.

Therefore λ_1 is the capital value, internal to the firm, of getting another unit of capital, recognizing that a new unit of capital gives rise to future (after tax) marginal value productivities and debt services. λ_2 is the capital value of all future tax savings from depreciation charges following upon an increase of the book value of capital by one unit.

Condition (1:3) above says then that the capital value of expected future cash flows, due to the investment of one unit of capital, $\lambda_1 + \lambda_2 P_K$, must equal the present loss of cash flow from the investment outlay, $(1-h)P_K$.

Noting that condition (1:3) must hold over time all along the optimal path of the firm, it follows that

$$\dot{\lambda}_1 = (1-h-\lambda_2)\dot{P}_K - P_K \dot{\lambda}_2 \qquad (1:6)$$

at all points in time. Introducing the assumption that the firm expects future tax rates τ (as well as rates of depreciation for tax purposes) to be constant makes $\dot{\lambda}_2$ in (1:6) equal zero. By substituting (1:4) into (1:3) and using (1:6) with the assumption $\dot{\lambda}_2 = 0$, we may then solve for PF'_K/P_K, which is the gross rate of return before tax on real investment on the optimal path

$$\frac{PF'_K}{P_K} = \delta - p + ih + \frac{k}{1-\tau}\left[1-h - \frac{\tau(\gamma - (\delta-p))}{k+\gamma}\right]. \qquad (1:7)$$

The formula (1:7) gives the minimum gross rate of return that the firm can afford to earn on new investment, leaving shareholders no worse off, i.e. the gross cost of capital.[1]

1.2 Real Cost of Capital

By subtracting from gross cost of capital, given by (1:7), the rate of economic depreciation, we get the net real cost of capital, here called r*. Economic depreciation, then, is defined as the depreciation charge that maintains intact the real value of the original amount invested. By our assumption of exponential decay, this depreciation charge is the rate of capacity depreciation, δ, times replacement cost.[2] This defines real net cost of capital:[3]

$$r^* = ih + \frac{k}{1-\tau}\left[1 - h - \frac{\tau(\gamma-(\delta-p))}{k + \gamma}\right] - p \qquad (1:8)$$

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[1] Letting $PF'_K/P_K = c$, $P_K c$ then stands for what has been called the nominal user cost or rental price of capital. Cf. Jorgenson and Siebert (1968).

[2] Cf Bergström (1976), p 446. By subtracting from gross cost of capital (1:7) the rate $(\delta-p)$ times replacement cost the <u>nominal</u> amount invested would be kept constant. This would define a <u>nominal</u> net cost of capital, directly comparable to (nominal) capital market interest rates, i and k.

[3] If the rate of debt amortization would be kept at δ instead of $\delta-p$ an extra term would be added to (1:8), namely

$$\frac{ph\left[\frac{k}{1-\tau} - i\right]}{k+\delta}$$

which means that the inflation induced fall in the average debt ratio would, <u>ceteris paribus</u>, increase, leave unaffected or <u>reduce capital</u> cost, depending on whether

$\frac{k}{1-\tau} \gtrless i$. Cf. page 239.

Now, for the interpretation of (1:8), let us first assume that the rate of depreciation for tax purposes, γ, equals δ-p. As explained in note 2, p.243 this is the rate of depreciation that would keep constant the nominal amount invested. Since h is the portion of the firm's investment financed by borrowing, (1-h) is the portion financed by equity capital, making the net cost of capital a weighted average of the cost of debt and the (before tax) cost of equity. If instead $\gamma > (\delta - p)$, i.e. the firm is allowed to defer taxes through acceleration of depreciation charges relative to what is needed to maintain the original nominal amount invested, the cost of equity is weighted by

$$1 - h - \frac{\tau[\gamma - (\delta - p)]}{k + \gamma} \qquad (1:9)$$

This weight, in turn, is the portion of the firm's investments financed by equity capital.

Thus $\gamma > (\delta - p)$ implies that a third part of capital growth, $\tau[\gamma - (\delta - p)]/(k+\gamma)$, is financed by deferred taxes, adding the weights up to one. However, this last cost of finance is zero and consequently it does not show up in (1:8).

