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MEASURABLE DYNAMIC GAINS FROM TRADE
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Productive factors such as human and physical capital are accumulated and trade can affect the steady-state levels of such factors. Consequently, trade liberalization will have dynamic effects on output and welfare as the economy moves to its new steady state, in addition to its usual static effects. The output impact of this dynamic effect is measurable and appears to be quite large. The welfare impact of this dynamic effect is also measurable. The size of this dynamic gain from trade depends on the importance of external scale economies.

Empirical researchers have consistently found that even major trade liberalizations raise aggregate income by an amount that is somewhere between negligible (0.1 percent: Deardorff and Stern 1978, 1981) and rather small (8.6 percent: Harris and Cox 1982). The oral tradition in international trade has long countered this "Harberger triangle problem" with the assertion that the most important gains from trade are dynamic, not static. Empirical studies of trade liberalizations ignore such factors since dynamic trade effects are poorly understood and supposedly impossible to measure.

This paper exposits and measures one type of dynamic effect of trade liberalization. The results confirm the oral tradition: Dynamic output effects are large — perhaps several times larger than the static allocation and strategic effects that existing studies have focused on. The source of this dynamic effect is simple. Trade liberalization may, ceteris paribus, raise the marginal productivity of capital. In virtually any model where the capital-labor ratio is endogenous, this will in turn raise the steady-state capital-labor ratio (even if it has no effect on the long-run growth rate). As the economy moves toward its new steady state, output rises more than the static effect alone would imply. The welfare gain from this additional output depends on the divergence between the social and private marginal productivity of capital.
Ricardian Dynamic Trade Effects

The Heckscher—Ohlin model explores the effects of factor supplies on trade. Ricardo (1815) focuses on the reverse causality. In Ricardo's model the steady—state growth rate is zero, due to diminishing returns in agriculture. Trade postpones the arrival date of the steady—state as, "England's agriculture is stationary but Manchester and Birmingham make her the workshop of the world which pays in food and primary products for the expanding output of the workshop."\(^1\)

Thus trade affects the steady—state supply of productive factors (wage labor and farm land) employed in steady—state, but not the steady—state growth rate. Ricardo's model has little direct relevance to the modern world. Yet the link between steady—state factor supplies and trade is important. Factors such as labor skill and physical capital are accumulated. Since trade can affect factor rewards, it almost surely affects the steady—state level of such factors.

This Ricardian dynamic trade effect is related to, but quite distinct from, the important dynamic effects stressed in the Grossman—Helpman literature on trade and growth.\(^2\) The Grossman—Helpman dynamic effect focuses on the link between trade and the rate of accumulation of factors of production (be it knowledge or varieties of specialized inputs). They show that trade may raise or lower this rate and thereby permanently raise or lower the long—run growth rate of output. By contrast, in the Ricardian model (and the neoclassical growth model) the rate of growth eventually returns to a steady—state rate determined by technology and tastes. Thus the Ricardian dynamic effect focuses on the link between trade and the steady—state level of factors of production. Another way to see the distinction between the two effects is to note that the Grossman—Helpman models are part of the new growth literature, in which the long—run growth rate is endogenously determined.\(^3\) The Ricardian effect is present even in the simplest Solow growth model.

More closely related is the extensive literature on trade and growth surveyed by Smith (1984) and Findlay (1984). Most of the models in this field assumes a constant savings rate. Stiglitz

Section 1 presents the basic model. Section 2 investigates existence, stability and convergence properties of the model. Section 3 presents the comparative steady-state analysis of a trade liberalization. Section 4 examines the welfare consequences. Section 5 quantifies the output and welfare effects for specific functional forms. Section 6 contains a summary, concluding remarks and directions for future research.

1. The Ricardian Dynamic Trade Effect

The Ricardian effect is first examined in a familiar trade model. The analysis focuses on the short and long run effects of protection on the capital rental rate and the steady-state capital–labor ratio.

Consider an integrated world equilibrium with two goods (1 and 2) produced with two factors (capital K and labor L) under constant returns to scale by price-taking firms. The fixed coefficients technology (identical in all countries) relates the output of the goods, \( x_1 \) and \( x_2 \), to inputs at all points in time (continuous time is employed; the time index is suppressed where clarity permits):

\[
\begin{align*}
    x_1 &= \min \left[ \frac{L_1}{(a_1 L / A)}, \frac{K_1}{a_1 K} \right], \\
    x_2 &= \min \left[ \frac{L_2}{(a_2 L / A)}, \frac{K_2}{a_2 K} \right]
\end{align*}
\]

Labor augmenting technology advances according to: \( A(t) = A(0)e^{\eta t} \), where \( \eta \) is the exogenous rate of technological progress. Good 2 is relatively capital intensive, so \( a_{2K} a_{1L} > a_{2L} a_{1K} \). Neither good is storable. There are no adjustment costs.

In the spirit of the Solow growth model, investment is forgone consumption, so:

\[
I = \left( I_1 \right)^{1/2} \left( I_2 \right)^{1/2}
\]
where $I$ is investment, $I_1$ and $I_2$ are the amounts of goods 1 and 2 devoted to making new capital instead of consumption. Depreciation is ignored.

