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Cournot Competition, Forward Markets and Efficiency

BLAISE ALLAZ AND JEAN-LUC VILA*

Massachusetts Institute of Technology, Cambridge, Massachusetts 02139; and
**Groupe HEC, 78350 Jouy-en-Josas, France*

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We build a model with two Cournot duopolists who produce at Time 0 wich is the date at which all demand is realized. N periods before time 0, the duopolists trade on a forward market for delivery at Time 0. Having made these contracts, they trade again at time $(-N+1)$ for delivery at Time 0, etc. We show that, in equilibrium, each of them will sell forward which makes them worse off and makes consumers better off than if the forward market did not exist. When N, the number of forward trading periods prior to production, tends to infinity, the outcome tends to the competitive solution. *Journal of Economic Literature* Classification Numbers: C72; D43; G10; G13; L13.

1. INTRODUCTION

The justification that one can typically find in the literature for the appearance of forward markets rests on the desire of some groups of agents to hedge risks. For example, grain processors who must hold inventories over long periods of time want to hedge against the possibility of an unfavorable price move during the period over which inventories are held. One could then believe that uncertainty about some variables should be a requirement for any forward market to exists. In this paper, we show that this need not be the case. We derive a rationale for the use of forward markets in the case of certainty and perfect foresight. Producers will deal on the forward market in an attempt to improve their situations on the spot market, i.e., they will use forward transactions as strategic variables.[1] This result is in the spirit of the Bulow, Geanakoplos and Klemperer [5]

[1] Williams [15], who shows that the presence of transaction costs in the spot market may induce risk neutral producers to use a futures market instead, is a notable exception to the view that risk aversion is necessary to explain the existence of futures markets.

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paper: "A firm's action in one market can change competitors' strategies in a second market by affecting its own marginal cost in that other market".[2] In our paper, the link will occur through the marginal revenue rather than the marginal cost. Forward sales will affect the marginal revenue from spot sales. We show that forward markets can improve the efficiency of output decisions in a Cournot duopoly. In the limit, when the number of forward trading periods becomes large, the duopolists will produce the competitive output level.

Most of the literature on futures and forward markets is developed within the framework of perfect competition. However, as Anderson [2] and Newbery [12] point out, many commodities traded on futures markets are produced in an environment which is far from being perfectly competitive. For example, Newbery [12] cites a study of his "which lists eight commodities for which single countries controlled more than 50 per cent of world trade, and another thirteen for which single countries controlled between 25 and 50 per cent (averaged over the 1977–79 period)." This fact has usually been overlooked in the literature.[3]

In a first attempt to correct this omission in the literature, Anderson and Sundaresan [4], Anderson [3], and Newbery [12] developed models where the good traded on the futures markets was produced by a monopolist (Anderson–Sundaresan and Anderson) or by a dominant producer facing a fringe of competitive producers (Newbery). However, their framework was not designed to study the interactions among a few large firms having access to a forward market.[4] In a more recent paper, Eldor and Zilcha [6] considered the case of an oligopoly of risk averse firms that could trade on a forward market. However, they assumed that spot and forward decisions were taken simultaneously, which ruled out any possible strategic link between the forward and the spot market.

This paper is organized in the following way: We start with a simple two-period model with linear costs and demand. Trading on a forward market occurs one period before production takes place. In order to focus on the strategic behavior of non-competitive producers that have access to

[2] Bulow *et al.* [5] p. 488.

[3] Except for Greenstone [8].

[4] In a paper related to ours, Anderson [3] considers a multi-period model of a durable good (that lasts forever) produced by a monopolist. Competitive buyers can also buy and sell the used good on a second-hand market. Anderson shows that, even without uncertainty, if there exists a forward market for the durable good, then the monopolist will use it for a strategic purpose. Typically, he will buy futures for an amount equal to his first period production, thereby credibly committing himself not to produce during subsequent periods and thus canceling the effects of the second-hand market (on his profits). This is similar to the idea of our paper. However, Anderson considers the case of a monopolist whereas our paper is concerned with firms' interactions.

a forward market, we ignore the risk hedging rationale for forward trading and conduct the analysis under the assumption of perfect foresight.

