A list of Working Papers on the last pages

No. 259, 1990

THE EFFECTS OF R&D EXTERNALITIES IN A SPATIAL MODEL

by

Jonas Häckner

This is a preliminary paper.

April 1990

THE EFFECTS OF R&D EXTERNALITIES IN A SPATIAL MODEL

by

JONAS HÄCKNER

The Industrial Institute for Economic and Social Research, Stockholm

ABSTRACT

This paper deals with the physical location of firms although other interpretations are also possible. It is a well known fact that firms in certain industries tend to cluster. However, since you would expect competition to be more intense when goods are less diversified in a locational sense there must be some explanation to these observations which is not usually dealt with in standard economic theory. One striking example is of course the American west coast high–tech industry clustering in Silicon Valley, but others are not difficult to find. There is a broad range of possible explanations to these phenomena. The focus of this paper is to explore the possibilities to explain clustering in terms of external effects in the R&D process. This is done through the introduction of R&D investments into a version of the Hotelling spatial duopoly model. We consider only cost reducing innovations (i.e process innovations). The investment decisions and the locational decisions are taken simultaneously. Then firms compete in prices, conditional on their choices in the first period. Marginal costs are reduced both through own investments and by spillovers from the competitor and these spillovers are decreasing in the distance between firms. The most surprising finding is that clustering will occur **only** if it is totally costless in terms of competition in the product market, that is, only when both firms act as unconstrained monopolists. Extending the model into an infinitely repeated game opens up the possibility to sustain locational equilibria characterized by clustering if the discount factor is large enough. This is so since the optimal collusive locational pattern implies at least some amount of clustering.

1 BACKGROUND

It is a well known fact that firms in certain industries tend to cluster geographically. However, since one would expect competition to be more intense when goods are less diversified in a locational sense there must be some explanation to these observations which is not usually dealt with in standard economic theory. One striking example is of course the American west coast high–tech industry clustering in Silicon Valley, but others are not difficult to find – the jewelry district of Amsterdam is one, and the optics industry cluster of Rochester, New York, is another. There is a broad range of possible explanations to these phenomena. Geographical concentration of the supply of certain inputs, such as skilled labor and raw materials is one. Another is when demand is highly concentrated in a

physical sense. Producers may also want to reduce consumers' search costs under conditions of uncertainty regarding prices and/or locations in order to increase the pie of surpluses. The focus of this paper is to explore the possibilities to explain clustering in terms of external effects in the R&D process. Of course, these kinds of externalities are only likely to be of importance in R&D intensive industries, but clustering is also a common feature in precisely these industries.

R&D investments could be given a very broad interpretation, including any cost reducing investment. For example, they could be interpreted as training costs in an industry characterized by a certain amount of turnover in the labor force (where turnover is negatively related to the physical distance between firms).

In passing, it should be mentioned that an equally interesting way of interpreting the locational problem is in terms of product differentiation in general. It seems likely that the cost of taking advantage of a rival's process innovation should be reduced when products are less diversified, but again, competition in the product market would then be intensified. (In this framework transportation costs should be thought of as a consumer's cost of not being able to buy a product with his favorite characteristics. Since we will assume transportation costs are quadratic in distance, this interpretation might be more realistic than if distance is thought of as being geographical. In many cases, there is reason to believe that travelling, in the geographical sense, is characterized by constant returns to scale, or even by increasing returns to scale but quadratic travelling costs of course imply decreasing returns to scale.) Also in this context, R&D investments could be thought of as training costs. These should be lower the more familiar a newly employed is with the type of product being manufactured. With a certain degree of labor turnover within the industry, training costs would therefore be lower if products are less diversified in product space.

2 THE MODEL

The most famous model of location, and basically the one to be used in this paper, is due to Hotelling (1929). However, as was pointed out by d'Aspremont, Gabszewicz and Thisse (1979), there is a problem with non quasiconcave profit functions in Hotelling's original paper (which led him wrong in his conclusion about minimal differentiation) and a slightly modified version due to these authors will be used instead. One reason why this model is chosen is its' implication of **maximal** differentiation in absence of externalities. This result seems to be extremely robust to changes in the specification of the model (of which there are quite a few). If clustering can be explained by R&D externalities in this very

"conservative" framework, they will seem a more convincing explanation then would otherwise be the case.

Consumers are assumed to be uniformly distributed along a line of unit length and there are two firms located at points a and (1–b) along this line. Firms will be denoted firm a and firm b (see figure 1.). Consumers buy at most one unit of a good which is homogeneous in all other respects than the distance between consumer and producer. The utility of a consumer located at k is:

$$
(1)\qquad
\begin{cases}
U_k = s - t(k-a)^2 - P_a & \text{if buying from firm a} \\
U_k = s - t(1-b-k)^2 - P_b & \text{if buying from firm b} \\
U_k = 0 & \text{otherwise}
\end{cases}
$$

where s is the reservation price before travel expenses are deducted, t times the squared distance gives total travelling expenses and P_a and P_b are the prices charged by the firms. Of course consumers make their purchases from the firm whose location and price gives them the highest utility, (if positive) or refuse to buy at all if prices are too high. A typical demand function is shown in figure 2.

Producers choose a location, an investment in R&D which reduces marginal cost (i.e a process innovation) and a price. There is no uncertainty.

