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A Theory of Gazelle Growth: Competition, Venture Capital Finance and Policy

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Abstract

This paper proposes a theory of gazelle growth in which gazelles can either grow organically or by acquisitions. In the model, there are three types of firms: incumbent, target, and gazelle. We show that the lower cost of organic growth can increase the incentives for acquisition growth. The reason for this is that the incumbent understands that if it acquires the target firm, the gazelle will then invest organically anyway to grow, and therefore, the acquisition will not be sufficient to protect the incumbent's market power. The gazelle could then acquire the target firm at a good price. We also show that financial support for the organic growth of gazelles can increase gazelles' growth by acquisitions since incumbents' preemptive motives are reduced.

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1 Introduction

Due to their substantial share in net employment growth and their role in providing society with new products and services, there is a rising interest in the research literature to understand and analyze the development of young high-growth firms, or so-called gazelles (Birch (1979), Henrekson and Johansson, (2010)).

The starting point of this paper is the observation that gazelles[1] face considerable problems when trying to fully exploit the potential value of an invention or a business idea. Complementary assets such as distribution networks, marketing channels, financial resources, manufacturing know-how and brand names are typically held by large established firms, but these assets are also often needed by gazelles. To obtain such assets, the gazelles could either set up a new plant or warehouse and grow organically or acquire a target firm with such assets in place (Delmar, Davidson and Gartner (2003); McKelvie, Wiklund and Davidson (2006)). However, many of the gazelles have to challenge large incumbent firms that are already market leaders competing in oligopolistic markets and possess market power. These incumbents' profits are diminished by the growth of the gazelles. The incumbents may have an incentive to acquire these target firms. Thereby, they can block the growth of gazelles and protect their market share.

The purpose of this paper is to propose a theory of gazelle growth that is consistent with these facts and that is tractable for policy analysis. To this end, we construct a model with the following components. There are three firms in a market. One of them is the market leader, which is labeled the incumbent. The other one is a firm that has been already in the market as a competitor to

[1]Gazelles and high-growth firms are not necessarily the same type of firms. Gazelles are often defined as young firms that are growing rapidly. The definition of high-growth firms, on the other hand, tends to be based on all firms (independent of firm age); see, e.g., Coad et al. (2014).

Our focus here is on gazelles, and a main focus of the analysis is that the gazelles need external financing to grow, which is typically the case for young firms.

the incumbent, but its assets are on sale, and it is going out of business. It is labeled the target. Finally, the last firm is a new firm entering the market. It is established by an entrepreneur and labeled the gazelle. The incumbent and gazelle are assumed to compete in an oligopolic fashion. The entrepreneur has invested in an innovative activity. This innovation could lead to the creation of a unique business idea or an invention, which could then be used to start a gazelle and make it grow. At the very beginning, the entrepreneur decides whether to invest in the innovative activity. If the entrepreneur is successful in the first period, the gazelle and the incumbent will compete to acquire the target firm's distribution network. If the gazelle does not acquire the target firm's distribution network, the gazelle could grow organically either on a small scale or on a large scale in the second stage. If it grows organically on a small scale, it will face higher distribution costs. The crucial feature of the model is that the gazelle lacks financial resources and needs to borrow when growing by acquisition or engaging in large-scale organic growth. On the other hand, the model assumes that the gazelle will not need any external funding for small-scale organic growth.

We first analyze the gazelle growth model. We show that if the gazelle faces sufficiently large distributional costs when investing on a small scale, the incumbent obtains the target firm's distribution network despite the fact that the gazelle would save considerably on the distribution cost by obtaining the target firm's distribution network. Once the distribution cost becomes sufficiently high, this distribution-cost saving effect is dominated by a preemptive anticompetitive effect. The reason is that when obtaining the target firm's distribution network in this situation, the incumbent firm gains great market power in the product market. It faces a competitor (the gazelle) with high distribution costs. Consequently, the incumbent firm obtains the target firm's assets. Therefore, the gazelle is left with the option of growing organically on a low scale. On the other hand, if the gazelle has a better position when growing on a small scale (it becomes a sufficiently large firm by doing so) or if the gazelle can grow on a large scale at a low cost, then the incumbent can no longer prevent the gazelle from enhancing its competitiveness in the product market. Therefore, the incumbent is not willing to pay a high price for the target firm's distribution network.

Access to finance for gazelle firms is often difficult due to their perceived riskiness and lack of collateral. Banks are usually reluctant to lend to gazelles. Therefore, venture capital (VC) is one of the primary sources of outside equity financing and support for gazelles. However, the amount of money that VC firms allocate for different stages of company development changes considerably. In the US, only around 4% of all VC deals are made with companies that are in the seed stage, whereas 50% of all deals are made with companies in the early stage.[2] VC firms prefer to finance early-stage companies that are ready to expand their operations, manufacturing capacities, sale and marketing teams or product development teams. An early-stage gazelle firm, which is generating revenue but is not necessarily profitable, is not as risky an investment for VC firms as a seed-stage company. Most of the time, gazelle firms have already proven a market need and have already found a good product-market fit. They are ready to increase their market share by enlarging their operations, i.e., acquiring the required assets from another firm for this purpose. Therefore, gazelles are attractive investments for VC firms.

Incorporating VC financing in the model, we show that VC financing indeed leads to higher gazelle growth. Moreover, and maybe more surprising, we show that VC financing increases acquisition growth more than organic growth. Why? The reason is that by securing VC financing, a gazelle could credibly threaten to

[2]Source: Yearbook 2016, National Venture Capital Association.

invest on a large scale organically if not growing by acquisitions. This reduces the incumbent's preemptive acquisition motive and decreases the acquisition price for the gazelle when growing by acquisition.

An example of a successful acquisition growth financed by VC is Flipkart, an Indian e-commerce company founded in 2007 by two entrepreneurs. Flipkart experienced high growth by becoming a USD 11 billion market-valued company in February 2016 from a start-up whose equity value was only USD 6000 when it was established. Flipkart managed this rapid growth through several acquisitions financed by VC funds. It was listed number one among top 10 e-commerce companies in India in terms of receiving funds. Flipkart obtained USD 542 million as of the end of 2014.[3] Interestingly, Flipkart's main competitor, Amazon, has tried to buy Flipkart, but instead, Walmart (an outsider to the market) made a staggering offer of USD 16 billion for a controlling stake of 77% in May 2018. An injection of USD 2 billion in fresh investment into Flipkart was also part of the deal.[4]

We then turn to policy issues. Entrepreneurship has emerged as a key issue in the policy arena in Europe and the US. An important factor in the process of commercialization and the growth of small businesses is their access to financing. In addition to bank loan financing, government policies aim at promoting alternative sources of financing, such as public listings for SMEs and venture capital investments. Recently, there has been a substantial increase in spending on such policies. For example, in 2015, the US Small Business Administration (SBA) approved USD 33 billion in loans, and it also provided more than $6 billion in capital to small businesses through investment funds licensed as SBICs. SBICs are privately owned and managed investment funds, licensed

[3] Online Source: www.businesstoday.in, https://en.wikipedia.org/wiki/Flipkart, www.indianexpress.com.

[4] Walmart is buying India's Flipkart, by Rishi Iyengar and Sherisse Pham @CNNMoney-Invest, May 9, 2018, http://money.cnn.com/2018/05/09/investing/walmart-flipkart-india-softbank/index.html

and regulated by the SBA. These investment funds use their own capital plus funds borrowed with an SBA guarantee to make equity and debt investments in qualifying small businesses.[5] Another government policy to improve SME access to financing is providing tax exemptions and deferments such as the provision of tax incentives for equity investors and businesses. Countries that provided tax subsidies and deferments targeted at SMEs during the period of 2007 and 2014 included Belgium, Finland, Italy, New Zealand, Norway, Spain, Sweden and Turkey.[6]

We start the policy analysis section by examining the effects of financial support to gazelles. We show that financial support such as a reduction in the interest rate tends to improve the possibility for gazelles to grow by acquisition rather than by organic growth. The reason is that a reduced interest improves gazelles' outside option (organic growth), which decreases the incumbent's preemptive motive to buy the target. This, in turn, implies that the price of the target firm is substantially reduced.

We then analyze the direct support for an organic growth, a subsidy. We show that a subsidy for large-scale organic growth might not increase the incentive for organic growth, which is the objective, but rather may actually decrease it. The reason is that the subsidy for organic growth reduces the possibility for preemption by the incumbent. The gazelle could credibly threaten the incumbent with large-scale organic growth. A subsidy helps the gazelle decrease the investment cutoff level determining the profitability and feasibility of large-scale organic growth. In short, by decreasing the investment cutoff level, a subsidy improves the position for the gazelle in the bidding competition against the incumbent while acquiring the target firm.

This paper can be viewed as a contribution to the literature on the growth

[5] Source: Summary of Performance and Financial Information Fiscal Year 2015, the U.S. Small Business Administration, 2015.

