

EC3.64 Microeconomic Theory 3

Problem set 1

1. Consider the following simultaneous-move game (where the payoffs in each cell are the von Neumann-Morgenstern utilities for that pair of strategies):

1\2	L	C	R
U	0,0	0,1	1,2
M	1,0	1,1	2,1
D	2,0	1,2	2,2

- Suppose player 2 plays strategy L. Compute player 1's expected payoff from playing a strategy that picks each of her strategies with equal probability.
- Repeat same exercise as in a) with player 1 playing U or M with equal probability and D with half that probability.
- Suppose that each player is equally likely to play each one of her strategies. Compute their expected payoffs.

2. Suppose you have £10,000 to invest. A broker phones you with the information you requested on certain junk bonds. If the company issuing the bonds posts a profit this year, it will pay you a 40% interest rate on the bond. If the company files for bankruptcy you will lose all you invested. If the company breaks even, you will earn a 10% interest rate. Your broker tells you there is a 50% chance that they will break even and a 20% chance that the company will file for bankruptcy. Your other option is to invest in a risk-free government bond that will guarantee 8% interest for a year.

- What is the expected interest rate for the junk bond investment?
- What investment will you choose if your utility function were given by $U(M) = M^2$? What is the certainty equivalent of the chosen investment?
- Which investment would you choose if your utility function were given by $U(M) = (M)^{1/2}$?

3. Jane has a utility function given by $U(M) = (M)^{1/2}$ where M is her income. If she becomes an economics professor, she will make $M = 81/\text{yr}$ with probability 1. If she becomes an attorney, she will make $M = 900/\text{yr}$ if she becomes a partner in a Wall Street firm, but only $M = 25/\text{yr}$ if she fails to make partner. The probability of her becoming a partner is 0.2. Mr. Smith is an infallible judge of legal talent. After a brief interview, he can state whether Jane will become a partner.

- What is most Jane will pay with the information?
- Assuming that the interview is costless for Mr. Smith to conduct, is he getting the highest possible expected return for himself by charging Jane the same fee regardless of the outcome of the interview? Explain.

4. Anna's utility function is $U(M) = \log(M+100)$. She would have an after tax income of 100 if she reported all her income. She is taxed at a rate of 50% of her earned income. If she is caught underreporting her income she will have to pay the taxes due, of course, but in addition she will pay a fine of F for unit of income she failed to report.

- a) how much income will she conceal if $F = 2$ and the probability of being caught is 0.10? Let C denote the amount of income concealed.
- b) Determine C as a function of the fine F and the probability of being caught " p ". Show that C fall when either F or " p " increase.
- c) In recent discussions of tax policy in both North America and Europe, proponents of reducing tax rates have argued that one of the benefits would be to reduce tax evasion. Explain this. Next, suppose you were advising the Treasury on drafting an appropriate bill to legislate a tax rate increase. If your concern is about implementation of the new tax rate what lesson from above analysis would you apply to the new proposal.

5. An investor may choose between investing $\text{£}W$ in financial option #1 and financial option #2. The returns to each option are subject to uncertainty: there are four possible states of the world $s_i, i=1,2,3,4$, and outcomes vary across the possible states. The data on outcomes (these show the gross outcomes obtained for every $\text{£}1$ invested) is summarized as follows:

	Possible states			
	s_1	s_2	s_3	s_4
Probability of each state	$(1-p)/2$	$(1-p)/2$	$p/2$	$p/2$
Outcome to option #1	x_1	0	0	x_1
Outcome to option #2	0	x_2	0	x_2

Suppose: $x_1 > x_2 > 0; 1 > p > 0$.

- a) "If the investor is risk neutral she will surely choose to have only option 1 in her portfolio." Explain whether you do or do not agree with the statement.
- b) Suppose the investor is risk averse and has an utility function $u = (x)^{1/2}$ where x is his final wealth. Show that the investor will include both the options in his portfolio. How does the proportion of option 2 in the portfolio depend on the value of p ? Explain the intuition of your answers the best you can.