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GENERAL EQUILIBRIUM WITHOUT AN AUCTIONEER*

by

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ABSTRACT

In order to construct a theory of general equilibrium without an auctioneer we extend models of "search market equilibrium" to incorporate general equilibrium considerations. The model we treat is one with a single product market and a single labor market. Imperfectly informed individuals follow optimal sequential strategies in searching for a suitably low price and high wage. For any distribution of price and wage offers across firms, these optimal strategies generate product demand and labor supply schedules. Firms then choose prices and wages to maximize expected profits, taking these schedules as given, and the resulting profits are paid out to individuals as dividends.

An equilibrium distribution of prices and wages is one which results from optimal price and wage setting behavior by firms given individuals' optimal search strategies. There are two possible equilibrium configurations, a degenerate equilibrium in which all firms charge the same price and wage, and a price and wage dispersion equilibrium. We prove the existence of a degenerate equilibrium and of a price and wage dispersion equilibrium.

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1 INTRODUCTION

This paper analyzes a model of a simple general equilibrium economy with a single product and a single factor of production (labor). The model has two crucial features. The first is that prices and wages are set by firms, i.e. there is no Walrasian auctioneer. The second is that individuals have incomplete information in the sense that if prices and wages follow non-degenerate distribution functions, then individuals know the forms of those distribution functions but don't know which firms are charging what prices and wages. These two features correspond to two fundamental (and related) problems of economic theory, namely, the logical foundations of competitive analysis and of search theory.

In competitive analysis individuals and firms are assumed to regard prices as exogenous. Demands and supplies are then treated as functions of the exogenously given prices, and equilibrium is determined by a vector of prices that equates supply and demand on all markets. This equilibrium should be locally stable if it is to be of any interest; that is, if prices are close to their equilibrium values, then the system should have a tendency to approach equilibrium. The usual way to ensure local stability is to assume a price adjustment mechanism. If there is excess demand for a good, then its price must rise; likewise excess supply must lead to a price decrease.

The idea of price adjustment in response to excess demand or supply is appealing since we believe that firms do in reality adjust prices in response to perceived profit opportunity. Unfortunately, this intuitive justification of the price adjustment mechanism faces a logical problem in the context of competitive analysis. (The classic statement of this problem is given in Arrow [2].) To derive a competitive equilibrium it is assumed, on the one hand, that firms regard prices as exogenously given while, on the other hand, the local stability of that equilibrium is ensured by a price adjustment mechanism that is intuitively justified by a story in which firms are active price-setters. Either firms set prices or they do not; they cannot be price-takers and price-makers simultaneously.

Of course the standard way to plug this logical hole is to introduce the fiction of the Walrasian auctioneer. Given the existence of the auctioneer, firms can be regarded as price-takers both in the derivation of equilibrium and in the analysis of the local stability of that equilibrium. The problem with this device is that it is so blatantly false. Almost no markets exhibit institutional arrangements that could be thought of as even remotely corresponding to the auctioneer. A much more satisfactory approach would thus be to assume from the beginning that prices are set by firms themselves.

What sort of equilibrium might one expect in a model with price-setting firms? If the market power of any one firm vis a vis other firms is negligible and if individuals are not completely ill-informed, then one might expect to find an equilibrium tolerably close to the one produced by competitive analysis. In that case one could accept the notion of equilibrium prices determined as if they were set by the auctioneer.

Unfortunately, there exist no well-formulated models with price-setting firms that generate the competitive outcome. On the contrary, a variety of models (eg, Diamond [5] and Axell [3]) have produced the monopoly outcome. More precisely, these models have shown in a single-market, partial equilibrium setting that if an equilibrium exists in which all firms charge the same price, then that price will be the one that would be charged by a monopolist controlling the entire market.

An even more interesting equilibrium possibility to consider is one in which not all firms charge the same price. The existence of such a dispersion equilibrium is of course essential for the logical foundations of search theory. This point has been forcefully made by Rothschild [7]. In that well-known survey paper the model in which consumers search from a known distribution of prices (or job-seekers search from a known distribution of wages) was criticized as being "partial-partial". The first "partial" refers to the fact that only one side of the market is analyzed; i.e. the price-setting behavior of firms that presumably generated the distribution from which individuals are searching is left untreated. The second "partial" refers to the fact that one market is analyzed in isolation. Consumer demand (or labor supply) is taken as given, usually at the level of one "unit" per period of analysis, which is equivalent to ignoring linkages between markets.