Now, decomposing the net real cost of capital, r* in (1:8), into a real part corresponding to capital cost without inflation and another part that is due to inflation, is the task of general equilibrium analysis, since the effects of inflation on market rates k and i need to be known.

These market rates will react to inflation in a complex way, reflecting both borrowers' and lenders' adjustments to inflation (and taxation).

This paper deals with one side of this market, borrowers' reactions to inflation when nominal interest --but not equity cost-- is deductible and when taxable profit is determined by deductions reflecting depreciations based upon historical investment costs.

On the supply side there are substitution effects between savings and consumption as well as between investment alternatives because inflation influences yield differentials--nominal before tax as well as real after tax--again because nominal interest is taxed and capital gains are taxed at relatively low marginal rates or not at all. These are the problems analyzed in a series of papers by Feldstein et al.[1] For our purposes it will suffice to simply assume that the nominal rates of return will rise with the rate of inflation. This means that we study what happens to the cost of capital when there is inflation but when real rates of return to equity and debt stay constant, i.e.:

$$k = k^* + p; \qquad i = i^* + p$$

where starred variables indicate cost of equity and debt, respectively, at zero inflation.[2]

Using our definition of the firm's real net cost of capital and the above assumptions regarding the

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[1] See Feldstein (1976), Feldstein, Green and Sheshinski (1978) and Feldstein and Summers (1978).

[2] It seems, in fact, that the adjustment of nominal interest rates due to inflation would be an approximate increase by the rate of inflation in the Fisherian tradition, although this is a net outcome of complex interactions due to taxation on both borrowers' and lenders' sides of the market. See Feldstein and Summers (1978).

effects of inflation on the nominal costs of equity and debt we get

$$r^* = i^*h + \frac{k^*}{1-\tau}\left[1-h-\frac{\tau(\gamma-\delta)}{k^*+\gamma}\right] + \frac{\tau\ p\ \gamma}{(1-\tau)(k^*+p+\gamma)}\left[\frac{k^*+\delta}{k^*+\gamma}\right] - \frac{\tau}{1-\tau}\ p + \frac{\tau}{1-\tau}\ p(1-h) \qquad (1:10)$$

The first two terms of r^* is net capital cost at zero inflation recognizing the possibility that the tax laws may provide for acceleration of depreciation charges ($\gamma > \delta$). Relative to this norm of constant prices, the effects of inflation on the firm's real net capital cost is captured by the last three terms.

The third term reflects that inflation brings about a real reduction in the base on which depreciation charges are taken, assuming that tax depreciation is calculated on historical cost. On the other hand, not taxing capital gains results in a reduction in real capital cost. This is shown by the fourth term. The last term of (1:10), $\frac{\tau p(1-h)}{1-\tau}$, reflects the assumption that the (after tax) cost of equity rises with p and that this increase is not deductible for tax purposes.

This last effect <u>partially</u> offsets the reduction in capital cost from not taxing capital gains. For a <u>complete</u> offset, however, tax laws should also provide for a restriction in the deductability of interest costs, allowing only deduction of real interest, i^*. This can be seen in the following way. The untaxed capital gain and the taxed increased cost of equity--the fourth and fifth terms added--result in a net lowering of capital cost by $\frac{\tau ph}{1-\tau}$ which can be interpreted as the effect of allowing the inflation increased interest on debt to be deductible. We see then, that the inflationary effects via the tax system can be described in two different ways.

The first one says that capital cost is lowered since capital gains are not taxed and raised because the inflation increased cost of equity is not a deductible cost to the firm. The other way, which states the net of these two effects, says that there is a fall in real capital cost because the firm can deduct full interest on debt when determining taxable profits.

Reformulating (1:10) in line with the last interpretation yields

$$r^* = i^*h + \frac{k^*}{1-\tau}\left[1-h-\frac{\tau(\gamma-\delta)}{k^*+\gamma}\right] + \frac{\tau\ p\ \gamma}{(1-\tau)(k^*+p+\gamma)}\left[\frac{k^*+\delta}{k^*+\gamma}\right] - \frac{\tau ph}{1-\tau} \qquad (1:11)$$

making it evident that the net effect of inflation on the firm's real cost of capital depends on two opposing forces: The current practice of basing depreciation charges on historical cost vs allowing the firm to deduct <u>nominal</u> cost of debt--including the part that constitutes compensation to lenders for inflation (p).