[6] Source: Financing SMEs and Entrepreneurs, OECD, 2016.

mode of young, high-growth firms. MacCann (1991) points out that young and small firms often do not have sufficient resources to grow aggressively via acquisitions. Several empirical studies provide evidence that organic growth is the dominant mode of growth for young and small firms (e.g., Davidsson and Delmar (1998), Kraemer and Venkataraman (1997), Levie (1997) and Delmar et al. (2003)). Whereas organic growth is the main mode for non-high-growth firms, acquisition growth seems more important for high-growth firms (HGFs). Clarysse et al. (2011) examine the growth paths of six young technology-based high-growth firms and document that two of these grew mainly via acquisitions. Brown and Mawson (2013) find that HGFs utilize overseas acquisitions to grow more aggressively in their international expansion than non-HGFs do. Some firms that grow very quickly and become superstar ventures use acquisitions; e.g., Cisco realized much of its rapid growth via acquisitions (Murmann et al. (2014)). We add to this literature by proposing a model with an endogenous growth mode for young firms. Our model shows that organic growth and acquisition growth are both substitutes and complements of each other. In particular, we illustrate through this model that if organic growth is very costly, then acquisition growth can be blocked by incumbent firms using preemptive acquisitions.

This paper also contributes to the literature studying when entrepreneurs will challenge incumbents or sell their ventures to the incumbents. To date, it has been found that a challenge is more likely to happen when entry costs are low, when the entrepreneurial firm has complementary assets, when brokers facilitating trade are not available, when incumbents' and entrepreneur's assets are substitutes, and when the intensity of product market competition is low (see, for instance, Gans and Stern (2000), Gans, Hsu and Stern (2002), Hellmann (2002), and Norbäck and Persson (2012)). We add to this literature by allowing entrepreneurs to challenge leading incumbents not only via organic growth but also through the acquisition of the nonleading incumbent firm's specific assets. This enables us to show that a lower cost for organic growth can increase the incentives challenging leading incumbents through acquisition growth. The leading incumbent understands that if it acquired the target firm, the gazelle would then invest organically anyway to grow, and therefore, the acquisition would not be sufficient to protect his market power as a leading incumbent firm from the gazelle. We also show that financial support for the organic growth of gazelles can increase gazelles' growth by acquisitions, rather than their organic growth, since incumbents' preemptive motives are reduced.

2 The Model

At the outset, an entrepreneur has an idea that has given rise to a viable business model. The entrepreneur will set up a gazelle firm denoted "g'', which will bring this novel business model to the market. The gazelle will then compete with an established incumbent firm "i'' in an oligopolistic market. There is also a target firm "t'' up for sale in the market. The target firm's business is no longer viable due to the emergence of the gazelle, but the target firm owns a distribution network, which may be an asset for the gazelle to serve the market. To make the analysis simple, we simply assume that there is a target firm for sale. In a more general setup, we could derive conditions under which the target firm would not be viable and how it would depend on cost asymmetries between the different firms. These cost asymmetries would then stem from different firm-specific assets, such as machinery, patents, management skills, etc.

The interaction between the gazelle and the incumbent occurs in three stages. In the first stage, the target firm sells its assets, denoted k_t. If the gazelle acquires the target firm's assets, it will gain a distribution cost advantage and will be able to distribute its good (or services) at zero distribution cost, i.e., $d = d_t = 0$; it will also hire L_t^g employees. However, the gazelle lacks financial resources and needs to borrow to be able to pay the acquisition price A. The total acquisition cost is $C_{acq} = A(1 + r)$, where r is the interest rate, i.e., the rate charged by the bank in the case of getting a loan.

In the second stage, if the gazelle does not acquire the target firm in the first stage, it has the option to grow organically on either a low scale or a large scale. If the gazelle grows organically on a small scale by investing in small-scale assets k_s, it will have a low fixed cost C_s, normalized to zero, but will face a high distribution cost, $d_s > 0$. The gazelle will then hire L_s^g employees. If the gazelle grows organically on a large scale, it will invest in large-scale assets, denoted k_N, at an investment cost of I. This investment, I, will reduce the gazelle's distribution cost to zero, i.e., $d_l = 0$. However, the gazelle needs to borrow to be able to pay the investment cost, I. The total cost of organic growth at a large scale is $C_l = I(1 + r)$. The gazelle will then hire L_l^g employees.

Finally, in the third stage, both the gazelle and the incumbent sell a homogeneous product in the market. The quantities produced and sold by the gazelle and the incumbent are, respectively, q_g and q_i. The gazelle may be exposed to an extra distribution cost d_s, unless it has invested in large-scale assets and has grown organically ($d_l = 0$) or it has acquired the target firm's assets ($d_t = 0$).[7] It should be noted that we do not study startups of small firms but of young firms that potentially will grow to be large firms. Most of these successful growing firm will then compete in situations where they and their rivals have market

[7] The theoretical model presented here builds on Norbäck's and Persson's model (2004, 2005, 2008). It relies on a similar structure of a game: an acquisition/entry stage using an auction with an externalities setup and an investment stage with an oligopolistic product market stage. The focus of the analysis by Norbäck and Persson (2004, 2005, 2008) is on how the acquisition-greenfield entry pattern of foreign firms depends on trade and greenfield investment costs. In this paper, the model framework is used to determine how the gazelle acquisition vs. organic growth patterns depend on **the** distribution cost, **the** interest rate and any subsidy targeted to a gazelle. A policy analysis for gazelle growth is added as a section later on.

power, and thus, oligopolistic interaction seems to be a reasonable assumption.

Note that we assume that the incumbent cannot acquire the gazelle. This could be because monopolization of the market would be blocked by antitrust authorities or that the gazelles business model would not fit into the incumbents' organization and thus the acquisition would not be profitable. A third reason would be that there are other viable incumbents present that would expand if the incumbent would acquire the gazelle and thereby make the acquisition unprofitable.

It should be noted that for some gazelles, bank financing might not be available since their ventures are characterized by high uncertainty. This should particular be true for high-tech ventures. If other financing, such as venture capital, is not available, the gazelle could then not grow. The case in which venture capital is available is examined in Section 4. It is shown there that the main analysis carries over to this case, as long as the VC firm's injection of capital covers all the investment costs. If so, the main analysis will hold for zero interest ($r = 0$).

The game is solved via backward induction; Section 2.1 describes the solution to the product market, and the following sections present the equilibrium organic growth decision and the solution to the acquisition game.

2.1 Period 3: The product market

The game is solved via backward induction. In the last stage, the gazelle and the incumbent compete in the product market in a Cournot fashion. The market demand function is given by $P(Q) = a - bQ$ and $a, b > 0$, where $Q = q_g + q_i$.[8] The incumbent already possesses one unit of distribution assets k_N, but the gazelle initially lacks such assets. Now, depending on which firm acquired the

[8] We assume that firms have a strictly concave inverse demand function, such that $P'(Q) < 0$ and $P''(Q) < 0$.

target's assets in period 2, we will have three different ownership structures to consider. To keep track of these, we denote the ownership structure by $M(k_g, L_g, k_i)$, where the gazelle possesses k_g units of assets and L_g numbers of employees, and the incumbent possesses k_i units. For example, $M(k_t, L_t, k_N)$ is then the duopoly in which the gazelle owns the target firm's assets k_t and hires L_t employees, whereas the incumbents has its initial assets of k_N, where $k_i = k_N$.

We make some assumptions regarding firms' marginal costs given different ownerships of the target's assets. A firm possessing at least one unit of large-scale assets is assumed to produce at zero marginal cost. Thus, the gazelle has a distribution cost disadvantage, d_s, per unit of output while serving the market when it grows organically at a small scale. We assume that the gazelle can avoid this extra cost d_s when it possesses large-scale assets (k_N). Having large-scale assets can be achieved by either acquiring the target firm's assets k_t or growing organically at a large scale k_N. In these two situations, the gazelle will have zero distribution cost as a variable cost, $d = 0$ ($d_t = 0$ or $d_l = 0$).

Let firm $j's$ profit function be defined by $\pi_j(q_g, q_i) = P(q_g + q_i)q_j - TC_j(q_j)$. Then, $\pi_j^D(d_s)$ indicates the duopoly profit for firm $j = g, i$ when the incumbent firm i has a variable cost of zero ($d_i = 0$) and the gazelle firm g has a variable cost of d_s. Moreover, $q_g(d_s)$ denotes the corresponding equilibrium quantity of the gazelle, and $d_s^{\max}$ represents d_s satisfying $q_g(d_s) = 0$, meaning that the gazelle does not produce or serve the goods in the market due to the high distribution cost. In addition, π^M denotes the monopoly profit when the incumbent firm becomes a monopolist and has zero distribution cost. We can make the following observation:

Observation 1: The incumbent will be hurt by the gazelle's acquisition growth or the gazelle's organic large-scale growth since the incumbent will then face tougher competitor. Thus, the incumbent will have an incentive to prevent the gazelle's expansion.

This preemptive motive is important for the growth pattern of the gazelle, as will be shown below.

2.2 Period 2: Organic growth: Small scale or large scale?

If the gazelle does not acquire the target firm in the first stage, it may grow organically in the second stage. The gazelle might either undertake large-scale organic growth investment at a cost of $C_l = I(1 + r)$, where I is the original investment cost and r is the interest rate (cost of finance), or grow on a small scale and have a zero fixed cost of $C_s = 0$, but it will have a variable distribution cost greater than zero ($d_s > 0$). In the case of large-scale organic growth, the gazelle reduces its distribution costs from d_s to 0.