The problem of removing the first "partial" was addressed by Axell [3] using a model in which each individual searches for one unit of a homogeneous good. His approach was to postulate a density function for prices, say $f(p)$, and a density function for consumer search costs, say $\gamma(c)$. Assuming that individuals follow an optimal sequential search rule, one can use the two postulated densities to derive the density function of reservation prices and of actual purchase or "stopping" prices, say $\omega(p)$. Next, he argues that a firm's expected demand will be proportional to $\omega(p)/f(p)$; then for a constant marginal cost function, he derives $\Pi(p)$, i.e. expected profits as a function of price. A price dispersion equilibrium is defined as a non-degenerate density, $f(p)$, such that $\Pi(p)$ is constant for all p in the support of $f(p)$. The basic result derived is a set of necessary and sufficient conditions on $\gamma(c)$ that ensure the existence of a price dispersion equilibrium. These are that $\gamma(c)$ must not be bounded away from zero, that $\gamma(c)$ must be decreasing and convex, and that the "degree of convexity" must satisfy certain conditions.

There are several other models of equilibrium price dispersion in the literature. Although almost none of these are based on the optimal sequential search strategy that is the essence of mainstream search theory, they are nonetheless supportive of the idea that the "law of one price" is quite capable of violation. (Burdett and Judd [**4**] give a good unified treatment of partial equilibrium price dispersion models based on non-sequential and "noisy" search.)

The current state of research on <u>search market equilibria</u>, i.e., equilibria in markets characterized by incomplete information and the absence of an external price-setting authority, can thus be broadly summarized as follows. In a single-market setting the equilibrium outcome of competition among firms will be either a degenerate equilibrium at the monopoly price or a price dispersion equilibrium. (See Hey [6], Chapter 25 for a good survey.)

In this paper we extend models of search market equilibrium to incorporate general equilibrium considerations. The motive for such an extension is of course to investigate whether the extremely anticompetitive (alternatively, pro-search theoretic) results of the existing literature are a partial equilibrium artifact. Simply stated, our results indicate that they are not.

2 THE GENERAL MODEL

We consider a simple general equilibrium economy with a product market and a labor market. There are u individuals and n firms in this economy. Both u and n are arbitrarily large, and $\mu \equiv u/n$ is also arbitrarily large.

Denote the distribution functions of prices and wages by F(p) and M(w), respectively. For convenience, we take F to be right-continuous and M to be left-continuous; i.e., $F(p) \equiv \Pr[\text{price} \leq p]$ and $M(w) \equiv \Pr[\text{wage} < w]$. Assume that individuals are following optimal search strategies (in a sense to be made precise below) given F and M. Then, conditional on F and M, each firm faces a product demand schedule q(p) and a labor supply schedule $\ell(w)$.

Assume each firm sets p and w to maximize expected profits. This maximization proceeds subject to the constraint that the offered wage elicits sufficient labor supply to produce the product demand induced by the offered price. Assume the simplest linear production function

$$q(p) = \ell(w). \tag{1}$$

Then the firm's decision problem is to choose p,w to maximize

$$\Pi(p,w) = pq(p) - w\ell(w) \tag{2}$$

subject to the production constraint (1). Assume that the profits earned by firms are paid out to individuals as dividends.

We want to characterize the Nash equilibria in this model. This means that we want to find distribution functions F and M such that:

(i) Each individual is following an optimal search strategy given F and M;

(ii) Each firm is setting (p,w) to maximize $\Pi(p,w)$ subject to the production constraint, where the optimal choice is taken conditional on F and M;

(iii) The outcome of firms' optimal choices of p and w generates the distribution functions F and M.

There are two possible types of equilibria to consider in this model:

(i) Degenerate equilibria, i.e., equilibria in which all firms charge the same price, p*, and offer the same wage, w*;

(ii) Dispersion equilibria in which both prices and wages follow non-degenerate distribution functions.

Individuals' search

We begin by characterizing optimal behavior for individuals. In each period there are u individuals in the economy. The individual's decision problem is to search in an optimal fashion for a suitably low price and high wage.

The individual is assumed to die with probability τ at the end of each period. This "constant death risk" assumption is a convenient means of combining the tractability of the "infinite horizon search model with discounting" with the introduction of a steady flow of new searchers into the economy.

The individual is assumed to decide whether or not to search based on the criterion of maximizing expected future lifetime consumption. Thus, if at the end of period t he faces the decision of whether or not to continue search, he chooses that alternative which maximizes the sum of expected consumptions over periods t+1, t+2,...

During each period of his existence the individual is endowed with a non-wage (dividend) income of Θ. Θ is assumed to be the same for all individuals and closes the economy (all profit is distributed to individuals as dividends).

At the beginning of an individual's existence he draws a "doubleton" price-wage offer, i.e., a price drawn at random from one firm and a wage drawn at random from another firm; and so long as he continues to search, he continues to draw a random price-wage offer at the end of each period. We assume that the individual's consumption during any period of search is $E(\theta/p)$, i.e., consumption out of dividends at the average price. The crucial point is that while engaged in search the individual consumes only out of non-wage income.

