# Urban Labor Economics

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July 8, 2006

## Part 2: Urban Efficiency Wages

## Chapter 4: Simple Models of Urban Efficiency Wages

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# 1. Introduction

We develop a model in which housing prices and workers' location (land market), as well as wages and unemployment (labor market) are determined in equilibrium. This paper constitutes the benchmark model of the urban efficiency wage theory that we first develop in section 2. In this simple model where workers' relocation is costless, firms set efficiency wages to prevent shirking and to compensate workers for commuting. The interaction between land and labor markets is here explicit since both wages and unemployment depend on commuting costs, and housing prices as well as location are in turn based on workers' wages.

We then consider some variations of this benchmark model. First, we introduce housing consumption and see how the results of the benchmark model are affected. Second, within the same model, we study two different cities and compare the two equilibria. Finally, we study the long-run properties of the model.

# 2. The benchmark model[1]

There is a continuum of ex ante identical workers whose mass is $N$ and a continuum of $M$ identical firms. Among the $N$ workers, there are $L$ employed and $U$ unemployed so that $N = L + U$. The workers are uniformly distributed along a *linear*, *closed* and *monocentric* city.[2] Their density at each location is taken to be 1. All land is owned by absentee landlords and all firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers are assumed to be infinitely lived, *risk neutral* and decide their optimal place of residence between the CBD and the city fringe. There are *no relocation costs*, either in terms of time or money. This is a simplifying assumption, which is quite standard in urban economics. It implies that workers change location as soon as they change employment status. In the context of labor markets in which workers tend to experience long unemployment spells (for example black workers), it is a rather good approximation since, when workers become unemployed, they will be less able to pay land rents and, after some time, they will have to relocate in cheaper places. We will relax this assumption in chapter 5.

[1] This section is based on Zenou and Smith (1995).

[2] See Appendix 1 at the end of this book for a precise definition of a linear, closed and monocentric city.

Each individual is identified with one unit of labor. As in the standard efficiency wage model without space (see Shapiro and Stiglitz, 1984), there are only two possible levels of effort that a worker can exert: either the worker shirks, has zero effort, $e = 0$ and contributes to zero production or he/she does not shirk, provides full effort, $e > 0$ and contributes to $e$ production. Each employed worker goes to the CBD to work and incurs a fixed monetary commuting cost $\tau$ per unit of distance. When living at a distance $x$ from the CBD, he/she also pays a land rent $R(x)$, consumes $h_L = 1$ unity of land and $z_L$ unities of the non-spatial composite good (which is taken as the numeraire so that its price is normalized to 1) and earns a wage $w_L$ (that will be determined at the labor market equilibrium). The budget constraints of non-shirker and shirker employed workers are respectively given by:

$$R(x) + \tau\, x + z_L = w_L - e \qquad (2.1)$$

$$R(x) + \tau\, x + z_L = w_L \qquad (2.2)$$

Because of risk neutrality, we assume that preferences of all workers (including the unemployed) are given by $\Omega(z_L) = z_L$ so that the *instantaneous* indirect utilities of an employed non-shirker and shirker residing at a distance $x$ from the CBD are respectively given by:[3]

$$W_L^{NS}(x) = w_L - e - \tau\, x - R(x) \qquad (2.3)$$

$$W_L^{S}(x) = w_L - \tau\, x - R(x) \qquad (2.4)$$

Concerning the unemployed, they commute less often to the CBD since they mainly go there to search for jobs. So, we assume that they incur a commuting cost $s\,\tau$ per unit of distance, with $0 < s < 1$. For example, $s = 1/2$ implies that the unemployed make only half as many CBD-trips as the employed workers. More generally, $s$ can be interpreted as the search intensity of workers since it indicates how often workers search for jobs. In this very simple framework, search intensity is only captured by the number of CBD-trips. In part 2 of this book, we will have a richer and broader definition of $s$.

Each unemployed worker earns a fixed unemployment benefit $w_U > 0$, pays a land rent $R(x)$, consumes $h_U = 1$ unit of land and $z_U$ units of the non-spatial composite good. In this context, because the preferences are given

[3] The subscript $L$ refers to the employed whereas the subscript $U$ refers to the unemployed. Among the employed workers, the superscripts $NS$ and $S$ refer respectively to non-shirkers and shirkers.

by $\Omega(z_U) = z_U$, the *instantaneous* (indirect) utility of an unemployed worker is equal to:

$$W_U(x) = w_U - s\,\tau\,x - R(x) \tag{2.5}$$

As stated in the introduction of Part 1, a steady-state equilibrium requires solving *simultaneously* an urban land use equilibrium and a labor market equilibrium. It is convenient to present first the former and then the latter.

### 2.1. Urban land use equilibrium

In equilibrium (this will become clear below), none of the employed workers will shirk so that we need analyze only the urban land use equilibrium with non-shirking workers. Since there are no relocation costs, the urban equilibrium is such that all the employed enjoy the same level of utility $W_L^{NS}(x) \equiv W_L$ while all the unemployed obtain $W_U$. Indeed, any utility differential within the city would lead to the relocation of some workers up to the point where all differences in utility disappear. We are now thus able to derive the bid rents of the (non-shirking) employed workers and the unemployed.[4] They are respectively given by:

$$\Psi_L(x, W_L) = w_L - e - \tau\,x - W_L \tag{2.6}$$

$$\Psi_U(x, W_U) = w_U - s\,\tau\,x - W_U \tag{2.7}$$

They are both linear and decreasing in $x$. We have the following straightforward result:

**Proposition 1.** *With workers' risk neutrality and fixed housing consumption, the employed reside close to jobs whereas the unemployed live at the periphery of the city.*

It is indeed easy to see that the bid rent of the employed is steeper than that of the unemployed and thus the employed will be able to outbid the unemployed to occupy the core of the city. This is because the employed do commute more often to the CBD than the unemployed and thus value more the accessibility to the center. This result is true whatever the preferences chosen as long as the housing consumption is fixed. As a result, since each worker consumes one unit of land, the employed reside between $x = 0$ and

[4] The bid rent is a standard concept in urban economics. It indicates the maximum land rent that a worker located at a distance $x$ from the CBD is ready to pay in order to achieve a utility level. See Appendix 1 for a formal definition.

$x = L$ whereas the unemployed reside between $x = L$ and $x = N$ (see Figure 4.1).

[*Insert Figure* 4.1 *here*]

Let us now define the urban-land use equilibrium. We denote the agricultural land rent (the rent outside the city or opportunity rent) by $R_A$ and, without loss of generality, we normalize it to zero. We have:

**Definition 1.** *An urban-land use equilibrium with no relocation costs and fixed-housing consumption is a 5-tuple* $(W_L^*, W_U^*, x_b^*, x_f^*, R^*(x))$ *such that:*

$$\Psi_L(x_b^*, W_L^*) = \Psi_U(x_b^*, W_U^*) \tag{2.8}$$

$$\Psi_U(x_f^*, W_U^*) = R_A = 0 \tag{2.9}$$

$$\int_0^{x_b^*} \frac{1}{h_L} dx = L \tag{2.10}$$

$$\int_{x_b^*}^{x_f^*} \frac{1}{h_U} dx = N - L \tag{2.11}$$

$$R^*(x) = \max\left\{\Psi_L(x, W_L^*), \Psi_U(x, W_U^*), 0\right\} \quad \text{at each } x \in \left(0, x_f^*\right] \tag{2.12}$$

Equations (2.8) and (2.9) reflect equilibrium conditions in the land market. Equation (2.8) says that, in the land market, at the frontier $x_b^*$, the bid rent offered by the employed is equal to the bid rent offered by the unemployed. Equation (2.9) in turn says that the bid rent of the unemployed must be equal to the agricultural land at the city fringe. Equations (2.10) and (2.11) give the two population constraints. Finally, equation (2.12) defines the equilibrium land rent as the upper envelope of the equilibrium bid rent curves of all workers' types and the agricultural rent line. Since all $N$ workers must consume 1 unit of housing each, and since there will be no vacant land inside the city in equilibrium, the distance from the CBD to the urban fringe must be given by $x_f^* = N$ and the border by $x_b^* = L$. As a result, the employed reside between 0 and $L$ whereas the unemployed reside between $L$ and $N$. Furthermore, for this equilibrium to exist (see e.g. Fujita, 1989), it has to be that the equilibrium land rent is everywhere continuous in the city.

By solving (2.8) and (2.9), we easily obtain the equilibrium values of the instantaneous utilities of the employed and unemployed workers in the city. They are given by:

$$\begin{aligned} W_L^* &= w_L - e - \tau\, x_b^* - s\tau\, \left(x_f^* - x_b^*\right) \\ &= w_L - e - \tau\, L - s\tau\, (N - L) \end{aligned} \tag{2.13}$$

$$W_U^* = w_U - s\tau\, x_f^* = w_U - s\,\tau\, N \tag{2.14}$$

The employment zone (i.e. the residential zone for the employed workers) is thus $(0, L]$ and the unemployment zone (i.e. the residential zone for the unemployed workers) is thus $[L, N]$.

Observe that, in equilibrium, all the unemployed obtain $W_U^*$ and all the employed $W_L^*$, whatever their location. This is because mobility is costless and the land rent compensate workers for different locations. By plugging (2.13) and (2.14) into (2.6) and (2.7), we easily obtain the land rent equilibrium $R^*(x)$. It is given by:

$$R^*(x) = \begin{cases} \tau\ (L - x) + s\tau\ (N - L) & \text{for} \quad 0 \leq x \leq L \\ s\,\tau\ (N - x) & \text{for} \quad L < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{2.15}$$

## 2.2. Steady-state equilibrium

We are now able to solve the labor market equilibrium and thus the steady-state equilibrium.

**Definition 2.** *A steady-state labor market equilibrium consists of a couple* $(w_L^*, L^*)$ *such that firms set an efficiency wage that prevents workers to shirk and determine a labor demand that maximizes their profit.*

Time is continuous and workers and firms live for ever. In the labor market, since shirkers add nothing to production, each firm is always motivated to discourage shirking behavior. Each firm is assumed to have only limited ability to monitor the productivity of its workers. The monitoring capability is taken to be characterized by a *detection rate*, $m$, which represents the rapidity at which the shirking behavior of any worker can be detected. Equivalently, $1/m$ represents the expected time required to detect shirking behavior. If a worker is caught shirking, he/she is automatically fired. We assume that changes in employment status are governed by a Poisson process in which $a$ is the (endogenous) job acquisition rate and $\delta$ the (exogenous) destruction rate. The standard (steady-state) Bellman equations for the non-shirkers, the shirkers and the unemployed are respectively given by:[5]

$$r\, I_L^{NS} = w_L - e - \tau\, x_b^* - s\tau\ \left(x_f^* - x_b^*\right) - \delta\,(I_L^{NS} - I_U) \tag{2.16}$$

$$r\, I_L^{S} = w_L - \tau\, x_b^* - s\tau\ \left(x_f^* - x_b^*\right) - (\delta + m)\,(I_L^{S} - I_U) \tag{2.17}$$

[5] See Appendix 2 for the derivation of the Bellman equations.

$$r\, I_U = w_U - s\tau\, x_f^* + a\,(I_L - I_U) \tag{2.18}$$

where $r$ is the discount rate, $I_L^{NS}$, $I_L^S$ and $I_U$ respectively represent the expected lifetime utility of a non-shirker, a shirker and an unemployed worker, and $W_L^{NS} \equiv W_L$ and $W_U$ are given by (2.13) and (2.14) and $W_L^S = W_L + e$. The first equation that determines $I_L^{NS}$ states that a non-shirker obtains today $W_L^{NS}$ but can loose his/her job with a probability $\delta$ and then obtains a negative surplus of $I_U - I_L^{NS}$. For $I_L^S$, we have the same interpretation, except for the fact that a shirker can lose his/her job for two reasons: either the job is destroyed or if he/she is caught shirking. The last equation has a similar interpretation.

As it can be seen by these equations, there is a trade off faced by a worker when deciding whether to shirk or not. There is a *short-run gain* of shirking because workers do not provide effort $e$ (the gain is thus $W_L^S - W_L^{NS} = e$) but there is a *long-run cost* of shirking because the rate at which workers lose their job is higher ($\delta + m$ instead of $\delta$). Thus firms must pay enough to prevent shirking, i.e. $I_L^{NS} \geq I_L^S$; otherwise workers will provide $e = 0$ and produce nothing. However, there is no need to pay more than the minimum needed to induce effort. Therefore, firms will choose a wage $w_L$ so that $I_L^{NS} = I_L^S = I_L$, i.e. the efficiency wage must be set to make workers indifferent between shirking and not shirking.

By using equations (2.16) and (2.17), the condition $I_L^{NS} = I_L^S = I_L$ can be written as:

$$I_L - I_U = \frac{e}{m} \tag{2.19}$$

This highlights the nature of our urban efficiency wage. The surplus of being employed is strictly positive and *does not depend on space*. As in Shapiro and Stiglitz (1984), this a pure incentive effect to deter shirking. This surplus only depends on the monitoring technology, since more monitoring implies less shirking, and on the effort level provided by workers.

