The Rotten Kid Problem

When choosing action *a*, a child affects his own revenue $c(a)$ and that of his parents $p(a)$. We assume that for any value of *a*, we have:

$$c(a) < p(a)$$

The child is selfish (« rotten kid »). He only cares about his *own* utility and thus the amount of money he obtains. On the contrary, his parents (the household) care about their own utility and that of their child.

Specifically, the parents' utility U_p is equal to the lowest amount of money between what they get and what their child obtains. Given this utility, parents must decide how much money to transfer to their offspring. This transfer is denoted by $t \geq 0$. In that case, the preferences of the child are given by:

$$U_e(a,t) = c(a) + t$$

While those of the parents are equal to:

$$U_p(a,t) = \min\{p(a) - t, c(a) + t\}$$

The timing of the game is as follow. First, the child decides action *a* and the parents decide how much to transfer (*t*) to their offspring.

2a) Model this situation as an extensive-form game.

2b) Solve the second stage of this game, that is the optimal choice of the parents in terms of transfer *t*. Denote by t^* the optimal choice.

2c) We assume that $c(a) = a$ et $p(a) = 2a$. We also assume that *a* can only take values between 0 and 1, i.e. $a \in [0,1]$. Calculate the subgame perfect Nash equilibrium of this game.

2d) We now assume that $c(a) = -a^2$ and $p(a) = a$, and *a* can take any positive value. Calculate the subgame perfect Nash equilibrium of this game. What kind of situation in the real-world this game can represent?