Suppose the individual has drawn a price-wage offer of (p,w). If he accepts (p,w), then he goes to work at the wage w in the next period and consumes $(w+\theta)/p$ per period so long as he continues to survive. Having accepted (p,w), the probability of surviving one period is $1-\tau$, of surviving two periods is $(1-\tau)^2$, etc., so the expected future lifetime consumption from an accepted (p,w) offer is $\frac{1-\tau}{\tau} \cdot \frac{w+\theta}{p}$.

Let V denote the expected lifetime consumption for a new entrant to the economy, given that he searches optimally. The value of accepting an offer (p,w) is $E(\theta/p) + \frac{1-\tau}{\tau} \cdot \frac{w+\theta}{p}$, whereas the value of rejecting it is $E(\theta/p)+(1-\tau)V$. Hence the optimal sequential strategy for an individual is to accept (p,w) iff

$$\frac{w+\theta}{p} \geq \tau V \equiv k. \tag{3}$$

That is, optimal sequential search behavior is characterized by the reservation rule (3) with reservation real income k. Now, V can be computed as

$$V = \theta E\left(\frac{1}{p}\right) + \frac{1-\tau}{\tau} E\left(\frac{w+\theta}{p} \,\middle|\, \frac{w+\theta}{p} \geq k\right) \cdot \Pr\left(\frac{w+\theta}{p} \geq k\right) +$$

$$(1-\tau)V \cdot \Pr\left(\frac{w+\theta}{p} < k\right).$$

Expressing this in terms of $F(p)$ and $M(w)$ and substituting $k/\tau = V$, gives the following equation for k

$$\frac{k}{\tau} = \theta \int_{-\infty}^{\infty} \frac{1}{p}\, dF(p) + \frac{1-\tau}{\tau} \int_{-\infty}^{\infty} \left(\int_{kp-\theta}^{\infty} \frac{w+\theta}{p}\, dM(w) \right) dF(p) +$$

$$\frac{1-\tau}{\tau}\, k \int_{-\infty}^{\infty} \left(\int_{-\infty}^{kp-\theta} dM(w) \right) dF(p). \tag{4}$$

(Note: Recall that F is defined to be right-continuous while M is defined to be left-continuous.)

Unemployment

Recall that each individual faces a constant death risk of τ. Therefore in a steady state τu individuals will enter and exit the system each period.

Let h denote the probability that a randomly drawn (p,w) offer will be acceptable, i.e.,

$$h \equiv \Pr\left(\frac{w+\theta}{p} \geq k\right) = \int_{-\infty}^{\infty} \left(\int_{kp-\theta}^{\infty} dM(w) \right) dF(p). \tag{5}$$

Then the number of searching individuals in the economy in any period t may be computed as follows. There are τu individuals entering the system at time t. There are $\tau u(1-\tau)(1-h)$ who entered at t-1 and neither died nor found their initial offer acceptable. There are $\tau u(1-\tau)^2(1-h)^2$ who entered at t-2 and who neither died nor found either of their first two offers acceptable, etc, etc. Thus, the fraction of individuals searching in any period, i.e., the unemployment rate, is

$$s \equiv \frac{\tau}{1-(1-\tau)(1-h)} = \frac{\tau}{\tau+(1-\tau)h} \quad . \tag{6}$$

Note there is a simple relationship that must hold between s and k, namely,

$$k = (1-s). \tag{7}$$

This follows as an accounting identity since (1-s) equals average production per individual, while k is average consumption. Equation (7) can be verified more formally once we have derived q(p), the demand schedule.

Product demand and labor supply

In any period there are μs searchers per firm in the economy, and the allocation of searchers across firms is random. Hence, if a firm charges a price p, it can expect a <u>demand of $\mu s\theta/p$ from searchers</u>. Among the μs searchers contacting a firm charging a price p in any given period, a fraction $1-M(kp-\Theta)$ will terminate search and accept p and the wage offer they simultaneously receive. In period t the firm will have $(1-\tau)\mu s(1-M(kp-\Theta))$ employed customers who terminated search at the end of period t-1, $(1-\tau)^2\mu s(1-M(kp-\Theta))$ who terminated search at the end of period t-2, etc. That is, a firm charging a price p will have $(1-\tau)/\tau\ \mu s\ (1-M(kp-\Theta))$ employed customers per period.