The timing is as follows:

1) Both firms simultaneously choose a location and an investment in R&D
2) Given the locations and marginal costs (which are now considered constant) the firms compete in prices and payoffs are realized

The equilibrium concept used is the subgame perfect Nash equilibrium. According to that concept, an equilibrium strategy must be optimal starting from any subgame. This rules out uncredible threats like "if you invest in period 1, I will set my price to minus infinity in period 2". Technically, we solve the problem backwards. Given **any** pair of marginal costs and locations, what are the Nash equilibrium prices and corresponding profits in period 2? Given these profit functions, what are the Nash equilibrium locations and investment levels in period 1 when account is taken of possible R&D externalities? Only symmetric equilibria are considered. With no loss of generality we assume $0 < a < 0.5$ and $0.5 < (1–b) < 1$. There is no discounting between the two periods.

Before going on to the main analysis it might be useful to work through to solution to the game in absence of any externalities. That is, both firms are assumed to have constant and identical marginal costs. Figure 3 shows the utility of consumers when there is only one firm, located at a and charging P_a. The consumer fortunate enough also to be located at a, has utility (s–P_a), but all others have lower utility levels, down to consumers (a–w) and (a+w) who are indifferent between buying and not buying. Consumers to the left of (a–w) and to the right of (a+w) choose not to buy. The shaded area represents total travel expenses in the market. It is usually assumed that all consumers earn a positive utility from buying in equilibrium as in figure 4a), where i^* denotes the consumer that is indifferent between the two firms. This, however, implicitly relies on the assumption that marginal costs are small compared to s and/or that travelling costs are relatively small. Given any s, and recognizing that equilibrium prices cannot be greater than costs it is easy to see that the top of the "utility mountain" in figure 3 approaches the horizontal axis as marginal costs increase, making full market coverage less likely. Increasing t leaves the maximum utility unchanged, but makes w increasingly smaller, also making full market coverage less likely. In all we have four types of potential equilibrium configurations in absence of fixed costs. These are illustrated in figure 4a)–d).(Equilibria without market coverage in the standard Hotelling model are rigorously treated in Economides (1984).)

3 THE EQUILIBRIA IN ABSENCE OF EXTERNALITIES

3.1 Type 1 situation

As said before, this is the standard setting, assuming full market coverage in equilibrium. Let us call the location of the indifferent consumer i^*. Then i^* is given by:

$$s - t(i^* - a)^2 - P_a = s - t(1 - b - i^*)^2 - P_b$$

Solving for i^* and noting that the demand functions, D_a and D_b, are given by i^* and $(1-i^*)$ respectively, we have:

$$(2) \qquad D_a = \frac{1 - b + a}{2} + \frac{P_b - P_a}{2t(1 - b - a)}$$

$$(3) \qquad D_b = \frac{1 - a + b}{2} + \frac{P_a - P_b}{2t(1 - b - a)}$$

Starting with period 2, firm a maximizes its profit, taking locations and its marginal cost

c_a as given. (Π_a is here clearly continuous in prices, which it was not in Hotelling's original paper where transportation costs were linear in distance. For a discussion of this, see d'Aspremont, Gabszewicz and Thisse (1979).)

$$(4) \qquad \Pi_a = (P_a - c_a)\left[\frac{1 - b + a}{2} + \frac{P_b - P_a}{2t(1 - b - a)}\right]$$

The first order condition is;

$$\frac{\partial \Pi_a}{\partial P_a} = \frac{a^2t - b^2t + 2bt - c_a - P_b - t + 2P_a}{2t(a + b - 1)} = 0$$

From this it is easy to see that the second order condition for a maximum is satisfied for $a < (1-b)$. From the first order condition we have the following reaction function for firm a:

$$P_a = \frac{1}{2}\left[\, t(1 - 2b + b^2 - a^2) + c_a + P_b \,\right]$$

and by symmetry, for firm b:

$$P_b = \frac{1}{2}\left[\, t(1 - 2a + a^2 - b^2) + c_b + P_a \,\right]$$

These reaction functions have all the expected properties of a Bertrand game with differentiated products. They are upward sloping, implying that prices are strategic complements, and their intercepts are increasing in cost. Solving for the equilibrium prices of period 2 we have:

$$P_a^* = \frac{1}{3}\left[\, t(3 - 4b - 2a + b^2 - a^2) + 2c_a + c_b \,\right]$$

$$P_b^* = \frac{1}{3}\left[\, t(3 - 4a - 2b + a^2 - b^2) + 2c_b + c_a \,\right]$$

Substituting the equilibrium prices into (4) and rearranging, we end up with:

$$(5) \qquad \Pi_a^* = \frac{\left[\, t(a^2 - b^2 + 2a + 4b - 3) + c_a - c_b \,\right]^2}{18t(1 - a - b)}$$

Knowing this will be the equilibrium payoff in period 2, firm a maximizes Π_a^* with respect to a in period 1, treating (1–b) as a constant. (The profit function is clearly continuous in location.) Letting $c_a = c_b$ = constant:

$$(6) \qquad \frac{\partial \Pi_a^*}{\partial a} = -\frac{t(a - b + 3)(3a + b + 1)}{18} < 0 \qquad ,\forall\, b$$

That is, no matter where firm b is located, firm a would want to keep as far away from him as possible. The same is true for firm b so the unique subgame perfect Nash equilibrium is $a = b = 0$. How could this be explained? There are two effects present in this context. The first effect can be called the business–stealing effect. It is tempting for both firms to locate close to the rival in order to steal customers from him. The second effect is the competition effect. The closer the firms, the more intense is competition, and the closer prices to marginal cost. Whenever $a = (1–b)$ and marginal costs are the same we know from the expressions for P_a^* and P_b^* that price equals marginal cost so profits are zero, as should also be expected in a Bertrand game when products are no longer differentiated. The maximum distance result indicates that the competition effect dominates over the business–stealing effect. If, on the other hand, prices were regulated, the business–stealing effect would of course dominate, changing the unique equilibrium to $a = (1–b) = 0.5$. In other words, we would have a minimum distance result.

