Taxes and Market Stability

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Much has been said and written by now about the possible stabilizing effects of public budgets on the effective demand in the total economy. On the following pages we are concerned with a hitherto seldom discussed topic, namely the possible destabilizing effects of taxes and subsidies in individual markets. Particular examples of these possibilities, for example in the labor markets and in the markets for housing, have lately aroused a good deal of public discussion in Sweden, whose world leadership when it comes to taxing ambitions, especially marked in the seventies, makes some of these problems particularly acute. Unfortunately we still lack a well-established analytical framework for dealing with these kinds of stability problems. The modest aim of the following discussion is merely, to point out some dimensions of the problem and to provide some illustrative examples of possible tax-induced instability.

The Changing Role of Taxation

Over the last half-century "taxation"--which in the following I take to include also negative taxes or subsidies--has not only been steeply increased in most market-economies, but has at the same time also changed character. Taxation once used to be dominated by the fiscal aim of financing the provision of certain basic collective goods, mainly the machinery of control--central administration, defense, justice, etc. The means, then, could be kept relatively few and simple--a low income tax with at most a mild form of progression and/or local estate rates. This, as it happens, is still the picture of the public sector often presented in economic equilibrium theory--the provision of collective goods being financed if not by lump-sum taxes then by some proportionate taxation on final goods. There is, then, no need to worry about taxes destabilizing individual markets. Apart from the problems of international adjustment, in a model economy without monetary markets proportionate price increases need not change the stability properties of individual product markets.

The aims and means of taxation today are very different. A drastic illustration of this is provided by Sweden, where the structural change in taxation has probably gone further and faster during postwar years than in any other industrialized market economy.

The provision of collective goods in the narrow definition of the word presented above, plays a steadily decreasing role in the public budgets and is now responsible for less than 15 per cent of total central government expenditure. Apart from social insurance the dominant expenditure items on the public budgets are, now, subsidies of social and private goods. In the national accounts these are classified either as public consumption or transfers depending on how production and distribution are organized.

The ways of financing public expenditure are also becoming more varied and complex. Although taxes on income and wealth, have been sharply increased and made more progressive in the early seventies, they now provide, in Sweden, less than half of central government income and are, to an increasing extent, being complemented by various forms of indirect taxation, including, V.A.T., obligatory social insurance fees and taxes on non-labor factors.

Today's public budgets, therefore, can be best characterized as huge instruments for central price and income regulation. By combining positive and negative taxation with various forms of tax rebates and subsidy rules a highly individualized and differentiated form of taxation can in principle be realized--given the necessary information. With the high general level of taxation--more than 2/3 of private disposable income being channeled through public budgets--the tax effects on individual markets are, in any case, becoming increasingly decisive for price-setting and profitability also in the private production sectors.

The differentiation of means are correlated to--and indeed to a large extent motivatived by--a differentiation of the aims of taxation. The central government's wish to fulfill increasingly differentiated aims concerning industrial and regional policy and income redistribution without undue centralization of market decisions, have put a great strain on the system. In the last few years the shrinking possibility for redistribution in Sweden by way of progressive income taxes has led to an increased use of differentiated price subsidies as a means of redistribution.

There are doubts as to whether we have - or will ever have - sufficiently precise tax instruments, and enough information on how to use them, to match the regulatory ambition of the government. Most tax instruments are still rather blunt in the sense that considerations of fairness and administrative simplicity force us into making tax rules so general that they usually hit rather widely or wildly compared with the aims of tax policy. The complex pattern of taxation and the decentralized handling of various policy areas also make it increasingly more difficult to discern or guess the combined impact of the various horizontal chains of taxation on individual markets and goods.

This raises several important questions concerning efficiency limits to economic control by way of taxation. The one we are going to deal with here is the problem of possible tax-induced market instability. What happens to "normal" price adjustment mechanisms when these are not only transformed by prevailing tax rates but also intercepted by a simultaneous process of tax adjustment with a quite different purpose? How do the "tax links" between different markets affect the stability of interrelated markets? What are the chances of attempted tax adjustments ever converging on the intended allocative or distributional targets?

Market Stability from An Equilibrium Point of View

In looking for an analytical framework for studying tax-induced market instability you are faced with two main alternatives. You can plunge directly into a disequilibrium scenario, which means paying the price of not being able to generalize and of not necessarily ever being in the neighborhood of equilibrium.

The other and more traditional way of studying stability problems is by looking at them from the point of view of an equilibrium position. The question will then roughly be the following: given that the agents behave as if they were constantly in an equilibrium and that the adjustment process follows some simple prescribed rules, what are the conditions for convergence? The results you attain this way are mostly of a rather formal and general nature, but may still provide some leads as to how to structure our approach to the problem of tax-induced instability.

The usual stability analysis aims at determining sufficient conditions under which a system of market price adjustments, each being a monotonic function of excess demand, will converge.[1] The results of these studies are by now well known (cf., for example, Karlin (1959), Lancaster (1968), Arrow-Hahn (1971)). To make sure of convergence three types of conditions are usually needed. One type of condition guarantees that the agents are willing to accept disequilibrium prices as if they stemmed from a final equilibrium (cf., "Walras' law"). A second type of condition--for discrete-time adjustments--is needed to ensure that the rate and/or stepsize of adjustment is not so big that you over-shoot the equilibrium target by too much.

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[1] See Appendix, note I, p. 225.

Finally you need some condition concerning the links between the adjustment in different markets to make sure that solving excess demand problems in one area does not inflate the same problems in other markets by too much.

This last condition can take many technical forms --"gross substitution", "aggregate revealed preference", "diagonal dominance", etc.-- all of which, unfortunately, appear rather restrictive and difficult to make intuitively plausible.

These conditions are suggestive when transplanted to our special problem of tax-induced instability. When agents become conscious of prices being to a large extent determined in government offices, they may be less willing to accept them as given data to which they passively adjust. The varying "tax multiplier" on price in different markets could increase the risk for excessive, destabilizing adjustment steps in some markets.

Taxes and tax adjustments tend to provide direct links between adjustments in different markets. The risk would consequently increase that an adjustment in one market might counteract overall stability by disrupting other markets.

There are other limitations of existing economic stability analysis apart from the restrictive conditions used. It tells us, in fact, little or nothing about those stability properties of the economic system that we are often most interested in when dealing with real-life economies.[1] One

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[1] See Appendix, notes II and III, pp. 226 ff.

such property, for example, is stability in the sense that prices (and volumes) originating from a point within a region will never move outside given boundaries. Another question has to do with the possibility of prices converging to an equilibrium "close" to the original one, after a shift in some coefficient. In as far as taxes tend to change even the behavioral structure of an economic system these stability questions are very pertinent and will be raised again later on in connection with some of the illustrative examples quoted.