In the first part, we show why a forward market could emerge in a world without uncertainty. When only one producer is given the opportunity to make forward sales, he actually benefits from a first mover advantage over his competitor and finds himself in the position of a Stackelberg leader on the spot market. When both firms can trade forward, the trading decisions give rise to a prisoner's dilemma: Each producer has incentives to trade forward but when they both do so, they end up worse off.

In the second part of the paper, we extend the analysis to N periods of trading. We give a formal definition of the equilibrium concept and its relation to the subgame perfect equilibrium concept. We show that when N gets large the outcome tends to the competitive outcome (price = marginal costs). We conclude by emphasizing the assumptions used to derive our results.

II. The Two-Period Model

To develop the intuition for our results, we start by considering only two periods. In the first period, the two producers can buy or sell forward contracts that call for delivery of the good they will produce in the second period. These forward contracts are *binding* and *observable* precommitments.[5] In the second period, the duopolists play a Cournot game in quantity but their payoff functions are modified by the positions that they have already taken on the forward market.

Under perfect foresight, equilibrium requires the forward market to be efficient: The forward price as a function of the forward positions must be equal to the price that will result from the Cournot competition on the spot market given these positions. Therefore, no arbitrage is possible.

We use the following notation:

x (resp. y) denotes the production of the first (resp. second) firm.

f (resp. g) denotes the first (resp. second) firm's forward sales in the first period.

p denotes the forward price in the first period.

$c(x)$ (resp. $d(y)$) denotes the cost function of the first (resp. second) firm.

$q(x+y)$ is the inverse demand function.

[5] For our analysis, the fact that forward contracts are binding is, of course, essential. We will also assume that all contracts are observable by forward market participants. In Section 3.2, we shall discuss the case where only the *aggregate* forward position is observable.

2.1. *The Production Game*

Given their forward positions f and g, the production game in the last period is defined by the two payoff functions $u^0(f, g)$ and $v^0(f, g)$[6]:

$$[u^0(f, g)](x, y) = q(x+y)(x-f) - c(x)$$

$$[v^0(f, g)](x, y) = q(x+y)(y-g) - d(y)$$

Indeed, given that the first firm has already sold out f, it can only sell the quantity $(x-f)$. If x is less than f then the firm must buy the good from its competitor to meet its commitment of sales or, equivalently, it can buy back its forward position at the cash price.

For technical simplicity, we now assume that both the demand and the cost functions are linear:[7]

$$c(x) = bx; \qquad d(y) = by$$

$$q(z) = a - z$$

$$0 < b < a^8.$$

Given f and g, the two competitors play the following game:

$$[u^0(f, g)](x, y) = (a-x-y)(x-f) - bx$$

$$[v^0(f, g)](x, y) = (a-x-y)(y-g) - by$$

The reaction function $x(y)$ is given by:

$$x(y) = \frac{a-b+f-y}{2}.$$

The key point is that the reaction function is increasing in f; Indeed when a competitor has a short position (i.e., $f>0$), then he is less "price sensitive" and therefore cares less about the price elasticity effect of an increase in production. Note that the marginal revenue from selling a further unit on the spot market is $q'(x+y)(x-f)+q(x+y)$ and not $q'(x+y)\,x+q(x+y)$, as is typical in Cournot models, because the decrease in price necessary to sell this additional unit does not affect the revenue from the forward sales. The next proposition presents the Nash equilibrium quantities and price.

[6] For further reference, we denote by $[u(f, g)](x, y)$, the value of the *function* $u(f, g)$ at the *point* (x, y).

[7] The two-period non-linear case has been solved in Allaz [1].

[8] We will not worry about the constraints $x \geqslant 0$, $y \geqslant 0$, $q \geqslant 0$; it is easy to check that they are verified in equilibrium. At the cost of significant complications, the analysis can be altered to take these constraints into account. The results are not affected.

