

No. 8, 1976

ESTIMATION AND ANALYSIS WITH
A WDI PRODUCTION FUNCTION

by

Göran Eriksson, Ulf Jakobsson and Leif Jansson

December, 1976

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Summary

By means of a new type of production function - the WDI function - an examination has been made of the production structure in thirteen branches of Swedish manufacturing industry. The function used, which is the same in all thirteen branches, allows for variable elasticity of substitution and gives us a possibility of distinguishing different types of bias in technological development. It was found for instance that the hypothesis on Hicks neutrality can be dismissed for most branches. The data material used contains both production data (i.e. observations regarding capital, labour and production) and observations of the factor income shares.

A "Full Information Maximum Likelihood (FIML)" method was used for estimation.

1. Introduction

In this paper complete estimates of production functions are presented for thirteen branches of the Swedish industry. The study is of interest as a systematic survey of the structure of industrial production, and as an application of a WDI production function. From both these viewpoints it is found essential to calculate the estimates using the same form of function for all branches.

The experiences gained from empirical work using the CES function (and its special case, the Cobb-Douglas function) is the demand for a more general production function. Commonly such functions are derived from properties of the elasticity of substitution (see Lovell [1968, 1973] and Revankar [1971]). Here instead emphasis is put on using a function that have properties reasonably for a production structure (see Shephard [1970] and Färe [1972]). In spite of these different approaches the function used in this paper is similar to one of those found in Lovell [1973]. The link between these functions are further investigated by Färe & Jansson [1975].

In empirical work on production functions the usual assumption is Hicks' neutral technological change is usually assumed. Against this background we have chosen to specify the autonomous technological development in such a manner that the type and degree of bias in it are estimated together with the other parameters of the production function. It then turns out that the hypothesis on Hicks neutrality can be rejected for most of the branches.

Econometric studies on production functions can be divided into three groups according to the data used: a) Studies which use production engineering data, i.e. observations regarding input of capital and labour and production (Douglas [1948], Aukrust & Bjerke [1959] och Walters [1962]), b) Studies which use observations of the factor distribution of income. The latter require an assumption on cost minimization in the companies. This type of observations does not yield sufficient information for an estimate of all parameters in the production function. In most cases the merit of these studies is limited to the value of the substitution elasticity (Brown & Cani [1963], Ferguson [1965]), c) Studies which use production data as well as distribution data. Our study belongs to this third group. Here the estimates are frequently done in two separate steps. First a group of parameters are estimated on the distribution data, after which the remaining ones are estimated on the production data (Klein & Preston [1967], Lovell [1968] and [1973]).

One reason for this "twostep method" is that the parameters sometimes can be estimated with linear regression. But the parameter estimates obtained are generally not efficient and anyhow there is no reason to consider this method here, as the advantage of linear regression is lost in our choice of production function. Instead the FIML ("Full Information Maximum Likelihood") method which gives both consistent and asymptotic efficient estimates is used. The likelihood function has to be maximized with non-linear regression, which has proved to be economically and practically feasible with today's computers and optimization programs. Among those who have earlier used the same method may be mentioned Bodkin & Klein [1967].

2. The theoretical model

2.1. The production function

The static version of the production function used in this study, called WDI-CD (Weak Disposability of Inputs, Cobb-Douglas type¹⁾) is of the form

$$Q = \begin{cases} AK^\alpha(L-bK)^{1-\alpha} & \text{if } K, L \in \{K, L \mid K \geq 0, L \geq 0, L-bK \geq 0\} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where

$$\left. \begin{array}{l} Q = \text{production volume} \\ K = \text{quantity of real capital} \\ L = \text{quantity of labour} \end{array} \right\} \in \mathbb{R}_+$$

A, α and b are parameters, where $A > 0$, $\alpha \in [0, 1]$ and $b \in \mathbb{R}$.