Now using (2.13), equation (2.16) can be written as:

$$\begin{aligned} w_L &= e + rI_L + \delta\,(I_L - I_U) + \tau\, x_b^* + s\tau\, \left(x_f^* - x_b^*\right) \\ &= e + rI_U + (\delta + r)(I_L - I_U) + \tau\, x_b^* + s\tau\, \left(x_f^* - x_b^*\right) \end{aligned}$$

Furthermore, using (2.18) and (2.19), this can be rewritten as:

$$w_L = w_U + e + \frac{e}{m}(a + \delta + r) + (1 - s)\,\tau\, x_b^*$$

Finally, at the steady state, flows out of unemployment equal flows into unemployment, i.e.

$$a\,(N - L) = \delta\, L$$

so that the efficiency wage is finally given by:

$$
\begin{aligned}
w_L^* &= w_U + e + \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau\, x_b^* \qquad (2.20)\\
&= w_U + e + \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau\, L
\end{aligned}
$$

This equation (2.20) is referred to as the Urban No-Shirking Condition (UNSC hereafter) since it is the lowest wage at each level of employment that is necessary to induce workers not to shirk and to stay in the city.

The following comments are in order. First, we have the standard results of the non-spatial efficiency wage literature (Shapiro and Stiglitz, 1984). Indeed, $w_L^*$ increases with $L$, goes to infinity when $L$ tends to $N$ and has a positive value when $L = 0$. This is because unemployment acts as a worker discipline device so that lower employment level, or equivalently higher unemployment level, reduces the efficiency wage because the outside option of workers is lower since it is more difficult to find a job. When there is full employment, $L = N$, then no efficiency wage can be implemented. Indeed, efficiency wages are not compatible with full employment since, in this case, workers will always shirk because, if caught and fired, they will immediately find another job. When there is full unemployment, $U = N$ or $L = 0$, the efficiency wage is strictly positive because firms have still to induce workers to take a job and leave unemployment. Second, we have the following comparative-statics effects of the efficiency wage. An increase in the unemployment benefit $w_U$, the job destruction rate $\delta$, the discount rate $r$, or the commuting cost $\tau$, or a decrease in the monitoring rate $\theta$ or the unemployed CBD-trips $s$ raises the efficiency wage. For the non-spatial elements, $w_U$, $\delta$, $r$ and $\theta$, the reason is that firms have to increase their wage to meet the UNSC in order to prevent shirking. To summarize, as in the standard efficiency wage model, firms must *induce workers not to shirk* and there thus is a positive surplus to work compared to unemployment, which is denoted by $\Delta SW$. It is given by $\Delta SW = \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right)$.

For the spatial elements, $\tau$ and $s$, firms have to compensate their employed workers for spatial costs. Indeed, when setting their (efficiency) wage, firms must compensate the spatial cost differential between the employed and the unemployed. For the employed and the unemployed who both live at $L$ (this is the border distance between the employed and the unemployed) and thus pay the same land rent, this differential is exactly equal to $(1-s)\tau L$. Now, since mobility is costless, all the employed and unemployed workers obtain respectively the same (both instantaneous and intertemporal) utility level whatever their location. Therefore, the spatial cost differential between any employed and unemployed worker is equal to $(1-s)\tau L$. All these elements imply that the efficiency wage has two roles: to prevent shirking (incentive component) and to ensure that workers are locationally indifferent (spatial compensation component).

In order to see this, we can calculate the *spatial costs* of each individual, which consist of transportation plus land rent. In equilibrium, using (2.13) and (2.14), they are given by:

$$SC_L = \tau\, x_b^* + s\tau\, \left(x_f^* - x_b^*\right)$$

for the employed and

$$SC_U = s\tau\, x_f^*$$

for the unemployed. We can thus calculate the spatial-cost differential between the employed and the unemployed. It is equal to:

$$\Delta SC \equiv SC_L - SC_U = (1-s)\,\tau\, x_b^* = (1-s)\,\tau\, L \tag{2.21}$$

It is easy to see that $\Delta SC$ is precisely the last term of (2.20) and its role is to compensate workers for spatial costs. This implies that the efficiency wage can be written as

$$w_L^* = w_U + e + \underbrace{SW}_{\text{Work Inducement}} + \underbrace{\Delta SC}_{\text{Spatial Compensation}} \tag{2.22}$$

where $SW = \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right)$ so that, compared to unemployment, working gives a wage premium of $\Delta SW + \Delta SC$. The first term, $w_U + e$, can be regarded as the *base wage level*, namely the minimum wage level required in order to induce any individual to leave welfare and expend effort level $e$ in working.

To close the model, let us now determine the labor demand $L$. There are $M$ identical firms in the economy. Each firm $j = 1...M$ has the same production function $f(el_j)$, which is assumed to be twice differentiable with $f(0) = 0$, $f'(el_j) > 0$ and $f''(el_j) \leq 0$ for all $l_j$, and to satisfy the Inada conditions, i.e. $\lim_{l_j \to 0} f'(el_j) = +\infty$ and $\lim_{l_j \to +\infty} f'(el_j) = 0$. All firms produce the same composite good and sell it at a fixed market price $p$ (this good is taken as the numeraire so that its price $p$ is set to 1). Each firm $j$ chooses $l_j$ that maximizes $f(el_j) - w_L l_j$ (remember that $w_L$ is the efficiency wage that is given by (2.20) and $e$ is the effort level in each firm and thus the average effort level in the economy)[6] so that $ef'(el_j) = w_L$. At the aggregate level, the total level of

[6]Since we only consider steady-state equilibria, we can treat fims' decision problems as essentially static, i.e., they maximize instantaneous profit.

employment is given by $L = \sum_{j=1}^{j=M} l_j = M\, l_j$ so that $f'(el_j)$ is equivalent to $F'(eL)$[7] and, using (2.20), the aggregate labor demand is given by:

$$eF'(eL^*) = w_U + e + \frac{e}{m}\left(\frac{\delta N}{N - L^*} + r\right) + (1 - s)\,\tau\, L^* \qquad (2.23)$$

**Definition 3.** *A steady-state equilibrium* $(R^*(x), x_b^*, x_f^*, w_L^*, L^*)$ *consists of a land rent function, a border between employed and unemployed workers, a city-fringe, wage and employment such that the urban land use equilibrium and the labor market equilibrium are solved for simultaneously.*

Because the labor demand is decreasing in wages and wages are increasing in employment (UNSC), it is easy to show that the steady-state equilibrium exists and is unique. Figure 4.2 displays this equilibrium: the intersection between the Urban No-Shirking Condition (UNSC) curve (equation (2.20)) and the labor demand curve gives the equilibrium values of wage $w_L(L)$ and employment $L$. Observe that at $L = N$, there is full employment and the corresponding wage, $w^{pc}$, is the wage that would be paid by firms in a perfectly competitive environment. Urban unemployment occurs here because wages are too high ($w_L(L) > w^{pc}$) and are downward rigid. *Urban unemployment is thus involuntary.* Indeed, even though the unemployed workers are ready to work for a lower wage in order to get a job, firms will never accept this offer because the UNSC will not be respected and all workers will shirk. Therefore it is the presence of high and sticky wages that create (involuntary) unemployment. In this context, *taking space into account increases the level of unemployment since spatial efficiency wages are higher than in the absence of commuting costs* (as for example in Shapiro and Stiglitz, 1984).

[*Insert Figure* 4.2 *here*]

## 2.3. Interaction between land and labor markets

We can now study the interaction between land and labor markets. In fact, at the steady-state equilibrium, the endogenous variables is $R^*(x)$ (land market)[8],

[7] Indeed, since $F(eL) = Mf(el_j)$ and $L = M\, l_j$, we have:

$$eF'(eL) = Mf'(eL/M)(e/M) = ef'(el_j)$$

[8] Because housing consumption is fixed, the border and the city fringe are basically exogenous variables and are given by $x_b = L$ and $x_f = N$.

determined by (2.15), $w_L^*$ and $L^*$ (labor market), determined by (2.20) and (2.23) respectively, and $I_L^*$ and $I_U^*$ (equations (2.16) and (2.18) respectively).[9]

For the sake of the exposition, let us rewrite the five equilibrium equations. We have:

$$R^*(x) = \begin{cases} \tau\,(L^* - x) + s\tau\,(N - L^*) & \text{for} \quad 0 \leq x \leq L^* \\ s\,\tau\,(N - x) & \text{for} \quad L^* < x \leq N \\ 0 & \text{for} \quad x > N \end{cases}$$

$$w_L^* = w_U + e + \frac{e}{m}\left(\frac{\delta\,N}{N - L^*} + r\right) + (1 - s)\,\tau\,L^*$$

$$eF'(eL^*) = w_U + e + \frac{e}{m}\left(\frac{\delta\,N}{N - L^*} + r\right) + (1 - s)\,\tau\,L^*$$

$$W_L^* = w_U + \frac{e}{m}\left(\frac{\delta\,N}{N - L^*} + r\right) - s\tau\,N$$

$$W_U^* = w_U - s\,\tau\,N$$

Each of these equilibrium equations is a function of only $L^*$. The key equation is thus (2.23) since once $L^*$ is calculated then all the other endogenous variables are determined. Even though, the labor market does not affect directly the land market and vice versa (indeed, the determination of one market equilibrium is totally independent of the other), spatial variables do affect labor variables and vice versa. Indeed, by totally differentiating (2.23), we obtain:[10]

$$L^*\left(\underset{-}{w_U}, \underset{?}{e}, \underset{-}{\delta}, \underset{+}{N}, \underset{-}{r}, \underset{+}{s}, \underset{-}{\tau}\right)$$

Therefore, spatial variables $s$ (the relative fraction of commuting of the unemployed) and $\tau$ (the pecuniary commuting cost per unit of distance) have respectively a positive (higher $s$ leads to lower wages and thus higher $L^*$) and negative (higher $\tau$ implies that wages have to increase and thus labor demand

[9] Of course the determination of $W_L$ and $W_U$ gives directly the values of $I_L$ and $I_U$.

[10] The impact of $e$ on $L^*$ is ambiguous because an increase in effort increases wages (thus reduces $L^*$) but rises productivity (which increases $L^*$). However, if

$$\eta \equiv -\frac{eLF''(eL)}{F'(eL)} > 1$$

then the latter effect dominates the former and

$$\frac{\partial L^*}{\partial e} < 0$$

decreases) impact on $L^*$. In this respect, the impact of the land market on the labor market in this simple model is only through the spatial compensating part of the efficiency wage.

Labor variables also affect the land market. If we differentiate (2.15), we obtain in the employment zone ($0 \leq x \leq L^*$):[11]

$$R^* \left( \underset{-}{w_U}, \underset{?}{e}, \underset{-}{\delta}, \underset{+}{N}, \underset{-}{r}, \underset{+}{s}, \underset{?}{\tau} \right)$$

whereas in the unemployment zone ($L^* < x \leq N$), we have:

$$R^* \left( \underset{+}{N}, \underset{+}{s}, \underset{+}{\tau} \right)$$

Thus, when for example the unemployment benefit $w_U$ increases, labor demand $L^*$ decreases and thus the employment zone is reduced. This in turn increases the instantaneous utility of the employed $W_L^*$ (see (2.13)) since the border $x_b^*$ between the employed and the unemployed is smaller and thus the employed reduce their bid rent (the higher the utility level to achieve the lower the bid rent). The same reasoning applies for the job destruction rate $\delta$ since, in a downturn economy, firms hire less people and, because there is less competition in the land market, the equilibrium land rent decreases. As a result, the impact of the labor market on the land market is through labor demand, which is also equal to the border between the employed and the unemployed.

## 3. Endogenous housing consumption

In the previous section, one of the key assumption was the fact that housing consumption was fixed and normalized to 1. This assumption limits the interaction between land and labor markets since both the border between the employed and the unemployed and the city fringe are exogenous. We now extend the benchmark model to the case of endogenous housing consumption.

[11] To be more precise, for $\tau$ we have:

$$\frac{\partial R^*}{\partial \tau} = L^* - x + s\,(N - L^*) + \tau\,\frac{\partial L^*}{\partial \tau}\,(1 - s) \gtrless 0$$

### 3.1. Urban land use equilibrium

In order for the model to be tractable,[12] we assume quasi-linear preferences for all workers. For worker with employment status $es = L, U$, we have

$$\Omega(h_{es}, z_{es}) = z_{es} + g(h_{es}) \tag{3.1}$$

where $h_{es}$ is the *housing* consumption for a worker with employment status $es = L, U$ and $g(\cdot)$ is any increasing function with $g''(\cdot) \leq 0$. It is also assumed that housing $h_{es}$ is a normal good so richer workers consume more land. The budget constraints for non-shirker employed and unemployed workers are respectively given by:

$$h_L R(x) + \tau\, x + z_L = w_L - e \tag{3.2}$$

$$h_U R(x) + s\tau\, x + z_U = w_U \tag{3.3}$$

where, as above, the composite good is taken as the numeraire good with unit price

Maximizing utility (3.1) subject to (3.2) yields the following *housing(land) demand for non-shirker employed workers* at $x$:

$$g'(h_L^{NS}) = R(x) \tag{3.4}$$

Similarly, maximizing (3.1) subject to (3.3) yields the following *housing (land) demand for unemployed workers* at $x$:

$$g'(h_U) = R(x) \tag{3.5}$$

This implies that

$$h_L^{NS}(x) = h_U(x) = h(x) \tag{3.6}$$

with

$$\frac{\partial h}{\partial R} < 0 \text{ and } h'(x) = \frac{\partial h}{\partial R} R'(x)$$

This result (3.6) is due to the nature of the quasi-linear preferences since housing consumption is independent of income and thus employment status. Not surprisingly, when land price increases, housing consumption decreases and, if $R'(x) < 0$ (which be shown below), then workers consume more land further away from the CBD since land is cheaper there.