The gazelle needs to borrow to be able to invest at a large scale. We define $\bar{I}(d_s, r)$ as the value of the investment cost such that the gazelle firm is indifferent between the alternatives of large-scale and small-scale organic growth. Formally, we find that large-scale organic growth takes place when the following holds: $\bar{I}(d_s, r) = \frac{\pi_g^D(0) - \pi_g^D(d_s)}{1+r}$ if $d_s < d_s^{\max}$, and $\bar{I}(d_s, r) = \frac{\pi_g^D(0)}{1+r}$ otherwise.
In the above expression, $\pi_g^D(0)$ stands for the gazelle's profit when it has zero distribution cost, and $\pi_g^D(d_s)$ stands for the gazelle's profit when it has the distribution cost $d_s > 0$.

Since the gazelle's small-scale growth profits $\pi_g^D(d_s)$ decrease monotonically in d_s, the critical investment cost $\bar{I}(d_s, r)$ is increasing in d_s and reaches its maximum at the point at which $d = d_s = d^{\max}$. When $d_s > d^{\max}$, the gazelle does not produce and sell the good unless it invests organically in large-scale assets with a critical investment cost of $\bar{I}(d_s, r) = \frac{\pi_g^D(0)}{1+r}$, where $\pi_g^D(0) = C_l = I(1 + r)$.

We can make the following observation:

Observation 2: If the cost of financing r is too high, large- scale organic growth will not be profitable for the gazelle.

Whether the gazelle will have an incentive for large-scale organic growth is an important factor in determining the outcome of the bidding competition over the target firm acquisition, as will be explained below.

2.3 Period 1: The acquisition game

The acquisition process is depicted as a perfect-information auction[9] in which the two firms simultaneously post bids, and the bidder with the highest bid obtains the target firm.[10] To solve the acquisition auction and determine bids, we need to determine the valuations of the bidders for obtaining the target firm's assets. To aid in this, we introduce the net gain function $N_g(A)$, which defines the net gain for the gazelle if the acquisition price is A. Furthermore, we assume that the acquiring firm borrows to be able to pay the acquisition price and has to pay an interest rate r.

A gazelle will have net gain functions defined as follows:

$$N_g(A) = \underbrace{[\pi_g^D(0) - (1+r)A]}_{\text{Acquire}} - \underbrace{[\pi_g^D(0) - (1+r)I]}_{\text{Do not acquire}}, \text{ if } I < \bar{I}(d_s, r) \quad (1)$$

$$N_g(A) = \underbrace{[\pi_g^D(0) - (1+r)A]}_{\text{Acquire}} - \underbrace{[\pi_g^D(d_s)]}_{\text{Do not acquire}}, \text{ if } I > \bar{I}(d_s, r). \quad (2)$$

Let us start with the first line, where the alternative to the acquisition for the

[9] Thus, we abstract from asymmetric information problems to focus on the competitive effects of the acquisition.

[10] If more than one firm posts such a bid, each such firm obtains the assets with equal probability. The winning buyer pays an amount equal to his bid. The auction will be solved for Nash equilibria in undominated pure strategies. There is assumed to be a smallest monetary unit, denoted ε. We assume ties to be randomly broken and all equalities in valuations to be ruled out. The smallest amount ε is chosen such that all inequalities are preserved if ε is added or subtracted.

gazelle is large-scale organic growth. The first term consists of product market profits net of the acquisition price including interest payments $\pi_g^D(0)-(1+r)A$. The second term is the product market profits in the case where the gazelle does not acquire the target firm and instead grows organically on a large scale, net of the investment cost including interest payments $\pi_g^D(0)-(1+r)I$.

We should note that a key insight is that a gazelle's maximum willingness to pay (v_g) for the target firm depends on what happens if it does not acquire the target firm, i.e., it might then invest organically on a large scale or not. The second line describes the valuations when the gazelle instead undergoes small-scale organic growth.

For the gazelle, the maximum willingness to pay, v_g, can be determined to be $v_g = \min A \ , \ s.t \ \ N_g(A) \geq 0$, where A is the acquisition price. Solving for $N_g(A)=0$, we obtain the maximum willingness to pay for each of the two net gain functions as follows:

$$v_g = I \text{ if } I < \bar{I}(d_s, r) \tag{3}$$

$$v_g = \frac{\pi_g^D(0)-\pi_g^D(d_s)}{1+r} \text{ if } I > \bar{I}(d_s, r). \tag{4}$$

Let us now turn to the incumbent's valuation of the target firm's assets. The maximum willingness to pay for the incumbent, v_i, to acquire the target firm can be directly written as

$$v_i = 0 \text{ if } I < \bar{I}(d_s, r) \tag{5}$$

$$v_i = \pi_i^D(d_s)-\pi_i^D(0) \text{ if } I > \bar{I}(d_s, r) \text{ and } d_s < d^{\max} \tag{6}$$

$$v_i = \pi_i^M-\pi_i^D(0) \text{ if } I > \bar{I}(d_s, r) \text{ and } d_s > d^{\max} \tag{7}$$

We note that the game is solved with the method of backward induction. We look for a subgame perfect equilibrium. Let us start with Equation 5, where the alternative to the acquisition of the incumbent itself is the large- scale organic growth of the gazelle firm. Then, the incumbent has no value for buying the target firm's assets. Its willingness to pay for the acquisition will be zero, $v_i = 0$. The reason is that the intensity of competition is not affected by the incumbent's acquisition of the target firm, and no cost savings occur when $I < \bar{I}(d_s, r)$.

However, Equation 6 shows that when the alternative to the acquisition of the target firm by the incumbent is the small-scale organic growth of the gazelle, the acquisition by the incumbent will have an effect on the competition in the product market. The incumbent will then ensure that it will face a weaker rival if it acquires the target firm's assets. This is captured by the term $\pi_i^D(d_s) - \pi_i^D(0) > 0 \Rightarrow v_i > 0$. The marginal willingness to pay in the acquisition will be greater than zero for the incumbent.

Finally, Equation 7 captures the case when the acquisition by the incumbent totally blocks the expansion of the gazelle firm and the incumbent becomes a monopolist. In this case, large-scale organic growth is not feasible for the gazelle due to the high investment cost, and production on a small scale is also not feasible for the gazelle due to the high distribution cost.

The analysis is straightforward since both firms will bid their maximum willingness to pay, and we can state the following result:

Lemma 1 *Let firm i be the firm with the highest valuation, v_i. The target firm's assets are then acquired by firm i at a price equal to the other firm's valuation: firm h's valuation of obtaining the target firm's assets instead of the firm i itself, v_h.*

3 The gazelle growth pattern

One more definition is required to proceed. To this end, we consider the situation where no large-scale organic growth investment takes place. The only way of decreasing the distribution cost to zero, $d = 0$, is an acquisition for the gazelle. We make use of the following definition:

Definition 2 *Preemption condition: As v_g is the maximum willingness to pay for the gazelle to acquire the target firm and v_i is the maximum willingness to pay for the incumbent firm, let d_s^* be the value of the distribution cost satisfying the following equality when $I > \bar{I}(d_s, r)$:*

$$v_g = \frac{\pi_g^D(0) - \pi_g^D(d_s^*)}{1+r} = \pi_i^D(d_s^*) - \pi_i^D(0) = v_i.$$

Therefore, d_s^* is the distribution cost at which the gazelle and the incumbent firms' valuations of the target firm's assets coincide with each other, given that no large-scale organic growth investment takes place. We are now set to derive the equilibrium ownership structure in the gazelle growth model presented above.

The game is solved backwards, and the following proposition identifies the equilibrium ownership structure and the equilibrium auction price.

Proposition 3 *The equilibrium ownership structure and the gazelle growth pattern are as follow:*

*(i) If $I > \bar{I}(d_s, r)$ and $d_s > d_s^*$, the incumbent firm obtains the target firm's assets and the gazelle firm grows organically on **a** small scale. The acquisition price is $A = v_g = \frac{\pi_g^D(0)}{1+r}$ when $d_s \geq d^{\max}$ and $A = v_g = \frac{\pi_g^D(0) - \pi_g^D(d_s)}{1+r}$ when $d_s < d^{\max}$. The ownership structure is $M(k_s, L_s^g, k_N + k_t)$.*

(ii) If $I > \bar{I}(d_s, r)$ and $d_s < d_s^*$, the gazelle grows by acquisition. It obtains the target firm's assets at a price of $A = v_i = \pi_i^D(d_s) - \pi_i^D(0)$. The ownership

structure is $M(k_t, L_t^g, k_N)$, where the gazelle owns the asset of the target firm, k_t.

(iii) If $I < \bar{I}(d,r)$, the gazelle grows by acquisition. It obtains the target firm's assets at a price of $A = v_i = 0$. The ownership structure is $M(k_t, L_t^g, k_N)$, where the gazelle owns the assets of the target firm, k_t.

Proof. See the Appendix.

To understand the logic of Proposition 3, we should note that firms' valuations depend on the gazelle's decision in stage 2, i.e., whether it makes investments and grows organically on a large scale. In the following subsections, we will examine the two separate cases more closely.

High cost of large-scale organic growth, i.e., $I > \bar{I}(d_s, r)$: Let us now characterize the difference between the valuations of the incumbent and the gazelle, $v_i - v_g$, when the gazelle will not grow on a large scale organically in stage 2, if it loses the auction in stage 1. This is the case when the investment cost is too high, i.e., $I > \bar{I}(d,r)$.