(1) is linear homogenous in K and L and if $b = 0$ (1) becomes an ordinary CD function. Next let us introduce the following time dependent version

$$Q = A(Ke^{\lambda_1 t})^\alpha (Le^{\lambda_2 t} - bKe^{\lambda_1 t})^{1-\alpha} \quad \text{if } K, L \in \{K, L \mid K \geq 0, L \geq 0, Le^{\lambda_2 t} - bKe^{\lambda_1 t} \geq 0\} \quad (2)$$

where $\lambda_1, \lambda_2 \in \mathbb{R}$.

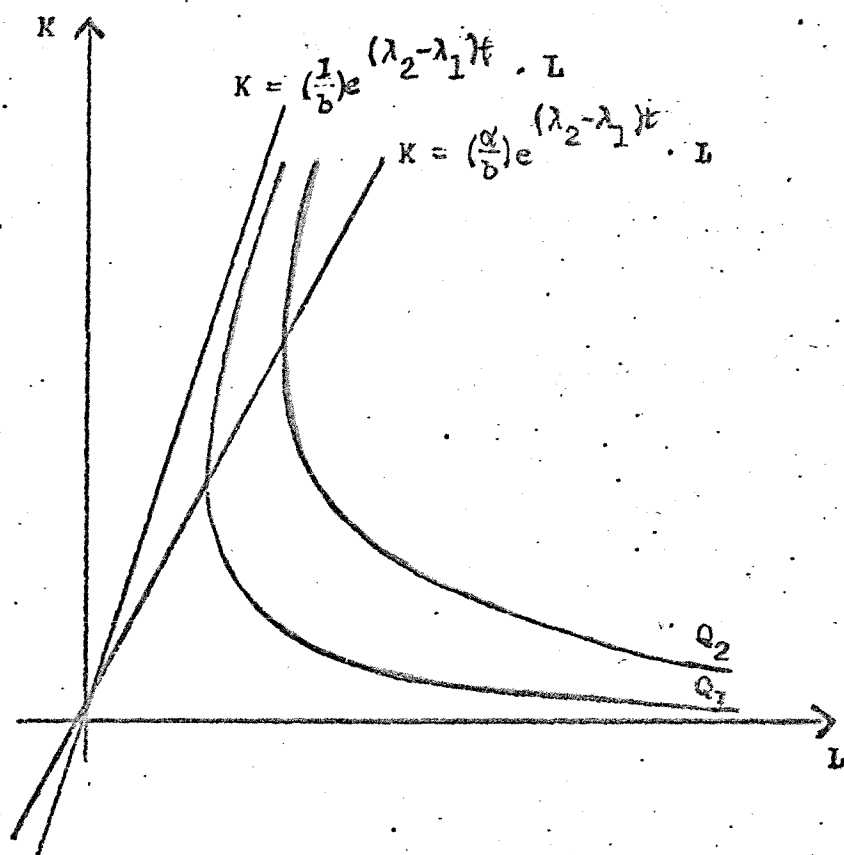
(2) allows the technological change to affect capital and labour differently. $e^{\lambda_1 t}$ and $e^{\lambda_2 t}$ are the capital and labour augmenting factors respectively.

The technological change is Hicks neutral if $\lambda_1 - \lambda_2 = 0$, Harrod neutral if $\lambda_1 = 0$ and Solow neutral if $\lambda_2 = 0$. Isoquants of the function is illustrated in fig. 1 with $b > 0$.

From (2) it appears that positive production is reached within the cone that is enclosed by the lines $K = 0$ and $K = (1/b)e^{(\lambda_2 - \lambda_1)t}L$. If $b > 0$ there is a non-economic zone of K and L values, where an increase in capital results in reduced production. If $b \leq 0$ there is no such zone. Between the lines $K = \frac{\alpha}{b}e^{(\lambda_2 - \lambda_1)t}L$ and $K = 0$ lies the economic zone of K and L values where the production increases at any increase of K and L . If b and the difference $\lambda_2 - \lambda_1$ have the same sign the zone expands over time whereas the zone shrinks if b and $\lambda_2 - \lambda_1$ have different signs.

1) A general form including this function is given in Färe & Jansson [1975].

Figure 1. Isoquants of the production function when $b > 0$.