The infinitely-lived representative consumer chooses consumption to maximize:

$$U = \left( \frac{1}{1-(1/\sigma)} \right) \int_0^\infty e^{-\rho t} \left[ c_1(t)^{1/2} c_2(t)^{1/2} \right]^{1-(1/\sigma)} dt,$$

subject to a lifetime budget constraint (a dot over a variable indicates a time derivative, e.g., $\dot{x} = dx/dt$):

$$\dot{K} = \left( \frac{1}{P(t)} \right) \left( \nu(t) A(t) L + r(t) K(t) \right) - c(t), \quad \text{subject to } \lim_{t \to \infty} K(t) = \chi \geq 0.$$ 

Here $\rho$ and $\sigma$ are the discount rate and intertemporal elasticity of substitution, $\chi$ is an arbitrary constant, $c(t)$ is defined as $(c_1(t))^{1/2} (c_2(t))^{1/2}$, and the index $P$ equals $2(p_1(t)p_2(t))^{1/2}$ where $p_1$ and $p_2$ are the prices of goods 1 and 2. It is useful to define indices for aggregate output, $X$, such that $X(t)$ is $(x_1(t))^{1/2} (x_2(t))^{1/2}$.

It is easily shown that utility maximization implies:

$$\dot{c}(t)/c(t) = \sigma \left[ r(t)/P(t) - \rho \right].$$

Also, defining the optimal expenditure level as $E(t)$, consumption and investment demand functions are:

$$c_1(t) = \left( p_1(t) \right)^{-1} \left( E(t)/2 \right), \quad c_2(t) = \left( p_2(t) \right)^{-1} \left( E(t)/2 \right),$$

$$I_1(t) = \left( p_1(t) \right)^{-1} \left( Y(t) - E(t) \right)/2, \quad \text{and} \quad I_2(t) = \left( p_2(t) \right)^{-1} \left( Y(t) - E(t) \right)/2.$$ 

Clearly, $p_1 x_1 = p_2 x_2$ at every instant and expenditure is exactly equal to $c(t)P(t)$. From (2) and (5), we have:

$$\dot{K} = X(t) - c(t).$$

Additionally world income, $Y(t)$, equals $w(t)A(t)L + r(t)K(t)$. Income equals output in equilibrium so $Y$ equals $x_1(t) + p_2(t)x_2(t)$ which equals $P(t)X(t)$.

Prices, factor rewards ($w$ for wages, $r$ for the rental rate) and outputs at all times satisfy:

$$1 \equiv p_1 = \left( a_{1L}/k \right) w + a_{1K} r.$$
The matrix of $a_{ij}$'s is assumed to be non-singular; good 1 is the numeraire.

Equations (1), (5), (7) and (8) define the instantaneous equilibrium prices and outputs. Equations (4) and (6) describe the evolution of the economy through time. For convenience we take $\eta$ equal to zero, and $A(0)L$ equal to one, so the two state variables are $c$ and $K$. $c$ can jump, $K$ cannot. Their steady-state values (denoted with a bar) are such that $r$ equals $\rho$, and consumption equals output. Namely, $\bar{K}$ is such that:

$$\rho = \left(\frac{a_{1L}}{\Delta}\right) \left(\frac{a_{2L} - a_{2L,K}}{a_{1L} - a_{1L,K}}\right) + \frac{a_{2L}}{\Delta}, \quad \text{where} \quad \Delta = a_{1k}a_{2L} - a_{2k}a_{1L}.$$

and $\bar{c}$ is such that:

$$\bar{c} = \left(\frac{1}{\Delta}\right) \left[\left(a_{2k} - a_{2L}\bar{K}\right)\left(a_{1L}\bar{K} - a_{1k}\right)\right]^{1/2}.$$

Note that $\bar{K}$ is unique, so that there is only one steady-state capital-labor ratio for which non-specialization occurs. Baldwin (1989b) shows that this system is characterized by saddle path stability and converges to $\bar{K}$ and $\bar{c}$.

**Trade and Protection**

Any division of factors among countries would reproduce the integrated world equilibrium, as long as the relative "endowments" are similar enough so that no country specializes. Any such division would be time-invariant due to factor price equalization. To be concrete we consider two such divisions. First suppose the home country is "endowed" with a capital-labor ratio, $K^0$, which is less than the world steady-state capital-labor ratio (call this $\bar{K}_w$), so the home country imports good 2. To keep the dynamics simple, we rely on the convenient fiction that the home country is small in the sense that its output does not affect world prices. The phase diagram describing this situation is given by Figure 1.
Consider the effects of a permanent home tariff. On impact the tariff raises $p_2$ and $r$ and lowers $w$. With fixed input coefficients, there is no immediate output response. The jump in $r$ raises the return to foregone consumption leading home consumers to optimally accumulate capital. This rise in $K$ increases good 2 production at the expense of good 1 production (Rybczynski effect) — reducing both imports and exports. Due to the small, open economy assumption the initial rise in $K$ has no effect on the return to foregone consumption. Therefore $K$ continues to increase. Indeed, as long as the tariff is effective, $r$ will be above $\beta$ so $K$ will continue to rise. When the home capital–labor ratio reaches $K_w^*$, imports cease and the tariff becomes irrelevant. This is a new steady-state. More formally, the economy jumps from $E^0$ to $B$, in Figure 1, and converges to $E'$ along SS.

A trivial implication of this is that the Stolper–Samuelson effect does not hold in the long run in this model. Instead the tariff induces what might be called factor endowment equalization. For the purposes of our analysis the only important points are that in this case the return on foregone consumption is ceteris paribus increasing in the tariff, and the tariff raises the steady–state home capital–labor ratio.

Next consider the case where the home country is 'endowed' with $\kappa$ greater than $K_w^*$ so it imports good 1. Again examine the effects of a home tariff. On impact the tariff lowers $r$ and leads to a fall in $K$. As before, $K$ continues falling until the home country's capital–labor ratio equals $K_w^*$. Parenthetically, we note that the Stolper–Samuelson effect again incorrectly predicts the long–run effect of protection on factor rewards. The relevant aspect, however, is that in this case the tariff lowers $r$ at the initial $K$, and reduces the home steady–state capital stock. It is a straightforward exercise to work out the exact adjustment with a phase diagram similar to Figure 1.