The problem with which we are concerned here--simultaneous price and tax adjustment in individual markets--can obviously be treated as an extension of the traditional market stability problem. The stability problem of decentralized policy, without involving simultaneous price adjustment, has been discussed by inter alia Mundell (1962) and Cooper (1967). They were concerned with the risks of instability with a decentralized policy arising from the inability of individual authorities to foresee and take into account the effects of policy instruments on markets or areas outside their own field of responsibility. The question of what happens if you combine the two problems--superimposing a tax adjustment on a market price adjustment--has, however, not been treated in economic literature, as far as we know. We hope the examples presented below will suffice to show that further work in this direction could be worthwhile and relevant to economic policy.

Tax-induced Instability in A Single Market

Let us start by looking at a general and very simple case --price- and tax-adjustment in continuous time in a single market. The "tax coefficient", T, is supposed to be defined in terms of the producer price, P. The product, TP, gives the demand price. The producer price is supposed to adjust in a simple way, changing in proportion to excess demand, while the tax rate is adjusted proportionate to some other function of market conditions. A straightforward tax target--relatively innocuous from a stability point of view--would be the volume of demand. The aim of the tax authorities could then simply be to make demand, d, adjust to a pre-set value d*. The purpose of such a tax target could be, for example, to limit the effect of environmental damage or some other collective externality or to keep down consumtion of some noxious commodity. Denoting the supply function by s(P) we would then have the following system:

$$P = \alpha E = \alpha\left(d(TP) - s(P)\right) \qquad (1)$$

$$P,\ T,\ \alpha,\ \lambda > 0$$

$$\dot{T} = \lambda G = \lambda\left(d(TP) - d^*\right) \qquad (2)$$

If we assume stability in the Liapunov sense, local asymptotic stability or resilience[1] is a necessary condition for global stability. With this assumption we can discover possibilities of

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[1] See Appendix, note I, p.225.

global instability by simply looking at local properties.[1]

If we assume E and G to be continuous functions and P*,T* to be an equilibrium point, we can use a linear approximation around this equilibrium;

$$E = E_p^* p + E_\tau^* \tau \qquad (3)$$

$$G = G_p^* p + G_\tau^* \tau \qquad (4)$$

where E_p^*, G_p^*, E_τ^* and G_τ^* denote the first partial derivatives of E and G with respect to P and T at the equilibrium, and p, τ stand for (P-P*) and (T-T*).

The linear adjustment system can then be written in vector form as:

$$(\dot{p}, \dot{\tau}) = A(p, \tau)^2 \qquad (5)$$

where A is the matrix

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[1] It should perhaps be emphasized that what we are, then, conditionally proving is only that the system will not tend to work back all the way to the equilibrium. To prove unconditionally that the system is unstable in the sense of Liapunov, that it will eventually tend to cross any preset boundary, would require, for example, the use of one of Liapunov's own instability theorems and would in the discussed examples be a difficult --and often impossible-- task.

[2] A tax adjustment similar from a stability point of view is implied by any progressive taxation of the supply price. This can be seen, for example, by writing the progressive rate as $T = \lambda P$ which gives $\dot{\tau} = \lambda \dot{p}$.

$$A = \begin{matrix} \alpha E_p^* & \alpha E_\tau^* \\ \lambda G_p^* & \lambda G_\tau^* \end{matrix} \qquad (6)$$

It may facilitate the understanding of the adjustment process if we rewrite (5) in terms of the slope o0 the demand curve, $\partial d^*/\partial(TP)$, and supply curve, $\partial s^*/\partial P$, respectively:

$$\dot{p} = \alpha(pT^*+\tau P^*)(\frac{\partial d}{\partial(TP)})^* - \alpha p(\frac{\partial s}{\partial P})^* = \alpha\Delta^*(d-s) \qquad (7)$$

$$\dot{\tau} = \lambda(pT^* + \tau P^*)(\frac{\partial d}{\partial(TP)})^* = \lambda\Delta^* d, \qquad (8)$$

where Δ is used to denote the differential. In comparison with a market situation without tax, two changes have occurred in the adjustment. The demand differential is now a function of two kinds of divergences instead of just one--in the producer's price and in the tax coefficient. Secondly, beside the price adjustment we now have the tax adjustment being proportionate to the change in demand as well.

The system (5) is a first order homogeneous linear vector differential equation. It will converge--showing local asymptotic stability--if and only if all roots of A have negative real parts.[1]

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[1] For a survey of the "mathematics of stability" cf. La Salle-Lefschetz (1961) and Murata (1977). See also Appendix, notes I-II, pp.225 ff.

The two roots, x_i, of A are:

$$x_i = \frac{\alpha E_p + \lambda G_\tau}{2} \pm \sqrt{\left(\frac{\alpha E_p + \lambda G_\tau}{2}\right)^2 - \alpha\lambda(E_p G_\tau - G_p E_\tau)} = \tag{9}$$

$$= \frac{\left[(\alpha T^* + \lambda P^*)\frac{\partial d}{\partial(TP)} - \alpha\frac{\partial s}{\partial P}\right]}{2} \pm$$

$$\pm\sqrt{\left[\frac{\left[(\alpha T^* + \lambda P^*)\frac{\partial d}{\partial(TP)} - \alpha\frac{\partial s}{\partial P}\right]}{2}\right]^2 - \alpha\lambda\left(-P^*\frac{\partial d}{\partial(TP)}\frac{\partial s}{\partial P}\right)} = \tag{10}$$

$$= a \pm\sqrt{a^2 - b}. \tag{11}$$

A closer inspection reveals that $a^2 > b$, i.e., the roots are real. No oscillatory price movements will occur owing to the fact that tax adjustment, as defined, follows and reinforces the price adjustment.

Given this, the convergence condition can be written as:

$$a^2 > b \rightarrow \left[\left(a \pm\sqrt{a^2 - b}\right) < 0 \equiv \begin{cases} a < 0 \\ b > 0 \end{cases}\right] \tag{12}$$

Written out in terms of the slopes of the demand and supply curves (12) acquires the following meaning:

$$\frac{\partial d}{\partial(TP)} < 0, \frac{\partial s}{\partial P} > 0 \tag{13}$$

This convergence condition should be compared with the condition for stability in the Walrasian sense in a market with only price adjustment:

$$\frac{\partial d}{\partial (TP)} < \frac{\partial s}{\partial P} \qquad (14)$$

In the "normal" case with a negatively sloping demand curve and a positive slope of the supply curve, we will have local stability both with and without tax adjustment. However, with supply price decreasing with scale, i.e. the supply curve having a negative slope,--and with the case of demand increasing with price--the risks of instability differ.