Real net cost of capital r*, therefore, will rise, remain unaffected or fall, depending on

$$h \lesseqgtr \frac{\gamma}{k^*+p+\gamma}\left[\frac{k^*+\delta}{k^*+\gamma}\right].$$

For instance, letting k* = 3%, p = 7%, γ = 20% and δ = 10%,--not unreasonable figures for Swedish industry in the mid 70's--a firm financing >37.6% of its capital growth by debt (h), would find investment incentives improve as a result of inflation. The advantage from deducting that part of the nominal cost of debt, constituting an inflationary compensation, would outweigh the loss from historical cost depreciation.

Table 1 extends this example to include several alternatives regarding rates of capacity depreciation (δ) and depreciation for tax purposes (γ) as well as the rate of inflation (p). The table indicates values of h above which inflation reduces real cost of capital. An indicated value of h in the table says that all firms with more of its total capital financed by debt will get a lower capital cost by inflation.

It may be noted that the critical values of h falls as the rate of inflation increases. Thus, at high rates of inflation even firms with low debt financing would find their real costs of capital fall as a result of inflation.

Table 1. Ratio of debt to total capital balancing counteracting effects on capital cost

p	δ = 0.05		δ = 0.10	
	γ = 0.05 (1)	γ = 0.10 (2)	γ = 0.10 (3)	γ = 0.20 (4)
0.02	0.50	0.41	0.67	0.45
0.05	0.38	0.34	0.56	0.40
0.07	0.33	0.31	0.50	0.38
0.10	0.28	0.26	0.43	0.34

Comparing the first and third columns of table 1 brings out another result regarding the effects of inflation on investment projects of different lengths. It takes a higher h to compensate for the loss due to historical cost depreciation the higher the rate of capacity depreciation (δ).[1] Therefore, in times of inflation, historical cost depreciation discriminates against short-lived investments (with a high δ).[2]

We can summarie the effects of inflation on real capital cost via the corporate tax system as follows:

(1) Inflation increases capital cost because depreciation charges are taken on historical cost. This effect is stronger, the shorter the investment period.

(2) Inflation decreases capital cost because deduction of the nominal cost of debt is allowed. The higher the debt ratio, the stronger is this capital cost decreasing effect of inflation.

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[1] By comparing the first column (δ = .05, γ = .05) with the third (δ = .10, γ = .10) we compare investments of different life lengths when there is no deferral of corporate taxes due to accelerated depreciations.

[2] This is due to our assumption of amortization.

2. BUSINESS TAXES AND HOUSEHOLD TAXES

2.1 Shareholder Taxation and Capital Cost

In section 1 of this paper we did not take into account that capital income in the corporate sector of the economy is taxed twice. On top the corporate profit tax dividends are taxed in the household sector at stockholders' marginal rate of income tax. To the extent that retained earnings lead to capital gains on corporate stocks these are also taxed in the household sector, albeit at a relatively low rate.[1]

In this section of the paper we pose the very same questions as we did in the first section of the paper, but we take into account the so called "double taxation" of corporate source income.

Now, let k represent stockholders' rate of return on alternative financial investments, exogenously given to the national economy by opportunities on capital markets in the world economy. This rate of return is assumed to be taxed as personal income at the marginal income tax rate, T, of the "representative" stockholder. Therefore stockholders' required net rate of return is $k(1-T)$.[2]

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[1] The analysis here draws upon Södersten (1977) and Bergström and Södersten (III:5 in this volume). It is not implied by our assumptions that there is a one-to-one relation between retained earnings and capital gains. This relation depends on the differential taxation of dividends and capital gains as explains in Bergström and Södersten, III:5 in this volume.

[2] For many countries this assumption may obviously be questioned, bearing in mind e.g. that capital gains on alternative investments open to households often receive a preferential tax treatment.