[12] Indeed, to calculate the efficiency wage, one needs to have a closed-form solution for the instantaneous utilities and $x_b$ and $x_f$. For example, with a Cobb-Douglas utility function, this is not possible. It seems that the quasi-linear utility function is the only one that allows us to calculate the efficiency wage.

Using (3.1) and (3.4), we can now derive the following indirect utility

$$W_L^{NS}(x) = w_L - e - \tau\, x - h(x)\, R(x) + g(h(x)) \tag{3.7}$$

for each *employed* worker at $x$, and, using (3.1) and (3.5), we have the following indirect utility

$$W_U(x) = w_U - s\tau\, x - h(x)\, R(x) + g(h(x)) \tag{3.8}$$

for each *unemployed* worker at $x$.

As in section 2, because there are no relocation costs, the urban equilibrium is such that all the employed enjoy the same level of utility $W_L^{NS}(x) \equiv W_L$ while all the unemployed obtain $W_U$. The bid rents of the (non-shirking) employed workers and the unemployed are thus equal to:

$$\Psi_L(x, W_L) = \frac{w_L - e - \tau\, x - W_L + g(h(x))}{h(x)} \tag{3.9}$$

$$\Psi_U(x, W_U) = \frac{w_U - s\tau\, x - W_U + g(h(x))}{h(x)} \tag{3.10}$$

Using the envelope theorem, we easily obtain:

$$\frac{\partial \Psi_L(x, W_L)}{\partial x} = \frac{-\tau}{h(x)} < 0 \text{ and } \frac{\partial^2 \Psi_L(x, W_L)}{\partial x^2} = \frac{\tau\, h'(x)}{[h(x)]^2} > 0$$

$$\frac{\partial \Psi_U(x, W_U)}{\partial x} = \frac{-s\tau}{h(x)} < 0 \text{ and } \frac{\partial^2 \Psi_U(x, W_U)}{\partial x^2} = \frac{s\tau\, h'(x)}{[h(x)]^2} > 0$$

They are both decreasing and convex in $x$. Indeed, workers living further away have higher commuting costs (direct effect) but larger lot sizes (indirect effect). But because, the envelope theorem indicates that this indirect effect is negligible when changes of $x$ are small, only the direct effect matters and we thus obtain a negative sign.

We can now implicitly define the housing consumption of each worker $h\,(x, W_L)$ and $h\,(x, W_U)$. Using (3.4) and (3.9) for the employed and (3.5) and (3.10) for the unemployed, we obtain:

$$g'(h\,(x, W_L)) = \frac{w_L - e - \tau\, x - W_L + g(h\,(x, W_L))}{h\,(x, W_L)} \tag{3.11}$$

$$g'(h\,(x, W_L)) = \frac{w_U - s\tau\, x - W_U + g(h\,(x, W_U))}{h\,(x, W_U)} \tag{3.12}$$

We have the following straightforward result:

**Proposition 2.** *With quasi-linear preferences such as (3.1) and endogenous housing consumption, the employed reside close to jobs whereas the unemployed live at the periphery of the city.*

The proof is given in Appendix A. When housing consumption is endogenous, then one would expect that no clear urban pattern would emerge (see Appendix 1). Indeed, on the one hand, the employed workers want to be closer to jobs than the unemployed because of higher commuting costs. On the other, because housing is a normal good, they consume more land and therefore prefer to be further away from the CBD since housing is cheaper there. However, because we assume quasi-linear preferences, there is *no income effect* and thus the second effect is nil since at $x_b$ both the employed and the unemployed consume the amount of land because housing consumption is independent of net income (see (3.4) and (3.5)). As a result, as in the previous section, there is only the commuting cost effect and thus the employed locate close to jobs. Interestingly, this result is robust if one uses any quasi-linear utility function in which the non-linearity is on $h$.

Because housing consumption is endogenous, we cannot as before equate $x_b$ to $L$ and $x_f$ to $N$, but we have to determine them. We focus on a closed-city model with absentee landlords (see Appendix 1). Since density is 1 at each location, we have the following definition

**Definition 4.** *An urban-land use equilibrium is a 5-tuple* $(W_L^*, W_U^*, x_b^*, x_f^*, R^*(x))$ *such that:*

$$\Psi_L(x_b^*, W_L^*) = \Psi_U(x_b^*, W_U^*) \tag{3.13}$$

$$\Psi_U(x_f^*, W_U^*) = R_A \tag{3.14}$$

$$\int_0^{x_b^*} \frac{1}{h_L(x, W_L^*)} dx = L \tag{3.15}$$

$$\int_{x_b^*}^{x_f^*} \frac{1}{h_U(x, W_U^*)} dx = N - L \tag{3.16}$$

$$R^*(x) = \begin{cases} \Psi_L(x, W_L^*) & \text{for} \quad x \leq x_b^* \\ \Psi_U(x, W_U^*) & \text{for} \quad x_b^* < x \leq x_f^* \\ R_A & \text{for} \quad x > x_f^* \end{cases} \tag{3.17}$$

Obviously, we cannot determine explicitly the values of the endogenous variables.

### 3.2. Steady-state equilibrium

As before, we are now able to solve the labor market equilibrium (the definition is exactly the same as before and is given by definition 2). The Bellman equations are still given by (2.16)-(2.18), with the difference that $W_L^{NS}$ and $W_U$ are now defined by (3.27) and (3.28) and that $W_L^S = W_L + e$. Observe that workers are not anymore risk neutral since their utility is not equivalent to their income. As above, to determine the efficiency wage, firms choose a wage $w_L$ such that $I_L^{NS} = I_L^S = I_L$. We easily obtain:

$$I_L - I_U = \frac{e}{m} \tag{3.18}$$

This is exactly as above. The surplus of being employed only depends on the monitoring technology and on the effort level provided by workers. We have the following result:

**Proposition 3.** *Assume no relocation costs. If the preferences of the employed and unemployed workers are quasi-linear with respect to $z_{es}$, i.e.*

$$\Omega(h_{es}, z_{es}) = z_{es} + g(h_{es}) \qquad , \qquad es = L, U$$

*where $g(\cdot)$ is any increasing and concave function in $h_{es}$, then the efficiency wage is given by*

$$w_L = w_U + e + \frac{e}{m}\left(\frac{\delta}{u} + r\right) + (1 - s)\,\tau x_b \tag{3.19}$$

*The value of $x_b$ depends on the specific form taken by $g(\cdot)$.*

This result is due to the fact that, when preferences are quasi-linear, the demand function for the good $h$ does not depend on the individual's wealth (see e.g. Mas-Colell, Whinston and Green, 1995) but only on the price of $h$. As a result, at the same location and thus at the same price $R(x)$, the consumption of $h$ will be the same for the employed and the unemployed. So, at the same location, which in our case can only be $x_b$ since the employed and the unemployed are totally separated in the city, the only spatial cost difference between the employed and the unemployed is the commuting cost difference, that is $(1 - s)\,\tau x_b$. Since the efficiency wage has two roles: to prevent shirking (which is not spatially related) and to compensate for spatial cost differences, the efficiency wage will always be given by (3.19). Observe that the case of fixed housing consumption normalized to 1 (section 2) is a special case of this proposition since it assumes that $g(h_k) = g(1) = 1$. In that case, $x_b = L$.

We can close the model exactly as in section 2.2 and obtain the labor demand in the following way:

$$w_U + e + \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau x_b = F'(L) \tag{3.20}$$

### 3.3. Closed-form solutions

Because we would like to calculate the exact value of the efficiency wage (and thus $x_b(L)$) and to study the interaction between the two markets (land and labor), we need to take a specific form for the function $g(h_{es})$. Assume that $g(h_{es}) = h_{es}^{1/2}$, which is increasing and concave in $h_{es}$. It is easy to show that

$$\Psi_L(x, W_L) = \frac{1}{4\,(W_L - w_L + e + \tau\, x)} \tag{3.21}$$

$$\Psi_U(x, W_U) = \frac{1}{4\,(W_U - w_U + s\tau\, x)} \tag{3.22}$$

and

$$h_L(x, W_L) = 4\,(W_L - w_L + e + \tau\, x)^2 \tag{3.23}$$

$$h_U(x, W_U) = 4\,(W_U - w_U + s\tau\, x)^2 \tag{3.24}$$

By plugging these values in Definition 4 and, without loss of generality, by normalizing $R_A$ to 1/4, we obtain:

$$x_b^* = \frac{4L^*}{[1 + 4s\tau\,(N - L^*)]\,(1 + 4L^*\tau)} \tag{3.25}$$

$$x_f^* = x_b^* + \frac{4\,(N-L)}{1 + 4s\tau\,(N-L)} \tag{3.26}$$

$$\begin{aligned} W_L^{NS} &= w_L - e + 1 - s\tau\,\left(x_f^* - x_b^*\right) - \tau\, x_b^* \\ &= w_L - e + \frac{1}{1 + 4\tau\,\left[sN + (1-s)\,L^*\right]} \end{aligned} \tag{3.27}$$

$$\begin{aligned} W_U &= w_U + 1 - s\tau\, x_f^* \\ &= w_U + 1 - \frac{4s\tau}{1 + 4s\tau\,\left(N - L^*\right)}\left[\frac{L^*}{1 + 4\tau\,\left[sN + (1-s)\,L^*\right]} + N - L^*\right] \end{aligned} \tag{3.28}$$

$$R(x) = \begin{cases} \frac{1}{4}\left(\frac{1}{1+4s\tau(N-L^*)+4\tau\, L^*} + \tau\, x\right)^{-1} & \text{for } \; x \le x_b^* \\ \frac{1}{4}\left(1 - \frac{4s\tau}{1+4s\tau(N-L^*)}\left[\frac{L^*}{1+4s\tau(N-L^*)+4\tau\, L^*} + N - L^*\right] + s\tau\, x\right)^{-1} & \text{for } \; x_b^* < x \le x_f^* \\ \frac{1}{4} & \text{for } \; x > x_f^* \end{cases} \tag{3.29}$$

It can be shown that $x_b^*(L=0)=0$ and $\partial x_b^*/\partial L>0$, and $\partial x_b^*/\partial s<0$ and

$$\frac{\partial x_b^*}{\partial \tau} \gtrless 0 \Leftrightarrow \tau \lessgtr \frac{1}{4\sqrt{s\,(N-L)\,[L+s\,(N-L)]}}$$

Thus if $L$ the level of employment in this economy increases, then more workers are employed and thus richer, and, therefore, the space they occupy increases. It also says that, at a given $L$ (at the labor equilibrium $L$ will itself depend on $s$ and $\tau$), the higher is the percentage of unemployed CBD-trips, the lower is $x_b$. This is because if the unemployed goes more often to the CBD, their commuting costs increase and, thus, their willingness to pay for land decreases (see (3.22)). Since $s$ does not affect the bid rent of the employed, the border $x_b$ decreases. Finally, when $\tau$ increases, the impact on $x_b$ is ambiguous. Indeed, the commuting cost $\tau$ negatively affects both bid rents (see (3.21) and (3.22)) so that an increase of $\tau$ reduces the willingness to pay for land for both the employed and the unemployed workers. However, if $\tau$ is small enough, then a rise in $\tau$ increases $x_b$, while we have the reverse if $\tau$ is large enough. This is because the employed have higher total commuting costs than the unemployed ($\tau x$ versus $s\tau x$) so, if $\tau$ is already large, then when $\tau$ increases, they will increase less their bid rent than the unemployed so that $x_b$ decreases. When $\tau$ is small, we have the reverse result.

Since $g(\cdot)$ has an explicit value, using (3.25)-(3.28), we can determine the efficiency wage. It is given by:

$$w_L^* = w_U + e + \frac{e}{m}\left(\frac{\delta\, N}{N-L} + r\right) + \Delta SC \tag{3.30}$$

where

$$\begin{aligned}\Delta SC &= (1-s)\,\tau x_b^* \\ &= (1-s)\left[\frac{1}{1+4s\tau\,(N-L)} - \frac{1}{1+4s\tau\,(N-L)+4\tau\,L}\right]\end{aligned}$$

is the spatial-cost differential between the employed and the unemployed.

Let us interpret $\Delta SC$ and the effects of the *spatial* variables, $s$ and $\tau$, on the efficiency wage $w_L^*$. In fact, the effects of $s$ and $\tau$ on $w_L^*$ are exactly the ones of $s$ and $\tau$ on $x_b$. Since $\Delta SC = (1-s)\,\tau x_b$ is the spatial compensation that firms must give to their employed workers, an increase in $s$, the percentage of unemployed CBD-trips relative to employed CBD-trips, will always reduce the efficiency wage. Indeed, firms need to compensate less the employed for spatial costs since $\Delta SC$ decreases with $s$. However, when the pecuniary commuting cost per unit of distance $\tau$ increases, the effect on wages in ambiguous. This is the effect mentioned above where an increase in $\tau$ raises $x_b$ only if $\tau$ is small enough.