The expected demand from each of these is

$$\frac{1}{p}\int_{kp-\Theta}^{\infty} (w+\Theta)dM(w)/(1-M(kp-\Theta)).$$

Thus, the expected demand from employed consumers for a firm charging p is

$$\frac{(1-\tau)\mu s}{\tau p} \int_{kp-\Theta}^{\infty} (w+\Theta)dM(w).$$

Adding together the expected demands from searchers and employed customers gives the firm's expected demand schedule:

$$q(p) = \frac{\mu s}{p} \left(\Theta + \frac{1-\tau}{\tau} \int_{kp-\Theta}^{\infty} (w+\Theta)dM(w)\right)^{1}. \tag{8}$$

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[1] To verify (7), use

$$(1-s) \equiv \frac{1}{\mu} \int_{-\infty}^{\infty} q(p)dF(p)$$

$$= s\theta \int_{-\infty}^{\infty} \frac{1}{p}\, dF(p) + s\, \frac{1-\tau}{\tau} \int_{-\infty}^{\infty} \left(\int_{kp-\theta}^{\infty} \frac{w+\theta}{p}\, dM(w)\right) dF(p)$$

Using equation (4), we then have

$$(1-s) = \frac{sk}{\tau} - \frac{1-\tau}{\tau}\, sk \int_{-\infty}^{\infty} \left(\int_{-\infty}^{kp-\theta} dM(w)\right) dF(p)$$

$$= sk\left(\frac{1}{\tau} - \frac{1-\tau}{\tau}\,(1-h)\right) = k. \qquad \text{QED.}$$

Likewise, among the μs searchers contacting a firm offering a wage w in any given period, a fraction $F((w+\theta)/k)$ will terminate search and become employees. In period t the firm will have $(1-\tau)\mu sF((w+\theta)/k)$ employees who terminated search at the end of period t-1, $(1-\tau)^2\mu sF((w+\theta)/k)$ who terminated search at the end of period t-2, etc. Each of these employees provides one unit of labor per period. Thus, the <u>firm's expected labor supply schedule</u> is:

$$\ell(w) = \frac{1-\tau}{\tau}\,\mu sF\left(\frac{w+\theta}{k}\right).\ ^{2} \tag{9}$$

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[2] Equation (9) can be used as the basis for an alternative derivation of (6). The employment rate is given by

$$1-s = \frac{1}{\mu}\int_{-\infty}^{\infty}\ell(w)dM(w) = s\,\frac{1-\tau}{\tau}\int_{-\infty}^{\infty}F\left(\frac{w+\theta}{k}\right)dM(w)$$

$$= s\,\frac{1-\tau}{\tau}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\frac{w+\theta}{k}}dF(p)\right)dM(w) = s\,\frac{1-\tau}{\tau}\int_{-\infty}^{\infty}\left(\int_{kp-\theta}^{\infty}dM(w)\right)dF(p)$$

$$= s\,\frac{1-\tau}{\tau}\,h, \text{ whence (6) follows.}$$

3 DEGENERATE EQUILIBRIUM

We begin by considering the possibility of a degenerate equilibrium, i.e., an equilibrium in which all firms are charging a common price p* and offering a common wage w*. Such a combination constitutes an equilibrium if

(i) no firm can increase its profits by deviating from (p*,w*)

(ii) individuals follow the optimal sequential search strategy

(iii) the production constraint is satisfied.

If all firms offer (p*,w*), then the only searchers in the market are new entrants, so that $s = \tau$. Individuals entering the economy will consume θ/p^* in their entry period and $(w^*+\theta)/p^*$ per period thereafter; hence

$$k = \tau\left(\frac{\theta}{p^*} + \frac{(1-\tau)}{\tau}\left(\frac{w^*+\theta}{p^*}\right)\right) = \frac{\theta}{p^*} + (1-\tau)\frac{w^*}{p^*} \tag{10}$$

Now consider the consequences of a single firm's deviation from (p*,w*). An individual will accept a price p in conjunction with the common wage w* so long as $\frac{w^*+\theta}{p} \geq k$, i.e., $p \leq p^* \cdot \frac{w^*+\theta}{(1-\tau)w^*+\theta}$. Hence, the individual firm faces the demand schedule

$$q(p) = \begin{cases} \frac{\mu}{p}\left(\theta + (1-\tau)w^*\right); & p \leq p^* \cdot \frac{w^*+\theta}{(1-\tau)w^*+\theta} \\ \frac{\mu\tau\theta}{p} & p > p^* \cdot \frac{w^*+\theta}{(1-\tau)w^*+\theta} \end{cases} \tag{11}$$

Similarly, an individual will accept a wage w in conjunction with the common price p* so long as $(w+\theta)/p^* \geq (\theta/p^*)+(1-\tau)w^*/p^*$., i.e. $w \geq (1-\tau)w^*$. Thus

$$\ell(w) = \begin{cases} 0 & w < (1-\tau)w^* \\ \mu(1-\tau) & w \geq (1-\tau)w^* \end{cases} \tag{12}$$

and it is clear that, so long as $w^* > 0$, the combination (p^*,w^*) cannot constitute an equilibrium, since each firm has both the incentive and the possibility to offer a wage of slightly less than w^*.