3.2 Type 2, 3 and 4 situations

In the type 2 situation, the firms are local monopolies and consequently, no strategic interaction is present. Hence, they could move inwards, gaining customers at the center without losing demand at the tails. In other words, this cannot be a Nash equilibrium. The incentive to deviate inwards would remain until the firms start competing, or until they do lose customers at the tails.

The type 3 situation can be a Nash equilibrium when costs are high. In this case, demand and profits are independent of locations. The locational choices are therefore unessential and the number of possible equilibrium configurations are infinite.

In the type 4 situation, each firm must be unhappy with it's choice of location. This is so,

simply because they would loose fewer customers from the center than they would win at the tails, when moving towards "their own" corner. In other words, there is no business stealing effect present here (quite the contrary) and hence the competition effect must dominate. Consequently this is not an equilibrium.

4 EQUILIBRIA WHEN INTRODUCING R&D EXTERNALITIES

4.1 Type 1 situation

In absence of externalities and with full market coverage we know that the only possible subgame perfect Nash equilibrium is $a = b = 0$. When introducing externalities, the game starting in period 2 is essentially unchanged since marginal costs and locations are treated as constants after period 1. Thus, firm a still maximizes equation (5), but now with an additional term representing R&D expenses, $X_a r$, where X_a is the amount of R&D conducted by firm a and r is the cost of R&D capital.

$$(5') \qquad \Pi_a^* = \frac{\left[\, t(a^2 - b^2 + 2a + 4b - 3) + c_a - c_b \,\right]^2}{18\,t\,(1 - a - b)} - X_a r$$

Of course, c_a and c_b are now functions of the R&D intensities of both firms as well as of the distance between firms. The general specification of the cost function is;

$$(7) \qquad c_a = f(\kappa, a, b, X_a, X_b, z) \qquad f_1 > 0 \qquad f_2, f_3, f_4, f_5, f_6 < 0$$

with c_b defined analogously. κ is the constant marginal cost in absence of any R&D activity. As explained above, X_a denotes the amount of firm a's own R&D, while X_b is firm b's R&D expenditure. z represents the intensity of R&D spillovers, and (a+b) measures the distance between the firms. This term is maximally equal to unity when $a = (1-b)$, that is when both firms locate at the same spot. Thus a firm can lower it's cost both by investing in R&D and by locating closer to his rival, taking advantage of spillovers. However, he must take into account that some of his own R&D will benefit the rival and that moving closer to the rival increases spillovers in both directions.

Differentiating equation (5') with respect to a, treating b as a constant, gives us,

$$(6') \qquad \frac{\partial \Pi_a^*}{\partial a} = -\frac{t(a - b + 3)(3a + b + 1)}{18} - \frac{1}{9}(a - b + 3)\left[\frac{\partial c_a}{\partial a} - \frac{\partial c_b}{\partial a}\right]$$

when evaluated at a point where $c_a = c_b$. (Due to symmetry, investment levels, and therefore marginal costs, are assumed to be equal in equilibrium.) Compared to (6), (6') has an additional term, partly consisting of the derivatives of the cost functions with respect to firm a's location. It is likely that shortening the distance between firms will affect spillovers in a symmetric way, so that it does not matter which one of the firms who initiated the move. Then the term in brackets will be zero so (6') equals (6). Then, since (6') is negative for any location, firm a would want to locate as far away as possible from firm b and vice versa. The only subgame perfect Nash equilibrium is $a = b = 0$. $X_a = X_b$ can be solved for recursively, differentiating (5') with respect to X_a and letting $a = b = 0$.

Introducing R&D spillovers obviously does **not** change the results from section 3.1. Moving towards your rival lowers costs symmetrically, and therefore lowers product prices in the second period. Moreover, it intensifies competition, leaving your products less differentiated, also lowering product prices. Here, the gains from lowering costs through increased spillovers and the business stealing gains are not sufficient to offset the losses from the price reductions in the second period. However, this finding is not all that surprising. The way the demand functions are specified it is implicitly assumed that the entire market will be covered no matter what prices are charged. If for instance $\kappa=\infty$ in equation (7) and $r=\infty$, both firms will locate at the tails, charging infinitely high prices without loosing any customers. In that sense, cost is basically unessential. This can also be seen from equation (5'). What drives the result is the fact that only cost **differences** are of importance in the reduced profit function. Thus, as long as externalities are symmetric they will have no influence on profits.

4.2 Type 2, 3 and 4 situations

Already in section 3.2 it was shown that a situation as described in figure 4b) (the type 2 situation) cannot be an equilibrium. Given any set of prices, deviating towards the center would unambiguously increase profits as long as firms are not competing directly and as long as no customers are lost at the tails. Introducing R&D externalities only gives an additional reason for wanting to deviate. Not only is demand increased but costs are also reduced. Thus, it cannot be an equilibrium in this context either.