In this case, $v_i - v_g = \pi_i^D(d) - \pi_i^D(0) - \left(\frac{\pi_g^D(0) - \pi_g^D(d)}{1+r}\right)$. It follows directly that $v_i - v_g = 0$ when $d = 0$, i.e., the value of avoiding high distribution costs equals the value of preemption. Let us now analyze how an increase in d affects the difference in valuations. At first sight, one might believe that an increase in d would make the value of avoiding the distribution cost, v_g, increase more than the value of preemption (the value of decreasing competition through target firm acquisition by the incumbent), v_i, because the gazelle has an incentive to avoid these higher distribution costs. However, more effects are involved. Following Norbäck and Persson (2004), we differentiate $v_i - v_g$ with respect to d and use the FOC's yields: [11]

$$\frac{d(v_i - v_g)}{dd} = \frac{dQ(d)}{dd} P' q_i(d) + \frac{dq_i(d)}{dd} d - q_g(d). \tag{8}$$

The first term in Equation (8) captures the *anticompetitive* effect due to the fact that an increased distribution cost induces the gazelle to be less aggressive in its market interaction, which softens competition and increases the revenues for the incumbent. The second term reflects the decrease in total distribution costs as the incumbent firm steals business from the gazelle. This effect is referred to as the *business-stealing effect*. The first two terms in Equation (8) increase the incumbent firm's valuation of the target firm's assets relative to that of the gazelle firm. The third term, the *direct distribution cost effect*, decreases the incumbent firm's valuation relative to that of the gazelle, as the gazelle firm faces higher distribution costs.

Let us now analyze how the difference in valuations $v_i - v_g$ depends on d. At $d = 0$, the *distribution cost effect* dominates the *anticompetitive effect*. At higher distribution costs, however, the direct distribution cost effect is weaker since the gazelle's sales are smaller. On the other hand, both the anticompetitive and the business-stealing effects are stronger since the incumbent has a larger market share, and each unit shifted from the gazelle to the incumbent implies larger cost savings. Hence, once the distribution costs become sufficiently high, $v_i - v_g$ will rise. When the distribution costs become sufficiently high at $d = d^{\max}$, the incumbent becomes a monopolist, and consequently, v_i is then greater than v_g.

In summary, when the distribution costs are high, the incumbent firm can prevent the gazelle from becoming a tough competitor, and thus, its willingness to pay for the target firm's asset increases. The preemptive value is getting higher. Consequently, the acquisition by the incumbent is more likely to happen.

[11] Please refer to Appendix H.3 for the derivation of Equation 8.

Low cost of large-scale organic growth, i.e., $I < \bar{I}(d_s, r)$**:** In this case, the gazelle will grow organically at a large scale in stage 2 upon losing the auction in stage 1 since the investment cost is sufficiently low, i.e., $I < \bar{I}(d, r)$. If the incumbent acquires the target firm's asset, the gazelle invests at a large scale organically, and no firm faces any distribution cost in the product market, but the gazelle faces an investment cost, I. If the gazelle acquires the target firm's asset, the investment cost is avoided. Consequently, since $v_g - v_i = I$ and $I > 0$, it follows that $v_g > v_i$.

Intuitively, the lower cost of large-scale organic growth decreases the incumbent firm's willingness to pay because the gazelle can credibly threaten to grow organically at a large scale if it does not obtain the target firm's asset. This implies that the value of preemption disappears. At the same time, the gazelle is still willing to pay I for the target firm's asset.

The above analysis thus contains several noteworthy features.

Observation 3: When the cost of large-scale organic growth is high and the distributional costs are high for small-scale entry, the incumbent firm obtains the target firm's assets despite the fact that the gazelle would save considerably on the distribution cost by obtaining the target firm. The reason for this is that when obtaining the target firm's assets, the incumbent firm gains high market power in the product market since it then faces a competitor with high distribution costs.

Observation 4: When the cost of growing organically on a large scale is sufficiently low, the gazelle grows by acquisition and obtains the target firm's asset. The reason is that the strategic position in the bidding competition in acquiring the target firm is improved for the gazelle since it can credibly threaten to invest organically on a large scale if it does not acquire the target firm.

4 Venture Capital

Empirical studies show that venture capitalists mostly concentrate their investment in early-stage entrepreneurial firms that have a potential to grow fast. For instance, Gompers (1995) finds from data on 794 venture capital-backed firms, as venture capital (VC) firms invest their funds mostly in high-technology startup companies, where the informational asymmetries are highest. Gompers also concludes from his data that the duration of VC financing is related to the nature of invested firms' assets: higher industry ratios of tangible assets to total assets, lower market-to-book ratios, and lower R&D intensities are associated with VC firms' longer funding duration. Therefore, as we see in the real-life business case of Flipkart, quickly growing start-ups are more likely to attract more VC funding in longer periods to finance their growth by acquisitions because they increase their tangible assets by acquiring other firms. When there is a specific asset to be acquired, venture capitalists are more eager to deal with information asymmetries in gazelles that naturally exist.

By providing capital funds to entrepreneurial firms, VC firms infuse financial strength and competence into ventures. Furthermore, VC firms improve the overall efficiency of entrepreneurial firms by helping them decrease their production costs and/or increase their sales (Chemmanur et al, 2011). At this point, we focus on how increased financial strength affects the growth pattern of an entrepreneurial firm, a gazelle.

Let us now incorporate VC financing into our model. Once an entrepreneurial firm makes a deal with a VC firm, it will obtain an additional equity in cash, F. This VC fund F can be used partly or totally in financing the acquisition or the large-scale organic growth. We assume that there are two symmetric VC firms competing to invest in the gazelle. We also assume that the VC firms are perfectly informed about the future profitability of the gazelle. Moreover, the gazelle is indifferent between these two symmetric VC firms. We also simplify the analysis by assuming that the gazelle needs a fixed amount of capital, F.

In equilibrium, one of these VC firms is investing F amounts of funds in the gazelle and getting an amount of shares valuing in total the equivalent of the amount F of the gazelle's ownership. This follows from the fact that the VC firms compete and make zero profit on their investment. Therefore, the investor VC firm will receive $\frac{F}{TSHE}$ amounts of shares in the gazelle's ownership, where TSHE stands for the gazelle's total shareholder's equity after the injection of VC equity. TSHE is the net present value of the gazelle's future product market profits, net-off investment costs.

We do not focus on any other detailed conditions, such as dividend policy, management, exit of the VC firm, etc., of the contract between the VC firm and the gazelle. We suppose that the level of VC ownership in the gazelle will not affect the gazelle's objective function. The gazelle will always maximize its profits.

We will now examine how the infusion of capital will affect the growth pattern of the gazelle. In our analysis, we once more distinguish between the cases when large-scale organic growth is feasible and when it is not feasible.

High cost of large-scale organic growth, i.e., $I > \bar{I}(d_s, r)$: Here, we assume that if the gazelle did not acquire the target in the auction in stage 1, it would not grow organically at a large scale in stage 2. From Lemma 1, it follows that a gazelle acquisition takes place if and only if $v_i - v_g < 0$.

Our analysis now concentrates on how the valuations of the target firm for the incumbent and the gazelle are affected by the introduction of the VC financing. First, note that the incumbent's valuation is not affected and is still given by expressions (6) and (7). However, the gazelle's valuation will change. To see this change, we rewrite the gazelle net gain function as follows:

$$N_g(A) = \underbrace{[\pi_g^D(0) - (1+r)(A-F)]}_{\text{Acquire}} - \underbrace{[\pi_g^D(d_s) + F]}_{\text{Do not acquire}}, \text{ if } I > \bar{I}(d_s, r). \quad (9)$$

The VC firm's infusion of capital reduces the acquisition cost, as captured by the term $(1+r)(A-F)$. Moreover, in the case of no expansion, the infusion of capital will come up as an additional capital F, where it is assumed that there is no interest on this capital.[12] Solving for $N_g(A) = 0$, we obtain the maximum willingness to pay for the gazelle:

$$v_g = \frac{\pi_g^D(0) - \pi_g^D(d_s) + rF}{1+r} \text{ if } I > \bar{I}(d_s, r). \quad (10)$$

As a result, the gazelle's valuation v_g will increase by the amount of $\frac{rF}{1+r}$, which captures the saved interest rate expense associated with the acquisition. Therefore, the difference between the maximum willingness to pay of the incumbent and the entrepreneurial firm, $v_i - v_g$, decreases when the entrepreneurial firm receives venture capital equity, F.

Low cost of large-scale organic growth, i.e., $I < \bar{I}(d_s, r)$: We examine the case in which $I < \bar{I}(d_s, r)$. The entrepreneurial firm can credibly threaten the incumbent with large-scale organic growth, and the value of preemption for the incumbent will be zero. Moreover, the gazelle's valuation will not change. After obtaining VC funds F, the gazelle has a net gain function defined as below:

[12] This assumption is crucial. When allowing the gazelle to earn the same interest on its bank account as the bank requires for lending, venture capital would have no effect on the valuations.

$$N_g(A) = \underbrace{[\pi_g^D(0) - (1+r)(A-F)]}_{\text{Acquire}} - \underbrace{[\pi_g^D(0) - (1+r)(I-F)]}_{\text{Do not acquire}}, \text{ if } I < \bar{I}(d_s, r). \tag{11}$$

Solving for $N_g(A) = 0$, we obtain

$$v_g = I \text{ if } I < \bar{I}(d_s, r). \tag{12}$$

Since the capital infusion can be used for financing either acquisitions or large-scale organic growth, the tradeoff between acquisitions and large-scale growth is not affected.