A further and important assumption here about the cost of equity to the firm, $k(1-T)$, is that k is independent of T. This means that personal taxation of equity income cannot be shifted. If investors have no alternatives, international or national, to avoid a general personal income tax that is applicable to all sources of household income this is a reasonable assumption. In this way, from the management (firm) point of view, an increased personal taxation lowers the cost of equity because the net rate of return to equity which shareholders apply when discounting expected cash flow in evaluating shares, is lowered.

Following Swedish (and U.S.) tax rules we let dividends from the corporate sector be taxed at the marginal income tax rate, T, and (accrued) capital gains, $dV(t)/dt$, at a lower rate, αT, $(\alpha<1)$.[1] The value of the firm's common stocks, $V(t)$, can now be formulated as the capital value of all future cash flow (expected with certainty):

$$V(t) = \int_{s=t}^{\infty} \{U(s)(1-T)-\alpha T\frac{dV(s)}{ds}\}e^{-k(1-T)(s-t)}ds \qquad (2:1)$$

where $U(t)$ is the sum of dividends and the second term under the integration sign is the assumed tax

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[1] The parameter α takes care of the fact that the rate of capital gains tax is lower than the marginal rate of income tax and further that in practice capital gains are taxed only upon realization, meaning that the effective rate is lower than the statutory rate when the latter is transformed to a tax on accruals (which in turn presupposes known holding periods). See Bailey (1969).

on accrued capital gains.[1]

The capital value (2:1) can be reformulated to a simpler form[2]

$$V(t) = \int_{s=t}^{\infty} \frac{U(s)(1-T)}{1-\alpha T} e^{-\frac{k(1-T)}{1-\alpha T}(s-t)} ds \qquad (2:2)$$

Dividends U(s) are already defined by the bracketed term in formula (1:1), page 240 of this paper. By insertion of this expression for U(s) in (2:2), we get an expression for the value of the firm in stockholders' portfolios with regard to the profit tax, the personal income tax and the capital gains tax.

Capital cost can now be derived in a manner similar to that of section 1 of this paper. The procedure will not be repeated here.

A complication should be mentioned, though. Even if investments are reversible there will now be a bound -- and upper bound -- on the volume of investment, due to our financial assumptions. With a

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[1] By this formulation we disregard new issues of common stocks. This requires $U(t)>0$, contrary to the case above with profit taxes only.

We assume here that all expectations are held with certainty and that shareholders are identical.

[2] Take the derivative of V(t) in (2:1) with respect to the lower limit of integration, giving

$$\frac{dV(t)}{dt} = -\left\{U(t)(1-T) - \alpha T \frac{dV(t)}{dt}\right\} + k(1-T)V(t)$$

which can be rewritten as

$$\frac{dV(t)}{dt} = \frac{k(1-T)}{1-\alpha T} V(t) - \frac{U(t)(1-T)}{1-\alpha T}.$$

From the solution of this differential equation we get (2:2).

constant debt ratio gross investments will be limited to the amount given by the volume that absorbs all retained earnings as the equity financed part. To invest more than this would call for new issues, a possibility we have excluded (here, but not in the case above of profit taxation only) in order to simplify the analysis.

Nevertheless, we treat the present problem as if there were no bound on the investment plan meaning that we study only free intervals where bounds are ineffective.[1]

We proceed, then, as if there were no bounds and after substitution for U(s) from (1:1) in (2:2) and using the same procedure as in part 1 of this paper we can compute the real net cost of capital (to be compared with (1:8)) as

$$r^* = ih + \frac{k(1-T)}{(1-\tau)(1-\alpha T)} \left[1 - h - \frac{\tau[\gamma - (\delta - p)]}{\frac{k(1-T)}{1-\alpha T} + \gamma} \right] - p \qquad (2:3)$$

2.2 Double taxation and real capital cost

The next step is to assume, again, that the nominal rate of interest, i, and stockholders' nominal required rate of return, k, increase with the rate of inflation such that $i = i^* + p$ and $k = k^* + p$, where again i^* and k^* express real rates. Note here that our assumption that the net rate of

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[1] Appelbaum and Harris (1978) have studied control problems with both upper and lower bounds on the investment plan. In free intervals "myopic rules" of the unbounded problem are still operative. See also Arrow (1964) and (1968).

return, k(1-T), is used in discounting means that the inflation compensating part of the nominal rate of return on stockholders' alternative investments, k, is also taxed at the marginal rate of income tax, T.