PROPOSITION 2.1. *The Nash Equilibrium is unique with equilibrium quantities and price given by*:

$$x = \frac{a-b+2f-g}{3}; \qquad y = \frac{a-b+2g-f}{3}; \qquad q = \frac{a+2b-f-g}{3}.$$

In the first period, when the orders f and g are submitted to the forward market, traders in this market know that the spot price in the subsequent production game will be $q(f, g)$ as given in Proposition 2.1. Now suppose that in the forward market, prices are set by a Bertrand auction where several buyers (at least two) bid for the aggregate supply $f+g$. The equilibrium outcome of this auction will generate a forward price $p(f, g)$ equal to the perfectly anticipated spot price $q(f, g)$. The complete game form is displayed in Appendix A.

2.2. *The Emergence of a Forward Market*

Suppose first that only one firm (firm 1) is allowed to trade forward ($g=0$). When making his trading decision, this producer knows that he will not make any arbitrage profit and that his payoff will be

$$q(x(f, 0)+y(f, 0))\, x(f, 0)-c(x(f, 0)).$$

Thus, producer 1 faces the following problem: Choose f such that the Nash equilibrium outcome in the second period is optimal for him.

PROPOSITION 2.2. *The equilibrium outcome is the Stackelberg outcome of the Cournot duopoly game without a forward market when player 1 is the leader*:

$$f = \frac{a-b}{4}; \qquad x = \frac{a-b}{2}; \qquad y = \frac{a-b}{4}; \qquad q = \frac{a+3b}{4}.$$

Proof. The Nash equilibrium in the second period belongs to the usual Cournot reaction curve of player 2 since $g=0$. On this curve, the best point for player 1 is the Stackelberg point which is given by the program:

$$\max_x q(x+y)\, x - c(x)$$

subject to

$$y = y(x).$$

The solution is

$$x = \frac{a-b}{2}; \qquad y = \frac{a-b}{4}.$$

By choosing $f = ((a-b)/4)$ player 1 can reach this point (see Proposition 2.1 with $g=0$); hence this is the forward market equilibrium.

Thus, when one player has the opportunity to trade forward, he can improve his profits. There is a strategic incentive for trading forward. Total output goes up from $(2(a-b)/3)$ to $(3(a-b)/4)$ (the fact that the output is greater in the Stackelberg equilibrium than in the Nash equilibrium is true in a more general context whenever $(\partial y/\partial x) \geqslant -1$).

When producer 1 can trade forward, he greatly benefits from doing so. Therefore, taking the actions of the first trader as given, producer 2 will want to trade forward too in an attempt to reap similar profits. Hence we can see that both producers (i.e., all producers) will sell forward part of their production. This will lead to the emergence of a forward market.

The preceding section has shown that producers have strong incentives to sell forward part of their production on a forward market. However, for a forward market to exist, there must be both sellers and buyers. In our model without uncertainty and with identical producers, all producers would like to be short on the forward market. Who then is going to take the opposite positions? Since there are no hedgers in this model (because of no uncertainty), it must be speculators. Why would they do it? Initially, with all producers short and no buyers, there must be a downward pressure on the forward price. However, under perfect foresight all agents know that the spot price is going to be $q(f, g)$. Hence, given our assumption that the forward market is perfectly competitive and that arbitrage is costless, as long as the forward price is lower than the (perfectly anticipated) spot price, there exists an arbitrage opportunity which will be exercised by the speculators. At the equilibrium, the forward price will be equal to the spot price and hence speculators' profits will be zero. Since speculators make zero profits on their forward purchases, they are indifferent about their levels and may as well choose them so that they are just equal to the short positions of the producers.[9, 10]

2.3. *Forward Market Equilibrium*

Given the positions f and g, the total profits of the first firm are:

$$u^1(f, g) = p(f, g) f + [u^0(f, g)](x(f, g), y(f, g))$$

which can rewritten as:

$$u^1(f, g) = \{q(f, g)\, x(f, g) - c(x(f, g))\} + \{(p(f, g) - q(f, g)\} f.$$

The expression for $u^1(f, g)$ contains two terms: The first is the standard

[9] A complete game form where the forward price equals the perfectly anticipated spot price is displayed in Appendix A.