However, suppose we normalize the unit of account for prices, wages and dividends by setting $\theta = 1$. Then there exists a degenerate equilibrium with $w^* = 0$ and $p^* = 1/(1-\tau)$. From equations (11) and (12), the indicated value for p* implies satisfaction of the production constraint. Equation (10) trivially indicates the optimality of accepting (p^*,w^*) for individuals. Optimality for firms follows from the discontinuities in q(p) precisely at $p^* = 1/(1-\tau)$ and in $\ell(w)$ precisely at $w^* = 0$.

Note that this degenerate equilibrium is very similar to the degenerate equilibrium at the monopoly price derived in a partial equilibrium setting. To see the analogy, imagine a single firm controlling both the labor market and the product market. Acting as a monopsonist on the labor market this firm would exploit the zero elasticity of labor supply (by employed workers) to drive the wage as low as possible, i.e., to $w^* = 0$. Once $w^* = 0$ and a normalization is chosen, a single price is determined by the economy-wide production constraint. Thus, the tendency towards the single-price "monopoly" outcome suggested by partial equilibrium analyses of search markets carries over to general equilibrium. It is interesting to note, however, that in a general equilibrium setting the "monopoly/monopsony" outcome is Pareto efficient. Any degenerate outcome avoids the wastage of resources on socially unproductive search, and the "monopoly/monopsony" equilibrium is the only self-sustaining degenerate outcome.

4 DISPERSION EQUILIBRIUM

We next consider non-degenerate equilibria. In this section we establish existence and investigate the properties of the simplest type of non-degenerate equilibrium, namely, a two-point joint distribution of prices and wages.

Consider a two-point wage offer distribution in which a lower wage w_0 is offered with probability γ_0 and a higher wage w_1 is offered with probability $\gamma_1 = 1-\gamma_0$. In equilibrium $\ell(w)$ cannot be constant in neighborhoods of either w_0 or w_1; otherwise firms could reduce wages without any loss in labor supply. Using equation (9) it follows that F must be increasing at $p_0 \equiv (w_0+\theta)/k$ and at $p_1 \equiv (w_1+\theta)/k$; that is in equilibrium the prices p_0 and p_1 are necessarily offered. There is a simple economic intuition behind this particular price- and wage-offer combination. If a searcher is unlucky and draws the low wage, w_0, then he will reject that wage unless it is offset by a sufficiently low price. But from the definition of k, p_0 is the highest such price; i.e., p_0 is the "reservation price" for those who draw the lower wage. Likewise, w_1 is the "reservation wage" for those unlucky enough to draw the higher price, p_1.

Demand and labor supply

If p_0 and p_1 are the only prices offered, then $q(p)$ and $\ell(w)$ are particularly simple functions. Applying equations (8) and (9),

$$q(p) = \begin{cases} \frac{\mu s}{p} \left\{\theta + \frac{1-\tau}{\tau}\left(\gamma_0(w_0+\theta)+\gamma_1(w_1+\theta)\right)\right\} & p \leq p_0 \\ \frac{\mu s}{p} \left(\theta + \frac{1-\tau}{\tau}\gamma_1(w_1+\theta)\right) & p_0 < p \leq p_1 \\ \frac{\mu s\theta}{p} & p_1 < p \end{cases} \qquad (13)$$

$$\ell(w) = \begin{cases} 0 & w < w_0 \\ \mu s \frac{1-\tau}{\tau}\gamma_1 & w_0 \leq w < w_1 \\ \mu s \frac{1-\tau}{\tau} & w_1 \leq w \end{cases} \qquad (14)$$

That is, $q(p)$ exhibits piecewise unitary elasticity and $\ell(w)$ is piecewise constant. The piecewise unitary elasticity of $q(p)$ implies that it must be that firms offering w_0 charge p_1 whereas firms offering w_1 charge p_0. Thus the candidate for equilibrium to consider is one in which the pair

(p_1,w_0) is offered with probability γ_0 and the pair (p_0,w_1) is offered with probability $\gamma_1=1-\gamma_0$.

Such a joint distribution is an equilibrium distribution if

(i) the offers (p_1,w_0) and (p_0,w_1) are profit-maximizing given the joint distribution of offers,

(ii) individuals follow an optimal sequential search strategy, i.e., k is optimal, conditional on the joint distribution of offers, and

(iii) the production constraint is satisfied at both (p_1,w_0) and (p_0,w_1).