2.2. Distribution of factor income and elasticity of substitution

It is assumed that there exists free competition in the factor markets and that the companies try to minimize their costs at each given production volume. We then get from (2)

$$\frac{rK}{wL} = \frac{\partial Q}{\partial K} K / \frac{\partial Q}{\partial L} L = \frac{\alpha}{1-\alpha} \left(1 - \frac{bke^{\lambda_3 t}}{\alpha} \right) \quad (3)$$

where $\lambda_3 = \lambda_1 - \lambda_2$

r = price of capital services

w = wage rate

$k = K/L$ = capital intensity.

For fixed t the ratio between capital and labor income is a linear function of capital intensity. It is further noticeable that changes in k over time may eliminate the changes in $e^{\lambda_3 t}$. The ratio between the income shares will then be constant over time. This ratio has been nearly constant for many branches in Sweden during the observation period, a fact that sometimes has been referred to as an indication that the underlying production structure should be a CD function.

(2) and (3) give the following relations

$$\beta = \frac{\partial Q}{\partial K} \frac{K}{Q} = \alpha - \frac{(1-\alpha)bke^{\lambda_3 t}}{1-bke^{\lambda_3 t}} \quad (4)$$

$$\sigma = \frac{\partial k}{\partial \left(\frac{r}{w}\right)} \frac{r/w}{k} = 1 - \frac{bke^{\lambda_3 t}}{\alpha} \quad (5)$$

$$\theta = \frac{\partial Q}{\partial t} \frac{1}{Q} = \gamma - \frac{(1-\alpha)\lambda_3 bke^{\lambda_3 t}}{1-bke^{\lambda_3 t}} \quad (6)$$

β = income share of capital

σ = elasticity of substitution

θ = rate of technological change

$\gamma = \alpha\lambda_1 + (1-\alpha)\lambda_2$.

The equations (4)-(6) summarize a few properties of our model which we will examine more closely. The effects of variations in capital intensity k and in the factor $e^{\lambda_3 t}$ are symmetrical. An equal increase of k or of $e^{\lambda_3 t}$ results in the same changes in these three company variables. How the β , σ and θ are affected by changes in the capital intensity is dependant on the value of b . We can discern three distinct cases:

Case 1. $b > 0$

Here $\beta < \alpha$ and $\sigma < 1$ for all capital intensities and ρ and σ decrease when capital intensity is raised. If capital intensity continues to increase so that $k = (\alpha/b)e^{-\lambda_3 t}$, β and σ will equal zero. The company has then chosen a factor ratio which lies on the line $K = (\alpha/b)e^{-\lambda_3 t} \cdot L$ enclosing the economic zone of K and L values. The effect on θ is moreover dependent on the sign of the technological parameter λ_3 . If for instance $\lambda_3 < 0$ then θ will grow at an accelerating rate with rising capital intensity.

Case 2. $b < 0$

Then instead $\beta > 0$ and $\sigma > 1$ for all capital intensities

β and σ increase when capital intensity is raised. There is no upper limit for the values of β and σ . When capital intensity goes to infinity the same applies to β and σ . If $\lambda_3 < 0$, θ decreases at a decreasing rate due to an increased capital intensity.

Case 3, $b = 0$

Then $\beta = \alpha$, $\sigma = 1$ and $\theta = \gamma$ and these three variables are not affected by variations in capital intensity. Our production function is then obviously identical with the $CD_{\lambda_3 t}$ function.

The factor $bke^{\lambda_3 t}$ determines how β, σ and θ develop over time and when $ke^{\lambda_3 t}$ is constant, these variables do not vary. The adaptation to changing factor prices by the companies thus does not lead to any alternations of income shares, substitution elasticity or rate of technological change if capital intensity grows at the same rate as the difference between labour-augmenting and capital-augmenting rate of technological change. Such an equally fast growth of capital intensity and of the factor-augmenting technological factors means that capital and labour, expressed in efficiency units ($Ke^{\lambda_1 t}$ and $Le^{\lambda_2 t}$ respectively) expand at the same rate.