To summarize, protection affects a country's rental rate and thereby its steady–state capital–labor ratio. The direction of the effect in this simple model depends solely on factor
intensities. It is well-known that in more general models the link between prices and factor rewards is ambiguous. Next we use these results to direct our investigation of the Ricardian effect in an implicit model.

1.1 An Implicit Model

Trade barriers may raise or lower \( r \), thereby inducing a Ricardian dynamic effect which exaggerates or mitigates the standard output effects of protection. The simple model above leads to the extreme result that protection raises home production of the imported good to the point of self-sufficiency. To demonstrate the generality of the Ricardian effect, we work with a more general model.

Suppose the world's real gross national product (GNP), \( y \), is given by (or at least can be well approximated by): \( F[K, L, \tau] \), where \( K \) and \( L \) are the world capital stock and labor force, and \( \tau \) is an index of global trade barriers. The dynamic effects we address involve the accumulation of capital. To highlight this, \( L \) is assumed to be time invariant. For notational simplicity we suppress \( L \) and work with:

\[
y(t) = f[K(t), \tau].
\]

The function is assumed to be increasing in \( K \) and decreasing in \( \tau \). Note that with \( L \) fixed \( K \) is proportional to the capital-labor ratio.

Investment is foregone consumption, so:

\[
\dot{K} = y - c,
\]

where \( c \) is consumption. Depreciation is ignored. Furthermore, assume that capital is the only means of carrying over income between periods. The real rate of return on foregone consumption is related to trade barriers and \( K \) by:

\[
r(t) = r[K, \tau]
\]

If we assume perfect competition and constant returns to scale, \( r[K, \tau] \) is the partial derivative of \( f \) with respect to \( K \). However, we wish to allow for a divergence between social and private rates of
return due to external economies of scale. Thus we assume only that \( r_c[K,r] \) (subscript denotes partial derivatives) is negative and the partial of \( r[K,r] \) with respect to \( r \) may be positive or negative (both cases are considered below).

The representative, infinitely-lived consumer maximizes:

\[
U = \left( \frac{1}{1-(1/\sigma)} \right) \int_0^\infty e^{-\rho t} c(t)^{1-(1/\sigma)} \, dt,
\]

subject to a lifetime budget constraint:

\[
\dot{K} = w(t)L + r(t)K(t) - c(t) \quad \text{where} \quad \lim_{t \to \infty} K(t) = \chi, \quad \text{s.t.} \quad x > \chi \geq 0.
\]

The Hamiltonian for this problem is: \( \left( e^{-\rho t} c^{1-(1/\sigma)}/(1-(1/\sigma)) \right) + \lambda(wL + rK) \). The optimal consumption path is characterized by (12) and the necessary conditions: \( e^{-\rho t} c^{1-(1/\sigma)} = \lambda \) and \( \dot{\lambda} = -\lambda r \). To make the analytics more intuitive, we work with \( K \) and \( c \) as the state variables, instead of \( K \) and the co-state variable, \( \lambda \). The necessary conditions imply:

\[
\dot{c}/c = \sigma \left( r(t) - \rho \right).
\]

Equations (12) and (16) describe the dynamics of the model.

2. Stability, Convergence and Existence of the Steady State in the World Economy

The dynamics of this system are simple and can be analyzed with Wilson-Dornbusch techniques. The steady-state \( c \) and \( K \) satisfy:

\[
r[K,r] = \rho \quad \text{and} \quad f[K,r] = \bar{c}.
\]

To characterize the dynamics out of steady state, we use a phase diagram (Figure 2). We plot the locus of \( c \) and \( K \) for which \( \dot{K} \) equals zero as \( K = 0 \). It is upward sloped since the marginal product of capital is positive. We plot the locus of \( c \) and \( K \) for which \( \dot{c} \) equals zero as \( c = 0 \). It is vertical since there is only one capital-labor ratio at which \( r \) equals the discount rate. Equations (12) and (16) describe the laws of motion off the \( \dot{c} = 0 \) and \( \dot{K} = 0 \) schedules. For all pairs of \( c \) and \( K \) to the left of \( \dot{c} = 0 \), \( c \) will be increasing; all points to the right correspond to falling \( c \). These
observations are depicted in Figure 2 with arrows. Points below $\dot{K} = 0$ correspond to rising $K$; points above it correspond to falling $K$. Again these laws of motion are shown with arrows.

This system is characterized by saddle path stability. That is, there is a unique locus of initial values of $c$ and $K$, drawn as SS, for which the economy will actually converge to the steady state. The capital stock changes continuously with time but the consumer can choose $c$ freely. Thus $c$ may make discrete jumps. The consumer would choose $c$ to be somewhere on the saddle path, since otherwise consumption will eventually fall to zero; if he chooses $c$ too low, capital accumulates forever as consumption trails off to zero; if he chooses $c$ too high, the capital stock is eventually run down to zero. In other words, any other choice would violate the transversality condition in (15).

More formally, the stability is analyzed by linearizing (12) and (16) around the steady state and investigating the sign of the eigenvalues of the resulting Jacobian. These are equal to:

$$\lambda = \frac{r}{K} \pm \left( \frac{r}{K} - 4 \sigma r_k \right)^{1/2} / 2.$$ 

Since $r$ is decreasing in $K$, there are two real roots of opposite sign. This ensures the existence of a unique saddle path.