Firms' behavior

We begin by examining the implications of profit-maximization. Since all firms are identical a necessary condition for profit-maximization is that the profits generated by the two pairs of offers be equal; that is, $p_1q(p_1) - w_0\ell(w_0) = p_0q(p_0) - w_1\ell(w_1)$. Applying (13) and (14) yields an equal-profit condition that must hold in equilibrium, viz.

$$w_1 - w_0 = \gamma_0\theta. \tag{15}$$

It is important to recognize that (15) is not only necessary but also sufficient for firms' optimizing behavior. To establish sufficiency consider the implications of a deviation from the 2-point distribution. The piecewise constancy of $\ell(w)$ and the piecewise unitary elasticity of $q(p)$ imply that any such deviation will result either in decreased profits (if p or w are increased) or in a violation of the production constraint (if p or w are decreased).

As an aside, it is interesting to note that (15) implies an equilibrium profit level of $\mu\theta$. That profits be equal to $\mu\theta$ is of course implied by the consistency condition that profits per firm equal dividend payments per firm. The argument uses

$$s = \tau/[1-(1-\tau)\gamma_0^2] \tag{16}$$

which follows as a special case of (6). The profits generated by, e.g., (p_1, w_0) are then

$$\begin{aligned} p_1 q(p_1) - w_0 \ell(w_0) &= \mu s\left(\theta + \frac{1-\tau}{\tau}\gamma_1(w_1+\theta) - \frac{1-\tau}{\tau} w_0\gamma_1\right) \\ &= \frac{\mu s}{\tau}\left(\tau\theta + (1-\tau)\gamma_1\left[\theta + (w_1 - w_0)\right]\right) \\ &= \frac{\mu s}{\tau}\left(\tau\theta + (1-\tau)(1-\gamma_0)(1+\gamma_0)\theta\right) \\ &= \frac{\mu s\theta}{\tau}\left(1-(1-\tau)\gamma_0^2\right) = \mu\theta \qquad \text{QED.} \end{aligned}$$

<u>Individuals' behavior</u>

Next we examine the implications of optimal sequential search behavior. Since individuals sample p and w from different firms there are four possible outcomes:

Income	Probability	
$(w_0+\theta)/p_1$	γ_0^2	not acceptable
$(w_0+\theta)/p_0$	$\gamma_0\gamma_1$	acceptable (=k)
$(w_1+\theta)/p_1$	$\gamma_0\gamma_1$	acceptable (=k)
$(w_1+\theta)/p_0$	γ_1^2	acceptable (>k)

Note once again the "reservation price" property of p_0 and the "reservation wage" property of w_1. Applying equation (4) to the 2-point distribution then gives

$$\frac{k}{\tau} = \theta\left(\frac{\gamma_0}{p_1} + \frac{\gamma_1}{p_0}\right) + \frac{1-\tau}{\tau}\,\gamma_1^2\,\frac{w_1+\theta}{p_0} + \frac{1-\tau}{\tau}(1-\gamma_1^2)k. \tag{17}$$

Production constraints

The final step is to apply the production constraints. These are $q(p_0) = \ell(w_1)$ and $q(p_1) = \ell(w_0)$. This implies in particular that

$q(p_1)/\ell(w_0) = q(p_0)/\ell(w_1)$, i.e.,

$$\frac{\frac{\tau}{1-\tau}\,\theta + \gamma_1(w_1+\theta)}{p_1\gamma_1} = \frac{\frac{\tau}{1-\tau}\,\theta + \gamma_0(w_0+\theta)+\gamma_1(w_1+\theta)}{p_0}\,.$$

After a bit of arithmetic this reduces to (recall that $p_i = (w_i+\theta)/k$)

$$w_1\,\frac{\tau}{1-\tau} = \gamma_1^2(w_1+\theta)\,. \tag{18}$$

One virtue of this formulation is that it allows a tremendous simplification of the equation characterizing k. Using $\gamma_1^2 = \frac{\tau}{1-\tau}\frac{w_1}{(w_1+\theta)}$, equation (17) becomes

$$\frac{k}{\tau} = \theta\left(\frac{\gamma_0}{p_1} + \frac{\gamma_1}{p_0}\right) + \frac{w_1}{p_0} + \frac{1-\tau}{\tau}k - \frac{w_1}{w_1+\theta}k, \text{ or}$$