When it comes to the type 3 situation there is again no strategic interaction taking place. Demand is not a function of location but so is cost now. As long as the firms can lower their costs by moving closer without competing for customers, they will want to do so. The only configuration that could be an equilibrium in this case is if the market segments of the two firms are tangent. Below, it will be shown that there will be no incentive to move

closer once the market segments are tangent so this is indeed the unique Nash equilibrium in absence of full market coverage and with external effects in the R&D process. $X_a = X_b$ can be found recursively by differentiating profits with respect to X_a, letting locations be given by the tangency condition and the unconstrained monopoly price.

Finally, we have the type 4 situation. In section 4.1 we saw that in the full market coverage case, the incentives to part were stronger than the incentives to cluster given any value of a=b. This was so even though demand was not increasing when moving outwards. In this case there is an **additional** incentive to part, namely that given any price, total demand will increase, just as was the case in section 3.2. In all other respects the situation is the same, so this configuration can not constitute an equilibrium.

5 DISCUSSION

In absence of R&D externalities, there is a unique subgame perfect Nash equilibrium with maximal differentiation in the case of full market coverage. However, this result rests on the assumption that marginal costs are small compared to the reservation price gross of travelling expenses and/or that travelling costs are relatively small. When the market is not fully covered, there is an infinite number of equilibria corresponding to the situation in figure 4c), where both firms act as local monopolies, unconstrained geographically.

When R&D externalities are introduced, the maximal differentiation result survives in the case of full market coverage. On the other hand, there is now a unique equilibrium when the market is not fully covered. There is a tendency wanting to deviate inwards if locations are as in figure 4c) while the firms would want to deviate outwards if locations were as in figure 4d). Thus, the unique equilibrium is having both firms acting as local monopolies with tangent market segments.

The situations described in figures 4b) and 4d) are no equilibria, regardless of what assumptions are made about R&D externalities.

The maximum distance result seems to be surprisingly robust in a setting with R&D spillovers. No matter what magnitude these externalities are, we would only expect clustering (in some **mild** sense) if it is totally costless in terms of competition in the product market. Otherwise, the gains from R&D spillovers and the business stealing gains are completely outweighed by the losses in terms lowered product prices when a firm is contemplating moving towards his rival. (Prices are lower both because costs will be lower and because products will be less differentiated.)

Partly, the explanation to these results must be found in the way product market competition is modeled. Having Bertrand competition, it is a priori possible to exclude total clustering from the set of potential equilibria. By definition, prices will equal marginal costs when products are not differentiated, as is the case when firms locate at the same spot. Having zero profits in the second period, no firm would make an R&D investment in the first period, so no question of externalities would arise. Knowing this, firms would prefer to maximize the distance as in the standard solution to this model. Hence, a minimum distance equilibrium is not possible. (For that we would need products to be differentiated in some other dimension as well.) Modeling product market competition in a Cournot fashion is possible in a spatial model, but it is generally a very complex task (see Salant (1986)). Nevertheless, it would be an interesting experiment since this would allow profits to be strictly positive even when products are no longer differentiated. My presumption is that clustering due to R&D spillovers would be a much more important feature in such a model.

An **approximate** minimum distance equilibrium would result if travel expenses are extremely high (or brand preferences are strong) and/or if the reservation price, gross of travel expenses, is very low compared to marginal costs. An empirical implication of the model would be to expect firms to compete in prices, without clustering, in industries where the willingness to pay is high. The opposite would be true in industries with a low willingness to pay.

6 EXTENSIONS

It is apparent that the mere introduction of R&D externalities will not effect the maximum distance result in any significant way. A natural question to ask is what alternative specifications are needed to make firms cluster. There are a number of possible extensions in this context. Introducing sequential entry adds an extra amount of commitment possibilities to the game which might change the behavior of firms. Exogenous cost asymmetries could make the high–cost firm more willing to cluster while the opposite might be true for the low–cost firm. This could possibly lead to the non–existence of equilibrium. The rules of entry are probably of great importance in this case. Finally, one has reason to believe that making the game dynamic would open up the possibilities to more cooperative solutions by the use of trigger strategies. This should increase the clustering tendencies since it would be in both firms' interest to lower costs as cheaply as possible.

For the rest of the paper we investigate the properties of a supergame version of the

original game. In other words, the two–period game is assumed to be repeated an infinite number of times. Throughout the remaining sections full market coverage will be assumed. Essentially, that is the same as assuming that the reservation price is "high enough". This is not too a restrictive assumption to make since the calculations are basically unchanged by changes in the reservation price once it is "high enough". Furthermore, this assumption guarantees that there will always be a certain amount of strategic interaction present and those situations are surely the most interesting ones.

6.1 The supergame version

When a game is repeated an infinite number of times the "Folk Theorem" assures us that any pair of average payoffs, between the worst one–shot Nash equilibrium payoffs and the best cooperative payoffs, can be sustained as a Nash equilibrium for a high enough discount factor. The intuition behind this is quite clear. The strategies that form the equilibrium is that you stick to the cooperative solution as long as your rival does and otherwise you play the one–shot Nash equilibrium (which will always also be a Nash equilibrium to the supergame when repeated an infinite number of times). You will not take advantage of the fact that the cooperative solution is not a one–shot Nash equilibrium if the one–shot gain by deviating (given the rival's action) is smaller then the losses in terms of reduced future profit streams. Furthermore, you can do nothing to avoid the punishment since the punishment strategies themselves form a Nash equilibrium of the entire game. Thus making the discount factor ,δ ,arbitrarily large will also make the discounted stream of profit reductions arbitrarily large so no deviation will take place for a δ high enough.