[10] One may object that since speculators make zero profits, they have no incentive to participate in the forward market. As usual, in the price formation process, we neglect the "scalping" profits which compensate the speculators for their provision of liquidity.

Cournot profit while the second is the arbitrage profit. Since arbitrageurs have perfect foresight, the second term is zero: $p(f, g) = q(f, g)$. Hence:

$$u^1(f, g) = q(f, g)\, x(f, g) - c(x(f, g))$$

and

$$v^1(f, g) = q(f, g)\, y(f, g) - c(y(f, g)).$$

PROPOSITION 2.3. *The only forward market equilibrium outcome is given by:*

$$x = y = \frac{2(a-b)}{5}; \qquad f = g = \frac{a-b}{5}; \qquad q = b + \frac{a-b}{5}.$$

Hence allowing forward trading decreases the firms' profits and increases social welfare.

Proof. By Proposition 2.1, the payoff of the first firm as a function of f and g is: $u^1(f, g) = (q-b)\,x = \frac{1}{9}(a-b-f-g)(a-b+2f-g)$. Now let $f(g)$ be the reaction function in the trading game. Hence f is given by $\partial u^1/\partial f = 0$ or $f(g) = \frac{1}{4}(a-b-g)$; similarly $g(f) = \frac{1}{4}(a-b-f)$. Hence, in equilibrium, $f = g = \frac{1}{5}(a-b)$, $x = y = \frac{2}{5}(a-b)$, and $q = b + \frac{1}{5}(a-b)$.

Actually, trading on the forward market represents a prisoner's dilemma for the two duopolists. When one of them succeeds in being the only producer to trade forward, he greatly benefits from doing so. However, when both producers trade forward, they both end up being worse off.

III. FORWARD MARKETS AND EFFICIENCY

Now, suppose that before production decisions are taken, there are N periods of forward trading.[11] Denote these trading periods by $N, .. j, .. 1$ and the production period by zero. Observe that period j occurs j periods before production.

We shall use the following notation:

x (resp. y) denotes the production of the first (resp. second) firm.

f^j (resp. g^j) denotes the first (resp. second) firm's forward sales in period j, $j = N, .. 1$.

$F^j = \sum_{k \geq j} f^k$ (resp. G^j) is the forward position of the first (resp. the second) firm in period j.

p^j denotes the forward price in period j.

$c(x)$ (resp. $d(y)$) denotes the cost function of the first (resp. second) firm.

$q(x+y)$ is the inverse demand function.

[11] Following our arguments in the previous section, we can show that if only $N-1$ trading periods are allowed, then each firm has an incentive to open a Nth trading period.

The profit equation for the first firm is given by:

$$U = q(x+y)\,x - c(x) + \sum_{j=1}^{N} (p^j - q)\,f^j.$$

Similarly, the second firm's objective function is:

$$V = q(x+y)\,y - d(y) + \sum_{j=1}^{N} (p^j - q)\,g^j.$$

The profit equations contain two terms: The first one is the standard Cournot profit term; The second one is the arbitrage profit. The description of the game played is the following:

— In period j, $j = N..1$, the firms choose f^j and g^j simultaneously, knowing the $f^{j'}$'s and the $g^{j'}$'s for $j' > j$. Then the market determines p^j.

— In the last period, they both choose their production levels, x and y, and then the market sets the price q according to the inverse demand function.

3.1. *Definition of an Equilibrium*

A *market equilibrium* is

- A sequence of trade functions $f^j(F^{j+1}, G^{j+1})$ and $g^j(F^{j+1}, G^{j+1})$;
- A sequence of market reactions $p^j(F^j, G^j)$;
- Two output functions $x(F^1, G^1)$, $y(F^1, G^1)$;

such that

[i] For every (F^1, G^1), $(x(F^1, G^1), y(F^1, G^1))$ is a Nash equilibrium in the production game.