Let us now compare our efficiency wage (3.30) with the efficiency wage obtained where housing consumption is fixed and equal to 1 (given by (2.20)). It is easy to see that the only difference between these two wages is on the spatial-cost differential between the employed and the unemployed $\Delta SC$, i.e. the spatial component of the wage that firms must set in order to compensate the employed workers for their spatial costs (commuting and land rent costs). In fact, since all employed workers obtain the same utility level and since all unemployed workers also obtain the same utility level, we can compare the employed and the unemployed who live exactly at a distance $x_b$ (i.e. the border between the employed and the unemployed) from the CBD since at $x_b$ they both pay the same land rent. As a result, in the case of fixed housing (normalized to 1), $x_b = L$ and the only difference between these two workers is the commuting cost differential, which is equal to $\Delta SC = (1 - s)\,\tau L$. However, in the case of endogenous housing consumption, the spatial difference is also $(1 - s)\,\tau x_b$ but the value of $x_b$ is now more complicated because it takes into account the competition between the employed and the unemployed in the land market.

### 3.4. Interaction between land and labor markets

Compared to the model with fixed housing consumption, the only difference here is that the city-fringe $x_f$ is now endogenous since a function of $L$ and is given by (3.26). It is easy to show that $\frac{\partial x_b^*}{\partial L^*} > 0$ but $\frac{\partial x_f^*}{\partial L^*} < 0$ for reasonable value of the employment rate (i.e. $L^*/N \geq 0.5$). So when employment increases, the employment zone $[0, x_b^*]$ is augmented but the city size $x_f^*$ decreases because the unemployment zone $\left]x_b^*, x_f^*\right]$ is reduced, this latter effect being stronger than the former.

## 4. Open cities and resident landlords

There are other aspects of cities that we would like to study now. They include open cities and non-absent landlords (Fujita, 1989). Indeed, in urban economics, the traditional distinction is between closed and open cities. In closed cities, as we have seen above, the utility of all agents (here the employed and the unemployed workers) is endogenous while their population levels are assumed to be exogenous. In open cities where there is free-mobility between cities, the utility obtained by city residents becomes exogenous (it is just their outside option) but the number of people living in the city is endogenous. This is what we study here. We will investigate further this issue in chapter 3 when we will analyze rural-urban as well as urban-urban migration. Another important issue in urban economics is whether landlords are absent or not. Indeed, in most cities, landlords are not absent and thus we would like to relax the assumption of absentee landlords. This will lead us to study the fulled closed city model (Pines and Sadka, 1986; Fujita, 1989, ch. 3), where urban land is rented from absentee landlords at a price equaling the agricultural rent.

### 4.1. Open cities

Let us study the case of an open city in the benchmark model of urban efficiency wage (section 2). The bid rents of the employed and unemployed workers are still respectively given by (2.6) and (2.7) and Proposition 1 is still valid so that the employed reside close to jobs whereas the unemployed live at the periphery of the city. We have to solve (2.8)−(2.12), given that the unknowns are now $N$ and $L$ while $W_L$ and $W_U$ are exogenous. By solving these equations, we easily obtain $x_f^* = N$ and $x_b^* = L$ as well as:

$$N^* = \frac{w_U - W_U}{s\,\tau} \tag{4.1}$$

$$L^* = \frac{w_L - e - w_U + W_U - W_L}{(1-s)\tau} \tag{4.2}$$

$$R^*(x) = \begin{cases} w_L - e - \tau\, x - W_L & \text{for} \quad 0 \leq x \leq L \\ w_U - s\,\tau\, x - W_U & \text{for} \quad L < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{4.3}$$

Let us now calculate the efficiency wage. The expected utilities of non-shirkers, shirkers and unemployed workers, (2.16)−(2.18), are now modified and equal to:

$$r\, I_L^{NS} = w_L - e - \tau\, L - R(L) - \delta\,(I_L^{NS} - I_U) \tag{4.4}$$

$$r\, I_L^{S} = w_L - \tau\, L - R(L) - (\delta + m)\,(I_L^{S} - I_U) \tag{4.5}$$

$$r\, I_U = w_U - s\tau\, L - R(L) + a(I_L - I_U) \tag{4.6}$$

Observe that the instantanous utilities of non-shirker, shirker and unemployed workers, which are here exogenous, are respectively given by (2.3), (2.4) and (2.5) and are all evaluated at $x = L$. Indeed, since in equilibrium all employed workers obtained the same (instantaneous) utility $W_L$ between $x = 0$ and

$x = L$ and all unemployed workers also obtained the same (instantaneous) utility $W_U$ between $x = L$ and $x = N$, it does not matter which location $x$ we choose. But since the only common location of the employed and unemployed workers is $x = L$, we have chosen to evaluate the utility at exactly this location. Now, because the optimal efficiency wage contract is such that $I_L^{NS} = I_L^S = I_L$, we easily obtain as in the benchmark case: $I_L - I_U = e/m$. Then, by solving the equations exactly as in the benchmark case, we easily obtain (using (4.1) for the second line):

$$\begin{aligned} w_L^* &= w_U + e + \frac{e}{m}\left(\frac{\delta N}{N - L^*} + r\right) + (1-s)\,\tau\,L^* \\ &= w_U + e + \frac{e}{m}\left[\frac{\delta\,(w_U - W_U)}{w_U - W_U - s\,\tau L^*} + r\right] + (1-s)\,\tau\,L^* \end{aligned} \tag{4.7}$$

which is exactly the efficiency wage (2.20). This is very intuitive since there is no reason for the wage setting to be affected by open or closed cities. Employers just set a wage that deter shirking and induce workers to stay in the city. There is however one crucial difference between (2.20) and (4.7) in that $N$ and $L$ are determined by (4.1) and (4.2) in (4.7) while they are parameters in (2.20). Because of that, we can use the value of $w_L^*$ in (4.7) and plug it in (4.2). We easily obtain:

$$W_L - W_U = \frac{e}{m}\left(\frac{\delta N}{N - L} + r\right) \tag{4.8}$$

Now by plugging back (4.8) in (4.7), we obtain another formulation of the efficiency wage expressed in (instantaneous) utility difference between the employed and the unemployed workers. We have:

$$w_L^* = w_U + e + W_L - W_U + (1-s)\,\tau\,L^* \tag{4.9}$$

Of course, here we are not capturing the essence of the efficiency wage model where unemployment "acts as a worker's discipline device", that is when $L$ approaches $N$, $w_L$ tends to infinity. So this is why we prefer to use (4.7). We can now close the model exactly as in the benchmark case by adding equation (2.23), which using (4.1) can be written as

$$eF'(eL^*) = w_U + e + \frac{e}{m}\left[\frac{\delta\,(w_U - W_U)}{w_U - W_U - s\,\tau L^*} + r\right] + (1-s)\,\tau\,L^* \tag{4.10}$$

The equilibrium can be calculated as follows. First, equation (2.23) gives the optimal $L^*$ as function of parameters only. Then, plugging this value of $L^*$ in (4.7) or (4.9) leads to the efficiency wage $w_L^*$. Finally plugging $w_L^*$ in (4.3) yields:

$$R^*(x) = \begin{cases} w_U - \tau\, x - W_U + (1-s)\,\tau\, L^* & \text{for} \quad 0 \leq x \leq L \\ w_U - s\,\tau\, x - W_U & \text{for} \quad L < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{4.11}$$

If we compare the equilibrium land rent in the closed-city model (equation (2.15)) and in the open-city case (equation (4.11)) models, it is quite different between $0 \leq x \leq L$. Indeed, apart from $\tau$, $s$, $L$ and $x$, the latter depends on $w_U$ and $W_U$ while the former is affected by $N$. This is because of the different nature of the two models where in the former the utility is endogenous while in the latter it is the size of the population that is endogenous.

As proved by Fujita (1989, Proposition 3.5, pages 62-63), the closed- and open-city models have to be consistent with one another. Fujita showed that the open-city model should give the same results as those obtained in the closed-city model using the equilibrium utilities $W_L^*$ and $W_U^*$. Let us check this now. From the closed-city model of section 2, by plugging the value of $w_L^*$ defined in (2.20) into (2.13), we obtain:

$$W_L^* = w_U + \frac{e}{m}\left(\frac{\delta\, N}{N-L} + r\right) - s\tau\, N$$

Also, using (2.14), we have:

$$W_U^* = w_U - s\,\tau\, N$$

Combining these two equations, we obtain:

$$W_L^* - W_U^* = \frac{e}{m}\left(\frac{\delta\, N}{N-L} + r\right)$$

which is exactly (4.8). The two models (closed and open cities) are thus totally equivalent.

### 4.2. Landlords are *not* absentee

In this section, we go back to the benchmark model with a closed city and extend it to the case of resident landlords. We will use the so-called public land-ownership model (Fujita, 1989, chapter 3). To be more precise, the city residents are now assumed to form a government, which rents the land for the city from rural landlords at agricultural rent $R_A$. The city government, in turn, subleases the land to city residents at the competitive rent $R(x)$ at each location $x$. We can define the total differential rent ($TDR$) from the city as:

$$\begin{aligned} TDR &= \int_0^{x_f} [R(x) - R_A]\, dx \\ &= \int_0^{x_f} R(x)dx \end{aligned}$$

since $R_A = 0$. The income of each individual is now given by $w_L + TDR/N$ and $w_U + TDR/N$ for the employed and unemployed workers, respectively. Observe that since employment and unemployment are not permanent states but rather transitional, the share of the land obtained by all workers is equal to $TDR/N$, independently of their employment status. Since $TDR/N$ is taken as given by each individual, the analysis is straightforward and follows closely that of section 2.

First, it is easy to verify that the equilibrium (instantanous) utilities are given by:

$$W_L^* = w_L + TDR/N - e - \tau\, L - s\tau\ (N - L) \tag{4.12}$$

$$W_U^* = w_U + TDR/N - s\, \tau\, N \tag{4.13}$$

while the equilibrium land rent is equal to:

$$R^*(x) = \begin{cases} \tau\ (L - x) - TDR/N + s\tau\ (N - L) & \text{for} \quad 0 \leq x \leq L \\ s\, \tau\ (N - x) - TDR/N & \text{for} \quad L < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{4.14}$$

Second, the efficiency wage is still given by (2.20) and $TDR/N$ does not affect this wage. This is mainly because employed shirkers and nonshirkers as well as unemployed workers have all the same land revenue $TDR/N$. Also, the labor demand $L^*$ is still determined by (2.23). Finally, the main innovation of this model is the fact that the equilibrium land rent is affected by the absentee landlords. We have:

$$\begin{aligned} TDR^* &= \int_0^N R^*(x)dx \\ &= \int_0^{L^*} \left[\tau\ (L - x) - \frac{TDR}{N} + s\tau\ (N - L)\right] dx + \int_{L^*}^N \left[s\, \tau\ (N - x) - \frac{TDR}{N}\right] dx \end{aligned}$$

This is equivalent to

$$TDR = \frac{(1 - s)\, \tau}{4} L^{*2} + \frac{s\, \tau}{4} N^2 \tag{4.15}$$

Plugging (4.15) in (4.12), (4.13), (4.14), and using (2.20), we finally obtain:

$$W_L^* = w_U + \frac{e}{m}\left(\frac{\delta\,N}{N-L^*}+r\right)+\frac{(1-s)\,\tau}{4N}L^{*2}-\frac{3}{4}s\,\tau N \tag{4.16}$$

$$W_U^* = w_U + \frac{(1-s)\,\tau}{4N}L^{*2}-\frac{3}{4}s\,\tau N \tag{4.17}$$

$$R^*(x)=\begin{cases}\tau\left(\frac{3}{4}sN-x\right)-(1-s)\left(\frac{L^*}{4N}-1\right)\tau\,L & \text{for} \quad 0\leq x\leq L\\ s\,\tau\left(\frac{3}{4}N-x\right)-\frac{(1-s)\tau}{4N}L^{*2} & \text{for} \quad L<x\leq N\\ 0 & \text{for} \quad x>N\end{cases} \tag{4.18}$$

where $L^*$ is defined by (2.23). Comparing (4.16) and (4.17), we see that;

$$W_L^* - W_U^* = \frac{e}{m}\left(\frac{\delta\,N}{N-L^*}+r\right)$$

which is in fact (4.8). This is not surprising since all models (closed city, open city, absentee landlords, public landownership models) have to be consistent with one another (see again Fujita, 1989, Proposition 3.5, pages 62-63). To conclude, when landlords are not absent but reside in the city, the efficiency wage as well as the employment level are not affected and are still given by (2.20) and (2.23), respectively. However, the instantaneous utilities are now equal to (4.16) and (4.17) (which also implies that the intertemporal utilities change accordingly) and the equilibrium land rent is now given by (4.18).

## 5. City structure

So far we have endogeneized housing consumption but because of quasi-linear preferences, we have precluded the possibility of differential housing consumption by employed and unemployed workers. Because it becomes very complicated, we go back to the assumption of risk neutral workers (utility equals income) and a closed city model but assume different housing consumption between the employed and the unemployed.