3. **Comparative Steady State Analysis of Multilateral Liberalization**

Consider the long-run output effects of lower global trade barriers as captured by the index $\tau$. In our model, a liberalization has two effects. A static effect on world GNP, and a dynamic effect via an induced change in the steady-state capital stock (i.e., capital-labor ratio since $L$ is fixed). To see this we totally differentiate the steady-state conditions (17), to get:

$$\frac{dy}{d\tau} = \left( \frac{\partial f}{\partial K} / \tau \right) \left( -\frac{\partial f}{\partial r} / \tau \right) \left( \frac{\partial f}{\partial r} / \tau \right) + \left( \frac{\partial f}{\partial r} / \tau \right).$$

The second term captures the usual static effects of a liberalization: by removing distortions, the same amount of capital and labor may be combined more efficiently, producing more output. In general a liberalization affects the rate of return on capital. This in turn, leads to the accumulation or decumulation of capital. If the trade barrier reduction leads to a ceteris paribus
rise in $r$, the dynamic effect amplifies the static effect. That is, consumers find it optimal to accumulate capital until the capital–labor ratio is sufficiently high to return $r$ back to its steady-state value, $\rho$. Alternatively if $r_T$ is negative, consumers will find it optimal to reduce the capital stock. In this case the Ricardian output effect tends to offset the static effect. This indirect effect of trade on factor endowments is captured by the first term. The quotient in large parentheses gives the proportional change in $K$ resulting from the liberalization. The output effect of this change in $K$ is determined by the capital–output elasticity of the GNP function.

The adjustment path can be seen in Figure 3. Here the liberalization shifts the new steady-state point from $E^0$ to $E'$. The new saddle path is shown as $SS'$. Consumption jumps from the old steady-state point $E^0$, to point B and the economy moves along $SS'$ to $E'$.

Since steady-state output equals consumption, (18) also gives the comparative steady-state increase in consumption. The welfare interpretation of this change is complicated. From the point of view of the infinitely-lived consumer, the rise in steady-state consumption due to the accumulation of capital is largely or entirely offset by the foregone consumption that was necessary to accumulate the capital. However stepping outside the model for a moment, note that if we take (14) as an approximation of the behavior of successive generations facing a complete, perfect capital market, then (18) does have a straightforward welfare interpretation for the generations who did not forego consumption to build up the capital stock. Namely, their consumption would be higher, yet they would not have had to forego consumption in order to build up the capital stock.

4. Welfare: Dynamic Gains from Trade

The most straightforward approach to gauging the welfare implications of trade liberalizations would be to solve explicitly for the adjustment path of $c$ and evaluate this with the utility function. The problem is that (12) and (16) are non-linear in the state variables. An
analytic solution for the saddle path is therefore impossible. We could linearize the system around the steady-state and work with the resulting system of linear differential equations. This is only correct for very small changes in \( c \) and \( y \). Since one of the points of this paper is to show that dynamic effects are large, the linearization approach is unsatisfactory.

As it turns out, we do not need to get an analytic solution to the consumption path in order

to find the welfare effects. To see this, note that the optimal consumption path is a function of time and implicitly of \( \tau \). Differentiating (14) (evaluated at \( \bar{c} \)) with respect to \( \tau \), we see that

d\( U/d\tau \) is: \( (c)^{-1/\sigma} \int_0^{\infty} e^{-\rho t} c(t) dt \). In other words, the welfare impact depends on the Laplace transform of the induced change in the consumption path. This comment is germane since Judd (1985) shows that it is much easier to deal with the Laplace transforms of state variables' paths than with the paths themselves.

To keep the analysis as general as possible, we consider a general form of changes in \( \tau \) over time. That is, we multiply \( \tau \) by \( 1 + ch(t) \) throughout (12) and (16), where \( h(t) \) is a known, arbitrary time path (usually a step function). This allows us to consider a broad class of trade policy changes. To determine the welfare effect of small changes in \( \tau \) over a time path described by \( h(t) \), we differentiate the altered differential equations with respect to \( \epsilon \) and evaluate the result at \( \epsilon \) equal to zero. In matrix form this yields:

\[
\begin{bmatrix}
\dot{c} \\
\dot{k} \\
\end{bmatrix} = J
\begin{bmatrix}
c \\
k \\
\end{bmatrix}
+ \begin{bmatrix}
\tau c \sigma \mu(t) r \\
\tau h(t) f \\
\end{bmatrix}
\]

where

\[
J = \begin{bmatrix}
0 & -c \sigma r_k \\
-1 & f_k \\
\end{bmatrix}
\]

The Jacobian matrix is evaluated at the steady-state levels of \( c \) and \( K \).

Next we multiply (12) and (16) by \( e^{\omega t} \), integrate over time, and then integrate the left hand side by parts. This yields the Laplace transform of (19):

\[
\begin{bmatrix}
C_c(\omega) \\
K_c(\omega) \\
\end{bmatrix} = (\omega I - J)^{-1}
\begin{bmatrix}
\tau c \sigma II_c(\omega) r \tau + c_c(0) \\
\tau II_c(\omega) f \tau \\
\end{bmatrix},
\]

where \( C_c(\omega), K_c(\omega) \) and \( II_c(\omega) \) are the Laplace transforms of \( c_c, K_c \) and \( h_c \) (e.g., \( C_c(\omega) \) equals
\[ 0^\infty e^{-\omega t} \epsilon(t) dt. \] Notice that the integration by parts has turned the system of differential equations into an algebraic system in Laplace transforms. The only unknown in (20) is the size of the consumption jump at time zero, \( c(0) \), since we used the fact that capital does not jump to set \( K(0) \) equal to zero.