[ii] (f^j, g^j) is a Nash equilibrium of the game $\Gamma^j(F^{j+1}, G^{j+1})$ where the games Γ^j are defined recursively:[12]

$$\begin{aligned}
&[U^1(F^2, G^2)](f^1, g^1)\\
&\quad = p^1(F^2 + f^1, G^2 + g^1)\,f^1 + [u^0(F^2 + f^1, G^2 + g^1)]\\
&\qquad \times (x(F^2 + f^1, G^2 + g^1), y(F^2 + f^1, G^2 + g^1))\\
&[U^j(F^{j+1}, G^{j+1})](f^j, g^j)\\
&\quad = p^j(F^{j+1} + f^j, G^{j+1} + g^j)\,f^j + [U^{j-1}(F^{j+1} + f^j, G^{j+1} + g^j)]\\
&\qquad \times (f^{j-1}(F^{j+1} + f^j, G^{j+1} + g^j), g^{j-1}(F^{j+1} + f^j, G^{j+1} + g^j))
\end{aligned}$$

(a similar expression holds for V^j).

U^j (resp. V^j) represents the forthcoming profits of the first (resp. second)

[12] Recall that $[U(F, G)](f, g)$ is the value of the *function* $U(F, G)$ at the point (f, g).

firm from period j onward (period j included). It has two components: The first is U^{j-1} (resp. V^{j-1}) and the second one is the sale on the forward markets. $u^0(\cdot)$ and $v^0(\cdot)$ are the profit outcomes of the production game (See Section 2).

$$\text{[iii]} \quad p^j(F^j, G^j) = p^{j-1}(F^j + f^{j-1}(F^j, G^j), G^j + g^{j-1}(F^j, G^j))$$

$$p^1(F^1, G^1) = q[x(F^1, G^1) + y(F^1, G^1)].$$ [13]

Condition [i] says that x and y have to be the Cournot outcome of the modified production game given the forward positions F^1 and G^1. Condition [ii] says that, for every period, the trades must be the Nash solution for the game starting at that period, and [iii] is the perfect foresight condition or efficient market condition (see Kyle [10] and Anderson and Sundaresan [4]).

3.2. *Discussion of the Equilibrium Concept*

First a word about Condition [iii]. The rationale for it is that there should be no arbitrage profits made in this model.[14] For example, this will be the case if two Bertrand speculators are bidding every period for the "open interest" $f^j + g^j$.[15] In this case, Condition [iii] is endogenous in any stationary perfect equilibrium of the 4-player game.

Second, a word about the informational structure of the model. We assume that the speculators can observe the forward positions of both producers. If, as this is the case on futures markets, positions are anonymous, then the market can only observe the aggregate open interest $F^j + G^j$. As we will see, in the linear case, the outcome of Cournot competition depends only on $F^j + G^j$, and thus, the results will not be altered if we assume that agents only observe the aggregate open interest. In the non-linear case, the assumption that only $F^j + G^j$ can be observed might lead to substantial complications.[16]

[13] Note that in our definition of an equilibrium, we have written the trade functions f^j and g^j only as functions of the forward positions F^{k-1} and G^{j-1} and not as functions of the complete history of the game at date j. Given our Cournot assumption, it is easy to show (by backward induction) that we are not losing any generality in doing so.

[14] In this model, we have not considered the case where the two players discount the future. One might argue that when N is large this assumption is no longer valid. The answer to that is that the payment of forward sales is usually received at delivery. Even if this is not the case, the discount rates are the market interest rates. The no arbitrage condition is now $p^{j+1} = p^j/(1+r)$.

[15] See Kyle [11] and Vila [14] for a discussion of the game theoretic foundations of imperfectly competitive rational expectations models. See also Appendix A.

[16] Even in the linear case, if speculators do not observe $F^j + G^j$ perfectly but instead observe $F^j + G^j + \varepsilon^j$ where ε^j is "noise trading," then our equilibrium will not hold. Hence *observability* of forward positions (at least in the aggregate) is essential to our analysis. Relaxing this assumption is an interesting but difficult extension.

3.3. *Equilibrium in the N-period forward markets*

In Section 2.3, we showed that allowing one period of forward trading improved the social welfare. When more periods are allowed the result is even more striking.