How is the large-scale organic growth cutoff investment amount, $\bar{I}(d_s, r)$, affected by capital injection of the VC firm? We evaluate how the incentive for undertaking large-scale organic growth changes after a VC equity injection to the gazelle, given that the gazelle does not acquire the target firm in the first period. Formally, we state that

$$\bar{I}(d_s, r) = \frac{\pi_g^D(0) - \pi_g^D(d_s) + rF}{1+r} \text{ if } d_s < d_s^{\max} \text{ and}$$

$$\bar{I}(d_s, r) = \frac{\pi_g^D(0) + rF}{1+r} \text{ otherwise.}$$

As can be observed from the above formulas, there is a positive relationship between the cutoff investment level and the VC funding. The higher the VC financing, the higher $\bar{I}(d_s, r)$ because the gazelle is willing to pay more for the large-scale organic growth investment since the cost of financing is lower.

$$\frac{d\bar{I}(d_s, r)}{dF} = \frac{r}{1+r} > 0$$

Therefore, the infusion of the VC fund makes investment in large-scale organic growth more feasible and thus the gazelle's threat more credible since the interest cost of financing large-scale organic growth can be reduced by the amount rF.

To summarize, we can state the following results:

Proposition 4 *A gazelle that receives venture capital funding is more likely to grow by acquisition for the following reasons:*

(i) The financing cost of an acquisition for the gazelle decreases due to saved interest payments.

(ii) The cutoff investment cost of large-scale organic growth for the gazelle increases due to decreased interest payments in financing the investment. This improves the gazelle's strategic position and reduces the incumbent's preemptive acquisition motive.

As a final remark, it can also be argued that increasing the level of equity in the gazelle's balance sheet will ease the bargaining conditions of the entrepreneurial firm with banks or other types of debtors. In the case in which the VC fund is not enough to finance the acquisition alone or the acquisition is partly financed via VC equity and partly via bank borrowing, the cost of debt, r, may decrease for the gazelle. Having a higher amount of equity, and thus a lower leverage ratio, will enable the gazelle to face a lower rate of interest in bargaining with the debtors. This will further strengthen the results derived above.

In practice, the VC also provides assets other than financial assets to the venture, such as knowledge and network assets. Unless these assets are much more useful for organic growth than for acquisition growth, our results will hold. Extending the analysis allowing for different type of investors with different assets and examining how this will affect the growth pattern seem to be interesting avenues for future research.

Additionally, note that if organic growth can only occur slowly because the needed resources are provided step-by-step by the venture capitalist, an incumbent acquisition may substantially slow down the market penetration of the gazelle. Hence, even if venture capital is available, the incumbent may still have a strong incentive to acquire the target.

5 Policy

Policymakers are concerned with gazelle development and growth. They would like to find an efficient means for gazelle development. The endogenous nature of the buyer's identity, the competition from the incumbent and the determination of the auction price in the present analysis, in addition to the different types of growth strategies available for the gazelle, all imply that the optimal policy is very complicated. Therefore, we will make a couple of remarks about policy. These remarks seem to open an interesting avenue of future investigations.

5.1 Reduction in the interest rate

One of the most discussed means of supporting gazelles is to provide them with financial resources under better financial conditions. To capture this in our model, we examine how a reduction in the interest rate, r, affects the equilibrium market structure and the gazelle growth pattern.

High cost of large-scale organic growth, i.e., $I > \bar{I}(d_s, r)$: Given that large-scale organic growth is not feasible, i.e., $I > \bar{I}(d_s, r)$, it follows that when r decreases, an acquisition by the gazelle becomes more likely. To analyze this, remember that according to Lemma 1, an acquisition by the gazelle takes place if and only if $v_i - v_g < 0$. In this expression, the incumbent's marginal willingness to pay, $v_i = \pi_i^D(d_s) - \pi_i^D(0)$, is independent of r. On the other hand, we show that the gazelle's marginal willingness to pay, $v_g = \frac{\pi_g^D(0) - \pi_g^D(d_s)}{1+r}$, decreases in r since

$$\frac{dv_g}{dr} = -\frac{\pi_g^D(0) - \pi_g^D(d_s)}{(1+r)^2} < 0$$

As a result, by reducing the interest rate, the government lessens the acquisition cost of the gazelle and thus increases the gazelle's marginal willingness to pay, v_g. As a result, a reduction in the interest rate makes an acquisition more likely for the gazelle.

Low cost of large-scale organic growth, i.e. $I < \bar{I}(d_s, r)$: We now analyze the situation after the reduction in the interest rate when $I < \bar{I}(d_s, r)$. The entrepreneurial firm can credibly threaten the incumbent with large-scale organic growth. Then, the value of preemption for the incumbent will be zero, as expected. Moreover, the gazelle's valuation will not change after the decrease in the interest rate. The gazelle has a net gain function defined as follows:

$$N_g(A) = \underbrace{[\pi_g^D(0) - (1+r)A]}_{\text{Acquire}} - \underbrace{[\pi_g^D(0) - (1+r)I]}_{\text{Do not acquire}}, \text{ if } I < \bar{I}(d_s, r). \tag{13}$$

Solving for $N_g(A) = 0$, we obtain

$$v_g = I \text{ if } I < \bar{I}(d_s, r). \tag{14}$$

As can be observed from the above, the gazelle's marginal willingness to pay in an acquisition is independent of the interest rate when large-scale organic growth is already feasible. Since the gazelle can benefit from a lower interest rate in both financing an acquisition and making an investment in large-scale organic growth, the tradeoff between an acquisition and large-scale growth is not influenced by the reduction in the interest rate.

How is the large-scale organic growth cutoff investment amount, $\bar{I}(d_s, r)$, influenced by a lower interest rate? Given that the gazelle does not acquire the target firm in the auction, we analyze how its incentive for making a large-scale organic growth investment will be influenced by a drop in the interest rate. The large-scale organic growth investment cutoff level, $\bar{I}(d_s, r)$, will increase as the interest rate r drops. The lower the interest rate is, the higher $\bar{I}(d_s, r)$ because a decreasing interest rate makes the gazelle more willing to make a large-scale organic growth investment. The financial capacity of the gazelle improves. We can view the negative relation between the cutoff investment level for large-scale organic growth, $\bar{I}(d_s, r) = \frac{\pi_g^D(0) - \pi_g^D(d_s)}{1+r}$, and the interest rate, r, as follows:

$$\bar{I}(d_s, r) = \frac{\pi_g^D(0) - \pi_g^D(d_s)}{1+r}$$
$$\frac{\partial \bar{I}(d_s, r)}{\partial r} = [\pi_g^D(0) - \pi_g^D(d_s)](-1)(1+r)^{-2}$$

Then we evaluate the sign of the derivative of $\bar{I}$ with respect to r calculated above,

$$\frac{\partial \bar{I}(d_s, r)}{\partial r} = \frac{-[\pi_g^D(0) - \pi_g^D(d_s)]}{(1+r)^2} < 0$$

$$\frac{d\bar{I}(d_s, r)}{dr} < 0 \text{ since we know that } \pi_g^D(0) > \pi_g^D(d_s).$$

It is a fact that increasing financial capacity due to the drop in the interest rate makes the gazelle more likely to undergo large-scale organic growth. However, this also means that the gazelle can credibly threaten the incumbent with a more probable large-scale organic growth scenario and can decrease the incumbent's preemptive motive in the acquisition in the first stage.

Furthermore, when the interest rate drops, the financial capacity of the gazelle increases for both large-scale organic growth and acquisitions. However, due to the competition, the gazelle is more willing to make an acquisition in the first stage rather than a large-scale investment in the second stage. Thereby, the gazelle will save time in the competition since making a large-scale investment one period later and an investment (e.g., constructing a warehouse from scratch) will take some time. In the first stage, with the drop in the interest rate, the gazelle's willingness to acquire increases more than that of the incumbent since the incumbent does not need to borrow under any circumstances. This implies that an acquisition by the gazelle becomes more likely when the interest rate in the economy decreases since the gazelle can also obtain the target firm's assets at a lower borrowing cost. As an interesting result, better financial conditions for organic growth can actually reduce organic growth and can instead increase acquisition growth.

Proposition 5 *A gazelle that receives better financial conditions (lower interest rates r) is more likely to grow by acquisition for the following reasons:*

(i) A decrease in the interest rate will motivate the gazelle to acquire the target firm by increasing the gazelle's marginal willingness to pay: $\frac{dv_g}{dr} < 0$.

(ii) A decrease in the interest rate will make the gazelle's acquisition more likely since the gazelle will have a larger chance to undergo large- scale organic growth in the second stage due to $\frac{d\bar{I}(d_s,r)}{dr} < 0$. *This enables the gazelle to pose a credible threat against the incumbent in the first stage. The incumbent's preemptive motive decreases with a credible threat.*

5.2 Subsidy for organic growth

Another means of providing financial support to gazelles is directly subsidizing the organic growth of these firms. In the United States, for example, one of the main programs to promote small businesses is the Technological Innovation Program (TIP), which subsidizes the commercialization of successful prototypes with up to USD 3 million. This support scheme is available only if the SME markets the product itself or is the leading company in a joint venture (OECD, 2010, p. 106). See OECD (2010) for a listing of similar support schemes for SMEs in all OECD member states. To analyze the effects of a subsidy targeted to gazelles in our model, we examine how a subsidy of S targeted at large-scale organic growth affects the equilibrium market structure and the gazelle growth pattern. In our analysis of the effect of a subsidy on the gazelle's growth pattern, we once more distinguish between the case in which large-scale organic growth is feasible and the case in which it is not feasible.