### 5.1. Urban land use equilibria

We now assume that the employed workers consume $h_L = h > 0$ units of land whereas the unemployed consume $h_U = \mu\,h > 0$, with $0 < \mu < 1$. This is a reasonable assumption since the employed workers, being richer, consume in general more land than the unemployed. Since all workers are risk neutral, the indirect utility of the (nonshirker) employed and unemployed are now respectively given by:

$$W_L^{NS}(x) = z_L = w_L - e - hR(x) - \tau\, x \tag{5.1}$$

$$W_U(x) = z_U = w_U - \mu\, hR(x) - s\tau\, x \tag{5.2}$$

We can compute the bid rents of all agents. They are given by:

$$\Psi_L(x, W_L) = \frac{w_L - e - \tau\, x - W_L^{NS}}{h} \tag{5.3}$$

$$\Psi_U(x, W_U) = \frac{w_U - s\tau\, x - W_U}{\mu\, h} \tag{5.4}$$

We easily obtain the following result:

**Proposition 4.** *With risk neutral agents and different housing consumption for the employed and the unemployed, we have:*

- $(i)$ *If* $\mu > s$, *then the employed live close to jobs while the unemployed reside at the periphery of the city. This is referred to as Equilibrium 1.*
- $(ii)$ *If* $\mu < s$, *then the unemployed live close to jobs while the employed reside at the periphery of the city. This is referred to as Equilibrium 2. The intuition of this result is straightforward. To determine the equilibrium location pattern, there is a trade off for the employed between the willingness to reside close to jobs (because they have higher commuting costs than the unemployed) and far away from jobs (because they consume more land than the unemployed and land is cheaper further away from the CBD). As a result, if* $\mu > s$, *then the second effect (housing consumption differential) dominates the first one (commuting cost differential) and the employed occupy the core of the city. If* $\mu < s$, *we have the reverse location pattern. Figure 4.3 describes the urban land use pattern of Equilibrium 2.*

[*Insert Figure* 4.3 *here*]

We can now give a formal definition for each urban equilibrium. Again, we normalize the agricultural land rent $R_A$ to zero. Given that $\mu > s$, the definition of Equilibrium 1 is exactly the same than that of Definition 1 in section 2, with $h_L = h$ and $h_U = \mu\, h$. It is easy to see that, by solving these equations, we obtain:[13]

$$\begin{aligned} W_L^{1*} &= w_L - e - \tau\, x_b^* - \frac{s\tau\, \left(x_f^* - x_b^*\right)}{\mu} \\ &= w_L^1 - e - (1-s)\,\tau\, h\, L^1 - s\tau\, hN \end{aligned} \quad (5.5)$$

$$\begin{aligned} W_U^{1*} &= w_U - s\tau\, x_f^* \\ &= w_U - s\tau\, h\, \left[L^1 + \mu\, \left(N - L^1\right)\right] \end{aligned} \quad (5.6)$$

$$x_b^{1*} = h\, L^1 \quad (5.7)$$

$$\begin{aligned} x_f^{1*} &= h\, L^1 + \mu\, h \left(N - L^1\right) \\ &= h\, (1-\mu)\, L^1 + \mu\, hN \end{aligned} \quad (5.8)$$

$$R^{1*}(x) = \begin{cases} \left[-x + (1-s)\, hL^1 + shN\right] \tau/h & \text{for } \; 0 \le x \le x_b^1 \\ \left[-x + hL^1 + \theta\, h\, (N - L^1)\right] s\tau/\theta\, h & \text{for } \; x_b^1 < x \le x_f^1 \\ 0 & \text{for } \; x > x_f^1 \end{cases} \quad (5.9)$$

For equilibrium 2, we have:

**Definition 5.** *Assume $\mu < s$, Then, urban-land use Equilibrium 2 is a 5-tuple $(W_L^{2*}, W_U^{2*}, x_b^{2*}, x_f^{2*}, R^{2*}(x))$ such that:*

$$\Psi_L(x_b^{2*}, W_L^{2*}) = \Psi_U(x_b^{2*}, W_U^{2*}) \quad (5.10)$$

$$\Psi_L(x_f^{2*}, W_L^{2*}) = 0 \quad (5.11)$$

$$\int_0^{x_b^{2*}} \frac{1}{\mu\, h} dx = N - L^2 \quad (5.12)$$

$$\int_{x_b^{2*}}^{x_f^{2*}} \frac{1}{h} dx = L^2 \quad (5.13)$$

$$R^{2*}(x) = \max\left\{\Psi_U(x, W_U^{2*}), \Psi_L(x, W_L^{2*}), 0\right\} \quad \textit{at each } x \in (0, x_f] \quad (5.14)$$

[13] Superscripts 1 and 2 refer respectively to Equilibrium 1 and Equilibrium 2.

Solving (5.10)-(5.14) gives:

$$\begin{aligned} W_L^{2*} &= w_L^2 - e - \tau\, x_f^{2*} \\ &= w_L^2 - e - \tau\, \left[hL^2 + \mu\, h\left(N - L^2\right)\right] \end{aligned} \tag{5.15}$$

$$\begin{aligned} W_U^{2*} &= w_U - s\tau\, x_b^{2*} - \mu\,\tau\, \left(x_f^{2*} - x_b^{2*}\right) \\ &= w_U - s\tau\, \mu\, h\left(N - L^2\right) - \tau\,\theta\, hL^2 \end{aligned} \tag{5.16}$$

$$x_b^{2*} = \mu\, h\left(N - L^2\right) \tag{5.17}$$

$$x_f^{2*} = \mu\, h\left(N - L^2\right) + hL^2 \tag{5.18}$$

$$R^{1*}(x) = \begin{cases} \left[-x + \theta\, h\, L^2/s + \theta\, h\, \left(N - L^2\right)\right] s\tau\, /\, (\theta\, h) & \text{for } \; 0 \le x \le L^1 \\ \left[-x + hL^2 + \theta\, h\,(N - L^2)\right] \tau/h & \text{for } \; L^1 < x \le N \\ 0 & \text{for } \; x > N \end{cases} \tag{5.19}$$

### 5.2. Steady-state equilibrium 1: The employed live close to jobs

Let us determine the labor-market equilibrium 1. We can write the steady-state Bellman equations for respectively the non-shirkers, the shirkers and the unemployed. Using (5.5) and (5.6), we obtain:

$$r\, I_L^{NS} = w_L - e - \tau\, x_b^* - \frac{s\tau\, \left(x_f^* - x_b^*\right)}{\mu} - \delta\left(I_L^{NS} - I_U\right) \tag{5.20}$$

$$rI_L^S = w_L - \tau\, x_b^* - \frac{s\tau\, \left(x_f^* - x_b^*\right)}{\mu} - (\delta + m)\left(I_L^S - I_U\right) \tag{5.21}$$

$$r\, I_U = w_U - s\tau\, x_f^* + a(I_L - I_U) \tag{5.22}$$

Let us now calculate the efficiency wage by having $I_L^{NS} = I_L^S = I_L$. We obtain:

$$w_L^1 = w_U + e + \frac{e}{m}\left(\frac{\delta\, N}{N - L} + r\right) + \Delta SC^1 \tag{5.23}$$

where

$$\begin{aligned} \Delta SC^1 &= (1 - s)\,\tau\, x_b^1 + s\tau\, (1 - \mu)\, h\left(N - L^1\right) \\ &= \tau\, h\left[(1 - s)\, L^1 + s\,(1 - \mu)\left(N - L^1\right)\right] \end{aligned}$$

is the spatial-cost differential between the employed and the unemployed.

In the models described above, where it was assumed *identical housing consumption for employed and unemployed workers*, the spatial-cost differential was equal to $\Delta SC = (1-s)\,\tau x_b^1$ (see (2.20) or (3.19)). Here, the spatial compensation is larger because housing consumption is higher for employed than unemployed workers. This is because firms must not only compensate for commuting costs (which are larger for employed workers) but also for housing consumption (which are also higher for employed workers). In fact, the difference in commuting costs is given by $(1-s)\,\tau\, x_b^1$ while the one in housing consumption is equal to $s\tau\, h\,(N-L^1)\,(1-\mu)$.

### 5.3. Steady-state equilibrium 2: The unemployed live close to jobs

Using exactly the same steps as in section 4.2, we can calculate the efficiency wage for Equilibrium 2. We obtain:

$$w_L^2 = w_U + e + \frac{e}{m}\left(\frac{\delta\, N}{N-L^2} + r\right) + \Delta SC^2 \tag{5.24}$$

where

$$\begin{aligned}\Delta SC^2 &= (1-s)\,\tau\, x_b^2 + \tau\, h\,\left(1-\theta\right) L^2 \\ &= \tau\, h\,\left[(1-s)\,\mu\left(N-L^2\right) + (1-\mu)\, L^2\right]\end{aligned}$$

is the spatial-cost differential between the employed and the unemployed workers.

### 5.4. Comparison between the two equilibria

We would like now to compare the two equilibria in terms of urban and labor aspects as well as welfare. Let us thus define the welfare as the sum of of all utilities (workers, firms and landlords) in each city. Since wages and land rents are pure transfers, the total welfare (which amounts to adding the utilities of all workers, firms and landlords) in equilibrium 1 is given by:

$$\mathcal{W}^1 = w_U\left(N-L^1\right) + F\left(L^1\right) - e\, L^1 - \int_0^{x_b^1} \tau\, x\, dx - \int_{x_b^k}^{x_f^1} s\,\tau\, x\, dx$$

while in equilibrium 2 it is equal to:

$$\mathcal{W}^2 = w_U\left(N-L^2\right) + F\left(L^2\right) - e\, L^2 - \int_0^{x_b^2} s\,\tau\, x\, dx - \int_{x_b^k}^{x_f^2} \tau\, x\, dx$$

To see how each welfare varies and if it can be compared across cities, let us proceed to a simple numerical resolution of the model. We use the following Cobb-Douglas function for the production function of equilibrium $k = 1, 2$: $f\left(l^k\right) = \left(l^k\right)^{0.5}$, which implies that

$$F\left(L^k\right) = M\, f\left(l^k\right) = M^{0.5}\,\left(L^k\right)^{0.5}$$

and thus

$$F'\left(L^k\right) = f'\left(l^k\right) = 0.5\, M^{0.5} L^{-0.5}$$

The values of the parameters, in yearly terms, are: $w_U = 80$, $e = 20$, $h = 20$, $\tau = 1$ and $r = 0.05$. The job destruction rate is $\delta = 0.2$ and the monitoring rate is $m = 1.8$, which mean that jobs last on average five years for a nonshirker and six months for a shirker. The number of firms $M = 70,000$ and the total population $N$ is normalized to 1.

We first fix $s$ and let $\mu$ vary. We take $s = 0.2$, which means that travel frequencies for the unemployed are assumed to be one-fifth of those for employed workers. Table 4.1a describes the results. We have the unemployment rate that is only due to space, $u_s^k = u^k - u_{ns}^k$, where $u^k$ is the unemployment rate defined in this paper, i.e. $\left(N - L^k\right)/N$ and $u_{ns}^k$ is the non-spatial unemployment rate, i.e. the unemployment rate in Shapiro and Stiglitz (1984). Similarly, the wage that is only due to space is $w_s^k = w_L^k - w_{ns}^k$, where $w_L^k$ is the wage defined in this paper and $w_{ns}^k$ is the non-spatial wage, i.e. the wage in Shapiro and Stiglitz (1984). Also, $TLR_L^k$, $TLR_L^k$ and $\Sigma\Pi^h$ signify respectively the total land rent paid by the employed, the total land rent paid by the unemployed and the sum of all profits in the city in equilibrium $k = 1, 2$. In this table, we have chosen to vary a key parameter $s/\mu$, the differential ratio of commuting cost and housing consumption between employed and unemployed workers. This parameter $s/\mu$ varies from a very low value 0.01 (where city 1 is the prevailing equilibrium) to a very large value 4 (where city 2 is the prevailing equilibrium). The cut-off point is equal to $s/\mu = 1$. The sign '$-$' indicates the 'limit to the left', whereas the sign '$+$' indicates the 'limit to the right'.

Our comments are as follows. First, on can see that switching from a city where the employed live close to jobs (city 1) to a city where the unemployed live close to jobs (city 2) does not alter any of the main variables. In particular, the total welfare is roughly the same. This is not surprising because, in this model, it is not harmful for the unemployed to be far away from jobs. In particular, the rate at which they leave unemployment is $a^k = \delta L^k/(N - L) =$

$\delta\left(1-u^k\right)/u^k$. Since, for both $s/\mu = 1-$ and $s/\mu = 1+$, $u^k = 0.094$, we have $a^1 = a^2 = 1.927$, which implies that the average duration of unemployment $1/a^k$ is six months. Second, the impact of the spatial variables on labor market outcomes are *not* negligible. If we take the two extreme cases, $s/\mu = 0.2$ ($\mu$ close to 1 so that employed and unemployed workers consume the same amount of land; city 1) and $s/\mu = 200$ ($\mu$ close to 0 so that unemployed workers consume negligible amount of land; city 2), the total unemployment rate increases from 9.32% to 10.42%, but this rise is only due to the spatial part of unemployment, which increases by 35 percent. This is because the city-size decreases by 11.6 percent (from 20 to 17.92), which affects wages and ultimately totally land prices, which decrease by 6.66 percent (from 8.5784 to 8.043).

Let us now fix $\mu = 0.2$, which means that housing consumptions for the unemployed are assumed to be one-fifth of those for employed workers, and let $s$ vary. Table 4.1b displays the results of the numerical simulations. The results are similar to the ones obtained in Table 4.1a, with the difference that the unemployment rate is now higher in city 1 than city 2 because $\mu$ is relatively small. If one again compares the two extreme cases, $s/\mu = 0.2$ ($s$ close to 0 so that the unemployed do not commute very often to the CBD; city 1) and $s/\mu = 5$ ($s$ close to 1 so that employed and unemployed workers have the same number of commuting trips; city 2), the decrease in unemployment is again only due to space. However, in this case, there is nearly no city-size effect but a very strong wage effect. Indeed, since the efficiency wage must compensate for the difference in spatial costs between the employed and the unemployed, a variation of $s$ has obviously large effects. In fact, from $s$ close to 0 to $s$ close to 1, the spatial compensation of wages decreases from 12.39% to 10.45%. This is the main effect that explains the decrease in spatial unemployment from 10.22% to 9.32%.