PROPOSITION 3.1. *The Market equilibrium with N trading periods is*

$$f^j = g^j = \frac{a-b}{3+2N}; \qquad x = y = \frac{a-b}{2}\left(1 - \frac{1}{3+2N}\right);$$

$$F^1 = G^1 = \frac{a-b}{2}\left(1 - \frac{3}{3+2N}\right); \qquad q = p^j = b + \frac{a-b}{3+2N}.$$

Hence when N tends to infinity, the unsold production $x - F^1$ *tends to* 0 *and the price q tends to the marginal cost b.*

In the limit, even with imperfect competition, Cournot markets with forward markets are efficient. Note that letting N go to infinity does *not* necessarily imply that forwards are traded over an infinite horizon but rather that trading occurs more frequently. Hence, the case $N = \infty$ can be interpreted as the case of continuous trading.

(The proof of Proposition 3.1 is in Appendix B).

3.4. *Comparison with the Monopoly Case*

The comparison with the case of monopoly provides some insight into Proposition 3.1. The introduction of a forward market has no effect on the monopoly output and hence on social welfare (see Anderson and Sundaresan [4]). This is clear since, without the possibility of any arbitrage profit, the best payoff that the monopolist can achieve is the monopolistic profit. This payoff is achieved by *not* trading forward.

To see this in the context of our model, let $x(F^1)$ be the monopolist's production if his final forward position is F^1. The trading strategy f^1 in period 1 is given by the program P^1:

$$\max\{q(x(F^2+f^1))(x(F^2+f^1)-F^2)-c(x(F^2+f^1))\} \quad \text{in } f^1.$$

But by definition $x(F^2)$ solves: $\max\{q(x)(x-F^2)-c(x)\}$ in x and thus $f^1=0$ achieves the maximum in P^1. Consequently, given that he has sold F^2 contract, the monopolist has no incentive to engage in any further trade. If his initial position is zero, then he will not trade on the forward market and will produce the monopolistic output.[17]

[17] The arguments above are also valid at any point in time ($j \neq 1, 2$).

3.5. *Intuition for Proposition* 3.1.

An intuitive explanation for the convergence result is that, when the number of periods tends to infinity the trading flow in each period must converge to zero. Therefore, let F and G be the limits of total forward positions at Date 0, F^1, and G^1, when N tends to infinity. F and G are also the limits of F^2 and G^2. Hence $F = F + f^1(F, G)$ and $G = G + g^1(F, G)$. In other words, the total "limiting positions" must be stationary in the sense that, in a two-period game starting with F and G, none of the competitors will trade forward in equilibrium. (0, 0) is a Nash equilibrium of the game:

$$U = q(x(F+f, G+g) + y(F+f, G+g))(x(F+f, G+g) - F)$$
$$- c(x(F+f, G+g)).$$
$$V = q(x(F+f, G+g) + y(F+f, G+g))(y(F+f, G+g) - G)$$
$$- d(y(F+f, G+G)).$$

Assuming differentiability, this yields:

$$\partial U/\partial f = 0 = \{q - c' + q'(x-F)\}\, \partial x/\partial F + q'(x-F)\, \partial y/\partial F$$
$$\text{at } f = g = 0.$$

But $q - c' + q'(x-F) = 0$ as (x, y) is the Nash equilibrium of the production game. Therefore $q'(x(F, G) - F)\, \partial y/\partial F = 0$ and $q'(y(F, G) - G)\, \partial x/\partial G = 0$. Since $\partial y/\partial F = (\partial y/\partial x)(\partial x/\partial F) < 0$ and, assuming no corner solution, i.e., $q' \neq 0$ it follows that the stationary positions satisfy $x(F, G) = F$ and $y(F, G) = G$ which implies $c'(x) = d'(y) = q(x+y)$. The reader will notice again the difference between the monopoly case and the duopoly case. For the monopoly, y is identically equal to zero and, as we have seen, any position is stationary. The intuition for the convergence result in the duopoly case is that, whenever profits are above their competitive levels, the competitors will trade forward if one more period of trade is allowed and their profits will decrease as in the two-period game. In the limit, profits are at their competitive levels. The intuitive argument given above gives us a good presumption that the convergence result may be true in a more general context.