(4)-(6) gives following relationship between σ, β and θ :

$$\sigma = \left(\frac{1-\alpha}{\alpha}\right) \frac{\beta}{1-\beta} \quad (7)$$

$$\theta = \lambda_1 - (1-\beta)(\lambda_1 - \lambda_2). \quad (8)^1)$$

From (7) and (8) is seen that

1) It can be shown that (8) is valid for all linearly homogeneous functions of the type $Q = F(Ke^{\lambda_1 t}, Le^{\lambda_2 t})$.

a) $\partial\sigma/\partial\beta > 0$ and $\partial^2\theta/\partial\beta^2 > 0$, i.e. the elasticity of substitution is an acceleratingly rising function of the production elasticity of the capital

b) when $\lambda_1 \neq \lambda_2$ the rate of technological change is a linear function of the production elasticity of capital. The functional dependence between θ and σ is also given by (7) and (8) to $\theta = \lambda_1 + (1-\alpha)(\lambda_2 - \lambda_1)/(1-\alpha(1-\sigma))$.

3. Stochastic model and calculation of parameter estimates

The data used is the national accounting statistics for Sweden covering the period 1950-1973. The following variables are used:

Q = Value added in constant prices

QP = Value added in current prices

L = Hours worked by entrepreneurs and employees

W = Total wages paid in current prices

KR = Stock of fixed capital in constant prices. (Both building and machine-capital are included)

U = utilization factor calculated as the ratio between actually used electricity energy per time unit and installed capacity of electrical machinery.

A problem is that stock of capital data does not refer to utilization of buildings and machinery, which may vary over time due to e.g. short term fluctuations in demand for products. To get a more correct measure of utilized stock of capital $K = U \cdot KR$ is used.

However, there are no observations on capital income or rate of interest so it is indirectly calculated by following accounting identity.

$$QP = rKR + W \quad (9)$$

The assumption of perfect competition and cost minimization and the imposed linear homogeneity give ^{the} following expression for the ratio of factor incomes

$$\frac{\partial Q}{\partial KR} \cdot KR / \frac{\partial Q}{\partial L} \cdot L = \frac{QP - W}{W} = S \quad (10)$$

Note that $KR \cdot \partial Q / \partial KR = K \cdot \partial Q / \partial K$ and thus utilized capital stock can be applied.

The statistical model is specified as

$$\frac{Q_t}{K_t} = A e^{\lambda_1 \alpha t} (e^{\lambda_2 t} / k_t - b e^{\lambda_1 t})^{1-\alpha} + \xi_t \quad (11)$$

$$S_t = \frac{\alpha}{1-\alpha} \left[1 - \frac{b k_t e^{(\lambda_1 - \lambda_2)t}}{\alpha} \right] + \mu_t \quad t = 1, \dots, T \quad (12)$$

where Q_t , K_t , k_t and R_t are the observed values of production in terms of value added the real capital, the capital intensity and the ratio between share of income from capital and share of income from labour respectively for each year t during the period 1950-1973. The error terms ξ , and μ , are assumed to be normally distributed with mean values equal to zero and independent between the years with the co-variance matrix Ω . In order to simplify the presentation write (11) and (12)

$$y_t = h_t(\phi) + \xi_t \quad (11')$$

$$x_t = g_t(\phi) + \mu_t \quad (12')$$

where ϕ is the parameter vector with the elements A , λ_1 , λ_2 , α and b .

Maximum likelihood-estimate of the parameters and Ω are obtained by maximizing the log likelihood function which with the assumptions made can be written (see Koopmann & Hood [1953] and Jonston [1963], p 399)

$$L(\Omega, \phi) = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \log \det \Omega - \frac{1}{2} \text{tr}(\Omega^{-1} V' V) \quad (13)$$

where V is the matrix with the residuals

$$V = \begin{bmatrix} \xi_1, \dots, \xi_T \\ \mu_1, \dots, \mu_T \end{bmatrix}$$

The Maximum Likelihood estimate of the parameters is simpler achieved by solving Ω as a function of V from the first order conditions

$$\frac{\partial L}{\partial \Omega} = 0 \quad (14)$$

which gives

$$\Omega = \frac{V'V}{T} \quad (15)$$

By (13) and (15) the concentrated likelihood function is obtained

$$L(\phi) = -T \log (2\pi) - T - \frac{T}{2} \log \det \frac{V'V}{T} \quad (16)$$

