“Game Theory"

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Lecture Notes 2

- Dominant and dominated strategies

- Iterated Deletion of Strictly Dominated Strategies (IDSDS)

Solution Concept 1: Dominance

- Games in **strategic forms**
- In solving these games, we determined that players don't ever play **strictly dominated strategies** (a strategy that will always be inferior to another strategy).
- Prisoner's Dilemma is an example where there isn't much strategic interaction because there are only two strategies and one is strictly dominated.

Solution concept 2: IDSDS

- When you're solving a game, once you eliminated a strictly dominated strategy, you can begin eliminating other strategies.
- This is called **Iterated Deletion of Strictly Dominated Strategies.**

Example: Hotel Location Problem

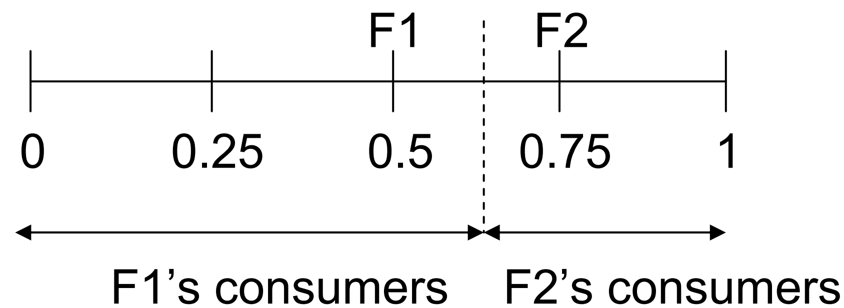
- There are two firms (Firm 1, Firm 2). They need to determine where to locate their facilities along a straight row.
- Once they build their facility, they cannot move.

Example: Hotel Location Problem

- These firms are identical, so consumers will buy from the seller who is closest to them.
- The firms' goal is to maximize the number of consumers.
- There are 5 locations where the firms can locate: 0, .25, .50, .75, 1.0.

Example: Hotel Location Problem

- Example: Firm 1 is at .50 and Firm 2 is at .75.



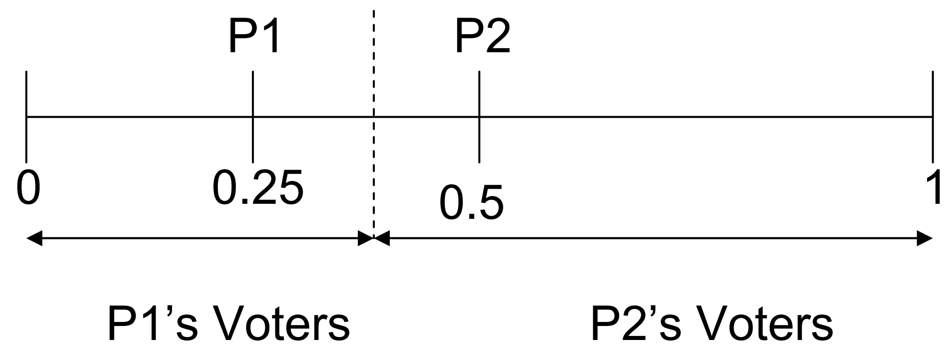
- Everyone from 0 to .50 will buy from Firm 1, and everyone from .75 to 1.0 will buy from Firm 2. Firms 1 and 2 will split consumers between .50 and .75.

Example: Voting model

- Let's change the set-up to Political Preferences/ Ideologies.
- 0 represents the far left and 1 represents the far right.
- Voters are spread out across the ideological spectrum.
- Parties compete for votes the same way Firms compete for locations—whoever is closest to them.

Example: Voting model

- Example: Party 1 is at .25 and Party 2 is at .50.



- If I, as a voter, am at 1.0, then I would vote for Party 2 over Party 1, even though Party 2 isn’t that close to my preference.

Hotelling Location Game

- The game in strategic form is called the Hotelling Location Game.