To determine, \( c(0) \) note that by the transversality conditions \( K(\omega) \) must remain finite for all values of \( \omega \). Consider \( \omega \) equal to the positive eigenvalue of \( J \) (call this \( \mu \)). Since \( \mu I - J \) is singular, it must be that (see Judd 1985 for details):
\[ c \sigma_T H(\mu) + c(0) - \mu h(\mu) f_T = 0. \]

Using this in (20), taking \( \omega \) equal to \( \rho \), yields the welfare impact:
\[ (21) \quad \left( \frac{dU}{dc} \right) \left( \frac{dc}{c} \right)^{-1/\sigma} = \frac{1}{\Delta} \left[ c \sigma_T H(\rho) + c(0) - \mu h(\rho) f_T \right]. \]
where \( \Delta \) is the determinant of \((\rho I - J)\), and all partials are evaluated at \( \tilde{c} \) and \( \tilde{K} \). For many policy changes, \( h(t) \), it is possible to obtain a closed form solution for \( H(\omega) \). For such \( h(t) \), it is a straightforward exercise to evaluate (21).

**Welfare Impact of a One-off Reduction in Trade Barriers**

Consider a one-off change in \( \tau \) (i.e., \( h(t) \) equal to unity for all \( t \)). In this case the proportional change in welfare, normalized by the marginal utility of consumption, is:
\[ (22) \quad \left( \frac{dU}{d\tau} \right) / \left( \frac{dU}{dc} \right) = \left[ \frac{1}{\rho - \rho k} \frac{\partial q}{\partial \tau} \right] + \left[ \frac{\rho - \rho k}{c \sigma_T + \rho^2 - \rho k} \right] \sigma \left( \frac{1}{\rho} - \frac{1}{\mu} \right) \left( \frac{\partial f_T}{\partial \tau} \right). \]

This expression is easy to interpret. The first term is equal to the present discounted value of the static gain. The second term captures the welfare effect of the Ricardian dynamic trade effect. If there are no external economies of scale in the employment of capital, then \( r[K, \tau] = f_k[K, \tau] = \rho \). Consequentially the dynamic welfare effect, (i.e., the dynamic gain from trade) is zero. In other words, although the Ricardian dynamic effect leads to a larger output effect, it does not contribute to welfare. Intuitively, think of this result as an application of the envelope theorem. The consumer is optimizing (taking \( \tau \) as a parameter) between consumption today and savings.
which will yield consumption in the future. The change in the objective function with respect to \( \tau \) is the same with and without reoptimizing on \( K \).

However, if there are external economies of scale, the social marginal product of capital may exceed the rental rate. Thus there will be dynamic gains from trade due to the Ricardian dynamic effect. To see this, note that with external economies \( r[K, \tau] \) need not equal the social marginal product of capital, \( f_k[K, \tau] \) (Section 5 considers an explicit example of this). Consequentially \( \rho \) can be less than \( f_k[K, \tau] \). The determinant of \((\rho I - J)\) is negative and the positive eigenvalue of \( J \) is greater than \( r[K, \tau] \), so the second term in (22) has the same sign as \( r[K, \tau] \). To summarize this discussion:

Proposition 1 (necessary condition for dynamic gains from trade): If the social and private marginal product of capital are identical, the Ricardian dynamic trade effect has no effect on welfare. If the social rate exceeds the private rate then the Ricardian dynamic effect has a positive welfare effect only if the liberalization raises the return to capital. If the liberalization lowers the return to capital, the Ricardian dynamic effect tends to offset the static gains from trade.

The result that the Ricardian effect may tend to lower welfare should be interpreted in the light of the theory of the second best. External economies drive a wedge between the private and social rates of return. In all such cases, many types of intervention may improve welfare. The best policy (ignoring the efficiency cost of government revenue) is to remove the wedge at its source. In other words, the best policy mix with external economies of scale is to subsidize capital formation directly and liberalize trade.

Dynamic Gains from Trade for Large Policy Changes

Equation (22) gives the exact welfare impact of small changes in trade barriers, and can be used to find a first-order approximation to the welfare impact of large policy changes. Evaluating the exact impact of a large policy change would involve solving for the Laplace transform of the actual adjustment path. The difficulty with this is that it would require us to solve non-linear differential equations. In the above procedure, we differentiated the dynamic
system with respect to \( \xi \), and evaluated it at \( \xi = 0, \ c = \bar{c} \) and \( y = \bar{y} \). Thus \( J \) was a matrix of scalars which posed no problem when we took the Laplace transform of the system.

What all this goes to say is that in general, it is not possible to solve for the exact welfare impact of large policy changes. Nevertheless, we can show that even when the social and private returns to capital coincide, a large liberalization may lead to dynamic gains from trade. The argument is illustrated in Figure 4. The outer curve in the diagram represents utility when the capital stock is optimally adjusted. The inner curve plots the utility when the capital stock is held constant. If \( \rho = f_k[K, \tau] \) then the curves are tangent at the initial \( \tau \) since the private and social planner's problems are identical. Small changes in the tariff lead to the same welfare impact with and without a re-optimization of the capital stock. This is the envelope theorem. Yet for a big change in \( \tau \), say to \( \tau' \) in Figure 4, the re-optimization of the capital stock is not negligible. In other words, the Ricardian dynamic trade effect would lead to positive dynamics gains from trade, even in the absence of external economies of scale.