High cost of large-scale organic growth, i.e., $I > \bar{I}(d_s, r)$**:** Here, we assume that if the gazelle were not to acquire the target firm in the auction in stage 1, it would not grow organically at a large scale in stage 2. From Lemma 1, it follows that a gazelle acquisition occurs if and only if $v_i - v_g < 0$. Since the subsidy of S affects neither v_i nor v_g, the support for organic growth, a subsidy, has no impact on the growth pattern of the gazelle when large-scale organic growth has already become infeasible.

Low cost of large-scale organic growth, i.e., $I < \bar{I}(d_s, r)$: Let us now examine the case in which $I < \bar{I}(d_s, r)$, i.e., large-scale organic growth is feasible. The gazelle firm can credibly threaten the incumbent with large-scale organic growth, and the value of preemption for the incumbent will be zero. However, the gazelle's valuation will change, as can be observed from the gazelle's net gain function:

$$N_g(A) = \underbrace{[\pi_g^D(0) - (1+r)A]}_{\text{Acquire}} - \underbrace{[\pi_g^D(0) - (1+r)(I-S)]}_{\text{Do not acquire}}, \text{ if } I < \bar{I}(d_s, r) \quad (15)$$

Solving for $N_g(A) = 0$, we obtain

$$v_g = I \ - S \text{ if } I < \bar{I}(d_s, r). \quad (16)$$

Since the subsidy S can only be used for saving from the financing cost of organic growth, the tradeoff between an acquisition and large-scale organic growth is affected. The gazelle will obtain the target firm. However, this result is valid as long as $I > S$.

How is the large-scale organic growth investment cutoff level, $\bar{I}(d_s, r)$, affected by a subsidy targeted at organic growth? Let us analyze how the incentive to undertake large-scale organic growth changes once a subsidy is given under the condition that the gazelle did not acquire the target firm in the first stage. When the gazelle is provided with a subsidy, an investment in large-scale organic growth will be more affordable for the gazelle. Formally, we know that after a subsidy, large- scale organic growth occurs when

$$\bar{I}(d_s, r) = \frac{\pi_g^D(0) - \pi_g^D(d_s) + rS}{1+r} \text{ if } d_s < d_s^{\max} \text{ and}$$

$$\bar{I}(d_s, r) = \frac{\pi_g^D(0) + rS}{1+r} \text{ otherwise.}$$

The gazelle will save some interest payment at the amount of rS by borrowing less from the debtors by the amount of S. The gazelle's financial capacity for large-scale organic growth improves after receiving a subsidy from the government. Therefore, $\bar{I}(d_s, r)$ increases due to the subsidy given to the gazelle:

$$\frac{\partial \bar{I}(d_s, r)}{\partial S} = \frac{r}{1+r} > 0 \text{ since } r > 0.$$

Since the subsidy gives a larger possibility of large-scale organic growth to the gazelle, the gazelle could more credibly threaten the incumbent. Therefore, in the first stage, the incumbent will not be interested in acquiring the target firm, and the value of the preemption move will disappear.

In conclusion, a subsidy for large-scale organic growth implies that large-scale organic growth becomes profitable for the gazelle in the case in which it has not obtained the target firm's assets. This, in turn, implies that an acquisition by the gazelle could become more likely since the gazelle could then credibly "threaten" the incumbent in the first stage to grow organically on a large scale in the second stage if it does not obtain the target firm's assets. As an interesting result, subsidies targeted to increase the organic growth of gazelles can actually reduce the likelihood of organic growth and instead increase the likelihood of acquisition growth of these firms.

Proposition 6 *A subsidy targeted to increase the gazelle's organic growth can,* *in fact, lead to an increase in the acquisition growth:*

(i) A subsidy, S, will make large-scale organic growth more likely for the gazelle in the second stage due to $\frac{\partial \bar{I}(d_s,r)}{\partial S} > 0$, and therefore, the gazelle will pose a more credible threat to the incumbent in the first stage. The incumbent's preemptive motive will decrease.

One might argue that the key to an efficient gazelle subsidy policy is to target these to specific firms. However, growth rates of gazelles tend to be very erratic, i.e., these firms are essentially 'one-hit wonders', unable to repeat their high growth rates.[13] This implies that targeted policies to specific firms seem difficult in practice.

6 Extension

6.1 Labor market effects

What would be the effects of gazelle growth on the demand for labor? This will depend on several characteristics of gazelle growth. First, it will depend on whether the gazelle's labor investment is a substitute or a complement to its capital investment. For instance, if the capital investment requires that many computer programmers or sales personnel need to be hired, then capital and labor are complements. In this case, L_i^g will be large, and employment will increase. However, if the capital investment is in machinery and it replaces low-skilled employees, then capital and labor are substitutes. In this case, L_i^g will be small, and employment will decrease.

Second, it will depend on whether the gazelle growth mostly replaces the incumbent's sale or if it creates new demand. If the gazelle growth creates new demand, then aggregate demand for labor will be high, and L_i^g will be large.

[13] See, for instance, Parker et al. (2010), Hölzl (2014) and Daunfeldt and Halvarsson (2015).

Third, it will depend on the type of acquisition growth of the gazelle. If the acquisition is mostly associated with using existing capital from the acquired firm and reducing existing employees, then employment will go down, i.e., L_i^g will be small. However, if there are complementary labor and capital investments associated with the gazelle's acquisition, employee hiring might actually increase, i.e., L_i^g will be large.

6.2 The incumbent's incentive and possibility to acquire the gazelle

What would be the effects of allowing the incumbent to acquire the gazelle? Such an acquisition would give rise to a stronger market power effect than the acquisition of the target firm. This implies that an acquisition of the gazelle by the incumbent might be the outcome of the game if such an alternative were incorporated into the model. However, there are a couple of reasons why an acquisition of the gazelle by the incumbent may not take place. First, in most jurisdictions, a merger that concentrates the market too much will not be allowed. In fact, it seems as this could be an explanation why Amazon could not acquire Flipkart in the example mentioned in the introduction.[14] Second, in the case in which there is more than one incumbent acquiring the gazelle, an acquisition might benefit the non-acquiring incumbents most since they will expand their business following the merger and thus benefit the most (Norbäck and Persson (2012)). Additionally, this aspect seems consistent with our example of Flipkart, in which Walmart (an outsider to the e-commerce market) was able to outbid Amazon (an incumbent) and where a large third

[14] An acquisition of Flipkart by Amazon is likely to be scrutinized by the Competition Commission of India (CCI), given the dominant market share these two entities have in the e-commerce space—around 70 percent collectively (see Flipkart likely to go with Walmart even as Amazon makes a last-ditch bid: report BusinessToday, May 3, 2018, www.businesstoday.in/current/corporate/flipkart-walmart-amazon-acquire-buy-deal-stake-softbank/story/276148.html).

player (Snapdeal) was present in the market. Thus, our identified mechanism seems in play also in the case that the possibility of acquiring the gazelle was included in the model. However, an interesting avenue for future research would be to extend the analysis to the case in which the possibility of acquiring the gazelle was included in the model and analyze how the incentives to start a gazelle firm are affected by such a possibility and also to study the timing of such acquisitions.

7 Conclusion

Gazelles can either grow organically by setting up a new facility or acquire a suitable target firm to grow. We show that the lower cost for organic growth can increase the incentives for acquisition growth. This seems counterintuitive at first sight but makes sense as soon as one takes into account how an acquisition price of a target firm is determined. When the cost of organic growth is high for the gazelle, the incumbent will benefit considerably from acquiring the target firm since the incumbent will thereby protect its market power in the product market. When the cost of organic growth drops, the incumbent takes into account that if it acquires the target firm, the gazelle will invest organically anyway to grow, and therefore, the acquisition will not be sufficient to protect the incumbent's market power. The gazelle could acquire the target firm at a good price, and it will prefer acquisition growth to organic growth.

One of the major obstacles to gazelle growth is the cost of financing. Receiving equity from a venture capital firm will reduce the financing burden of the gazelle for growth. The financial strength gained from the venture capital fund will reduce the incumbent's possibility and motivation to impede the gazelle's growth by making the acquisition itself. Therefore, venture capital support will induce the gazelle's acquisition.

These results suggest that financial policies supporting the growth of gazelles may, in particular, ignite gazelle growth by acquisitions. The reason is that these policies not only reduce the financial cost for the gazelles but also reduce the possibility for incumbents to use strategic gazelle growth acquisitions. Policies improving both the financial market for gazelles and the M&A market could then spur gazelle growth substantially. In contrast, the existing policies do, to a large extent, exclusively focus on stimulating the organic growth of small firms, and there is a lack of policies that stimulate the ownership transfers.