Without tax, the price will be instable only if the negative slope of the supply curve is less steep than that of the demand curve. This traditional condition for stability means that the convergent price change via the demand term should in absolute terms dominate an eventual counteracting supply term.

With the tax being determined as in (2), any negatively sloping supply curve will, however, make system (5) instable. This can be intuitively understood from the expressions (7) and (8). We see that divergences in demand price ($pT^* + \tau P^*$) determine the tax change, and also affect the change in the producer's price. The tax in other words, acting as a wedge between supply and demand prices, keeps the demand price from diverging too fast, which in turn makes it possible for the supply price to outrun the demand price.

Without taxes this cannot happen even when supply tends to decrease slightly with price. Suppose supplies are too big, with supply prices being too low. This in itself will tend to lower the price further. Demand, however, will act in the opposite, stabilizing direction. Being more price-sensitive, it will dominate. Introducing a tax wedge means that the demand price can be controlled by way of increased taxation allowing the supply price to slide further without being effectively checked by a demand expansion, etc. The tax has made both prices instable.

Other tax targets may, however, introduce new and potentially larger risks of instability. Local government price subsidies for utilities, housing, etc., in Sweden seem to aim at keeping the household expenditures for these "necessities" constant relative to household income. Let us assume prices to be expressed in some representative numeraire and neglect income changes. This tax target would then mean that current expenditure on the item in question has to be adjusted to some prescribed amount M. In a wider political interpretation this tax rule could be thought of as implying that political decision-makers allocate the subsidies to the big expenditure items so as to maximize appreciation and votes. With this interpretation the rule approximates subsidizing policies within a wide range of state and local areas, from adult education and recreational activities to fringe services on health and old-age care. Keeping the denotations as above, the adjustment system can be written as:

$$\dot{P} = \alpha E = \alpha(d-s) \qquad (15)$$

$$P,\ T,\ \alpha,\ \lambda > 0$$

$$\dot{T} = \lambda G = \lambda(M - PTd). \qquad (16)$$

Using the same reasoning as before, we find that the real parts of the corresponding matrix roots have to be negative for the adjustment system to converge.

The matrix roots are:

$$x_i = \frac{1}{2}\left[\alpha T^*\frac{\partial d}{\partial (TP)} - \alpha\frac{\partial s}{\delta P} - \lambda P^* d(1+e_P)\right] \pm \sqrt{\left(\frac{1}{2}\left[\alpha T^*\frac{\delta d}{\partial (TP)} - \alpha\frac{\partial s}{\delta P} - \lambda P^* d(1+e_P)\right]\right)^2 - \alpha\lambda P^* d(1+e_P)\,\frac{\partial s}{\delta P}} = \tag{17}$$

$$= a \pm \sqrt{a^2 - b}, \tag{18}$$

where e_P denotes the price elasticity of demand. As before, all derivatives are evaluated in equilibrium.

In this case, complex roots may appear giving rise to oscillatory price movements, which is what we would expect since tax and price adjustment in (15-16) tend to counteract each other.

We thus have the following two possibilities of convergence:

I. Dampened oscillation $\begin{cases} a < 0 \\ b > a_2 \end{cases}$ (19)

II. Straight convergence $\begin{cases} a < 0 \\ a_2 > b > 0 \end{cases}$ (20)

The common necessary conditions for convergence, $a < 0$, $b > 0$, can be derived directly from (17):

$$T^* \frac{\partial d}{\partial(TP)} - \frac{\partial s}{\partial P} < \frac{\lambda}{\alpha} P^* d(1+e_P); \; \frac{\partial s}{\partial P}(1+e_P) > 0 \qquad (21)$$

Let us finally also have a closer look at the condition that differentiates between dampened oscillation (19) and straight convergence (20). We will get oscillatory convergence if:

$$4\alpha^2 T^* \frac{\partial d}{\partial(TP)} \frac{\partial s}{\partial P} > \left[\alpha\left(T \frac{\partial d}{\partial(TP)} + \frac{\partial s}{\partial P}\right) - \lambda P^* d(1+e_P)\right]^2 > 0 \qquad (22)$$

One simple implication of (22) is that:

$$\frac{\partial d}{\partial(TP)} \frac{\partial s}{\partial P} > 0.$$

In other words we will get oscillatory convergence only if the supply or demand curve behaves "abnormally", when we have, for example, a negatively sloping supply curve. If condition (22) is fulfilled, the movement of both the supply price and the tax coefficient will be described by:

$$[P(t),\ T(t)] = k_1 \ e^{\rho t}[\cos(\nu t+\Phi)r - \sin(\nu t+\Phi)\upsilon]$$

$$+ k_2 \ e^{\rho t}[\cos(-\nu t+\Phi)r + \sin(-\nu t+\Phi)\upsilon] \qquad (23)$$

where $\rho \pm \nu i$ = the roots, $r \pm \upsilon$ = the characteristic vectors associated with the roots, and where both the conjugate constants k_1 and k_2 and the phase constant, Φ, depend on initial conditions.

From (21) we see that with an elastic demand, ($e_P < -1$), and a positive supply curve, subsidies aimed at stabilizing expenditure will introduce instability of price. This is also easy to understand intuitively. While, in the first example, producer prices and the tax coefficient are adjusted in the same direction, thereby slowing down the adjustment of each other we now have a reversed situation. Suppose the producer's price has been set too low. This gives rise to excess demand, moving the supply price upwards. At the same time, however, with elastic demand, expenditures are too big, which means that the tax coefficient moves down. Hence, subsidies grow, counteracting the effect of the producer's price on demand price. This, obviously, leads to a decreasing demand price followed by an increasing supply price, etc.

Taking a gradual increase of both income and of the expenditure target, M, into account does not change this conclusion. A too low supply price then means an increased potential risk of instability compared to a too high supply price. If the subsidy rule is changed to mean that subsidies vary in a fixed proportion to demand, the conclusion is in fact strengthened --holding for an inelastic demand as well. Political expediency may often seem to require the use of such "explosive" subsidy rules. This is illustrated by the Swedish experience in some areas of health and recreation.