It is further true that $\max_{\phi} L(\phi)$ is equivalent to $\min_{\phi} \det V'V$, which here can be written

$$\min_{\phi} \det V'V = \min_{\phi} \left[\sum_{t=1}^T (y_t - g_t)^2 \sum_{t=1}^T (x_t - h_t)^2 - \left[\sum_{t=1}^T (y_t - g_t)(x_t - h_t) \right]^2 \right] \quad (17)$$

(17) is the expression used to obtain the estimates.

4. The regression estimates

The estimated parameters of the production function for the separate branches of industry and for the industry as a whole are given in table 1. The α coefficient varies between 0.16 for the chemical industry and 0.54 for the wood and paper industry. The values of α thus lie within the $[0, 1]$ interval and are significant on the 5 % level except for the chemical branch. A similar branch pattern is shown in Lovell [1968]. During the 1949-1963 period he got for food, textile, wood and paper, and engineering industries $\alpha = 0.44, 0.29, 0.54$ and 0.32 respectively. Lovell [1973] estimated $\alpha = 0.47$ for the whole manufacturing industry.

The estimates of the technological parameters of the input factors show that $\lambda_2 > 0$ for all branches while five branches have $\lambda_1 < 0$. λ_1 is not significantly different from zero in six, and λ_2 in three branches.

Three branches have negative b values. The generalized CD function in Lovell [1968] includes a β parameter which has an influence on elasticity of substitution similar to that of our b parameter. If for instance $\beta < 0$ this means that the elasticity of substitution decreases with rising capital intensity. Lovell got $\beta < 0$ in all his 16 branches except the tobacco industry and Lovell [1973] got $\beta < 0$ for industry as a whole.

The $\bar{R}^2$ values given are calculated in analogy with the definition of the multiple regression coefficient adjusted for degrees of freedom for the least

Table 1. Parameter estimates

Branch of manufacturing industry	A	α	λ_1	λ_2	b	$\bar{R}^2$	DW
Sheltered food	2.73	0.47	-0.062	0.034	0.0031	0.93	0.42 1.78
Import-competing food	1.60	0.51	0.039*	0.014*	0.00018*	0.94	1.19 1.31
Beverages and tobacco	2.13	0.46	-0.033	0.070	0.0077*	0.95	0.57 1.33
Textile, wearing apparel & leather	2.18	0.39	-0.0027	0.047	0.0124	0.40	1.41 0.28
Wood and paper and wood and paper products	0.64	0.54	0.023*	0.056	0.0031	0.52	0.28 1.62
Printing, publishing & allied industries	7.15	0.22	-0.053	0.017*	-0.0061	0.92	0.30 1.36
Rubber products	1.40	0.53	0.000*	0.039	0.0131	0.40	0.06 0.96
Chemicals and chemicals and plastic products	2.81	0.16*	0.0059*	0.061*	-0.0033	0.98	0.76 1.92
Petroleum and coal	2.42	0.30	0.000*	0.008	-0.0036	0.89	0.06 0.79
Other non metallic mineral products	1.60	0.38	0.028	0.037	0.0006*	0.96	0.27 0.99
Basic metal	1.03	0.24	0.316*	0.0041	0.000*	0.64	0.56 1.17
Metal products machinery and equipment except ship building	2.77	0.36	0.027	0.039	0.0033	0.44	0.81 0.70
Other manufacturing industry	3.55	0.45	0.045	-0.076	0.0423	0.83	0.10 0.02
Total manufacturing industry	1.44	0.42	0.024	0.045	0.0021	0.88	1.13 1.26

Estimates marked with a * are not significant different from 0 on 5 % level.

squares method for one equation. Note that $\bar{R}^2$ is no longer limited to values between 0 and 1 as in OLS, but may assume also negative values. The upper limit, however, is still 1 which occurs at perfect fit. For each branch DW statistics are given first for the production function and then for the factor income distribution function.