$$0 = \frac{\theta\gamma_0 - w_1}{p_1} + \frac{\theta\gamma_1 + w_1}{p_0} - k.$$

Using $w_1 = w_0 + \gamma_0\theta$ gives

$$0 = \frac{-w_0}{p_1} + \frac{\theta + w_0}{p_0} - k, \quad \text{or}$$

$$w_0 = 0.^3 \qquad (19)$$

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[3] If one assumes decreasing returns to scale, exactly the same argument establishes $w_1(\tau/(1-\tau)) > \gamma_1^2(w_1+\theta)$ and $w_0 < 0$; whereas if increasing returns to scale are assumed, $w_1(\tau/(1-\tau)) < \gamma_1^2(w_1+\theta)$ and $w_0 > 0$. The acceptance of a wage $w_0 < 0$ need not be inconsistent with optimal search by individuals. Individuals accept w_0 when drawn together with p_0 in order to secure "permanent access" to the lower price. However, we find it interesting to note that this example and others we have worked with in models of this genre seem to suggest the relative plausibility of nonconvex production technologies, at least in the sense of generating more "plausible" model outcomes.

Existence

We are now prepared for the existence proof. We seek values for the variables γ_0, γ_1, w_0, w_1, p_0, p_1, θ and k such that $\gamma_0 + \gamma_1 = 1$ $(0<\gamma_1<1)$, $p_i = (w_i+\theta)/k$, i=1,2 and such that the four equilibrium conditions are satisfied, viz.,

(i) the two production constraints,
$q(p_0) = \ell(w_1)$ and
$q(p_1) = \ell(w_0)$

(ii) the condition (15) for firms' optimization,
$w_1 - w_0 = \gamma_0\theta$;

(iii) the condition (17) for individuals' optimization.

In addition, some unit of account (a numeraire) must be chosen, and for convenience we again set $\theta=1$. This leaves us with 8 variables and 8 equations, which we now proceed to prove have a unique solution, given a value for τ.

From eq.(18) we have

$$w_1 \frac{\tau}{1-\tau} = \gamma_1^2(w_1+\theta),$$

which together with firms optimization $\left(w_1 - w_0 = (1-\gamma_1)\theta\right)$ implies

$$\gamma_1^3 - 2\gamma_1^2 + (1-\gamma_1)\frac{\tau}{1-\tau} = 0 \qquad (20)$$

For $\gamma_1=0$, the LHS of (20) is positive, and for $\gamma_1=1$ the LHS is negative. Further, the LHS of (20) is strictly decreasing in γ_1 for $0 \leq \gamma_1 \leq 1$; hence there exists a unique solution for γ_1 in the range $0<\gamma<1$.

We define γ_1 to have this value, and also define consecutively
$\theta = 1,\ \gamma_0 = 1-\gamma_1,\ w_0 = 0,\ w_1 = w_0 + \gamma_0 \theta$.

We now define p_1 by,

$$p_1 = w_1 + \theta + \frac{\tau\theta}{\gamma_1(1-\tau)},$$

which in view of (13) and (14) implies that

$$q(p_1) = \ell(w_0). \tag{21}$$

Finally, we define k and p_0 by;

$$k = \frac{w_1 + \theta}{p_1} \quad \text{and} \quad p_0 = \frac{w_0 + \theta}{k}.$$

We now establish that this does indeed yield a solution to our system. The only equations that are not obviously satisfied are condition (iii) and the first condition of (i). However, (20) together with (ii) and $w_0 = 0$ implies (18), and this in turn has been shown to be equivalent to

$$q(p_1)/\ell(w_0) = q(p_0)/\ell(w_1).$$

This, together with (21), establishes condition (i). It remains to show (iii) But we have seen that (17), given the production constraints, firms optimization and the relations $p_i = (w_i+\theta)/k$ reduces to (19), i.e. $w_0 = 0$. This settles condition (iii).

Thus we have shown that there exists a 2-point Nash equilibrium in which some firms offer a low wage (equal to zero) and charge a high price while all other firms offer a higher wage and charge a lower price.

This proposition asserts the existence of an equilibrium in which identical firms are indifferent between a high margin/low volume operation on the one hand and a low margin/high volume operation on the other hand. The mechanism that allows the high margin firms, i.e., firms offering the high price and the low wage, to attract any employees and customers is a coupling of the labor and product markets in the search process. Thus, for example, individuals who have drawn a high price will be willing to accept that price if acceptance ensures "permanent access" to a high wage. Existence has been proven for constant returns to scale only, but it is not difficult to construct numerical examples of equilibria using decreasing or increasing returns to scale production functions. Nor is it difficult to construct a general existence proof for such production functions, providing we do not allow "too much" deviation from the constant returns to scale case.