[*Insert Tables* 4.1*a and* 4.1*b here*]

## 6. Long-run equilibrium with free entry

We now study the long-run properties of the urban efficiency wage model in the benchmark model. Firms freely enter the labor market up to point where profits equal to zero. If $c$ denotes the fixed-entry cost, then, for each equilibrium $k = 1, 2$, the zero-profit condition can be written as:

$$f(l^k) - w_L^k l^k - c = 0$$

As a result, for each equilibrium $k = 1, 2$, the long-run equilibrium is characterized by four unknowns, $l^k, w^k, u^k, M^k$ and the four following equations:

$$w_L^k = f'(l^k) \tag{6.1}$$

$$f(l^k) - w_L^k l^k - c = 0 \tag{6.2}$$

$$w_L^k = w_U + e + \frac{e}{m}\left(\frac{\delta}{u^k} + r\right) + \Delta SC^k \tag{6.3}$$

$$1 - u^k = M^k l^k \tag{6.4}$$

Here is the way each equilibrium is solved. First, (6.1) gives the labor demand as a function of wage, $l^k(w_L^k) = f'^{-1}(w_L^k)$. Then, we plug this value in (6.2) and we obtain the wage $w_L^k(c)$. As already noted by Albrecht and Vroman (1996, 1999), the nice properties of the long-run equilibrium of the shirking model is that, because of the zero-profit condition, wages are solely determined by the parameters of the production function and the value of $c$. So, for example, if $t$, the commuting cost increases, because the free-entry condition has to hold, aggregate labor demand shifts in tandem with the urban non-shirking curve in such a way as to keep the equilibrium efficiency wage constant. Then, by plugging $w_L^k(c)$ in (6.3), we obtain $u^k(c, w_U, e, m, \delta, s, \tau)$ and finally by using (6.4), we get $M^k(c, b, e, m, \delta, s, \tau)$. It is easy to show that

$$\frac{\partial M^k}{\partial \tau} < 0 \ , \ \frac{\partial M^k}{\partial s} > 0 \text{ and } \frac{\partial M^k}{\partial \mu} > 0 \tag{6.5}$$

that is higher commuting costs and/or lower CBD-trips for the unemployed implies less entry since it increases unemployment.

For example, if we take $f\left(l^k\right) = \left(l^k\right)^{0.5}$, then (6.1) gives

$$l^k(w_L^k) = \frac{1}{4\left(w_L^k\right)^2}$$

and, using the zero-profit condition (6.2), we obtain:

$$w_L^k = \frac{1}{4c}$$

This makes the comparative statics calculations very simple. In particular, in the long-run, the wage with and without space is exactly the same and equal to $1/4c$. In other words, in the short-run, space affects both wages and unemployment, whereas in the long-run it does not affect wages but affect unemployment and the number of firms. In fact, it has a stronger effect than in the short run.

Let us perform again numerical simulations. We use exactly the same parameters as in the previous example. However, now $M$ is endogenous and the fixed-entry cost is $c = 0.0018$. Because the efficiency wage is exactly the same in the short-run and in the long-run, we do not reproduce the wage effects in the tables. Also, we differentiate between the number of firms when there is no space $M_{ns}^k$ (i.e. in the Shapiro's and Stiglitz's model) and when space is introduced $M^k - M_{ns}^k$.

As above, we first fix $s = 0.2$ and let $\mu$ vary. Table 4.2a displays the results. First, the free-entry case shows that the short-run results are quite robust since there is no real change when one switches from city 1 ($s/\mu = 1-$) to city 2 ($s/\mu = 1+$) and the values of unemployment rates, welfare, ... are roughly the same. The only difference with the short-run case is really the number of firms that enter the market. Of course, when $s/\mu$ varies, $M_{ns}^k$ is not affected but $M^k - M_{ns}^k$ increases by 42.6 percent. Indeed, when $\mu$ decreases from 1 to 0, both the entry cost $c$ and the wage $w$ are not affected but the unemployment is and thus entry is reduced (see (6.5)). If we now fix $\mu = 0.2$ and let $s$ vary (Table 4.2b), we obtain the reverse results since, for a given $\mu$, $\frac{\partial M^k}{\partial s} > 0$.

[*Insert Tables* 4.2*a and* 4.2*b here*]

## 7. Endogenous unemployment benefit

To close the benchmark model, let us endogeneize the unemployment benefit $w_U$. Indeed, in the model so far, it was assumed that all welfare payments were distributed by the (local) government, and that no income taxes were paid. To add realism to this "manna from heaven" scenario, we now introduce endogenous taxation. For sake of simplicity, we consider the case in which all city welfare payments are financed by employee taxes on local firms. In particular, for any given employment level, $L$, the employee welfare tax, $\Delta$, must satisfy the condition that

$$\Delta(L) = \frac{w_U\,(N - L)}{L} \tag{7.1}$$

Indeed, the total benefit of taxation for the government is $\Delta\, L$, which finances the total cost of welfare payments equals to $w_U\,(N-L)$. It is easy to see that

$$\Delta'(L) < 0\ ,\ \lim_{L\to 0}\Delta(L) = +\infty\ ,\ \lim_{L\to N}\Delta(L) = 0 \tag{7.2}$$

Indeed, more employment means less individual tax and if nobody works then the tax has to be infinite to finance the unemployment benefit of everybody. Finally, if on the contrary all workers are employed ($L = N$), then there is nobody to finance and $\Delta = 0$.

In this new setting, it follows that the profit function of each firm $j$ is now given by:

$$\Pi_j = f(el_j) - [w_j + \Delta(L)]\, l_j$$

Given this modification, the labor-market equilibrium condition is now equal to:

$$\begin{aligned} eF'(eL^*_\Delta) &= w_L(L^*_\Delta) + \Delta(L^*_\Delta) \qquad (7.3) \\ &= w_U + e + \frac{e}{m}\left(\frac{\delta\, N}{N - L^*_\Delta} + r\right) + (1-s)\,\tau\, L^*_\Delta + \Delta(L^*_\Delta) \end{aligned}$$

where $L^*_\Delta$ is the new equilibrium value for employment. This new equilibrium situation is described in Figure 4.4. Notice in particular from (7.2) that when $L \to 0$, the right-hand side of (7.3) approaches infinity at the origin. For simplicity, we do not assume the Inada conditions on the production function and hence in the endogenous taxation case there will generally be *multiple equilibria*. However, it should be clear from the monotonicity properties of $F'(.)$ that the equilibrium with highest employment level (such as $L^*_\Delta$ in Figure 4.4) must yield the lowest value of marginal cost, $w_L(L_\Delta) + \Delta(L_\Delta)$, and hence yield the unique *maximum-profit equilibrium* for firms. Moreover, it is easy to see that this maximum-profit equilibrium is the unique *stable* equilibrium since any reasonable adjustment process will move the system from the low-employment to the high-employment equilibrium level because of the monotonicity properties of $F'(.)$.

[*Insert Figure* 4.4 *here*]

## 8. Non-technical summary and notes on the literature

We have exposed the basic urban efficiency wage model with different variations. This model will constitute the core of part one of this book and will be used to explained urban ghettos and labor-market outcomes in part three of this book.

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# A. Appendix A.4.1. Proof of Proposition 2

Assume that the two bid-rent functions intersect at $x_b$ (this will be shown below). Then, it has to be that:

$$\Psi_L(x_b, W_L) = \Psi_U(x_b, W_U)$$

$$\Leftrightarrow \frac{w_L - e - \tau\, x_b - W_L + g(h(x_b))}{h(x_b)} = \frac{w_U - s\tau\, x - W_U + g(h(x_b))}{h(x_b)}$$

$$\Leftrightarrow W_L - W_U + (1-s)\,\tau\, x_b = w_L - e - w_U \tag{A.1}$$

This implies in particular that:

$$W_L - w_L + e < W_U - w_U \tag{A.2}$$

Let us now show that the employee's bid rent is steeper than the unemployed worker's bid rent. For that,we have to show that:

$$-\frac{\partial \Psi_L(x, W_L)}{\partial x}\mid_{x=x_b} > -\frac{\partial \Psi_U(x_b, W_L)}{\partial x}\mid_{x=x_b}$$

$$\Leftrightarrow [W_U - w_U - (W_L - w_L + e) - (1-s)\,\tau\, x_b]\, h'(x_b) + (1-s)\,\tau h(x_b) > 0$$

Using (A.2), it is easy to see that $W_U - w_U - (W_L - w_L + e) - (1-s)\,\tau\, x_b > 0$ and since $h'(x) > 0$, $\forall x$, then this inequality is always true and thus the employee's bid rent is steeper than the unemployed worker's bid rent at $x = x_b$. We have now to verify that indeed the two curves intersect at $x_b$. For that, we have to show that

$$\Psi_L(0, W_L) > \Psi_U(0, W_L)$$

$$\Leftrightarrow w_L - e - W_L > w_U - W_U$$

$$\Leftrightarrow W_U - w_U > W_L - w_L + e$$

which is true by (A.2). Hence, we have shown that:

$(i)$ $\Psi_L(0, W_L) > \Psi_U(0, W_L)$;

$(ii)$ the employee's bid rent function $\Psi_L(x, W_L)$ is steeper than the unemployed worker's bid rent function $\Psi_U(x, W_U)$ at $x = x_b$;

$(iii)$ both functions $\Psi_L(x, W_L)$ and $\Psi_U(x, W_U)$ are decreasing and convex with respect with $x$.

Since these two functions $\Psi_L(x, W_L)$ and $\Psi_U(x, W_U)$ are well-behaved (in particular continuous and smooth) because the utility function is itself well-behaved, then it has to be that they intersect at some point. ▮