In building the model the emphasis has been on explaining the trend development in the period 1950-1973. No attempts have thus been made to explain short time fluctuations in the production, with e.g. lag structures. It is therefore to be expected that $\bar{R}^2$ values and especially the DW statistics will be poor. The DW statistics are throughout lower for the production function than for the distribution function with the exception of metal products and other manufacturing industry.

A strong point of the above estimates is that 80 % of the parameters estimated are significantly different from 0, and for all branches every observation lies within the field where the model assumes that increased input of labour and capital will generate increased production.

5. Factor shares of income and the elasticity of substitution

No matter what type of production function the income shares of capital and labour are always equal to the production elasticity of these two factors given perfect competition and marginalistic behaviour of the companies. In contrast to many other production models the income shares and the elasticity of substitution in our model are functions of capital intensity and of time. With the estimated parameters inserted in the equations (4) and (5) and measures on capital intensity we have calculated the income share of capital (β) and the substitution elasticity (σ). In table 2 are presented the average value and the differences between the initial and end values of these variables for our industrial branches in the 1950-1973 period.

The capital income shares, β , are well within the $[0,1]$ interval. Fairly great variations exist between the branches. For all industry the average capital income share amounts to 0.31. Solow [1957] and Åberg [1969], who have made aggregated time series estimates on US resp. Swedish data with a CD-function obtained, for the periods 1909-1949, and 1946-1964, values of the production elasticity of capital of 0.35 and 0.43 respectively.

Most of the branches have a substitution elasticity $\sigma < 1$. Ferguson [1965] who estimated elasticities of substitution with a CES function directly on time series data during the period 1949-1961 got results which agree remarkably well with those we have obtained. He found, as we did, that $\sigma > 1$ for beverage and tobacco, printing, chemical and petroleum industry.¹⁾ Lovell [1968] further found that $\sigma > 1$

1) Note that Ferguson got $\sigma > 1$ for three more of his in all 19 branches.

Table 2. Average values and differences in β , σ , θ

Branch of manufacturing industry	β	$\Delta\beta$	σ	$\Delta\sigma$	θ	$\Delta\theta$
Sheltered food	0.42	0.05	0.83	0.18	-0.71	-0.52
Import-competing food	0.50	-0.02	0.96	-0.08	2.63	-0.35
Beverages and tobacco	0.55	-0.02	1.36	-0.14	1.51	0.24
Textile, wearing apparel & leather	0.22	-0.15	0.51	-0.39	3.49	0.75
Wood and paper and wood and paper products	0.36	-0.15	0.52	-0.32	4.35	0.50
Printing, publishing allied industries	0.32	0.01	1.65	0.06	-0.57	-1.15
Rubber products	0.22	-0.22	0.54	-0.55	2.86	0.60
Chemicals and chemicals and plastic products	0.47	-0.11	4.57	-2.01	3.54	0.59
Petroleum and coal	0.60	0.39	3.21	5.91	0.33	0.20
Other nonmetallic mineral products	0.33	-0.05	0.84	-0.19	3.40	0.64
Basic metal	0.16	-0.17	0.90	-0.75	7.23	-5.15
Metal products machinery and equipment except ship building	0.26	-0.10	0.69	-0.33	3.55	0.11
Other manufacturing industry	0.29	0.13	0.67	0.31	3.42	-1.51
Total manufacturing industry	0.31	-0.10	0.66	-0.29	3.84	0.21

for tobacco industry. For the rest of his 15 branches σ was significantly less than 1.

If $b \neq 0$ in our production function then is $\sigma \neq 1$. If for instance $b > 0$ then $\sigma < 1$. It is only the industry branches of import competing food, beverage and tobacco, non-metallic mineral products and basic metal that have got b that do not significantly differ from zero. The CD function (with $b=0$) can thus be rejected for all the other branches. The same is applicable to industry as a whole.

It is evident from the values of $\Delta\beta$ and $\Delta\sigma$ (differences between initial and end values of β and σ) in the table that the income share of capital and the elasticity of substitution have varied in the same direction and that these two company variables have decreased in most of the branches. That β and σ vary in the same direction is a consequence of the special form we have chosen for the production function.