It is easy to imagine the existence of other dispersion equilibria, as well. The most obvious extension would be to an N-point dispersion equilibrium, i.e., an equilibrium in which N wages, $w_1 < \ldots < w_N$, are offered together with the corresponding prices, $p_N \equiv \frac{w_N+\theta}{k} > \ldots > p_1 \equiv \frac{w_1+\theta}{k}$. Another intriguing possibility to consider is a continuous dispersion equilibrium, i.e., an equilibrium in which the supports of F and M are intervals, say $[p_0, p_1]$ and $[w_0, w_1]$. We have been unable to prove the existence of a continuous dispersion equilibrium; however, if such an equilibrium exists, then we can demonstrate that it has two very interesting properties. These are, again conditional on existence, (i) the supports of F and M must be bounded and (ii) F must have a mass point at p_0, the lowest price offered, while M must have a corresponding mass point at w_1, the highest wage offered.

The argument that demonstrates that the supports of F and M must be bounded is as follows. First, the existence of a minimum price, p_0, and a maximum wage, w_1, follows from the requirement that profits be non-negative in equilibrium. Second, for any joint distribution of prices and wages there is a minimum wage, $w_0=kp_0-\theta$, below which no labor supply is elicited. Hence there is a lower bound on the wage offer distribution. Likewise, there is a maximum price, $p_1 = \frac{w_1+\theta}{k}$, above which no firm can attract "permanent" customers. The revenues of a firm offering $p>p_1$ are $\mu s\theta$ and costs are non-negative. But in equilibrium profits must be given by $\mu\theta$; hence p_1 must be an upper bound on the price offer distribution. The above argument also gives the essence of the proof that F and M must have mass points at p_0 and w_1, respectively. In equilibrium profits, $(p-w)q(p)$, equal $\mu\theta$ for all p in the support of F, $[p_0,p_1]$, i.e., $q(p) \geq \mu\theta/p$ for these values of p, whereas $q(p) = \mu s\theta/p$ for all $p>p_1$. Hence, q(p) must be discontinuous at p_1. But, from (8), this implies a corresponding discontinuity in M at $w_1=kp_1-\theta$; i.e., M must have a mass point at w_1 and F a corresponding mass point at p_0. QED.

5 CONCLUSIONS

In this paper we have constructed a general equilibrium model of an economy in which (i) prices and wages are actively set by firms and (ii) imperfectly informed individuals follow an optimal sequential strategy in searching for a suitably low price and high wage. We have demonstrated the existence of both degenerate ("monopoly/monopsony") and dispersion equilibria and in the process have confirmed a basic insight suggested by partial equilibrium models of search; namely, that in the absence of costless information the conclusions of competitive analysis depend crucially on the fiction of an external price-setting authority ("the auctioneer").

On the other hand, our results contradict a second basic "insight" suggested by the partial equilibrium approach. Partial equilibrium analysis seems to indicate that dispersion equilibria and sequential search are difficult bed-fellows at best, that one needs to resort to non-sequential or "noisy" search in order to attain a dispersion equilibrium with a minimum of cumbersome assumptions. This turns out to be not at all the case in general equilibrium. The reason is that in general equilibrium the cost of search, i.e., the foregone wage, is endogenously determined. Each firm has the incentive to set its wage offer such that the net cost of search associated with that wage offer and one of the prices that is offered is zero.

Note further that in general equilibrium there is no need for heterogeneity among individuals and/or firms to generate dispersion. This again contradicts the flavor of results suggested by partial equilibrium models of sequential search. In our model differences in "reservation prices" across individuals needed to sustain price dispersion are generated by differences in outcomes of labor market search; likewise differences in "reservation wages" are generated by differences in outcomes of product market search. The linkage between markets that is the essence of general equilibrium analysis can thus create sustainable dispersion "out of thin air".

Our basic objective in this paper was to study existence questions; however, it turns out that the model we have constructed can be easily modified to study more concrete issues. Thus, in Albrecht and Axell [**1**] we have used the basic structure of this model to examine the effects of unemployment compensation when the endogeneity of the wage offer distribution is taken into account. Considering the extent to which "partial- partial" search analysis has been employed in labor and macroeconomics, the list of further potential applications would seem to be large indeed.

APPENDIX

Notation

The general model

Θ : Dividends

k : Optimal reservation real income $\frac{w+\Theta}{p}$.

τ : The death risk (constant)

u : Number of individuals

n : Number of firms

μ : u/n = individuals per firm

s : Unemployment rate

$\ell(w)$: The supply of labor a firm faces offering the wage w

$q(p)$: The demand for products a firm faces charging the price p

$F(p)$: Distribution function of prices

$M(w)$: Distribution function of wages

The degenerated case

w^* : The single wage

p^* : The single price

The two-point case

w_0 : The low wage

w_1 : The high wage

p_0 : The low price

p_1 : The high price

γ_0 : Frequency of low wage - high price firms

γ_1 : Frequence of high wage - low price firms

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