The model exemplified above can be generalized to the multi-market case. Without individual specification of the tax rules involved little more can, however, be learned from such a generalization except the important, but obvious, conclusion that none of the ususal sets of sufficient stability conditions retain any credibility when extended to involve also tax adjustment rules.[1]

The step-size of tax adjustment

Real life adjustment is seldom a continuous process. This is true both for price-setting producers and, perhaps even more, for tax authorities.

If we make the realistic assumption that adjustments take place in discrete steps, the size of these steps or the rate of adjustment becomes important for stability.[2]

Since there is, no longer, an immediate feed-back from market reaction to adjustment, you now run the risk of over-shooting your targets. If your "over-correction" is even bigger than the needed correction, the adjustment will obviously become unstable.

This is true already when there is only a price adjustment to deal with. Formulated as a difference equation with $\Delta p(t) = p(t+1)-p(t)$ and $p(t)$ representing the divergence from equilibrium, the price adjustment can be written:

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[1] Cf. Ysander (1980), where sufficient conditions for the multi-market case are discussed.

[2] In actual life you may, of course, decide independently how often to adjust and how much to adjust. In the analytical example above, however, the time period is taken as given, restricting the possible variation to the rate of adjustment.

$$\Delta p(t) = \alpha E_p^* p(t) \qquad \alpha > 0 \tag{24}$$

By iteration, this can be solved as:

$$p(t+1) = (1 + \alpha E_p^*)^t p(0). \tag{25}$$

The wellknown condition for convergence is:

$$-2<\alpha E_p<0 \text{ (with alternating values for } -2<\alpha E_p<-1) \tag{26}$$

This simply expresses that any "over-correction" must be less than the needed correction. The Walrasian condition for market stability being fulfilled, (26) can be expressed as limits for the rate of adjustment:

$$0 < \alpha < \frac{2}{-\left(\frac{\partial d}{\partial p} - \frac{\partial s}{\partial \bar{p}}\right)} \tag{27}$$

Since any fixed positive tax, T, will increase the step-size of demand-induced adjustment by $(T-1)\alpha$, by definition it follows that even without tax adjustments all proportional market taxes will narrow the safety margins for stable price adjustment.

Let us now take a further step and introduce a tax that is adjusted at the same intervals as price and has the same simple aim as that in our first example above, i.e., to keep demand at a pre-determined value d*. In vector form the adjustment system (neglecting again the asterisks when possible) can be written as:

$$(\Delta p(t),\ \Delta\tau(t)) = A(p(t),\ \tau(t)), \tag{28}$$

where, A, stands for the same matrix as in (6) above and $\Delta(t) = \Delta(t+1) - \tau(t)$ with $\tau(t)$ representing the divergence from an equilibrium tax coefficient, T*. A necessary condition for convergence of a simple difference system of this kind is that:

$$|1 + x_i| < 1; \qquad i=1,\ 2 \tag{29}$$

where x_i is a root of A.

We already know the roots from (9-11) above, and know that they are real. Thus:

$$-2 < a \pm \sqrt{a^2-b} < 0 \tag{30}$$

It was shown in (13) above that the second part of this condition requires that the demand slope be negative and the supply slope positive, i.e., a "normal" market situation. The first part of (30) is the now added restriction on step-size. Given the second part of (30) we can spell out the first part in the following manner:

$$\begin{matrix} a^2 > b \\ a<0,\ b>0 \end{matrix} \rightarrow \left[(a \pm \sqrt{a^2-b} >) -2 \equiv \begin{matrix} a > -2 \\ \\ b > -4(1+a) \end{matrix} \right] \tag{31}$$

The two inequalities to the right in (31) express constraints on the rates of adjustment, α and λ.

$$(\alpha T^* + \lambda P^*) \frac{\partial d}{\partial (TP)} - \alpha \frac{\partial s}{\partial P} > -4 \tag{32}$$

$$\alpha\lambda P^* \frac{\partial d}{\partial (TP)} \frac{\partial s}{\partial P} - 2\left[(\alpha T^*+\lambda P^*) \frac{\partial d}{\partial (TP)} - \alpha \frac{\partial s}{\partial P}\right] < 4 \tag{33}$$

After some reshuffling (32) and (33) yield the following limits --now expressed in terms of the decision variables p and τ-- for the rate of price adjustment, α:

$$\frac{\frac{\partial d}{\partial \tau}}{\frac{2}{\lambda}\left(\frac{\partial d}{\partial p} - \frac{\partial s}{\partial p}\right) - \frac{\partial d}{\partial \tau}\frac{\partial s}{\partial p} + 4} < \alpha < \frac{4 + \lambda\frac{\partial d}{\partial \tau}}{-\left(\frac{\partial d}{\partial p} - \frac{\partial s}{\partial p}\right)} \qquad (34)$$

Comparing (34) with the restriction on α without taxes in (27) (and remembering that the slope of the original demand curve corresponds to $\partial d/\partial(TP = 1/T^*(\partial d/\partial p))$ we see that the introduction of tax means that α is now bounded also from below and that both bounds are functions of the rate of tax adjustment, λ. The right-side inequality shows, for example, that the more price-sensitive demand is, the slower the tax adjustment has to be, given α. Increasing the relative tax adjustment rate, λ/α, will always lead to instability.

Taxes and Structural Stability

Our examples so far have dealt with stability in the usual sense, i.e., we have discussed price developments in a market characterized by given coefficient values.

Of at least equal interest, but more difficult to exemplify formally, is the case where a tax adjustment rule renders the market structurally instable, in the sense that even small changes in the parameters will change the behavior of the system, establishing a quite different set of equilibria or regions of stability.

When we are discussing the stability of an economic system in the face of large quantitative or qualitative changes, say, a big hike in oil prices or drastic changes in the laws governing ownership of firms, the myopic study of local stability properties is seldom of much use. The kind of instability we are then interested in means that we are far from the original equilibrium or the established growth-path. If the initial disturbance concerned the size of an endogenous variable in our model of the economy, we would say that the size of the change had been "out of bounds" for the stability region within which we had, so far, been operating. With the change occurring in an exogenous variable or a behavioral parameter we would, instead, interpret the result as evidence of "structural instability" in the sense that shifts in the parameters can lead to changed stability properties, a new topography for the phase space of the system.[1]

The introduction of taxing procedures on various markets is, in itself, an important change that could modify the structural stability properties of the entire system. Taxes may, moreover, often induce changes in the behavior of the economic actors as well as alter the system's ability to adjust to and absorb other institutional or environmental changes that occur.

The Swedish economy abounds with illustrative examples of tax-adjusted behavior and tax-induced changes in market structure.

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[1] See Appendix, note III, p.228.