5. **Measuring the Ricardian Dynamic Trade Effect**

This section adopts simple functional forms that enable quantification of the positive and welfare impact of the Ricardian dynamic trade effect. The functional form for the GNP function implies that \( r \) is everywhere decreasing in \( \tau \). We think of this as capturing the effect of a worldwide liberalization of intra-industry trade in capital intensive goods (say manufacturing). Here \( \tau \) captures foreign and domestic tariffs, and we presume that a multilateral reduction in trade barriers in the sector raises the rate of return on capital. Of course, one can construct models where a multilateral liberalization of manufacturing would have exactly the opposite effect on \( r \). As Section 1 and 3 pointed out the effects work in the opposite direction is \( r \) is increasing in \( \tau \).

**A Specific Functional Form**

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Suppose the GNP and \( r \) functions are:

\[
y(t) = \beta A(t) \left( K(t)^{\alpha_i} L^{1-\alpha_i} \right), \quad \text{and} \quad r(t) = \alpha \beta A(t) \left( K(t)^{\alpha_i} L^{1-\alpha_i} \right) / K(t),
\]

where \( \beta \) equals \((-\frac{1}{1+\tau})\) and \( A \) is total factor productivity. The true determinants of total factor productivity are not well understood. On one hand, the neoclassical growth model assumes it is driven by exogenous technological progress. On the other hand, the new theory attempts to endogenize the advancement of primary factors productivity (e.g., Romer 1983, Grossman and Helpman 1989b). The Ricardian dynamic effect demonstrated by this paper does not depend on the exact source of the productivity growth. Rather than tie our model to a specific school of thought, we assumed that:

\[
A(t) = B(t) K(t)^{\theta} L^{\psi}, \quad \text{where} \quad B(t) = B(0) e^{\eta t}.
\]

Here \( B \) represents the state of basic scientific knowledge, and \( \eta \) is the exogenous rate at which disembodied technology advances (due, say, to human curiosity). \( \theta \) captures the external economies in the usage of capital. Firms are assumed to take the path of \( A \) as given.

There are several possible interpretations of equation (24). The most straightforward is that \( K^{\theta} L^{\psi} \) represents the standard external economies of scale. Thus the production function for a typical firm employing \( K_i \) and \( L_i \) units of capital and labor is:

\[
y = \psi K_i^{\alpha_i} L_i^{1-\alpha_i}, \quad \text{where} \quad \psi \text{ is a measure of the external scale economies.}
\]

In this case, \( \alpha + \theta = \alpha / (1-\psi) \) and \( 1-\alpha + \psi = 1-\alpha / (1-\psi) \). Alternatively, Romer (1987) argues that external economies are entirely captured by \( K_{\theta} \) and \( \eta \) and \( \varphi \) are zero. Lastly the Solow model is where \( \theta \) and \( \varphi \) equal zero.

Unless \( \alpha + \theta < 1 \), steady-state \( K \) and \( c \) do not exist. We therefore restrict our attention to \( \alpha + \theta < 1 \). For convenience we take \( L \) equal to one and set \( \eta \) equal to zero. Allowing for exogenous technological progress is a straightforward exercise (define new state variables, \( e/B \) and \( K/B \), and proceed as before).

**Comparative Steady-State Analysis: Output Effects**

With these additional assumptions, the proportional rise in \( y \) due to a liberalization is (using
Jonesian hat notation, i.e., \( \dot{x} \equiv \frac{dx}{x} \):

\[
(25) \quad \dot{y} = \left( \frac{1}{\alpha + \theta} - 1 \right) \hat{\beta} + \hat{\beta}.
\]

where \( \hat{\beta} \) equals the static output effect of the liberalization considered (the increase in GNP with no change in the capital stock). Clearly it is extremely simple to measure the size of this output effect. Only two readily available estimates are required. The capital—output elasticity of the GNP function (i.e., \( \alpha + \theta \)), and an estimate of the size of the static gain (i.e., \( \hat{\beta} \)). To illustrate the measurement of the Ricardian dynamic trade effect, we take the EC's 1992 program as an example.

The size of \( \alpha + \theta \) is an unsettled empirical question. Prior to the new growth literature, it was widely assumed that \( \alpha + \theta \) equaled capital's share of income (or one minus labor's share of income). This is an implication of perfect competition and constant returns to scale much exploited by the growth accounting literature. Table 1 reproduces a number of such estimates for France, Germany, the Netherlands and the UK. The numbers range from 0.446 to 0.222. A recent survey, Maddison (1987), takes 0.3 as the consensus figure.

Econometric estimation of the GNP function is problematic due to simultaneity between optimal factor choice and random productivity shocks. Hall (1989), and Caballero and Lyons (1989a, b) have pioneered new techniques to skirt this problem. Using these techniques, Caballero and Lyons (1989b) estimate the sum of capital and labor output elasticities for France, Germany, Belgium and the UK. To recover \( \alpha + \theta \) from the Caballero and Lyons numbers, we must multiply their aggregate number by capital's cost share. Since the authors use panel data on capital's cost share, it is not possible to recover the exact \( \alpha + \theta \). We get a rough approximation by multiplying the Caballero and Lyons' aggregate number by Maddison's consensus 0.3. To test the results for sensitivity to the estimates, we do the same calculation for their points estimates plus and minus one standard error. Table 1 lists the resulting numbers.
Equation (25) shows that Ricardian dynamic output effect can be thought of as a multiplier on the static effect. The size of this Ricardian output multiplier can by itself tell us how important the Ricardian dynamic effect is. For instance take the low estimate of $\alpha + \theta$ for France from Table 1, 0.23. In this case the multiplier equals about 0.3. In other words, by ignoring the fact that the capital stock is endogenously determined, empirical estimates of the static effect alone underestimates the total output effect by at least 30 percent. Table 2 presents the multipliers that correspond to the high and low values of $\alpha + \theta$ from Table 1 for each country. They range from 24 to 136 percent.