We now proceed by calculating the best cooperative solution in terms of maximizing joint profits with respect to R&D investments, locations and prices. Solving for the endogenous variables as functions of the exogenous variables only seems to be a very complex task so we will have to illustrate the solution by means of a numerical example. Having done that, we go on calculating the gains from deviating and the punishment payoffs, still in the context of the numerical example. Finally we calculate what restrictions have to be made on the value of the discount factor for the optimal cooperative solution to be a Nash equilibrium.

The assumptions of the supergame must now be specified more carefully. First, locations are fixed only in the very short run. Thus, it is the two–stage game that is repeated an infinite number of times and not just the price game. Second, deviations take place in the price–game and not in the location/R&D investment period. This is not too a restrictive assumption to make. If a deviation took place in the first period the strategy would be to

play the one–shot Nash equilibrium prices in the second period. The resulting non–cooperative payoff would in most cases be lower then the payoff of an optimal deviation in the cooperative price game. Certainly, there are cases in which it would be more appropriate to model locations as fixed also in the very long run but this will not be done in this paper.

In absence of externalities and assuming that full market coverage is profitable (s large) it is obvious that the joint profit is maximized when firms are located at (0.25, 0.75) since the average surplus of the consumers is then minimized. (This would also be the social optimum since travel expenses are minimized.) Introducing externalities can only imply this amount of clustering or more. Thus a,b $\geq$ 0.25. Due to symmetry it is also obvious that actions that are optimal for one firm will also be optimal for the other when maximizing joint profits. Otherwise the more profitable action could be replicated. We can therefore ignore the subscripts, treating P, X and a as the endogenous variables. The cost function, convex in all arguments, is specified in equation (8);

$$(8) \qquad c_a = \frac{1}{k + X_a + z(a+b)X_b} = \frac{1}{k + X + 2zaX} = c$$

with c_b defined analogously. The general description of the function is found i section 4.1. (κ corresponds to (1/k) in this specification.) The firms maximize,

$$\Pi = P - \frac{1}{k + X + 2zaX} - 2Xr \qquad \text{subject to}$$

$$P + ta^2 \leq s$$

$$P + ta^2 - ta \leq s - t\tfrac{1}{4}$$

As before, r denotes the cost of R&D capital. The restrictions guarantee that the entire market will be served. It will not be profitable to leave a strictly positive surplus to every consumer since, given locations, you could always increase the price without violating the full market coverage restrictions. Hence, at least one of the restrictions will be binding in equilibrium. Since we know that a $\geq$ 0.25 we see that the first restriction imply the second one so this one only (or both) will in fact be binding, leaving the customers at the end–points with a zero surplus and everybody else better off (The second restriction makes the consumers in the middle have a weakly positive surplus.) The lagrangean is given by;

$$L = P - \frac{1}{k + X + 2zaX} - 2Xr + \gamma_1(s - P - ta^2) + \gamma_2(s - P - ta^2 + t(a - \tfrac{1}{4}))$$

where γ_1 and γ_2 are the lagrange multipliers. The first–order conditions are (9)–(13) (with P, X, a, γ_1, $\gamma_2 \geq 0$).

(9) $$\frac{\partial L}{\partial P} = 1 - \gamma_1 - \gamma_2 \leq 0 \qquad \frac{\partial L}{\partial P} P = 0$$

(10) $$\frac{\partial L}{\partial X} = \frac{1+2za}{(k+X+2zaX)^2} - 2r \leq 0 \qquad \frac{\partial L}{\partial X} X = 0$$

(11) $$\frac{\partial L}{\partial a} = \frac{2zX}{(k+X+2zaX)^2} - \gamma_1 2ta + \gamma_2(t-2ta) \leq 0 \qquad \frac{\partial L}{\partial a} a = 0$$

(12) $$\frac{\partial L}{\partial \gamma_1} = s - P - ta^2 \geq 0 \qquad \frac{\partial L}{\partial \gamma_1} \gamma_1 = 0$$

(13) $$\frac{\partial L}{\partial \gamma_2} = s - P - ta^2 + t(a - \tfrac{1}{4}) \geq 0 \qquad \frac{\partial L}{\partial \gamma_2} \gamma_2 = 0$$

Note that the objective function is concave in the endogenous variables while the restrictions are convex so the Kuhn–Tucker sufficiency conditions for a maximum are satisfied. Before turning to the numerical example, some general properties of the optimal solution will be discussed. First, we know that $a \geq 0.25$ so compared to the non–cooperative solution there will at least be some clustering here. It is also possible to show that the investment level will be lower than in the non–cooperative case whenever spillovers are large (z close to one or even larger than one) and initial marginal costs are high (k close to zero). That is, if the investment incentives are large, there will be less investment in the collusive case than in the competitive case. Intuitively, it is not surprising that the lack of externalities in the non–cooperative case has to be compensated for by a greater amount of R&D. Another interesting feature of the collusive solution is that "extra" clustering ($a > 0.25$) will only occur for intermediate values of z and r. If spillovers are non–existent ($z=0$), $a=0.25$ will surely be optimal, but this will also be true for $z=\infty$ since then marginal costs are zero for $a=0.25$ and X arbitrarily small. Thus, spillovers are of no importance. Similarly, if $r=0$ marginal costs can be reduced to zero at no cost at all, so again, spillovers do not matter. For large values of r, no R&D investments will be made so marginal costs are independent of locations. Consequently, $a=0.25$ is optimal. Finally, if travelling costs