Substituting $k^* + p$ and $i^* + p$ for i and k in (2:3) gives after some manipulations the basic result of our analysis:

$$r^* = i^*h + \frac{k^*(1-T)}{(1-\tau)(1-\alpha T)}\left[1-h-\frac{\tau(\gamma-\delta)}{\frac{k^*(1-T)}{1-\alpha T}+\gamma}\right] +$$

$$\frac{\tau p\gamma}{(1-\tau)\left[\frac{(k^*+p)(1-T)}{1-\alpha T}+\gamma\right]}\left[\frac{\frac{k^*(1-T)}{1-\alpha T}+\delta}{\frac{k^*(1-T)}{1-\alpha T}+\gamma}\right] - \frac{\tau ph}{1-\tau} -$$

$$\frac{(T-\alpha T)p}{(1-\tau)(1-\alpha T)}\left[1-h-\frac{\tau\gamma}{\left[\frac{(k^*+p)(1-T)}{1-\alpha T}+\gamma\right]}\,\frac{(\gamma-\delta)}{\left[\frac{k^*(1-T)}{1-\alpha T}+\gamma\right]}\right] \qquad (2:4)$$

This is the real net cost of capital with regard to both profit taxation and personal income and capital gains taxes. We see that the personal taxes have substantially complicated the expression for real capital cost compared to that with regard to profit taxation only (compare (2:4) to (1:11)). The different terms of (2:4), however, still have an intuitively clear economic interpretation.

The <u>first two</u> terms represent the net cost of capital without inflation. This real net cost of capital at zero inflation is our norm of comparison for the further analysis. The <u>third</u> term represents the capital cost increasing effect, in times of inflation, due to historical cost depreciations (as compared to replacement cost depreciation, inherent in the inflation free cost of capital. Cf the third term of (1:11)).

The fourth term shows that capital cost is reduced, because the full nominal interest on debt is deductible against corporate profits, whereby in fact the "real rate of amortization", p, is deductible for taxation.

The fifth awkward-looking term has to do with stockholders' taxation. It represents, on the one hand, a reduction of capital cost due to the fact that stockholders are taxed at marginal income tax rate T also for that part of the nominal rate of return, k, on alternative financial investments that is a compensation for inflation, p. Stockholders' real rate of return net of tax is then $k(1-T) - p = k^*(1-T) - pT$, implying a reduced cost of equity to the firm. On the other hand, there is an increase of capital cost following from the fact that nominal capital gains on stockholdings are taxed at the rate αT.

It may be noted that the term added by the introduction of personal taxes tends to lower real capital cost, provided capital gains receive a preferential tax treatment (i.e. $\alpha T < T$). In other words, taxing stockholders' nominal rate of return on alternative financial investments at marginal tax rate T, outweights the capital cost increasing effect of taxing nominal capital gains on corporate stock.[1]

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[1] This is not the whole story, however, since personal taxation also affects the third term of (2:4), reflecting the increase in capital cost due to historical cost depreciation.

Expression (2:4) makes it evident that the net effect of inflation on real capital cost depends on four opposing forces. These include current practice of basing depreciation allowances on historical costs, of allowing the firm to deduct nominal costs of debt, of taxing shareholders' nominal rates of return on alternative financial investments and of taxing nominal capital gains on corporate stock.

After some rearranging of (2:4), it can be demonstrated that if

$$T \geq \tau + \alpha T(1-\tau) \qquad (2:5)$$

i.e. stockholders' marginal income tax rate is greater than or equal to the total tax burden on retained profits, then net real capital cost r* will fall as a result of inflation. Assuming the corporate tax rate (τ) to be 50% and α, i.e. that part of (accrued) capital gains that must be declared as taxable income, to be 15%, this condition means that the firm would find real capital cost fall when shareholders' marginal tax rate T exceeds 54%. Assuming, instead, $\alpha = 0.4$, capital cost will fall when T > 62.5%.[1]