Policymakers are interested in promoting gazelle development since gazelles create many jobs through organic growth. However, if gazelles grow via acquisition, it is possible that no new jobs are directly created, so policymakers might be less interested in promoting the growth of gazelles via acquisition. In contrast, if there are complementary labor investments associated with the gazelle's acquisition, the hiring of new employees might actually increase. In fact, using a panel data of Swedish firms over a 10-year period, Lockett, Wiklund, Davidsson and Girma (2011) find that previous acquisition growth has a positive effect on current organic growth, which will in turn increase new employment in the future. Thus, policymakers might consider the possibility that acquisition and organic growth by gazelle firms are complements of each other.

The model has several limitations. The first limitation is that there is only one incumbent, one gazelle, and one asset that belongs a target firm and is up for acquisition. If we were to add multiples of any of these three categories, the model would be more complicated. However, the key assumption is that there are sufficiently few of these categories to have the mechanisms identified here in play. If there were many assets (target firms) to acquire, the incumbent could not so easily pre-empt the gazelle; if there were many gazelles, the incumbent presumably would not be able to pre-empt them all; and if there were many incumbents, the strategic motive to acquire would be much lower. Thus, the model would not apply to markets with very low entry cost, small network effects and small values of marketing.

Second, we have assumed that the incumbent has no use of the target's asset to highlight the strategic motive. If the incumbent has use of the target's asset, the analysis will be more involved, but the main insight will be that the value for the incumbent of acquiring the target will increase, and we will predict less acquisitions made by the gazelle. However, unless the synergies between the incumbent's and target's are too large, our analysis above will be valid.

In this study, we have not taken asymmetric information problems into account in detail, such as in the venture capital financing of the gazelle. It is likely that the gazelle owner has an informational advantage by knowing the quality of the business idea better than the venture capitalist. In future research, it would be interesting to extend the theoretical model by allowing for asymmetric information.

Moreover, gazelle growth is often characterized by uncertainty both on the demand side and on the technology development side. This might affect the choice of growth mode. In particular, large-scale organic growth might be costly since the time between the onset of the investment and the use of the new plant might be long. Examining these aspects seems to be a fruitful avenue for future research.

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H Appendix

H.1 Cournot Model:

In the Cournot oligopoly model, we consider two firms in an industry: an established market-leading firm, called the incumbent, and an entrepreneurial firm that has been recently established and has the potential to grow quickly, called the gazelle. We characterize each firm with zero total cost except the distribution cost of the gazelle, d, in small-scale organic growth. The firms are producing identical goods, and they are price takers. The market-demand function is given by:

$P(Q) = a - bQ$ and $a, b > 0$, $a > d$, where $Q = q_g + q_i$.

In this two-seller game, each firm's action is defined as choosing its production level. First, we assume here that both the incumbent and the gazelle choose their actions simultaneously. Thus, each firm j chooses $q_j \in A_j \equiv [0, \infty)$, $j = 1, 2$. Then, the payoff function of each firm j is its profit function defined by:

$\pi_j(q_g, q_i) = P(q_g + q_i)q_j - TC_j(q_j)$, where $TC_j(q_j) = 0$, except that $TC_g(q_g) = dq_g$ under the circumstance of small-scale organic growth. Based on the simultaneous move assumption, we first calculate the Cournot-Nash equilibrium. The triplet $\{P^c, q_i^c, q_g^c\}$ is a Cournot-Nash equilibrium if the following conditions held:

(1) Given $q_g = q_g^c$, q_i^c solves

$max_{q_i} \pi_i(q_i, q_g^c) = P(q_i + q_g^c)q_i - TC_i(q_i) = [a - b(q_i + q_g^c)]q_i - 0$

Given $q_i = q_i^c$, q_g^c solves

$max_{q_g} \pi_g(q_i^c, q_g) = P(q_i^c + q_g)q_g - TC_g(q_g) = [a - b(q_i^c + q_g)]q_g - dq_g$

(2) $p^c = a - b(_i^c + q_g^c), P^c, q_i^c, q_g^c \geqslant 0$.

The incumbent firm's profit maximization problem yields the first-order condition given by $\frac{\partial \pi_i(q_i,q_g)}{\partial q_i} = a - 2bq_i - bq_g = 0$. The second-order condition guaranteeing a global maximum is satisfied since $\frac{\partial^2 \pi_i}{\partial (q_i)^2} = -2b < 0$ for every q_i and q_g. If we solve for q_i as a function of q_g, we will obtain the best-response function of the incumbent, denoted as $R_i(q_g)$.

$q_i = R_i(q_g) = \frac{a-bq_g}{2b} = \frac{a}{2b} - \frac{1}{2}q_g$

.

Similarly, we can obtain the gazelle's best-response function from the first-order condition $\frac{\partial \pi_g(q_i,q_g)}{\partial q_g} = a - 2bq_g - bq_i - d = 0$. $R_g(q_i)$ is given by

$q_g = R_g(q_i) = \frac{a-bq_i-d}{2b} = \frac{a-d}{2b} - \frac{1}{2}q_i$

.

Due to the difference in the cost structure between the incumbent and the gazelle, their best-response functions do not look exactly the same. The Cournot equilibrium output levels can be calculated by solving these two best-response functions. The Cournot equilibrium quantity for the incumbent is given by

$$q_i^c = \frac{a}{2b} - \frac{1}{2}\left(\frac{a-d}{2b} - \frac{1}{2}q_i\right)$$

$$q_i^c = \frac{a}{2b} - \frac{a}{4b} + \frac{d}{4b} + \frac{1}{4}q_i$$

$$q_i^c = \frac{a+d}{3b}$$

The Cournot equilibrium quantity for the gazelle is given by

$$q_g^c = \frac{a-d}{2b} - \frac{1}{2}\left(\frac{a}{2b} - \frac{1}{2}q_g\right)$$

$$q_g^c = \frac{a-d}{2b} - \frac{a}{4b} + \frac{1}{4}q_g$$

$$q_g^c = \frac{a-2d}{3b}$$

Hence, the aggregate industry-output level is $Q^c = q_i^c + q_g^c = \frac{2a-d}{3b}$, and the Cournot equilibrium price is $P^c = a - bQ^c = a - b(\frac{2a-d}{3b}) = \frac{a+d}{3}$.

On the other hand, in our model, the game between the incumbent and the gazelle is a sequential-moves game, called a Stackelberg game in the literature. The incumbent is the leader and moves first, whereas the gazelle is the follower and moves after observing the output level chosen by the incumbent. This game has a continuum of subgames indexed by the output level chosen by the incumbent in the first stage. A finite-horizon dynamic game is generally solved backward. We look for a subgame perfect equilibrium for this game. Therefore, we first analyze the gazelle's action in the last period, assuming that the actions played in the previous period are given. Then, we go one period backward and analyze the incumbent's action given the strategy of how the gazelle chooses its output level based on the first-period action. In the second period, only the gazelle moves and chooses q_g to maximize its profit, taking the incumbent's quantity produced, q_i, as given. The second-period problem of the gazelle is identical to the problem solved by the gazelle in a Cournot market structure. The maximization gives the same best-response function for the gazelle: $R_g(q_i) = \frac{a-bq_i-d}{2b} = \frac{a-d}{2b} - \frac{1}{2}q_i$. This function $R_g(q_i)$ constitutes the gazelle's strategy for this game. On the other hand, in the first period, the incumbent is able to calculate how the gazelle will best reply to its choice of quantity. The incumbent chooses q_i^s to maximize its payoff function, π_i^s.

$$\begin{aligned}
max_{q_i}\pi_i^s &= P\big(q_i + R_g(q_i)\big)q_i - TC_i(q_i) \\
\pi_i^s &= \left[a - \left(\frac{a-d-bq_i}{2b} + q_i\right)\right]q_i - 0 \\
\pi_i^s &= aq_i - bq_i^2 - \frac{a}{2}q_i + \frac{d}{2}q_i + \frac{b}{2}q_i^2 \\
\frac{\partial \pi_i^s}{\partial q_i} &= \frac{a}{2} - bq_i^s + \frac{d}{2} = 0 \\
q_i^s &= \frac{a+d}{2b}
\end{aligned}$$

As expected, the incumbent produces a higher level of quantity than the Cournot equilibrium quantity: $q_i^s > q_i^c$.

Under the sequential-moves market structure, we calculate the quantity chosen by the gazelle by substituting q_i^s into $R_g(q_i)$:

$$\begin{aligned}
q_g^s &= \frac{a-d}{2b} - \frac{1}{2}q_i^s = \frac{a-d}{2b} - \frac{1}{2}\left(\frac{a+d}{2b}\right) \\
q_g^s &= \frac{a-3d}{4b}
\end{aligned}$$

In turn, the quantity level of the gazelle drops compared to the Cournot equilibrium quantity: $q_g^s < q_g^c$.

H.2 Proof of Proposition 3:

Lemma 1 implies that the firm i with the highest valuation obtains the assets at a price of v_i.

(i) If $I > \bar{I}(d_s, r)$ and $d_s > d^*$: In this interval, the gazelle g will not undertake large-scale organic growth. It will grow organically at a small scale. On the other hand, it then follows from Lemma 1 that the incumbent i acquires the target firm and its distribution network at a price of $\frac{\pi_g^D(0)}{1+r}$ when $d_s \geq d^{\max}$ and at a price of $\frac{\pi_g^D(0)-\pi_g^D(d_s)}{1+r}$ when $d_s < d^{\max}$.