IV. Concluding Remarks

Our main result, Proposition 3, says that, in the limit, as the number of forward trading periods becomes very large, the total output of the duopoly will be equal to that of a perfectly competitive industry.

In the long discussion of whether forward and futures markets are beneficial or not to the consumer, this results speaks in favor of forward markets. However, we must stress the assumptions under which such a powerful result has been obtained. (i) The sequential nature of the trading process is essential. There could not be any strategic play in a one-period model where spot and forward transactions would take place simultaneously. (ii) The assumption of Cournot behavior is also crucial. Allaz [1] has shown that the results are sensitive to the type of conjectural variation that is used. (iii) The assumption of linearity does not seem too restrictive. Needless to say a generalization to nonlinear cost and demand functions would be very cumbersome. (iv) The assumption of no uncertainty is obviously unrealistic. It is used in order to focus exclusively on the strategic rationale for forward trading. It also greatly simplifies the analysis. When uncertainty is introduced, e.g., by assuming that the spot price is not known at Time 1 when trading takes place on the forward market, strategic purposes interact with risk hedging if the producers are risk averse. In the case of Cournot competition, we can expect that the risk hedging motive will reinforce the strategic motive in leading risk averse firms to sell forward. However, if both producers and speculators are risk neutral, then the results (in expected value) will be identical to those derived under perfect foresight.[18] Hence, the no uncertainty case can be reinterpreted as the risk neutral case.

Finally it is interesting to briefly compare our results with those of the monopolist's durable good pricing. The Coase conjecture (see reference in Tirole [13]) states that a monopolist producing a durable good will not be able to sell his production at a price above marginal cost because of the intertemporal substitution: Sales today reduce demand tomorrow and therefore the monopolist creates his own competition. More precisely, "when price adjustments become more and more frequent the monopolist's profit converges to zero" (Tirole [13]), p. 73). This result looks similar to ours. In both cases, intertemporal trading reduces the non-competitive behavior. There are, however, significant differences. First, we do not assume the good to be durable. Second, in our model, intertemporal trading *enhances already existing competition among producers* but does not affect a monopolist (see Section 3.4.). Third, it is interesting to note that in the case of a durable good monopolist, the existence of a forward market actually eliminates the intertemporal substitution effect. Anderson [3] shows that by taking *long* forward positions the monopolist can credibly commit himself not to produce again and therefore will not compete with himself.

[18] This is because the demand and cost functions are linear.

We hope that these results will stimulate further research on the role of forward markets in the economy.

APPENDIX A: The Complete Game Form of the Two-Period Model

There are four players: the two firms and two speculators. The game unfolds as follows:

1. The two firms choose their forward sales (respectively f and g) simultaneously.

2. The two speculators (1 and 2) announce their bids p^1 and p_2 simultaneously. The highest bidder is called the winner (W), the lowest bidder the loser (L). In case of a tie, 1 is the winner. The forward price is: $p = \max(p_1, p_2)$.

3. The two firms produce x and y simultaneously. The spot price is given by the inverse demand: $q = a - x - y$.

The payoffs are as follows:

$u^0 = pf + q(x - f) - bx$	for the first firm.
$v^0 = pg + q(y - g) - by$	for the second firm.
$U^W = (q - p)(f + g)$	for the winner.
$U^L = 0$	for the loser.

In Section 2.3, we compute the subgame perfect equilibrium in this game. The condition that the winner and the loser must have the same utility implies that $p = q$.