High tax rates have, in many cases, led to the establishment of "grey" or "black" markets. Competition from these often modifies behavior in the "official" markets considerably. In the fifties and sixties market structure also tended to change as a result of taxation laws being generally unfavorable to small family businesses. In recent years the combination of complex tax laws, mostly written in nominal terms, and a high rate of inflation have led to huge unintended discrepancies betweeen the tax treatment of various kinds of real and financial investment. Since these discrepancies are quickly discounted in capital values they tend to make the whole economic system increasingly vulnerable to changed expectations of inflation or of tax adjustment.[1]

Any attempt to discuss these structural stability problems in substance would take us far beyond the scope and ambition of this paper. Let us, however, try to clarify the formal stability concepts involved by giving an example from oil price-setting, couched in the same terms of market adjustment as our preceding analysis. The example chosen may fill this function, although it can claim no immediate relevance for policy.

Suppose there are two kinds of oil prices, P_r which is an index of the US producer price of refined oil and, p_o, which stands for an index of the Saudi government's unit charge for crude oil.

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[1] For an assessment and a discussion of these assymetries and discrepancies in the tax treatment of different kinds of investment cf. Johansson (1978).

It is assumed, here, that the U.S. oil companies try to reduce any eventual gap between their domestic price increase and that of the Saudi government. The Saudis on their side are considered to have an idea of what constitutes a "fair" proportion, r, between the price increase they get and that of the U.S. companies. The price adjustments can then be described by the following:

$$\dot{p}_r = \alpha(p_o - p_r) \tag{35}$$

$$\dot{p}_o = rp_r - p_o. \tag{36}$$

The stability properties of this system obviously depend crucially on r, the Saudi's preset idea of a fair proportion. r=1, for example, means that any point with $p_o = p_r$ is a stable equilibrium. With r>1 no equilibria exist and prices will explode.

The U.S. government now interferes in the game, trying to curb the inflationary impulses of the oil parties by taxing away domestic demand whenever oil price hikes increase. The oil tax rate, τ, expressed as a multiple of p_r, is raised in proportion to the product of both oil prices, although at a decreasing rate. The Saudis now have to take the tax into account in calculating the "fair" proportion. The total adjustment can be written as follows:

$$\dot{p}_r = \alpha(p_o - p_r) \tag{37}$$

$$\dot{p}_o = (r - \tau)p_r - p_o \tag{38}$$

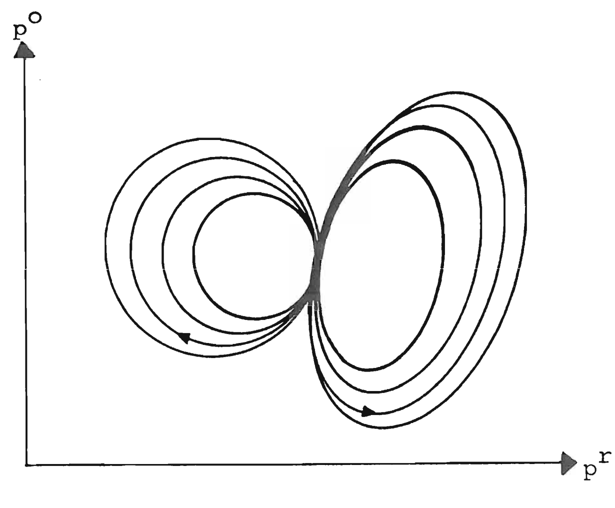
$$\dot{\tau} = p_r p_o - \beta\tau. \tag{39}$$

The behavior of this system is very different. For small values of r the system globally tends to a simple equilibrium. Should the Saudis, unlikely enough, consider it "fair" that the crude price develops much slower than the U.S. domestic price, the effect of the tax may be, in fact, to accelerate the downsliding of both prices towards zero. For a somewhat larger r, there is one stable equilibrium (two, if negative prices are allowed), denoting an equal price increase, with a positive tax to balance off the Saudi's claim for a "fair" price edge.

If r gets even larger--magnifying the Saudi's idea of a fair relation of price--it suddenly leads to a completely new mode of behavior. Wherever the development starts off (excepting some isolated points of equilibrium) it will eventually be drawn into a circular motion of prices and tax. The crude price leads, due to the Saudi's high price ambition, with the U.S. price following. Both are, however, outrun by a fast although decelerating tax change. The high tax then turns the movement downwards, again with the crude price in the lead, followed by tax and U.S. domestic price until the shift in relative oil price is enough to offset the tax and the crude price starts increasing again. The relative oil price will thus vary around 1 while the tax rate moves around (r-1). The development is, however, very sensitive to small differences in the values of the variables. After a certain number of "orbits" (the rotation numbers being a Markov sequence) the system will suddenly branch off into another but similar "orbit", only to return again after a while to the first "orbit", etc. Looking at the system from outside we would observe sudden shifts in the price- and tax-cycles occurring according to a seemingly stochastic schedule. The movement could --projected on the price plane--look like figure A.

This rather "exotic" example[1] illustrates the fact that taxes may not only change the stability properties around equilibria; they can also change the whole nature of equilibria and their structural stability in the face of parameter changes.

Figure A. Alternating price cycles



[1] The quoted model is an instance of the so-called Lorentz model, originally invented to solve a problem in aerodynamics (Lorentz, 1963). It has later been shown to give a good description also of the reversals of Earth's magnetic field over geological times (Ruelle and Takens, 1971). Continued work with this kind of attractor system has been reported by Grümm(1976a-b).

Tax Uncertainty and Market Stability--the Housing Market

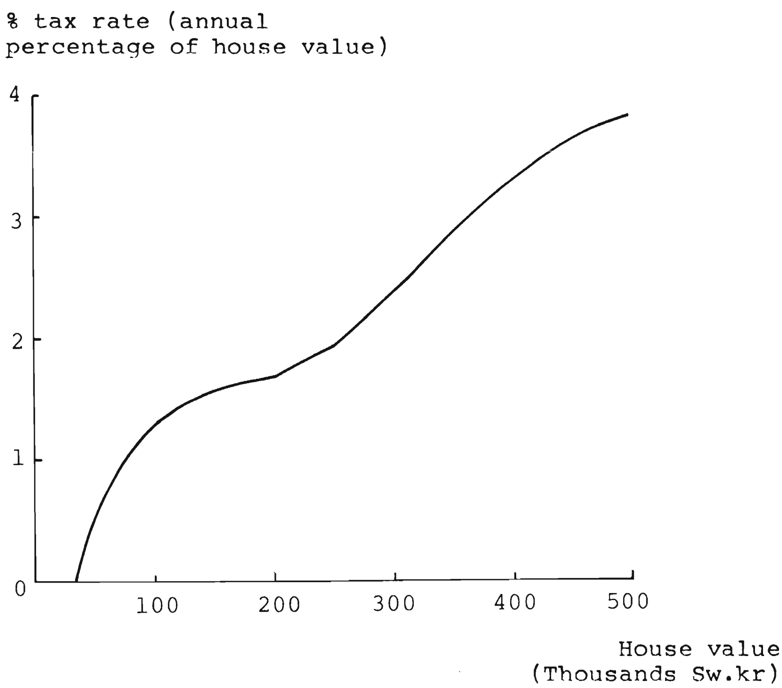
So far we have dealt explicitly only with tax-induced instability under full information. However, if tax adjustment is hard to predict for the parties concerned, the induced uncertainty may give rise to stability problems in the form of highly erratic price movements. A striking example of this is provided by the Swedish market for owner-occupied houses.