(ii) If $I > \bar{I}(d_s, r)$ and $d_s < d^*$: In this interval, the gazelle g will not be able to undertake large-scale organic growth. However, it then follows from Lemma 1 that the gazelle g will obtain the target firm's assets at a price of $[\pi_i^D(d_s) - \pi_i^D(0)]$ when it makes the acquisition.

(iii) If $I < \bar{I}(d_s, r)$: In this interval, the gazelle g will grow by acquisition, although it could also choose to grow organically on a large scale. It then follows from Lemma 1 that the gazelle g obtains the assets at a price of 0 in the case of the acquisition.

H.3 Derivation of Equation 8:

First, note the following:

$$
\begin{aligned}
v_i(d) - v_g(d) &= \pi_i^D(d) - \pi_i^D(0) - \left(\frac{\pi_g^D(0) - \pi_g^D(d)}{1+r}\right) \qquad \text{(H.1)} \\
&= \pi_i^D(d) + \frac{\pi_g^D(d)}{1+r} - \left(\pi_i^D(0) + \frac{\pi_g^D(0)}{1+r}\right) \\
&= \Pi_i(d) - \Pi_g(0)
\end{aligned}
$$

where $\Pi_i(d) = \pi_i^D(d) + \frac{\pi_g^D(d)}{1+r}$ is the aggregate profit under the incumbent's ownership of the assets and $\Pi_g(0) = \pi_i^D(0) + \frac{\pi_g^D(0)}{1+r}$ is the aggregate profit under the gazelle's ownership of the assets. Hence, distribution costs d only affect profits under the incumbent's ownership. To study how $v_i - v_g$ reacts to changes in d, we can simply explore aggregate profit $\Pi_i(d)$ when the incumbent buys the target firm. The firm's profits under an incumbent's acquisition of the target firm are then as follow:

$$\Pi_i(d) = \pi_i^D(d) + \frac{1}{1+r}\left(\pi_g^D(d)\right)$$

$$\pi_i^D(d) = P(q_i + q_g)q_i \tag{H.2}$$

$$\frac{1}{1+r}\big(\pi_g^D(d)\big) = \frac{1}{1+r}\big[P(q_i + q_g)q_g - dq_g\big]. \tag{H.3}$$

The FOCs are as follows:

$$\frac{\partial \pi_i^D}{\partial q_i} = P + P'q_i = 0 \tag{H.4}$$

$$\frac{\partial \pi_g^D}{\partial q_g} = \frac{1}{1+r}(P + P'q_g - d) = 0 \tag{H.5}$$

$$\Rightarrow P + P'q_g - d = 0.$$

We know that $Q = q_i + q_g$ and $\frac{dQ}{dd} = \frac{dq_i}{dd} + \frac{dq_g}{dd}$. Differentiating (H.4) with respect to q_i, q_g, and d and solving for $\frac{dq_i}{dd}$, $\frac{dq_g}{dd}$ and $\frac{dQ}{dd}$ imply the following:

$$\frac{d}{dd}(P + P'q_i) = 0$$

$$\frac{dP}{dQ}\frac{dQ}{dd} + \frac{dP'}{dQ}\frac{dQ}{dd}q_i + P'\frac{dq_i}{dd} = 0$$

$$P'\left(\frac{dq_i}{dd} + \frac{dq_g}{dd}\right) + P''q_i\left(\frac{dq_i}{dd} + \frac{dq_g}{dd}\right) + P'\frac{dq_i}{dd} = 0$$

$$(P' + P''q_i + P')\frac{dq_i}{dd} + (P' + P''q_i)\frac{dq_g}{dd} = 0$$

$$(P' + P' + P''q_i)dq_i + (P' + P''q_i)dq_g = 0$$

$$\Rightarrow dq_g = \frac{-(2P' + P''q_i)dq_i}{(P' + P''q_i)} \tag{H.6}$$

Differentiating (H.5) with respect to q_i, q_g, and d and solving for $\frac{dq_i}{dd}$, $\frac{dq_g}{dd}$ and $\frac{dQ}{dd}$ imply the following:

$$\frac{d}{dd}(P + P'q_g - d) = 0$$

$$\frac{dP}{dQ}\frac{dQ}{dd} + \frac{dP'}{dQ}\frac{dQ}{dd}q_g + P'\frac{dq_g}{dd} - 1\frac{dd}{dd} = 0$$

$$P'\left(\frac{dq_i}{dd} + \frac{dq_g}{dd}\right) + P''q_g\left(\frac{dq_i}{dd} + \frac{dq_g}{dd}\right) + P'\frac{dq_g}{dd} - 1\frac{dd}{dd} = 0$$

$$\big(P' + P''q_g\big)\frac{dq_i}{dd} + \big(P' + P''q_g + P'\big)\frac{dq_g}{dd} - 1\frac{dd}{dd} = 0$$

$$\big(P' + P''q_g\big)dq_i + \big(P' + P' + P''q_g\big)dq_g - 1dd = 0$$

$$\Rightarrow \big(P' + P''q_g\big)dq_i + \big(P' + P' + P''q_g\big)dq_g = dd \tag{H.7}$$

Inserting (H.6) into (H.7), we obtain the following:

$$(P' + P''q_g)dq_i + (2P' + P''q_g)\frac{-(2P' + P''q_i)dq_i}{(P' + P''q_i)} = dd$$

$$(P' + P''q_i)(P' + P''q_g)dq_i + (2P' + P''q_g)[-(2P' + P''q_i)]dq_i = (P' + P''q_i)dd$$

$$\Big(\big[(P')^2 + P''P'q_g + P''P'q_i + (P'')^2q_iq_g\big] - \big[4(P')^2 + 2P'P''q_i + 2P''q_gP' + (P'')^2q_iq_g\big]\Big)dq_i$$
$$= (P' + P''q_i)dd$$

$$\Big(- 3(P')^2 - P''P'q_g - P''P'q_i\Big)dq_i = (P' + P''q_i)dd$$

$$\frac{dq_i}{dd} = \frac{(P' + P''q_i)}{-3(P')^2 - P''P'q_g - P''P'q_i}$$

$$\frac{dq_i}{dd} = (-1)\frac{(P' + P''q_i)}{P'[3(P') + P''q_g + P''q_i]} = (-1)\frac{(P' + P''q_i)}{P'[3P' + P''(q_g + q_i)]} = (-1)\frac{(P' + P''q_i)}{P'[3P' + P''Q]}$$

$$\Rightarrow \frac{dq_i}{dd} = -\frac{(P' + P''q_i)}{D} > 0 \text{ since we know that } P' < 0 \text{ and } P'' < 0$$

and $D = P'\,[3P' + P''Q] > 0$.

In summary, when we solve for $\frac{dq_i}{dd}$, $\frac{dq_g}{dd}$ and $\frac{dQ}{dd}$, we obtain the following inequalities:

$$\frac{dq_i}{dd} = -\frac{P' + P''q_i}{D} > 0, \qquad \frac{dq_g}{dd} = \frac{2P' + P''q_i}{D} < 0, \qquad \text{(H.8)}$$
$$\frac{dQ}{dd} = \frac{P'}{D} < 0,$$

where $D = P'[3P' + P''Q] > 0$ and $Q = q_i + q_g$. We can then define the (reduced-form) aggregate profits under incumbent ownership of the assets as a function of d:

$$\Pi_i(d) = \pi_i^D(q_i(d), q_g(d), d) + \pi_g^D(q_i(d), q_g(d), d) \tag{H.9}$$

Taking the total derivative in d and using (H.2), (H.3), (H.4) and (H.5), (H.9) can be written as follows:

$$\frac{d\Pi_i}{dd} = P'q_i\frac{dq_g}{dd} + P'q_g\frac{dq_i}{dd} - q_g \tag{H.10}$$

Using the first-order conditions (H.4) and (H.5) and that $\frac{dQ}{dd} = \frac{dq_i}{dd} + \frac{dq_g}{dd}$ must hold, (H.10) can be rewritten as follows:

$$\frac{d\Pi_i}{dd} = P'q_i(\frac{dQ}{dd} - \frac{dq_i}{dd}) + \underbrace{[P'q_g]}_{d-P}\frac{dq_i}{dd} - q_g$$

$$\frac{d\Pi_i}{dd} = P'q_i\frac{dQ}{dd} - \underbrace{[P'q_i]}_{-P}\frac{dq_i}{dd} + d\frac{dq_i}{dd} - P\frac{dq_i}{dd} - q_g$$

$$\frac{d\Pi_i}{dd} = P'q_i\frac{dQ}{dd} + P\frac{dq_i}{dd} - P\frac{dq_i}{dd} + d\frac{dq_i}{dd} - q_g$$

$$\Rightarrow \frac{d\Pi_i}{dd} = P'q_i\frac{dQ}{dd} + d\frac{dq_i}{dd} - q_g. \tag{H.11}$$

Finally, from (H.1), $v_i(d) - v_g(d) = \Pi_i(d) - \Pi_g(0)$. It then follows that

$$\frac{d\,(v_i - v_g)}{dd} = P'q_i\frac{dQ}{dd} + d\frac{dq_i}{dd} - q_g$$

$$\frac{d(v_i - v_g)}{dd} = \frac{dQ(d)}{dd}P'q_i(d) + \frac{dq_i(d)}{dd}d - q_g(d).$$

This above equation is equation 8 in Section 3.