- Are any strategies strictly dominated?

Hotelling Location Game

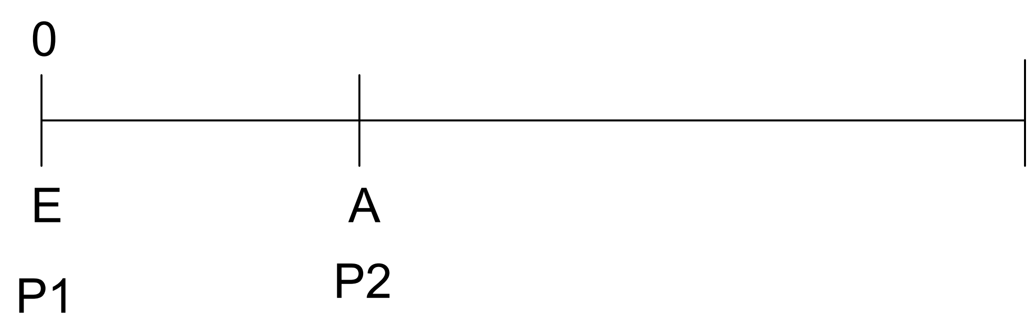
- Rule of thumb: extreme strategies are usually eliminated as strictly dominated.

- Let's call strategy 0 (left extreme) E.

- Would Player 1 ever play this strategy?
- No matter what Player 2 plays?

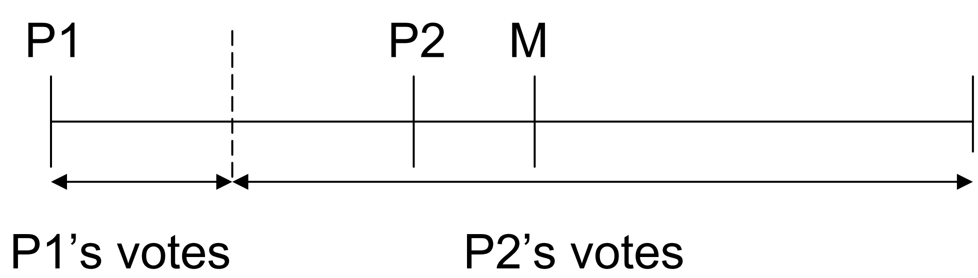
Hotelling Location Game

- What if they both locate themselves on E?
- If there is a tie, people are indifferent, they both get half the votes.
- If Player 1 plays on E, Player 2 does better by locating on A, one position to the right of E.



Hotelling Location Game

- What if Player 2 moves closer to the Center?
- Does Player 1 do better by locating on E or A?
- Player 1 will do better by locating on A because they will get a larger portion of the voters.



Hotelling Location Game

- Strategy E is strictly dominated by A.
- Because of symmetry, the same thing will happen at the other extreme end of the spectrum.
- Also, Player 1 and Player 2 will act identically.

Hotelling Location Game

- Repeated elimination of strategies through this method, we see that both players will be pushed to the Center of the spectrum.

- Elections are generally considered a race to the center.

Definition

- Definition (**Strict domination**): In a strategic form game, player *i*'s action a_i'' **strictly dominates** her action a_i' if:

  $u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for every list a_{-i} of the other players' actions.
- We say that the action a_i' is strictly dominated.

Definition

- Definition (**Weak domination**): In a strategic form game, player *i*'s action a_i'' **weakly dominates** her action a_i' if:

  $u_i(a_i'', a_{-i}) \geq u_i(a_i', a_{-i})$ for *every* list a_{-i} of the other players' actions

and

  $u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for *some* list a_{-i} of the other players' actions.

- We say that the action a_i' is weakly dominated.