Pricing, in this market, is to a large extent determined by the tax authorities. This is done firstly by assessing the taxable value of the property--supposedly at 3/4 of market value--and secondly by applying to this value a progressive scale of imputed taxable income, which is then superimposed on the already steeply progressive income tax. The outcome in many cases is that the owner pays more to the government than to his bank and that what the tax authorities evaluate is in fact the result of previous tax decisions. Especially when tax scales and tax norms are changing rapidly and at an unpredictable rate this can give rise to cyclic price fluctuations and demand instability. In recent years, inflationary gains have dominated houseowners' expectations. Tax instability--which increases with inflation--could soon, however, become a serious problem especially if inflationary expectations also become unstable. A relatively advantageous taxing of capital gains on private houses compares favorably with the level of taxation on more rigidly taxed markets, for example, the stock market and bank deposits. Fluctuating capital gains from private real estate find their way back to other markets and there contribute to intermittent swings in demand.

Let us take a closer look at the way in which unpredictable tax adjustments create instability problems.

The theoretical impact of current property tax rates is shown in Fig. B. We have computed the curve for a recently assessed house whose owner has, on average, a marginal income tax rate of 75 per cent. The curve is "theoretical" in so far as it presupposes that the prescribed assessment norm --3/4 of market value-- is strictly adhered

Figure B. <u>Current Swedish tax rates on owner-occupied houses</u>



to. Actually, this has not been the case in recent years. By systematically lowering the norm for more expensive houses in the most recent assessment (1975), the tax authorities seem to have, to a certain degree, counteracted the effects of progression.

To see what the progressive rates might do to the prices of houses, one can compute and compare price curves for proportionate and progressive tax rates respectively, as shown in Fig. C.

If we use the following notations:

$V(t)$ = market value of house at time t
a_0 = net annual user value (rent value) at time 0
p = rate of growth of user value
s = tax coefficient (tax paid in percentage of market value of house) (40)
r = discount rate
$n-t$ = remaining economic life of house
b = parameter of tax progression, $s(t) = b\,V(t)$

The market value of the house computed as the discounted value of future incomes and tax payments can then, with a constant proportionate tax coefficient, be written as:

$$V(t) = \int_t^n (a_0 e^{pu} - sV(u))e^{-r(u-t)}du \qquad (41)$$

which resolves into:

$$V(t) = \frac{a_0}{p-r-s} e^{pt}\left(e^{(p-r-s)(n-t)}-1\right) \qquad (42)$$

We now use the following parameter values:

$$
\begin{aligned}
a_0 &= 6 \\
p &= 0.08 \\
s &= 0.01 \\
t &= 0.06 \\
n &= 40
\end{aligned} \tag{43}
$$

A computation of (42) with these parameter values gives the price curve I, in Fig. C. As expected the elasticity of price to changes in the tax coefficient is relatively low, -0,2, at the start and -0,1 at half-life.

Let us now introduce progression by setting $s = 0.00007V(t)$. Compared to the current formal tax scales these rates are relatively low, both as to level and progression. Thus, they take some account of the effect of intermittent assessment. The market value of the house can now be written as:

$$V(t) = \int_t^n \left(a_0 e^{pu} - bV^2(u)\right) e^{-r(u-t)} du \tag{44}$$

The explicit solution--which the common buyer is supposedly following in his evaluation--turns out to be a rather tortuous and long-winded expression.[1] The numerical result for the chosen parameters is shown as price curve III, in Fig. C.

The elasticity of price to changes in the tax parameter is now very much higher, given a high rate of growth in user value.

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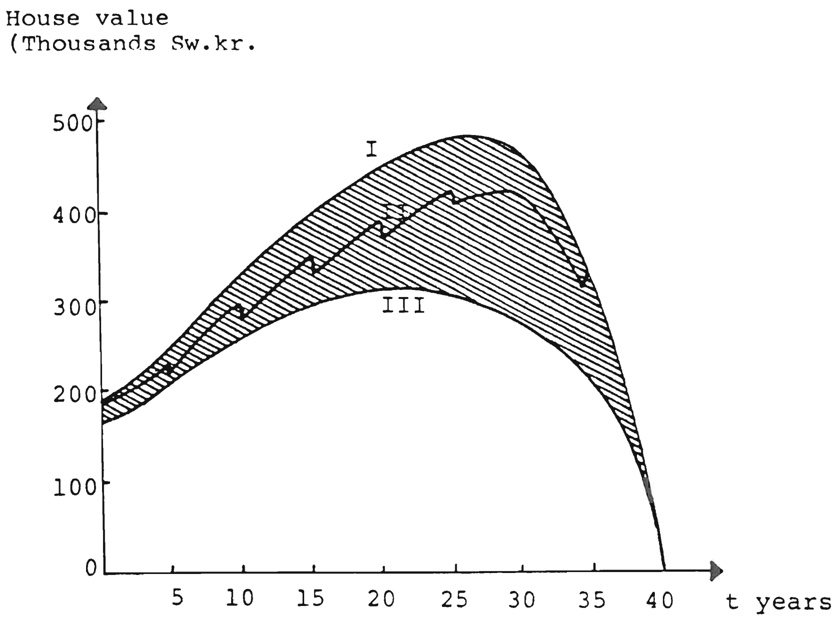
[1] An account and discussion of the complete solution is given in Ysander (1976).

Figure C. Development of house price for different taxing and market behavior



Comparing the curves I and III we see that an increase of the tax yield is not likely to be the main effect of applying a progressive scale. First, and foremost, the price difference between the various categories of houses diminishes. Bigger and/or more comfortable houses become less profitable to build and sell.