are small, the solution will be characterized by full clustering. This can be seen if (10) and (11) are used to solve for a as a function of X and the parameters. It is easy to show that $\partial a/\partial t < 0$ in equilibrium and that a=0.5 for some t>0. When it comes to the high–tech industry, it seems plausible that transportation costs **are** small (compared to total costs). Thus, collusive agreements could be able to explain phenomena of the Silicon Valley type, both when externalities are thought of as turnover in the labor force or as traditional R&D spillovers.

Let z=1, k=0, t=0.5, r=0.1 and s=2. Assuming a>0.25, we will have $\gamma_1=1$ and $\gamma_2=0$ (from (9)). Furthermore (9)–(12) will hold with equality since investments have to be positive for a>0.25 to be optimal. Then the solution is;

The optimally collusive outcome

a = 0.382	c = 0.336
X = 1.684	P = 1.927
D_a= 0.5	D_b= 0.5

$\Pi = \Pi_a + \Pi_b = 2 \times 0.627 = 1.254$

Thus, the assumption a>0.25 proved to be right given these parameter values. Just as should be expected, the solution is characterized by some amount of clustering, with firms taking advantage of externalities. Total clustering does not occur since the spillover gains are not sufficient to compensate for the price reduction which is necessary to keep the market covered.

To calculate under which conditions these payoffs are possible to sustain as a subgame perfect Nash equilibrium we also have to compute the payoff resulting from deviation in a single period and the payoffs that follow in the punishment phase. When deviating, firm a maximizes (4) taking everything exogenous except for P_a. It is easy to show that in this case $\partial\Pi_a/P_a < 0$ for all $P_a > 1.19$. But for $P_a < 1.809$ demand is totally inelastic because then firm a has captured the entire market as shown in figure 5. (The problem is that (4), and its counterpart for firm b, does not exclude the possibility of a negative demand for one firm and a demand greater then one for the other.) The optimal deviation strategy therefore yields;

The optimal deviation outcome

P_a= 1.809	P_b= 1.927
D_a= 1.0	D_b= 0.0
Π_a= 1.305	Π_b=–0.168

$$\Pi = \Pi_a + \Pi_b = 1.137$$

It is worth noting that the deviation profit is in fact **greater** than the total collusive profit in this case. This might seem counter–intuitive but the explanation is of course that firm b subsidizes the production cost of firm a through R&D spillovers without sharing the profits. **Industry** profits are certainly lower when the cooperation breaks down as it should be by definition of optimal collusion.

Finally, the payoffs of the punishment phase are computed. These are just the payoffs of the two–period game analyzed in section 3. Differentiating (5') with respect to X_a, letting $a=b=0$, $X_a=X_b$ and $c_a=c_b$, yields $X_a= 1.826$. Plugging this into (8) together with the parameter values, gives us c_a. Then it is easy to solve for P_a from the expression for P_a^* on page 5 and to solve for Π_a from (5').

The one–shot Nash equilibrium outcome

$a = b = 0$	$c = 0.548$
$X = 1.826$	$P = 1.048$
$D_a = 0.5$	$D_b = 0.5$

$\Pi = \Pi_a + \Pi_b = 2 \times 0.067 = 0.135$

So, compared to the optimal cooperative solution, investment levels are higher, but this does not fully compensate for the lower degree of spillovers so marginal costs are also higher in this case. Profits are considerably lower for those reasons and also because prices are lower.

Let us denote the optimally collusive (per firm) profit by Π^c, the optimal deviation profit by Π^d and the punishment Nash equilibrium (per firm) profit by Π^{nc}. For Π^c to be a sustainable as a subgame perfect Nash equilibrium payoff, then the gains from deviating must be smaller than the gains from continuing the cooperation. With an infinite time horizon and with a discount factor δ, this means that;

$$(14) \qquad \Pi^d + \frac{\delta \Pi^{nc}}{1-\delta} \leq \frac{\Pi^c}{1-\delta} \quad \text{or} \quad \delta \geq \frac{\Pi^d - \Pi^c}{\Pi^d - \Pi^{nc}} = 0.55$$

Hence, for a discount factor greater than 0.55 (an interest rate less than 0.83) we will in this case be able to sustain any pair of average payoffs between Π^{nc} and Π^c as a subgame perfect Nash equilibrium of the repeated two–period game.

There are of course limits on what can be learnt from numerical examples and a more general treatment of the problem would certainly have been preferred. However, there are are at least some conclusions that can be made. First, the intuition that introducing an infinite time horizon (making trigger strategies possible) could lead to a greater amount of clustering is verified. One could argue that it is even likely that the firms try to coordinate on an equilibrium maximizing joint profits (which implies a=b=0.5 for small travelling costs). However, lots of other equilibria are of course also possible to sustain. It is self–evident that there is a social waste of both R&D resources and travelling expenses when the distance between firms is maximal. In the example we saw that the maximum distance equilibrium had both higher research costs **and** a higher marginal cost. Second, we saw that externalities could lead to the counter–intuitive result that the payoff from deviating from the optimally cooperative equilibrium could in fact be greater than the industry profits of that equilibrium. A priori, this would mean that collusive outcomes could be more difficult to sustain in industries characterized by a great deal of positive externalities.