APPENDIX B

Proof of Proposition 3.1. Let $x(j, F^j, G^j)$ and $y(j, F^j, G^j)$ be the outputs as functions of the stocks held j periods before production. From Section 2:

$$x(1, F^1, G^1) = \frac{a-b}{3} + \frac{2F^1}{3} - \frac{G^1}{3}.$$

We assume that $x(j, \cdot, \cdot)$ and $y(j, \cdot, \cdot)$ are linear functions of F^j and G^j, i.e.,

$$x(j, F^j, G^j) = \alpha_j(a-b) + \beta_j F^j - \gamma_j G^j$$

$$y(j, F^j, G^j) = \alpha_j(a-b) - \gamma_j F^j + \beta_j G^j$$

and prove that a similar functional form holds for $j+1$. From Conditions [ii] and [iii] in the definition of an equilibrium, it follows that:

$$[U^j(F^{j+1}, G^{j+1})](f^j, g^j) = q[x(F^j, G^j) + y(F^j, G^j))]$$
$$\times (x(F^j, G^j) - F^{j+1}) - c(x(F^j, G^j)).$$

In equilibrium: $\partial U^j/\partial F^j = 0 = (q-b)\,\partial x/\partial F^j + (x - F^{j+1})\,\partial q/\partial F^j$ which yields:

$$(q-b)\,\beta_j + (x - F^{j+1})(\gamma_j - \beta_j) = 0. \tag{B1}$$

Similarly:

$$(q-b)\,\beta_j + (y - G^{j+1})(\gamma_j - \beta_j) = 0. \tag{B2}$$

Together with $q = a - x - y$, (B1) and (B2) yield:

$$q - b = \frac{\beta_j - \gamma_j}{3\beta_j - \gamma_j}(a - b - F^{j+1} - G^{j+1}). \tag{B3}$$

From (B3) and (B1) and since $x(j, F^j, G^j) = x(j+1, F^{j+1}, G^{j+1})$, and $y(j, F^j, G^j) = y(j+1, F^{j+1}, G^{j+1})$ we get:

$$\alpha_{j+1} = \frac{\beta_j}{3\beta_j - \gamma_j},\ \alpha_1 = \frac{1}{3}; \qquad \beta_{j+1} = \frac{2\beta_j - \gamma_j}{3\beta_j - \gamma_j},\ \beta_1 = \frac{2}{3}; \qquad \gamma_{j+1} = \alpha_{j+1},\ \gamma_1 = \frac{1}{3}.$$

Hence:

$$\alpha_j = \gamma_j,\ \alpha_j + \beta_j = 1 \qquad \text{for all } j. \tag{B4}$$

Therefore, $\alpha_{j+1} = ((1-\alpha_j)/(3 - 4\alpha_j)) = f(\alpha_j)$; the only fixed point of $f(\cdot)$ is 1/2 and thus,

$$(\tfrac{1}{2} - \alpha_{j+1})^{-1} = 4 + (\tfrac{1}{2} - \alpha_j)^{-1}, \tag{B5}$$

and hence, $\alpha_j = (j/1 + 2j)$. It follows that

$$x(j, F^j, G^j) = \alpha_j(a-b) + (1-\alpha_j)\,F_j - \alpha_j G^j \tag{B6}$$
$$q(j, F^j, G^j) - b = (1 - 2\alpha_j)(a - b - F^j - G^j). \tag{B7}$$

To derive the forward trading strategies, we write that,

$$x(j+1, F^{j+1}, G^{j+1}) = x(j, F^{j+1} + f^j(F^{j+1}, G^{j+1}), G^{j+1} + g^j(F^{j+1}, G^{j+1}))$$

and

$$y(j+1, F^{j+1}, G^{j+1}) = y(j, F^{j+1} + f^j(F^{j+1}, G^{j+1}), G^{j+1} + g^j(F^{j+1}, G^{j+1})).$$

This yields:

$$f^j(F^{j+1}, G^{j+1}) = \frac{a - b - F^{j+1} - G^{j+1}}{3 + 2j}. \tag{B8}$$

When N periods of futures trading are allowed, i.e. $F^{N+1} = G^{N+1} = 0$, the equilibrium path is as follows:

— The forward trading flows are constant over time $f^j = g^j = ((a-b)/(3+2N))$.

— The forward price is constant and equal to the cash price: $p^j = q = b + ((a-b)/(3+2N))$.

— The output levels are $x = y = ((a-b)/(2))(1 - (1/(3+2N)))$.

— The net spot market supplies are $x - F^1 = y - G^1 = ((a-b)/(3+2N))$.

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