Figure 4.1: Urban equilibrium in the benchmark model

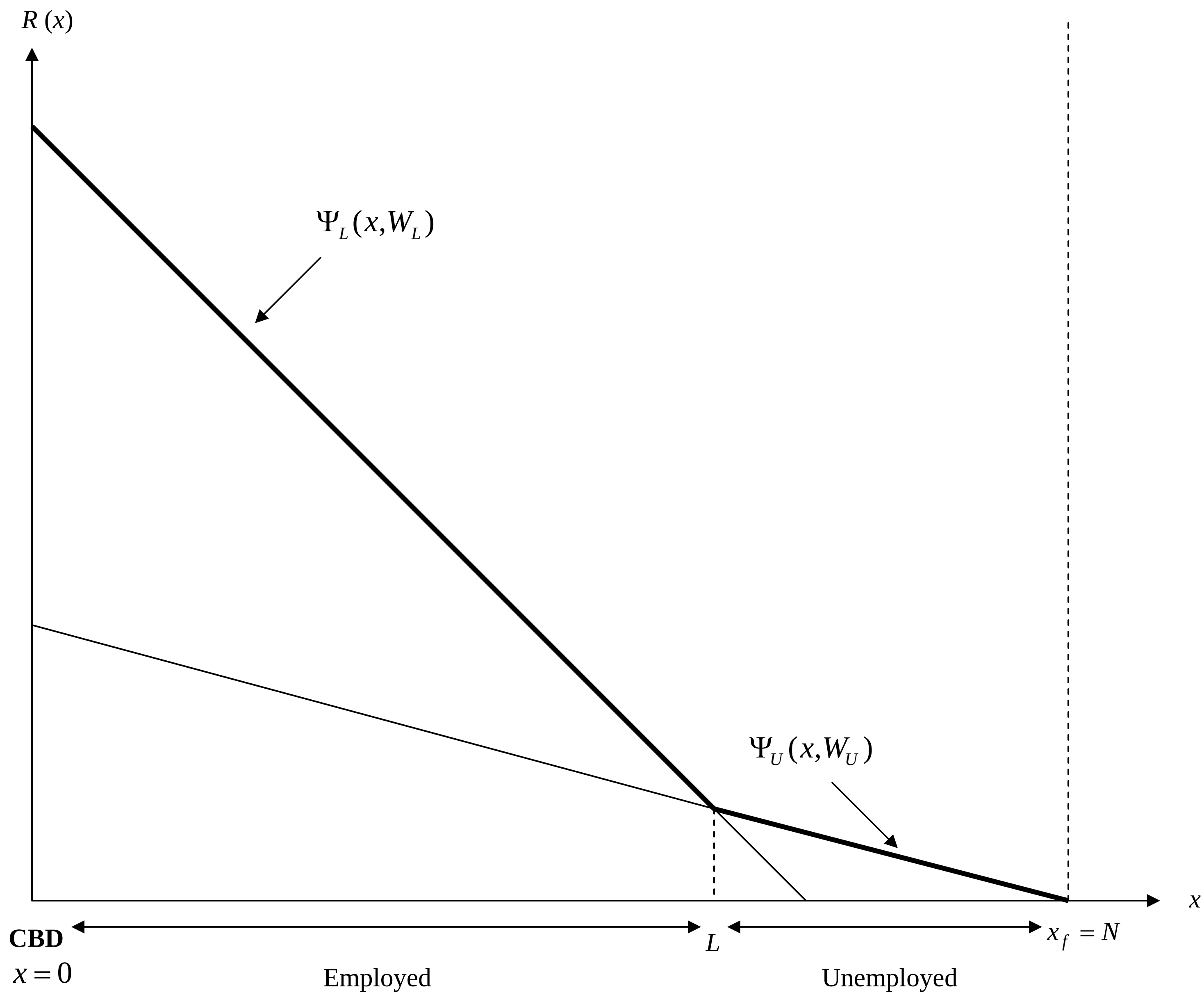

Figure 4.2: Labor equilibrium in the benchmark model

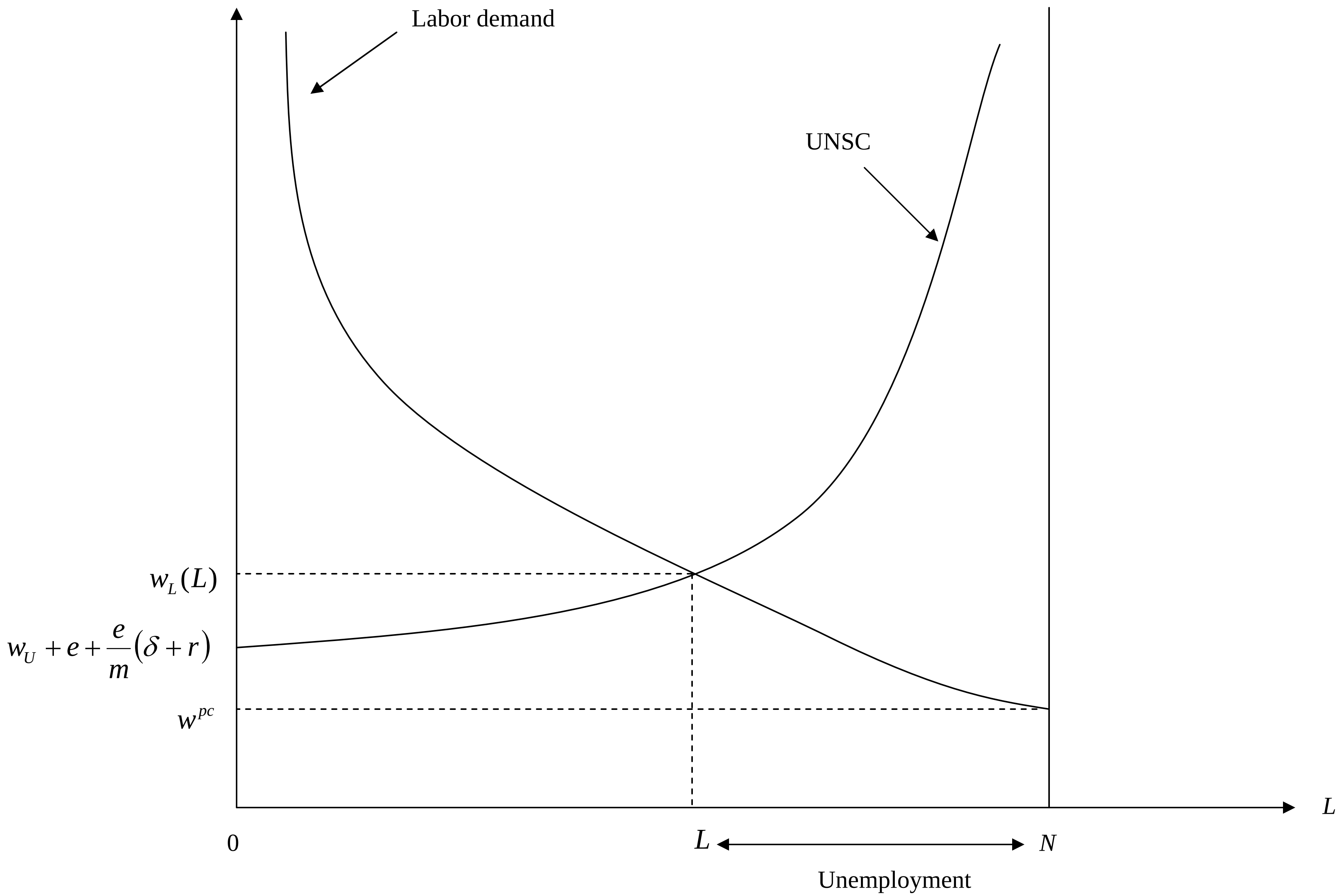

Figure 4.3: Urban equilibrium 2

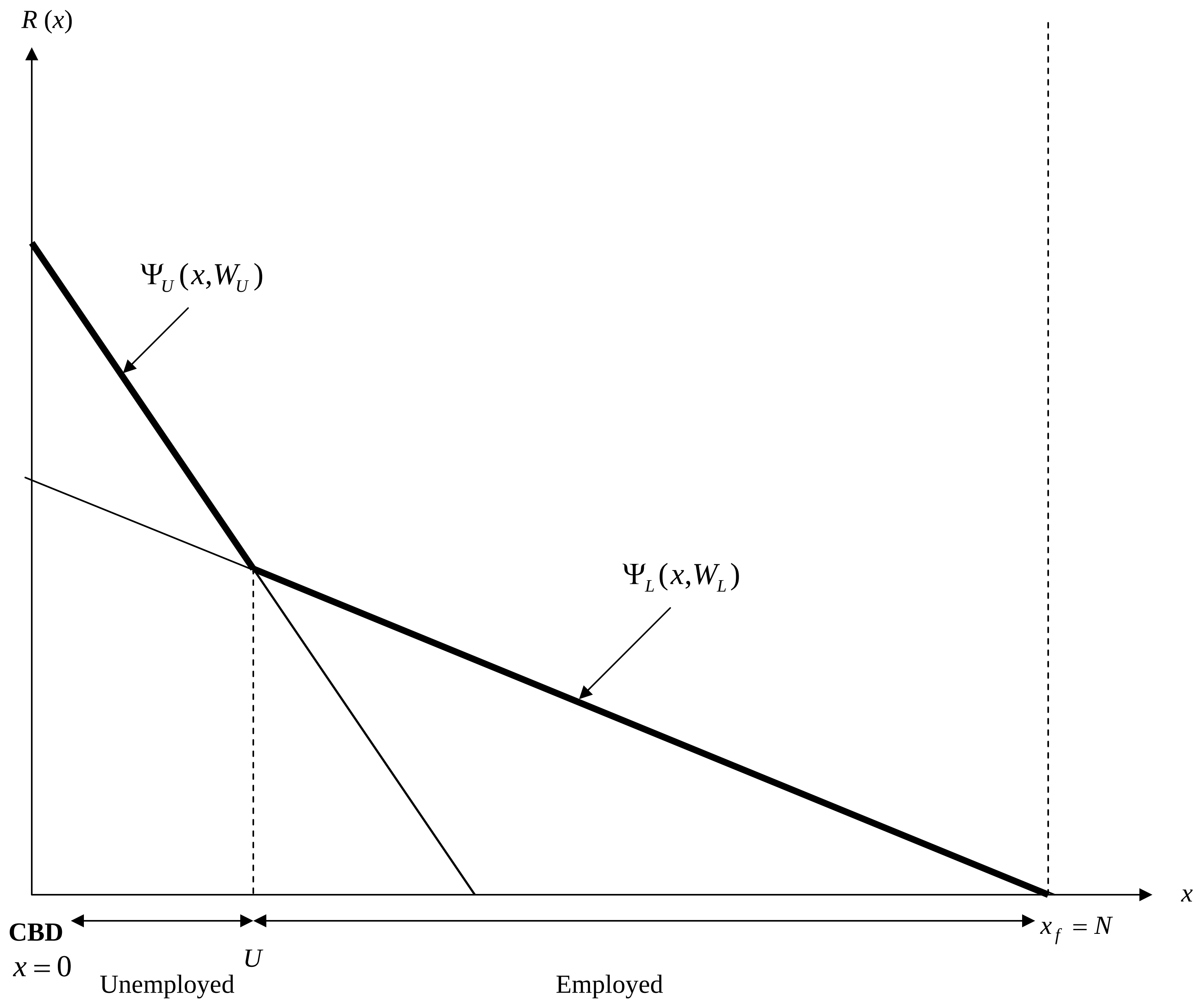

Figure 4.4: Labor equilibrium with tax and subsidy

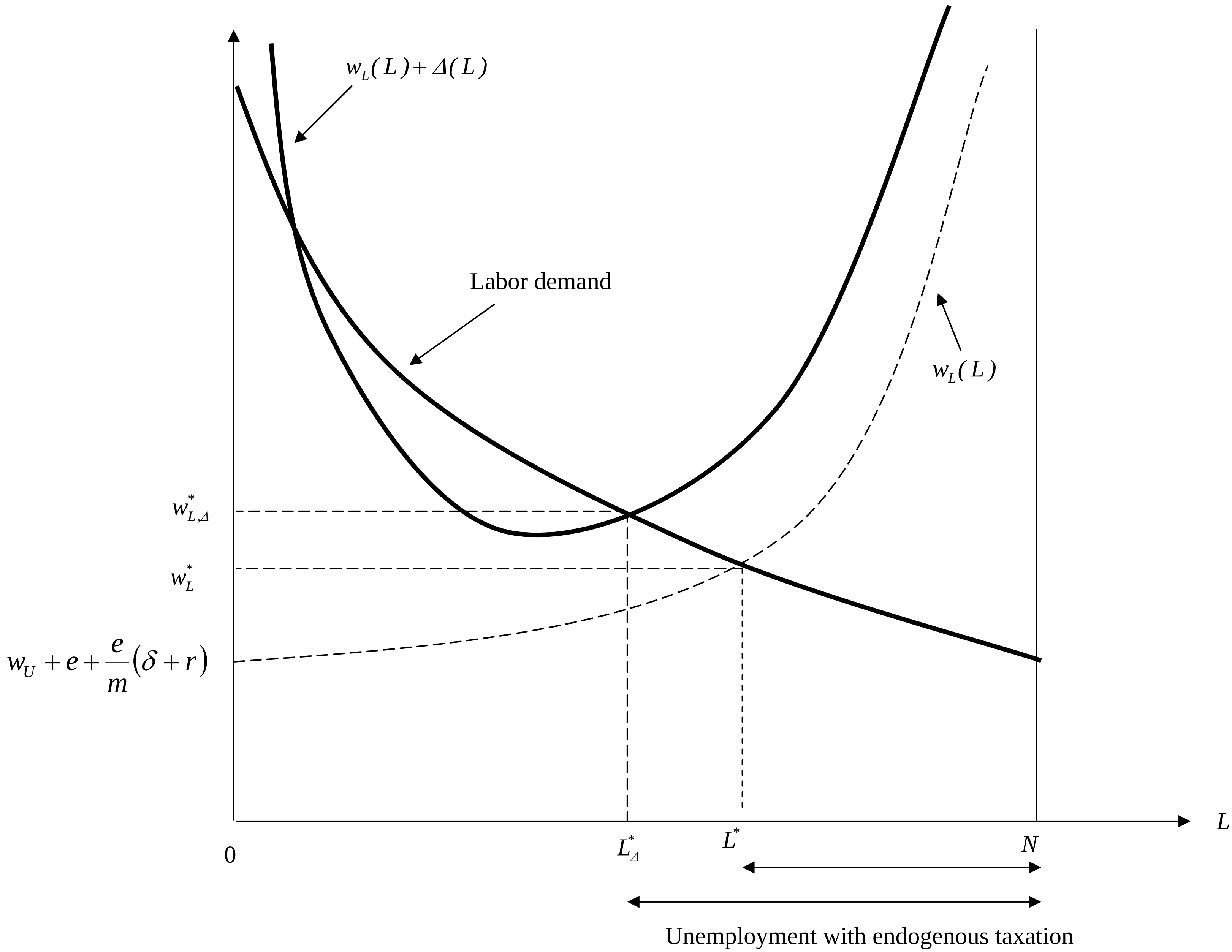

Table 4.1a: Short-run equilibria when $s = 0.2$ and $\mu$ varies

| $s/\mu$ (with $s$ =0.2) | City | $u^k$ | $u^k_{ns}$ | $u^k_s$ | $\frac{u^k_s}{u^k}$ | $\frac{w^k_s}{w^k_L}$ | $\frac{\Delta SC^k}{w^k_L}$ | $\frac{\Delta WI^k}{w^k_L}$ | $TLR^k_L$ | $TLR^k_U$ | $x^k_b$ | $x^k_f$ | $\Sigma\Pi^k$ | $\mathcal{W}^k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2001 ($\mu$ =0.9999) | 1 | 9.32 | 6.17 | 3.15 | 33.77 | 1.69 | 10.44 | 31.97 | 8.561 | 0.0174 | 18.137 | 20 | 125.98 | 66477 |
| 0.3 ($\mu$ =0.6666) | 1 | 9.35 | 6.17 | 3.18 | 34.02 | 1.71 | 10.53 | 31.89 | 8.556 | 0.0117 | 18.130 | 19.38 | 125.95 | 66467 |
| 0.4 ($\mu$ =0.5) | 1 | 9.37 | 6.17 | 3.20 | 34.15 | 1.72 | 10.57 | 31.86 | 8.553 | 0.0088 | 18.126 | 19.06 | 125.94 | 66461 |
| 0.6 ($\mu$ =0.3333) | 1 | 9.39 | 6.17 | 3.22 | 34.28 | 1.73 | 10.61 | 31.82 | 8.551 | 0.0059 | 18.122 | 18.75 | 125.92 | 66456 |
| 0.9 ($\mu$ =0.22222) | 1 | 9.40 | 6.17 | 3.23 | 34.36 | 1.74 | 10.64 | 31.80 | 8.549 | 0.0039 | 18.120 | 18.54 | 125.92 | 66452 |
| 1− ($\mu$ =0.20001) | 1 | 9.40 | 6.17 | 3.23 | 34.38 | 1.74 | 10.65 | 31.79 | 8.548 | 0.0035 | 18.119 | 18.50 | 125.91 | 66451 |
| 1+ ($\mu$ =0.19999) | 2 | 9.40 | 6.17 | 3.23 | 34.38 | 1.74 | 10.65 | 31.79 | 8.208 | 0.3440 | 0.376 | 18.50 | 125.91 | 66446 |
| 1.2 ($\mu$ =0.16666) | 2 | 9.57 | 6.17 | 3.40 | 35.50 | 1.83 | 11.02 | 31.47 | 8.178 | 0.2914 | 0.319 | 18.41 | 125.80 | 66387 |
| 1.4 ($\mu$ =0.143) | 2 | 9.69 | 6.17 | 3.52 | 36.29 | 1.89 | 11.28 | 31.25 | 8.157 | 0.2529 | 0.277 | 18.34 | 125.72 | 66345 |
| 1.6 ($\mu$ =0.125) | 2 | 9.78 | 6.17 | 3.61 | 36.88 | 1.94 | 11.48 | 31.08 | 8.140 | 0.2229 | 0.244 | 18.29 | 125.66 | 66313 |
| 1.8 ($\mu$ =0.1111) | 2 | 9.85 | 6.17 | 3.68 | 37.34 | 1.98 | 11.63 | 30.95 | 8.128 | 0.1994 | 0.219 | 18.25 | 125.61 | 66288 |
| 4 ($\mu$ =0.05) | 2 | 10.16 | 6.17 | 3.99 | 39.29 | 2.15 | 12.29 | 30.39 | 8.071 | 0.0923 | 0.102 | 18.07 | 125.39 | 66174 |
| 200 ($\mu$ =0.001) | 2 | 10.42 | 6.17 | 4.25 | 40.80 | 2.29 | 12.81 | 29.96 | 8.024 | 0.0019 | 0.0021 | 17.92 | 125.20 | 66081 |

Table 4.1b: Short-run equilibria when $\mu = 0.2$ and $s$ varies

| $s/\mu$ (with $\mu$ =0.2) | City | $u^k$ | $u^k_{ns}$ | $u^k_s$ | $\frac{u^k_s}{u^k}$ | $\frac{w^k_s}{w^k_L}$ | $\frac{\Delta SC^k}{w^k_L}$ | $\frac{\Delta WI^k}{w^k_L}$ | $TLR^k_L$ | $TLR^k_U$ | $x^k_b$ | $x^k_f$ | $\Sigma\Pi^k$ | $\mathcal{W}^k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2001 ($s$ =0.04) | 1 | 10.22 | 6.17 | 4.05 | 39.60 | 2.18 | 12.39 | 30.30 | 8.135 | 0.0008 | 17.957 | 18.37 | 125.35 | 66157 |
| 0.3 ($s$ =0.06) | 1 | 10.11 | 6.17 | 3.94 | 38.97 | 2.12 | 12.18 | 30.48 | 8.189 | 0.0012 | 17.978 | 18.38 | 125.42 | 66195 |
| 0.4 ($s$ =0.08) | 1 | 10.01 | 6.17 | 3.84 | 38.34 | 2.07 | 11.97 | 30.67 | 8.243 | 0.0016 | 18.000 | 18.40 | 125.49 | 66233 |
| 0.6 ($s$ =0.12) | 1 | 9.80 | 6.17 | 3.63 | 37.05 | 1.95 | 11.53 | 31.03 | 8.348 | 0.0023 | 18.040 | 18.43 | 125.64 | 66307 |
| 0.9 ($s$ =0.18) | 1 | 9.50 | 6.17 | 3.33 | 35.06 | 1.79 | 10.87 | 31.60 | 8.500 | 0.0033 | 18.100 | 18.48 | 125.85 | 66416 |
| 1− ($s$ =0.19999) | 1 | 9.40 | 6.17 | 3.23 | 34.38 | 1.74 | 10.65 | 31.79 | 8.548 | 0.0035 | 18.119 | 18.50 | 125.91 | 66451 |
| 1+ ($s$ =0.20001) | 2 | 9.40 | 6.17 | 3.23 | 34.38 | 1.74 | 10.65 | 31.79 | 8.208 | 0.3443 | 0.376 | 18.50 | 125.91 | 66446 |
| 1.2 ($s$ =0.24) | 2 | 9.40 | 6.17 | 3.23 | 34.35 | 1.74 | 10.64 | 31.80 | 8.209 | 0.3449 | 0.376 | 18.50 | 125.92 | 66448 |
| 1.4 ($s$ =0.28) | 2 | 9.39 | 6.17 | 3.22 | 34.32 | 1.73 | 10.63 | 31.81 | 8.209 | 0.3454 | 0.376 | 18.50 | 125.92 | 66449 |
| 1.6 ($s$ =0.32) | 2 | 9.39 | 6.17 | 3.22 | 34.29 | 1.73 | 10.62 | 31.82 | 8.210 | 0.3460 | 0.376 | 18.50 | 125.92 | 66451 |
| 1.8 ($s$ =0.36) | 2 | 9.39 | 6.17 | 3.22 | 34.26 | 1.73 | 10.61 | 31.83 | 8.211 | 0.3465 | 0.375 | 18.50 | 125.93 | 66452 |