It is interesting that Brown and Cani [1963] have formulated and found empirical support for the following prediction. A sinking (rising) substitution elasticity decreases (increases) the income share of capital provided that labour is scarce. This hypothesis is consistent with our results. The supply of capital has increased faster than that of labour during the period 1950-73, and the income share of capital has varied in the same direction as the elasticity of substitution.

6. The technological change

Both embodied technological changes, such as introduction of better equipment use of more skilled labour and disembodied changes are brought together in the capital and labour augmenting factors $e^{\lambda_1 t}$ respectively $e^{\lambda_2 t}$.

To our knowledge only David and Klundert [1965] have directly estimated these factors in essentially the same manner as we have done. They used a CES function on data for USA for the period 1899-1960 and got the result that $\lambda_1 = 0.014 - 0.015$ and $\lambda_2 = 0.022 - 0.023$.

Our concept of factor-augmenting technological change can be related to the classification of technological changes introduced by Hicks. He defined technical progress as labour-saving (capital-saving) if it for a given capital intensity increases (decreases) the ratio between the marginal productivity of capital $\partial Q/\partial K$ and the marginal productivity of labour $\partial Q/\partial L$. It is evident from equation (3) that same signs of b and the difference $(\lambda_2 - \lambda_1)$ makes $(\partial Q/\partial K)/(\partial Q/\partial L)$ an increasing function of time. It is only within import competing food, printing, chemical, petroleum and coal and other manufacturing industries that b and $(\lambda_2 - \lambda_1)$ have different signs. So the technological change has mainly been labour-saving.

(3) gives that $(\partial Q/\partial K)/(\partial Q/\partial L)$ are independent of time when $\lambda_2 = \lambda_1$. The difference $\lambda_2 - \lambda_1$ is significantly different from zero on the 5 % level for all branches except three: import competing food, rubber and non-metallic mineral products. The results lead us to reject the hypothesis that the technological development merely has been Hicks-neutral.

Another consequence of $\lambda_1 \neq \lambda_2$ is that the rate of technological change (θ) becomes a function of both capital intensity and time. θ values increases either because $\lambda_2 > \lambda_1$ at the same time as $bke^{(\lambda_1 - \lambda_2)t}$ increases, or because $\lambda_2 < \lambda_1$ as the $(\lambda_1 - \lambda_2)t$ decreases - see equation (6).

Inserting estimated parameters in equation (6) gives the average value and the difference between the initial and end values of rate of technological change (θ and $\Delta\theta$) for the period 1950-73. The calculated values of θ and $\Delta\theta$ are given in table 2. We see that θ varies quite substantially among the branches from a low of -0.7- -0.6 % for import competing food and printing industry to a high of 7.2 for the basic metal. For industry as a whole the annual average amounts to 3.8 %.

7. Growth of production

Knowing the elasticities of labour and real capital we can calculate the contribution of these factors to growth of production. We have further split the contribution of capital into two components, where one relates to changes in the existing real capital and the other to variations in its utilization ratio. The results of these calculations for the different branches of industry are compiled in table 3. Note that the contributions from the increase of the input factors plus the rate of technological change are equal to the estimated growth rate of production.

The rate of technological change has been the most significant factor in explaining the increase of production in most of the branches. This is also the case for the entire industry since about 70 % of the production growth is due to change in the technology. The fact that most of the growth of production is due to the "unexplained part", namely the technological change, is a finding often encountered in time series studies, see e g the classical study by Solow [1957].

It is clear that the contribution of capital has been larger than that of labour, which is not surprising since the input of manhours has not increased except for chemical and machinery and equipment industry and basic metals where the growth has been small and of no significance. In nine branches and in industry as a whole labour has decreased. The branches that have had decided drops in input of labour are textile, beverage and tobacco and non-metallic products.