To get estimates of this dynamic effect of the 1992 program, we multiply the various estimates of the multiplier by an estimate of the static output impact of 1992. Here we employ the Cecchini Report’s estimate that 1992 will lead to a once-off increase in EC GNP of between 2.5 and 6.5 percent. We take the high and low estimates of the multiplier for each country from Table 2, and multiplied these by the high and low estimates of the static effect from the Cecchini Report (2.5 to 6.5 percent). The results are listed in Table 3. The first and second rows in Table 3 presents 1992’s effect on EC GNP (in percentage points) due solely to the indirect effect. Of course there would be no indirect effect without the static gain, so the total effect (the static range of 2.5 to 6.5 plus the high and low ranges from the first row) of 1992 on EC GNP is presented in the third and fourth rows of Table 3.

The most robust conclusion from Table 3 is that the indirect effect is considerable in all cases. At the very least, it means the endogenous rise in capital will boost EC GNP by an extra 0.6 percent. The largest numbers in this table are large by comparison with the Cecchini Report range. They are all about twice the size of the high end of the Cecchini Report range. Baldwin (1989a) attempts to establish an upper bound on this type of gain by using an ordinary least squares (OLS) estimate of the GNP function. The OLS estimate of $\alpha + \theta$ (which is obviously upward biased) is 0.975. This value of $\alpha + \theta$ yields a multiplier of 38.
Ricardian Welfare Multiplier

For the functional form adopted the proportional change of $r$ and $y$ with respect to $\tau$ are identical, so:

\begin{equation}
\frac{dU}{dc} \frac{dU}{dc} = \left[ \left( \frac{1}{\rho} \frac{\partial y}{\partial r} \right) + \phi \frac{\partial y}{\partial r} \right],
\end{equation}

where

$$
\phi \equiv \left( \frac{\theta}{1-\alpha-\theta} + \frac{\theta}{\sigma} \right) \left( \frac{1}{\rho} - \frac{1}{\mu} \right),
$$

and the positive eigenvalue of $J$, $\mu$, equals:

$$
\frac{\rho}{2\alpha} \left[ \alpha + \theta + \left( (\alpha+\theta)^2 + 4\alpha\sigma(1-\alpha-\theta) \right)^{1/2} \right].
$$

The term, $\frac{\partial y}{\partial r}$, represents the static impact of trade liberalization on GNP (this is what empirical studies of trade liberalizations typically measure). Consequently, it may be useful to think of $\phi$ as a multiplier. That is, in addition to the well-known static gains from trade, the Ricardian effect leads to a further welfare gain that is proportional to the static gain. We now turn to approximating the size of this Ricardian welfare multiplier.

Estimates of all the parameters in the multiplier are readily available in the literature. Table 4 presents the calculated values of the Ricardian welfare multiplier for the Caballero and Lyons capital-output elasticities (and these estimates plus one standard error). In all cases, we take the discount rate equal to 0.05, the intertemporal elasticity of substitution as 0.1 (this is the consensus figure from Hall 1988), and $\alpha$ equal to Maddison's consensus 0.3.

The main point to emerge from Table 4 is that this dynamic gain from trade is not insignificant. For France, Germany, the UK and Belgium the multiplier ranges from 0.17 to 0.87. That is, the Ricardian effect accounts for an extra rise in welfare of that is somewhere between 15 and 90 percent of the static output effect of the liberalization. However, the increase in welfare due to the Ricardian effect is small relative to the welfare contribution of the static effect. The welfare impact of the static effect is the percent rise in output (holding $k$ constant) multiplied by

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something like 20 \( (\text{for } \rho = 0.05) \). The welfare impact of the dynamic effect is the output effect multiplied by a number that is close to unity. Intuitively, this reflects the fact that the static gain is 'for free' while the dynamic gain is largely offset by the foregone consumption necessary to build the capital stock.

6. Conclusion and Directions for Further Research

Productive factors such as human and physical capital are accumulated. Since the steady-state levels of such factors are determined endogenously, trade policy can affect these levels. A trade liberalization therefore has a dynamic effect on output and welfare as the economy moves to its new steady state. This paper show that both the positive and normative impact of this dynamic effect are measurable. The extra output change due to this dynamic effect appears to be quite large. The size of the welfare impact depends on the degree of external economies of scale in the economy. Note that this dynamic effect is not dependent on the new growth models; it is present even in the Solow growth model. Ricardo (1815) first explored the effect of trade on steady-state factor supplies.

This paper suggests that further work be done on estimating the aggregate capital-output elasticity. This is not an easy task (see Caballero and Lyons 1989a,b). From the theory standpoint, it may be worthwhile allowing more than one factor to accumulate. Since a country's 'endowment' of skilled labor has played an important role in the standard trade model, this is probably a reasonable candidate. Moreover, if it turns out that external economies are important empirically, it would be useful to explicitly model the externalities as in the Grossman–Helpman literature. Such theoretical refinements are important subjects for future research.
FOOTNOTES

I gratefully acknowledge the comments and suggestions of Elhanan Helpman, Peter Pedersen, Victor Norman, Jon Markussen and Forsken Persson. Avinash Dixit suggested the use of Laplace transforms in gauging welfare effects. I thank the Institute for International Economic Studies in Stockholm for providing a fertile environment for this work.