| 2.5 ($s$ =0.5) | 2 | 9.37 | 6.17 | 3.20 | 34.15 | 1.72 | 10.57 | 31.86 | 8.214 | 0.3485 | 0.375 | 18.50 | 125.94 | 66458 |
| 4 ($s$ =0.8) | 2 | 9.34 | 6.17 | 3.17 | 33.92 | 1.70 | 10.49 | 31.92 | 8.21957 | 0.3526 | 0.374 | 18.51 | 125.96 | 66470 |
| 4.99 ($s$ =0.998) | 2 | 9.32 | 6.17 | 3.15 | 33.77 | 1.69 | 10.45 | 31.97 | 8.223 | 0.3553 | 0.373 | 18.51 | 125.97 | 66477 |

Table 4.2a: Long-run equilibria when $s = 0.2$ and $\mu$ varies

| $s/\mu$ (with $s$ =0.2) | City | $u^k$ | $u^k_{ns}$ | $u^k_s$ | $\frac{u^k_s}{u^k}$ | $M^k$ | $M^k_{ns}$ | $\frac{M^k - M^k_{ns}}{M^k}$ | $TLR^k_L$ | $TLR^k_U$ | $x^k_b$ | $x^k_f$ | $\mathcal{W}^k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2001 ($\mu$ =0.99999) | 1 | 9.33 | 5.80 | 3.53 | 37.85 | 69964 | 72687 | 3.89 | 8.560 | 0.017 | 18.135 | 20 | 66439 |
| 0.3 ($\mu$ =0.66666) | 1 | 9.37 | 5.80 | 3.58 | 38.15 | 69928 | 72687 | 3.95 | 8.553 | 0.012 | 18.125 | 19.375 | 66391 |
| 0.4 ($\mu$ =0.5) | 1 | 9.40 | 5.80 | 3.60 | 38.31 | 69910 | 72687 | 3.97 | 8.550 | 0.009 | 18.121 | 19.060 | 66366 |
| 0.6 ($\mu$ =0.33333) | 1 | 9.42 | 5.80 | 3.62 | 38.46 | 69892 | 72687 | 4.00 | 8.546 | 0.006 | 18.116 | 18.744 | 66341 |
| 0.9 ($\mu$ =0.22222) | 1 | 9.44 | 5.80 | 3.64 | 38.57 | 69879 | 72687 | 4.02 | 8.544 | 0.004 | 18.113 | 18.532 | 66325 |
| 1− ($\mu$ =0.20001) | 1 | 9.44 | 5.80 | 3.64 | 38.59 | 69877 | 72687 | 4.02 | 8.543 | 0.004 | 18.112 | 18.490 | 66321 |
| 1+ ($\mu$ =0.19999) | 2 | 9.44 | 5.80 | 3.64 | 38.59 | 69877 | 72687 | 4.02 | 8.201 | 0.345 | 0.378 | 18.490 | 66316 |
| 1.2 ($\mu$ =0.16666) | 2 | 9.65 | 5.80 | 3.86 | 39.95 | 69711 | 72687 | 4.27 | 8.162 | 0.294 | 0.322 | 18.391 | 66081 |
| 1.4 ($\mu$ =0.143) | 2 | 9.81 | 5.80 | 4.01 | 40.91 | 69590 | 72687 | 4.45 | 8.134 | 0.256 | 0.281 | 18.318 | 65911 |
| 1.6 ($\mu$ =0.125) | 2 | 9.93 | 5.80 | 4.14 | 41.64 | 69496 | 72687 | 4.59 | 8.112 | 0.226 | 0.248 | 18.262 | 65778 |
| 1.8 ($\mu$ =0.1111) | 2 | 10.03 | 5.80 | 4.23 | 42.19 | 69423 | 72687 | 4.70 | 8.095 | 0.203 | 0.223 | 18.217 | 65674 |
| 4 ($\mu$ =0.05) | 2 | 10.46 | 5.80 | 4.67 | 44.60 | 69087 | 72687 | 5.21 | 8.017 | 0.095 | 0.105 | 18.012 | 65201 |
| 200 ($\mu$ =0.001) | 2 | 10.83 | 5.80 | 5.03 | 46.48 | 68803 | 72687 | 5.65 | 7.951 | 0.002 | 0.002 | 17.836 | 64801 |

Table 4.2b: Long-run equilibria when $\mu = 0.2$ and $s$ varies

| $s/\mu$ (with $\mu$ =0.2) | City | $u^k$ | $u^k_{ns}$ | $u^k_s$ | $\frac{u^k_s}{u^k}$ | $M^k$ | $M^k_{ns}$ | $\frac{M^k_{ns}-M^k}{M^k_{ns}}$ | $TLR^k_L$ | $TLR^k_U$ | $x^k_b$ | $x^k_f$ | $\mathcal{W}^k$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2001 ($s$ =0.04) | 1 | 10.54 | 5.80 | 4.74 | 44.79 | 69030 | 72687 | 5.30 | 8.079 | 0.00089 | 17.893 | 18.31 | 65122 |
| 0.3 ($s$ =0.06) | 1 | 10.39 | 5.80 | 4.59 | 44.21 | 69143 | 72687 | 5.13 | 8.142 | 0.00130 | 17.922 | 18.34 | 65282 |
| 0.4 ($s$ =0.08) | 1 | 10.25 | 5.80 | 4.45 | 43.42 | 69254 | 72687 | 4.96 | 8.203 | 0.00168 | 17.951 | 18.36 | 65439 |
| 0.6 ($s$ =0.12) | 1 | 9.97 | 5.80 | 4.17 | 41.84 | 69470 | 72687 | 4.63 | 8.321 | 0.00238 | 18.007 | 18.41 | 65744 |
| 0.9 ($s$ =0.18) | 1 | 9.57 | 5.80 | 3.77 | 39.41 | 69778 | 72687 | 4.17 | 8.490 | 0.00330 | 18.087 | 18.47 | 66181 |
| 1− ($s$ =0.19999) | 1 | 9.44 | 5.80 | 3.64 | 38.59 | 69877 | 72687 | 4.02 | 8.543 | 0.00356 | 18.112 | 18.49 | 66321 |
| 1+ ($s$ =0.20001) | 2 | 9.44 | 5.80 | 3.64 | 38.59 | 69877 | 72687 | 4.02 | 8.201 | 0.34550 | 0.378 | 18.49 | 66316 |
| 1.2 ($s$ =0.24) | 2 | 9.43 | 5.80 | 3.64 | 38.55 | 69881 | 72687 | 4.02 | 8.206 | 0.34217 | 0.377 | 18.49 | 66322 |
| 1.4 ($s$ =0.28) | 2 | 9.43 | 5.80 | 3.63 | 38.51 | 69886 | 72687 | 4.01 | 8.203 | 0.34655 | 0.377 | 18.49 | 66328 |
| 1.6 ($s$ =0.32) | 2 | 9.42 | 5.80 | 3.63 | 38.47 | 69890 | 72687 | 4.00 | 8.204 | 0.34706 | 0.377 | 18.49 | 66335 |
| 1.8 ($s$ =0.36) | 2 | 9.42 | 5.80 | 3.62 | 38.44 | 69895 | 72687 | 4.00 | 8.205 | 0.34758 | 0.377 | 18.49 | 66341 |
| 2.5 ($s$ =0.5) | 2 | 9.40 | 5.80 | 3.60 | 38.31 | 69910 | 72687 | 3.97 | 8.209 | 0.34938 | 0.376 | 18.50 | 66363 |
| 4 ($s$ =0.8) | 2 | 9.35 | 5.80 | 3.56 | 38.03 | 69942 | 72687 | 3.92 | 8.217 | 0.35318 | 0.374 | 18.50 | 66409 |
| 200 ($s$ =0.998) | 2 | 9.33 | 5.80 | 3.53 | 37.85 | 69964 | 72687 | 3.89 | 8.222 | 0.35566 | 0.373 | 18.51 | 66439 |