If, on the other hand, (2:5) does not hold, capital cost will still fall provided

$$h > 1 - \frac{\tau[(1-Z)(1-\alpha T) + \tau ZQ(1-T)]}{\tau + \alpha T(1-\tau) - T} \qquad (2:6)$$

where

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[1] Cf. Bailey (1969) for empirical estimates of α for the U.S.

$$Z = \frac{\gamma}{[(k^*+p)\,\frac{(1-T)}{1-\alpha T} + \gamma]}$$

and

$$Q = \frac{\gamma - \delta}{\frac{k^*(1-T)}{T-\alpha T} + \gamma} \, .$$

To explore the meaning of this requirement for the firm's debt ratio we have calculated some numerical examples including several alternatives of T, α, γ, and p. Tables 2A and 2B, which assume the corporate income tax rate τ to be 50%, the rate of capacity depreciation δ to be 10% and stockholders' real required rate of return k* to be 3%, indicate values of h above which inflation will <u>reduce</u> real cost of capital. A certain value of h in the tables, says then that all firms with <u>more</u> of its total capital financed by debt will get a lower cost of capital as a result of inflation.

It may be noted that the critical values of h falls as the rate of inflation and the marginal rate of income rax rise. Also, h falls when the corporate income tax is lowered by way of accelerated depreciation ($\gamma>\delta$) or the capital gains tax parameter α is reduced. The most important result emerging from Tables 2A and 2B, however, is that for reasonable values of the parameters real cost of capital falls as a result of inflation. This conclusion presumes -- realistically -- that most stockholders are located in income brackets with high marginal tax rates and / or that the corporate tax system provides for acceleration of depreciation allowances ($\gamma>\delta$). Taking into account personal taxes on dividends and capital gains, therefore, reinforces the tendencies noticed in the first part of the paper, namely that under certain circumstances, inflation will lower real capital cost.

Table 2. Ratio of debt to total capital above which inflation will reduce real cost of capital

Table 2A: α = 0.4

	T							
	0.3		0.4		0.5		0.6	
p	γ=0.1	γ=0.2	γ=0.1	γ=0.2	γ=0.1	γ=0.2	γ=0.1	γ=0.2
0.02	0.52	0.22	0.39	0.01	0.05	0	0	0
0.07	0.25	0.09	0.03	0	0	0	0	0
0.10	0.14	0.03	0	0	0	0	0	0

Table 2B: α = 0.15

	T							
	0.3		0.4		0.5		0.6	
p	γ=0.1	γ=0.2	γ=0.1	γ=0.2	γ=0.1	γ=0.2	γ=0.1	γ=0.2
0.02	0.42	0.07	0.13	0	0	0	0	0
0.07	0.09	0	0	0	0	0	0	0
0.10	0	0	0	0	0	0	0	0

2.3 Eliminating Distortions with Profit Taxes and Personal Taxes on Dividends and Capital Gains

The results presented in previous sections lead us to the question of indexing. How can the inflationary distortions via the tax system be eliminated?

The standard norm of comparison in the literature on inflation and taxation is capital cost at zero inflation and no distortions from the tax system. Recognizing, however, that governments in many countries, e.g. Sweden, consciously intervene in resource allocation promoting in particular industrial growth by various means of accelerating depreciation allowances[1], another norm is of great interest: The norm of capital cost at zero inflation given the distorting system of taxation. We will first state ways of eliminating distortions relative to this last mentioned norm.

1. (i) Change the system of corporate taxation so that the book value on which depreciation charges are taken may be adjusted for price changes. This makes the third term of (2:4) vanish.[2]

(ii) Furthermore, let only the real interest rate i^* be deducted against corporate profits. This eliminates the fourth term of (2:4).

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[1] See Bergström (1977) and Södersten (1978).

[2] This can be seen by substituting $\gamma\tau P_K(s)D(s)$ for $\gamma\tau D(s)$ in (1:1), page 240 and then performing the analysis as we have done it in the paper.