In some cases increase in production has been strongly influenced by a changed degree of utilization in the real capital. According to our calculations

more than half of the contribution from the capital factor in food exposed to competition has been gained from increase in the degree of utilization. However, the measure of utilization is far from perfect. First it only relates to the utilization intensity of the machinery capital. Secondly it cannot be excluded that the measure influenced by factors other than variations in the utilization intensity, such as for instance a change in the energy effectiveness of the machinery and in the use of electric power in proportion to other forms of energy.

The factor contributions in the table relate to variations of the production factors in quantitative measures. This means that we have not taken into account qualitative improvements of the factors. For the sake of simplicity let us assume that the entire technological development has been factor embodied and then define labour and capital in terms of efficiency units $L_e = L e^{\lambda_2 t}$ and $K_e = K e^{\lambda_1 t}$. But that the difference between the contributions to production by capital and labour is considerably reduced. For five branches increases in L_e will then account for a larger share of production growth than increases in K_e . The faster increase of efficiency of labour relative to capital may have been due to rising relative wage rate. The case would then be that the companies have primarily tried to make the production more effective by economizing on labour, thereby reducing the effects of rapidly rising labour costs.

Concluding remarks

It is possible to distinguish three restrictions often made a priori on the production function that has been relaxed in this study, namely

- 1) Hicks neutral technological change,
- 2) Constant elasticity of substitution and
- 3) Strong disposability of inputs, i.e. the production function is nondecreasing over the whole input space.

The hypothesis of Hicks neutral technology change is rejected for the main part of the manufacturing industry as well as for the overall. The technological change mainly estimated as labour saving.

Whether the elasticity of substitution is constant or not cannot be tested explicitly since either is $\sigma = 1$ and constant or $\sigma \neq 1$ and varies with respect to capital labour ratio and time. Apparently variations in σ to a great extent help to explain the changes over time. On the other hand it can be argued that a CES function where labour and capital augmenting also are inferred with

Table 3. The Average Production growth rate and its components in per cent during 1950-73

Branch of manu- facturing industry	Observed Produc- tion growth rate	Calculated Contribution to total growth rate from growth rate of				Residuals
		Techno- logical change	Labour force	Stock of capital	Utili- zation factors	
Sheltered food	-1.2	-0.8	-1.2	1.1	0.3	-0.6
Import - competing food	5.9	2.6	-0.6	2.1	1.2	0.6
Beverages and tobacco	3.5	1.6	-1.7	2.1	0.6	0.9
Textile, wearing apparel & leather	0.4	3.6	-3.8	0.5	0.4	-0.3
Wood and paper and wood and paper products	5.2	4.3	-0.6	2.3	-0.2	-0.6
Printing, publishing & allied industries	1.8	-0.1	-0.2	1.5	0.7	-0.1
Rubber products	5.8	2.9	-0.2	1.2	0.2	1.7
Chemicals and chemi- cals and plastic products	7.8	3.6	0.8	9.5	-0.5	1.4
Petroleum and coal	5.9	0.4	-1.1	5.4	0.4	0.3
Other nonmetallic mineral products	3.2	3.4	-1.4	1.0	0.3	-0.1
Basic metal	9.3	7.7	0.1	1.3	-0.3	-0.5
Metal products & machinery and equip- ment except ship building	7.0	3.6	0.6	1.5	0.5	-0.8
Other manufacturing industry	5.3	3.2	0.0	1.0	1.4	-0.3
Total manufacturing industry	4.9	3.5	-0.4	1.6	0.2	-0.3

$\sigma \neq 1$ but constant might as well give reasonable estimates. But we did not want to impose strong disposability of inputs a priori as the case is with the CES function for the following reason. Violation of 3) together with quasi-concavity of the production function implies that the cone where positive output can be obtained is a proper subset of the input space, see Färe & Jansson [1975]. That implies that the values of the capital labour ratio k that give positive production cannot take all values between zero and infinity. That fact is of importance e.g. when effects of price changes are studied or when production possibilities are assessed when inputs are restricted. For the sake of completeness it should be pointed out that with the function used restriction 2 and 3 above are linked as follows, if 2 is fulfilled so is 3, if 2 is violated then 3 is fulfilled when $b \leq 0$ and violated when $b > 0$.

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