Rather than taking full account of future progression, buyers and sellers may expect the current total tax coefficient to remain constant. The result would be a jumpy price development as demonstrated by price curve II in the figure. As shown by the Swedish experience in the seventies it is very difficult to predict when and how far tax rates will be adjusted for inflation or counteracted by assessing practices. The shaded area between curves I and III, in Fig. C, can be interpreted as a margin for the price uncertainty arising from progressive taxation. This margin will, moreover, tend to increase with inflation. The instability normally associated with changing inflation rates will thus be multiplied by this "tax uncertainty".

Taxation and wage inflation

Up to now we have dealt exclusively with isolated adjustments in a single market. However, the most widely observed and best known example of tax-induced instability relates to the adjustment of heavily taxed wage markets to price increases in the product markets, i.e., to inflation. This has been an acute problem in Sweden during most of the seventies.

In contrast to our previous examples we are faced, here, with annual tax adjustments aiming, mainly and explicitly, to compensate for the stability problems created by the tax structure itself.

The rates of income tax in Sweden are highly progressive -- and changing rapidly. Even excluding the various kinds of employers' social insurance fees, etc.--adding up to about 40 per cent of paid out wages--the marginal income tax rate for an average skilled industrial worker in Sweden now approaches 70 per cent, the average rate being some twenty per cent lower--all measured in terms of taxable personal income. The progression is steeper for high-income earners--and for low-income earners receiving subsidies.

If the worker, cited above, should be compensated for say a 10 per cent of inflation--with tax-scales not being automatically adjusted for inflation--he would have to receive a wage increase of some 17 per cent--starting off a run-away wage inflation spiral.

Negotiations are further aggravated by the variance in marginal tax rates between different groups of labor. Since gross wages are what is negotiated any compromise between the unions is likely to add further inflationary pressure.

Continuous tax revisions or an indexing of the tax scales provide the standard answer to the first problem--that of eliminating the "tax multipliers of inflation".

The second part of the problem however does not disappear so easily. Support for a tax redistribution of today's income does not automatically mean acquiescence in the further leveling of tomorrow's income implied by the marginal tax rates necessary to carry through the redistribution. To ward off this cause of wage inflation, annual revisions of relative total tax rates for various income-groups have, in recent years, become an important part of collective wage negotiations in Sweden. The structure of any progressive income tax is unfortunately such that every attempt to use tax revisions to satisfy claims for further leveling of net wages is apt to aggravate the "locking-in" effects and stability problems for the next round of wage negotiations.

There is another side of this instability problem that should be mentioned here, although it falls somewhat outside the model context of the previous discussion. Introducing progressive taxation, applied to gross market price, definitionally means, ceteris paribus, a lowering of the gross price elasticity of supply in the market. In terms of the labor market this means making labor less inclined to move in response to certain given wage inducements.

When this weakened pull effect is compounded, as in Sweden, with an institutionally and legislatively restricted push effect--by restrictions on how and when and why labor can be laid off--the possible consequences on market stability are apparent. The adjustment to shifts in foreign demand and/or to relative price changes will be slowed down and the competition for labor from expanding firms could either result in more inflationary wage increases or a petering out of expansion with inflated wage demand working as a damper.

Instead of Conclusions

Our previous discussion has involved a rather varied collection of examples of possible tax-induced instability. Our focus on individual market adjustment however means, that we have not treated the equally important problems of the impact of taxation on macro-economic stability.

The examples presented earlier do not readily lend themselves to any general interpretation or conclusion. They do however illustrate two important points.

The first one concerns policy. When raising the "technical" ambitions of tax-policy, gradually using it for more differentiated regulatory aims, the risk of disrupting the "normal" market adjustment processes grows.[1]

Stability problems are thus added to the more widely discussed problems of the long-term allocative effects of tax-induced changes in relative prices. The Swedish experience in the seventies seems to suggest that, also from the stability point of view, there are severe limitations to what you can safely hope to accomplish by tax policy.

The second point has to do with research. We have by now a fairly well-developed literature on "optimal taxation" and the welfare effects of a fixed tax structure from an "equilibrium point of view". Our examples demonstrate that there is now good reason to take one further step and investigate the impact of taxes and tax adjustment on market stability as well. Unfortunately, any thorough investigation into these problems will have to work with disequilibrium models, which makes points of departure harder to find. The results will also be less general and theoretically convincing. That may be an explanation for our being late to start but it is hardly an excuse for further delay.

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[1] Alternative ways of pursuing these policy aims may of course be even worse from a stability point of view. The use of more direct intervention or regulation by definition makes the economy more rigid and hence less shock-proof. Having more "fixtures" and less free variability tends to narrow the margins of adjustment in the economy.

APPENDIX

THREE NOTES ON THE CONCEPT OF STABILITY

I Some basic stability concepts

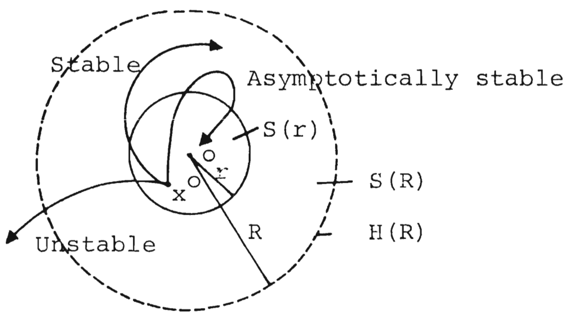
To facilitate reading the paper the reader may want to recall some basic stability concepts.

The concepts can be illustrated as in Fig. 1. We assume that we are dealing with an autonomous system, i.e., a system in which time, t, is not an essential variable but only used as a parametrization variable. We further assume that we are working in some open region of phase space, through each point, x, of which there goes a unique path of the differential system:

$\dot{x} = X(x),\ X(o) = 0;$

where x and $\dot{x}$ denote vectors.

Figure 1. Some basic stability concepts



We shall designate by S(r), S(R) the spherical region $\|x\| < r$ and $\|x\| < R$, respectively, and by H(R) the sphere $\|x\| = R$ itself.

We now say that the origin o is:

1) Stable (or stable in the Liapunov sense) whenever for each R there is an $r \leq R$ such that a path initiated in S(r) always remains within S(R).

2) Asymptotically stable or resilient[1] whenever it is stable, and, in addition, every path starting inside some $S(R_o)$, $R_o>0$, tends to the origin as time increases indefinitely.

3) Unstable whenever for some R and r, no matter how small, there is always in S(r) a point x such that the path through x reaches the boundary H(R).