(iii) Change personal taxation so that stockholders are taxed only for the real rate of return on alternative financial investments. In this way nominal after tax cost of equity becomes $k-T(k-p)= k(1-T)+pT$. This in turn means that the real after tax cost of equity is $k^*(1-T)$.[1]

(iv) Finally, let stockholders be taxed only for real capital gains on corporate stock. Capital gains tax at time t would then equal

$$\alpha T\left\{\frac{dV(t)}{dt} - pV(t)\right\}.$$

With all these adjustments net capital cost becomes

$$r_1^* = i^*h + \frac{k^*(1-T)}{(1-\tau)(1-\alpha T)}\left[1 - h - \frac{\tau(\gamma-\delta)}{\frac{k^*(1-T)}{1-\alpha T} + \gamma}\right]$$

where capital cost is still a function of the tax system (in a way intended by the government) but independent of the rate of inflation.

2. As a special case of the above procedure, free depreciation can be allowed.[2] In our model, this would require γ, the rate of tax depreciation to be infinitely large.[3] Rewriting (2.4) under this condition gives

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[1] Since $k = k^*+p$, then $k(1-T)+pT-p = k^*(1- T)$.

[2] This was the case in Sweden during the years 1938-51.

[3] To make an investment "evaporate" immediately γ must go to infinity.

$$r_2^* = i^*h + \frac{k^*(1-T)}{(1-\tau)(1-\alpha T)}(1-h-\tau) -$$

$$\frac{p\tau h}{1-\tau} - \frac{pT(1-h-\tau)}{(1-\tau)(1-\alpha T)} + \frac{p\alpha T(1-h-\tau)}{(1-\tau)(1-\alpha T)}.$$

The first two terms again represent net cost of capital at zero rate of inflation. By applying then the last three rules of case 1) above capital cost becomes independent of inflation (but not of taxation). Thus, investment incentives would be preserved at zero inflation standards.

3. Finally, let us look at the over all norm of no inflationary and no tax distortions. By letting tax depreciations be taken on replacement cost at a rate coinciding with capacity depreciation, (i.e. $\gamma = \delta$), the third term of (2:4) disappears as well as the ratios within the brackets of the second and fifth terms.

As above allowing only real interest to be deductible takes away the fourth term. If, on top of this, the real cost of equity, k^*, is deducted for tax purposes the corporate tax system would be "corrected".

For personal taxation, capital gains on corporate shareholdings should be taxed at the same rate as other capital income ($\alpha = 1$). For the final corrections on the personal taxation side there are two ways to choose between, one real and the other nominal. Remaining distortions from personal taxation may be eliminated either by taxing <u>real</u> capital gains and <u>real</u> rates of return on alternative investments or by taxing <u>nominal</u> gains (at the same rate as other capital income) as well as nominal rates of return on alternative investments. This last alternative means that the two components of the last term of (2:4) cancel out, whereas the first alternative means that both these components are zero.

With all these adjustments capital cost would be

$$r_3^* = i^*h + k^*(1-h).$$

This procedure would thus result in a distortion-free tax system, untouched by inflation. Capital cost would be invariant both with respect to taxes and inflation.

The latter results stated above make it clear that to have a neutral tax system, it is not necessary to have a real norm of taxation. Even a nominal norm will do as long as the norm is consequently stuck to. The principle of real taxation described above could be substituted by nominal taxation--both corporate and personal.

We have already described the choice between real and nominal personal taxation above. To see that there is a similar choice also for profit taxation let the firm deduct nominal rates k and i and tax the capital gains on real corporate capital in the firm. This last rule eliminates the fourth term of (2:4) and the net result is again r_3^* above.

2.4 Concluding Remarks

It seems evident that the most rational and most simple way of indexing the tax system is the first way, described under alternative 1) above. This alternative of indexing results in just that cost of capital intended by the government by the construction of the tax system (in an inflation-free world). Furthermore, it is an easy correction to undertake as the only information needed is the rate of inflation. This rate of inflation is used to adjust book values, nominal costs of debt, nominal rates of return on alternative investments, and the values of common stocks. In practice it would be conceivable to define broad price indices of capital goods to be used for approximate corrections of existing tax systems.

The other two alternatives would change the present tax laws also at zero rate of inflation. The third alternative --alternative 3 --would furthermore require knowledge of capacity depreciations to be applied to replacement cost as the basis for tax depreciations.

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