Example 1: **The Prisonner's Dilemma**

	Not confess	Confess
Not Confess	(-1,-1)	(-9,0)
Confess	(0,-9)	(-6,-6)

Identify the primitives of the game…

$I=\{1,2\}$; $S_1 \times S_2 = \{(NC,NC), (C,C), (C,NC), (NC,C)\}$

$u_i(NC,NC) = -1 \quad i = 1,2$ $\qquad$ $u_i(C,C) = -6 \quad i = 1,2$
$u_1(C,NC) = 0 = u_2(NC,C)$ $\qquad$ $u_1(NC,C) = -9 = u_2(C,NC)$

Solution Concept 1: Dominance

Def. A *strictly dominant strategy* is the best choice for a player *regardless* of what the others are doing.

s_i is a *strictly dominant* strategy for i if for all $s'_i \neq s_i$ and all $s_{-i} \in S_{-i}$:

$$u_i(s_i, s_{-i}) > u_i(s'_i, s_{-i})$$

Def. A strategy is weakly dominant if does as least as well as any other of my strategies against *all* of my opponents' strategies, and it does strictly better for *some* of them.

s_i is a *weakly dominant* strategy for i if for all $s'_i \neq s_i$:

$$u_i(s_i, s_{-i}) \geq u_i(s'_i, s_{-i}) \text{ for all } s_{-i} \in S_{-i}:$$
$$u_i(s_i, s'_{-i}) > u_i(s'_i, s'_{-i}) \text{ for some } s'_{-i} \in S_{-i}:$$

Dominant Strategy Equilibrium (DSE): a strategy profile is A DSE if each player's strategy is a dominant strategy.

Prisoner's Dilemma: (C,C) is the unique DSE
Pareto efficient outcome: (NC,NC)

Applications of the Prisoner's Dilemma Framework:

Cooperation issues in environmental economics
Price-setting in oligopoly
Free-riding in the provision of public goods
Arms races
Theoretical sociology
…

Solution concept 2: Iterated elimination of <u>strictly</u> dominated strategies

Example 2:

	$L^{(3)}$	M	$R^{(1)}$
U	(1,0)	(1,2)	(0,1)
$D^{(2)}$	(0,3)	(0,1)	(2,0)

Def. A *strategy* is *strictly dominated* for player i if there is at least another strategy he can play that does strictly better *regardless* of what the others are doing.

s_i is *strictly dominated* if there exist an $s'_i \neq s_i$ such that for all $s_{-i} \in S_{-i}$:

$$u_i(s_i, s_{-i}) < u_i(s'_i, s_{-i})$$

A rational player will not play strictly dominated strategies…(U,M) unique equilibrium.

Iterated elimination of weakly dominated strategies…?
…very imprecise solution concept

a) layers of rationality need to be assumed
b) order/speed of deletion matters (multiple outcomes possible)

Example 3 : iterated elimination of weakly dominated strategies
Layers of rationality (Dutta p. 57)

(4,5)	(1,6)	(5,6)
(3,5)	(2,5)	(5,4)
(2,5)	(2,0)	(7,0)

Example 4: iterated elimination of weakly dominated strategies
Order of deletion matters…multiple outcomes (Dutta p.58)

1/2	L	R
T	(0,0)	(0,1)
B	(1,0)	(0,0)

Features of **dominance** as a solution concept:

a) Knowledge of actions taken by players not required.
b) Need to assume that it is common knowledge that players are rational
c) It does not imply Pareto optimality
d) May fail to provide a solution (nonexistence):

Example 6: **The battle of the Sexes**

H/W	Football	Ballet
Football	(2,1)	(0,0)
Ballet	(0,0)	(1,2)

Exercise on IDSDS

- Find the strategy profiles that survive iterated elimination of dominated strategies.

	x2	y2	z2
x1	(4,3)	(5,1)	(6,2)
y1	(2,1)	(8,4)	(3,6)
z1	(3,0)	(9,6)	(2,8)

Exercise on IDSDS

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Exercise on IDSDS

- The strategy profile that survives IDSDS is: $(x1; x2)$