REFERENCES

d'Aspremont, C., J. Jaskold Gabszewicz, and J.–F. Thisse (1979) "On Hotelling's Stability in Competition." Econometrica 17, 1145–1151

Economides, N. (1984) "The Principle of Minimum Differentiation Revisited." European Economic Review 24, 345–368

Hotelling, H. (1929) "Stability in Competition." Economic Journal 39, 41–57

Salant, D. (1986) "Equilibrium in a Spatial Model of Imperfect Competition with Sequential Choice of Locations and Quantities." Canadian Journal of Economics XIX, 685–715

FIGURE 1

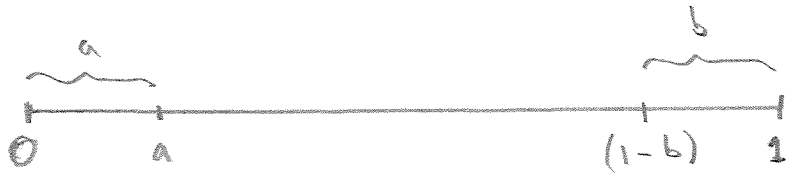


FIGURE 2

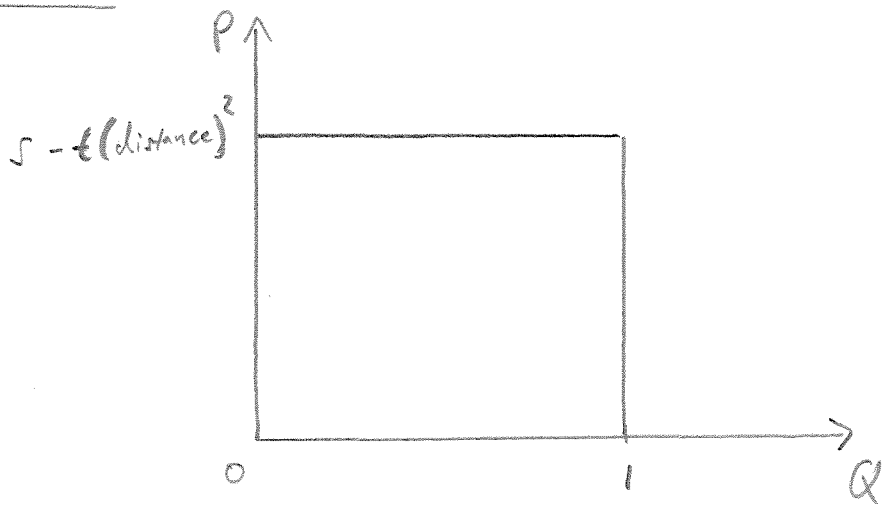


FIGURE 3

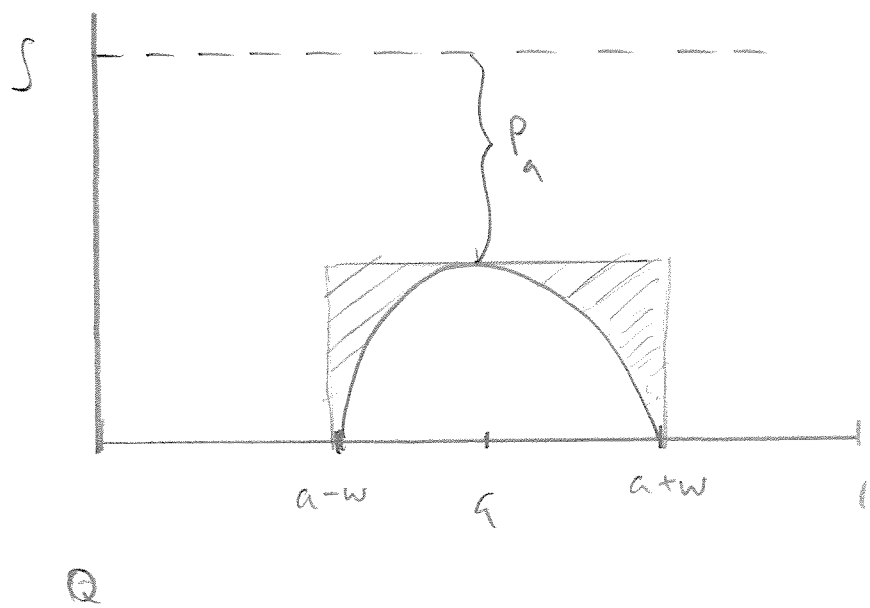
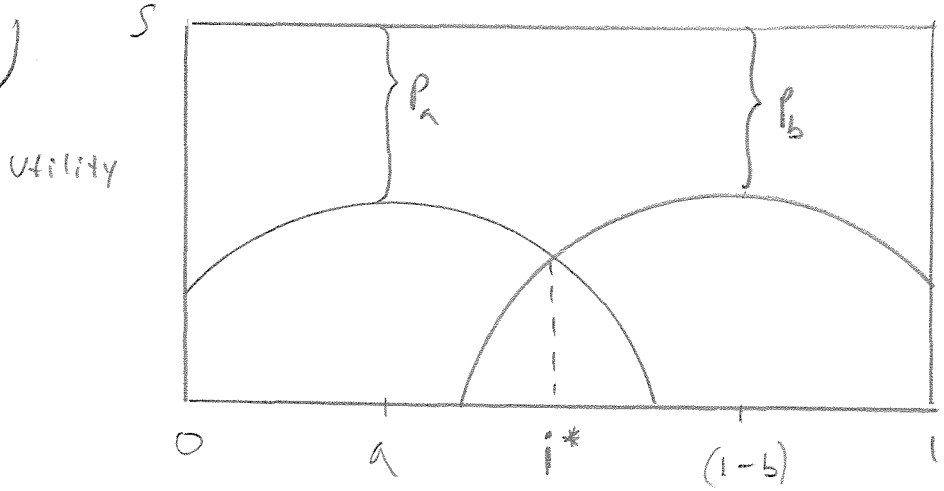