II Boundedness, Practical and Ultimate Stability

The usual basic concepts of stability analysis unfortunately turn out to be of little practical use when applied to price developments in real life economics. There are, in particular, four further problems that must be taken into account in any attempt at measuring stability in actual price movements.

In real economics time is an essential variable, i. e., the systems are non-autonomous. In theory, a generalization of the stability concepts to non-autonomous systems is straightforward although proofs tend to get more laborious. In practice we almost never know enough to analyze explicitly the time-dependence.

Resilience and stability are empirically indeterminate properties as long as we are talking in terms of some neighborhood which may be arbitrarily close to the origin. To acquire an empirical content the concepts must be quantified by measuring the extent of the regions involved in the stability definitions.

In most economic as well as physical systems, stability problems usually arise, not primarily because of initial conditions being far from equilibrium, but because of various kinds of persistent disturbances or perturbations. Any useful stability concept must therefore refer to the movements of such a perturbed system.

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[1] Different authors use "resilience" to cover various shades or aspects of stability. We have chosen, here, to use the word when the system tends to become more narrowly confined within some neighborhood of an equilibrium.

Finally, we are often less interested in ascertaining the return to origin than we are in making sure that the system stays within bounds. Stability in the sense of Lagrange means just this, viz., that all solutions are bounded. Again this definition needs to be quantified to make empirical sense.

In trying to meet these four empirical requirements we could end up with the following two stability definitions that are illustrated by Fig. 2. Our starting-point is a system:

$$\dot{x} = X(x,t)+p(x,t), \quad t>0;\ X(0,t)=0 \text{ for all } t>0$$

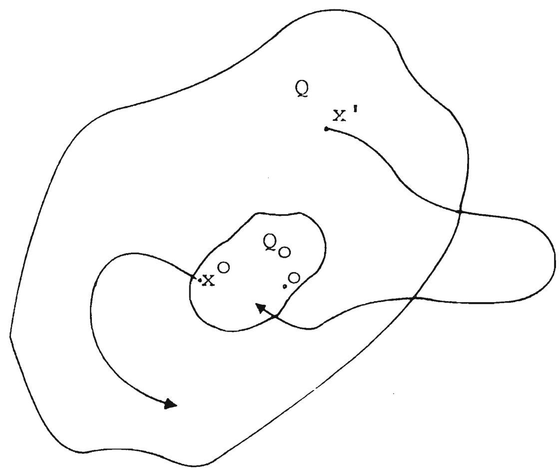
where p denotes perturbations satisfying $p<\delta$. We have, also, in the figure two sets: Q which is a closed and bounded set containing the origin, and Q_o which is a subset of Q. We could then, following LaSalle-Lefschetz (1961) define:

Practical stability of the origin as the property requiring that for given Q, Q_o and δ, any solution starting in Q_o will remain in Q for $T>t>0$ (cf. x' in Fig. 2).

Somewhat analogous to the concept of asymptotic stability or resilience would be:

Practical resilience: requiring that, for given Q, Q_o and δ, any path going through Q will be in Q_o for all $t > T$ (cf. x' in Fig. 2).

Figure 2. Practical stability and practical resilience



III. Structural and Comprehensive Stability

In most economic discussions of stability we deal with a system with fixed parameters where the path of prices, for example, can be completely described as a function of the state variables: $dx = f(x)dt$.

In real economies parameters do change. This is obviously the case with the parameters representing the state of the external world, such as world market prices for a national economy. Even if we simplify by ignoring these exogenously determined parameters we will still be faced with changing parameters.

In a widened or lengthened perspective we must take account of the fact that the behavior or the institutionally determined parameters of an economic system change according to some rule. Denoting the vector of parameters p, such a generalized explanation of change could be written as: $dx = f(x, p)dt$.

To avoid making the analysis too unwieldy economists usually try to discuss time developments in two stages - sometimes identified as a short and a long run. In the short run, parameters can be treated as given and the total change can thus be split into two parts:

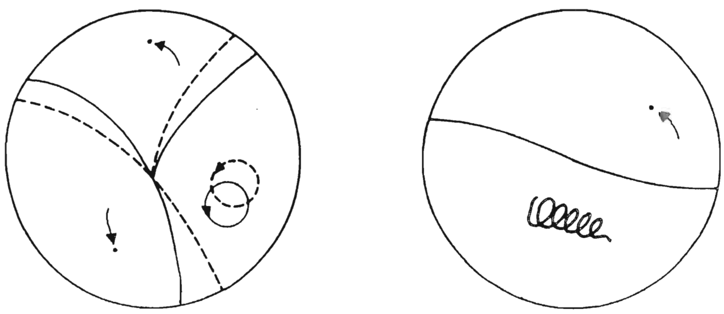
$$dx = f_1(x, p)dt + f_2(x,p)dp.$$

To be able to separate the impact of parameter change, f_2, from the "short-run" developments with given parameters, f_1, it is obviously necessary to assume that parameter changes are measured in time scales quite different from those used to define "short run" changes. This could be done by assuming parameter changes to be extremely "sudden". Usually however, economists go the opposite way, making the "comparative static" assumption that parameter changes occur slowly enough so that the "short run" system always has time to reach its asymptotic equilibria.

Instead of discussing stability as a property of the "phase-portrait", f_1, of a system with given parameters one may want to treat stability as a question of how big or how continuously the change in "phase-portrait" is, that results from certain parameter changes. This is roughly what is meant to be measured by "structural stability" in the sense of Smale (1967) or of the "catastrophy theory".

Fig. 3 may help to give some intuitive idea of this concept. Drawn with full lines in Fig.3a is the original "phase-portrait", which is supposed to be fairly simple--three basins, each with an attractor.

Figure 3. Change in "phase-portraits" caused by change in parameters



a b

We now make a slight variation of the parameters and watch for results. The dotted lines in Fig. 3a show what could happen if the structure of the system is relatively stable. The parameter variation does not change the dynamic structure but only causes a continuous shifting of basins and limit-cycles. Fig. 3b illustrates a structurally unstable case where the same variation completely remodels the phase-portrait, reducing the number of basins and changing the character of attractors.

Once you include parameter changes in the framework of analysis there is one further question of stability to be considered. What causes parameters to change and does that kind of "system change" tend to counteract or reinforce instability "within the system"? Do institutions and economic behavior adapt in the long run so as to reduce or to maintain long-term imbalances? These questions concerning comprehensive stability --central to the current discussion of stagflation-- can, however, seldom be usefully analyzed within our economic models. The inability of our models to deal with "structural change" is indeed probably a major explanation for their poor showing during recent years.

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