3. Romer's 1983 PhD thesis is considered the seminal paper. Also see Shleifer (1986), Romer (1986, 1987a, b).

4. The slope of $\hat{K} = 0$ depends on whether $X$ is increasing or decreasing in $K$. This curve will have a bell shaped. More formally note that the $dX(t)/dK$ equals

$$(1/2\Delta)((a_{2K} - a_{2L}K)(a_{1L}X - a_{1K}))^{-1/2}times (a_{1L}a_{2K} + a_{2L}a_{1K} - a_{1L}a_{2L}2K),$$

where $\Delta$ is the determinant of the $a_{ij}$ matrix. Define a range of $K$ equal to $(a_{1K}/a_{1L}) + v$, $v \geq 0$. The range of $K$ for which this derivative is positive, for any given set of $a_{ij}$'s, is defined by those $v$ which satisfy:

$$(1/2)^{a_{2K} - a_{2L}} > v.$$ 

Note that this set is not empty since if the integrated world equilibrium is to be non-specialized, $a_{2L} > a_{2L}$.

The range of $K$'s for which the derivative is positive (and $K$ is in the diversification cone) is given by those $v$'s for which $(1/2)(a_{2K} - a_{2L}) < v$ and $v < (a_{2K} - a_{2L})$. The saddle path is positively sloped whether the steady state is located on the rising or falling portion of $\hat{K}=0$.

5. Capital and labor are often taken as the two factors in the Heckscher-Ohlin model.

6. Stolper and Samuelson (1941) were careful to refer to their factors as labor and land.
REFERENCES


Deardorff, A. and Stern, R. (1978),


Harris, R. and Cox (1982)


Romer, Paul (1987b) "Growth Based on Increasing Returns to Scale Due to Specialization," *American Economic Review*.


Table 1: Estimates of Aggregate Capital—Output Elasticity ($\alpha + \beta$):

<table>
<thead>
<tr>
<th>Source</th>
<th>France</th>
<th>Germany</th>
<th>Netherlands</th>
<th>UK</th>
<th>Belgium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denison</td>
<td>.23</td>
<td>.263</td>
<td>.26</td>
<td>.222</td>
<td></td>
</tr>
<tr>
<td>Denison and Chung</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maddison (1987)</td>
<td>.305</td>
<td>.3</td>
<td>.296</td>
<td>.255</td>
<td></td>
</tr>
<tr>
<td>Kendrick</td>
<td>.382</td>
<td>.349</td>
<td></td>
<td>.348</td>
<td></td>
</tr>
<tr>
<td>Christensen, Cummins and Jorgenson</td>
<td>.403</td>
<td>.386</td>
<td>.446</td>
<td>.385</td>
<td></td>
</tr>
<tr>
<td>Caballero and Lyons (1989)</td>
<td>.365</td>
<td>.477</td>
<td>.339</td>
<td>.426</td>
<td></td>
</tr>
<tr>
<td>Minus one Std Error</td>
<td>.288</td>
<td>.39</td>
<td>.195</td>
<td>.276</td>
<td></td>
</tr>
<tr>
<td>Plus one Std Error</td>
<td>.444</td>
<td>.564</td>
<td>.483</td>
<td>.576</td>
<td></td>
</tr>
</tbody>
</table>

Source: First four rows reproduced from Maddison (1987), Table 8; see same for references. Fifth row reproduced from Caballero and Lyons (1989) taking 0.3 as capital's share of income.
Table 2: Underestimate of GDP Rise by Ignoring Indirect Effect
(Percent)

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>Netherlands</th>
<th>UK</th>
<th>Belgium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>30</td>
<td>36</td>
<td>35</td>
<td>24</td>
<td>38</td>
</tr>
<tr>
<td>Hi</td>
<td>80</td>
<td>129</td>
<td>124</td>
<td>93</td>
<td>136</td>
</tr>
</tbody>
</table>

Source: Author’s calculation.

The percent underestimate is 100 times \((\alpha+\theta)/(1-\alpha-\theta)\).

Table 3: Eventual Increase in GDP due to 1992

<table>
<thead>
<tr>
<th>Indirect Effect on GDP due to rise in Steady—State Capital Stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Percentage Points to be Added to Static Range)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Lo</td>
</tr>
<tr>
<td>Hi</td>
</tr>
</tbody>
</table>

Total Effect (Static plus Dynamic) (Percent rise in GDP due to 1992)

| Lo | 3.3 to 8.5 | 3.4 to 8.8 | 3.4 to 8.8 | 3.1 to 8.1 | 3.5 to 9 |
| Hi | 4.5 to 11.7| 5.7 to 14.9 | 5.6 to 14.2| 5.8 to 12.5| 5.9 to 15. |

Source: Author's calculation based on Table 2 and Cecchini Report's estimate of static effect.
Table 4: Size of the Ricardian Welfare Multiplier

<table>
<thead>
<tr>
<th></th>
<th>France</th>
<th>Germany</th>
<th>UK</th>
<th>Belgium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare Multiplier due to Ricardian dynamic effect (Numbers to be multiplied by static effect on GNP)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caballero and Lyons estimates of $\alpha + \theta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>multiplier</td>
<td>.29</td>
<td>.64</td>
<td>.17</td>
<td>.50</td>
</tr>
<tr>
<td>$(\alpha + \theta)$</td>
<td>.37</td>
<td>.48</td>
<td>.34</td>
<td>.43</td>
</tr>
<tr>
<td>Caballero and Lyons estimates plus one standard error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>multiplier</td>
<td>.53</td>
<td>.83</td>
<td>.64</td>
<td>.87</td>
</tr>
<tr>
<td>$(\alpha + \theta)$</td>
<td>.44</td>
<td>.56</td>
<td>.48</td>
<td>.58</td>
</tr>
</tbody>
</table>

Source: Author's calculation and rows 5 and 6 from Table 1.