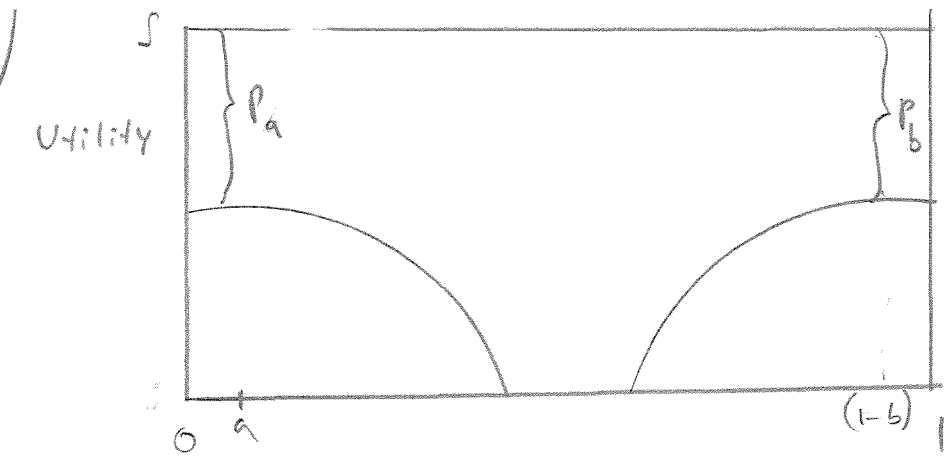
FIGURE 4

a)



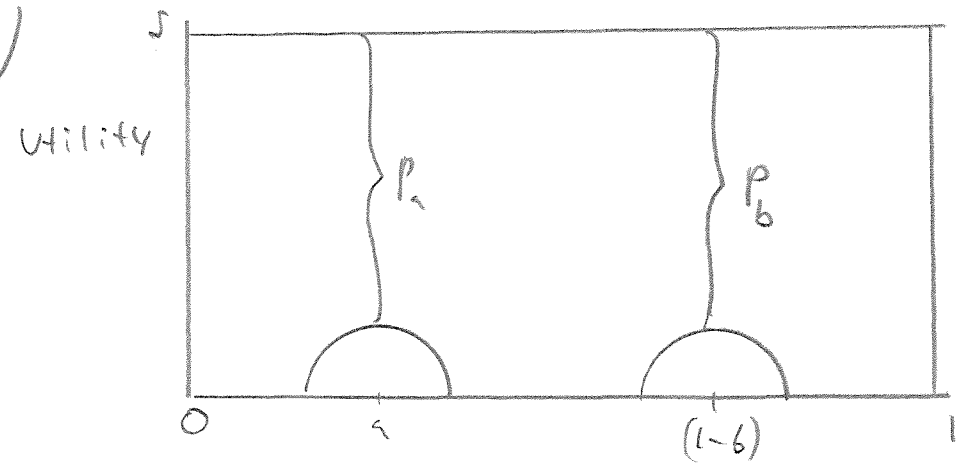
TYPE 1 EQ.

b)



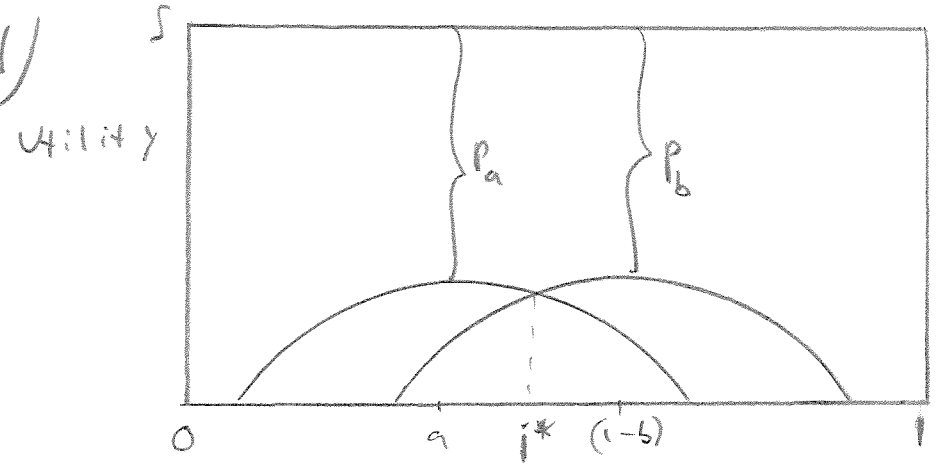
TYPE 2 EQ.

c)



TYPE 3 EQ

d)



TYPE 4 EQ