Does a pure strategies NE always exist? (Dutta Ch.8)

By the definition of NE given, there is no NE in pure strategies in the following game :

Example: **Matching Pennies**

1/2	H	T
H	(-1,1)	(1,-1)
T	(1,-1)	(-1,1)

Each player would like to outguess the other….

Nash (1950): in any finite game (N and strategy sets are finite) there exists *at least one* NE. This equilibrium may involve *mixed strategies*.

Def. A *mixed strategy* is a probability distribution over a player's pure strategies.

In the example:

player 1: Play H with probability r.
Play T with probability 1-r.

player 2: Play H with probability q.
Play T with probability 1-q.

Strategies are now *continuous* variables…$r \in [0,1]$, $q \in [0,1]$.

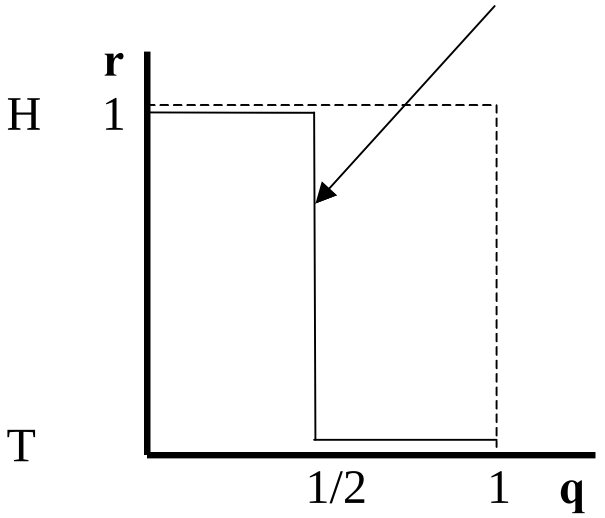
In a (mixed strategies) NE each player's mixed strategy is a best response to the other player's mixed strategy.

Player 1' s **expected payoff** from playing the *mixed strategy* (r,1- r) **given** his *belief* (q,1- q) about player 2 strategy:

$$
\begin{aligned}
Eu_1(r,q) &= r[-q + (1-q)] + (1-r)[q - (1-q)] \\
&= r[1-2q] + (1-r)[2q-1] \\
&= r[2-4q] + 2q-1
\end{aligned}
$$

$q = 1/2$ $Eu_1(r,1/2) = 0$ for every choice of $r \in [0,1]$
$q < 1/2$ $Eu_1(r,q)$ maximised for $r = 1$ (play H)
$q > 1/2$ $Eu_1(r,q)$ maximised for $r = 0$ (play T)

Best Response Mapping r*(q):



Similarly for player 2:

r = 1/2 q*(1/2) = [0,1]
r < 1/2 q* (r) = 0 (play T)
r > 1/2 q* (r) = 1 (play H)

…If it is a NE for BOTH players to randomise, it must be that:
q* = 1/2 = p*, where q* (r) and r* (q) intersect.

N-players:

Set of mixed strategies for player i: $\sigma_i \in \Sigma_i$

σ_i **:** S_i **® [0,1]** $\sigma_{i1} + \ldots + \sigma_{ik}$ **= 1**

Def. (Nash Equilibrium) A mixed strategy profile $\sigma^* = (\sigma_1^*, \ldots, \sigma_N^*)$ is a NE of game $G = \{I, (u_1,\ldots, u_N), (\Sigma_1,\ldots, \Sigma_N)\}$ if for every i:

$$u_i(\sigma_i^*, \sigma_{-i}^*) \geq u_i(\sigma_i, \sigma_{-i}^*) \qquad \text{for all } \sigma_i \in \Sigma_i.$$

Mixed strategies can be used to model situations where any pure strategy is a losing one:

Tennis (left or right?)
Goal Keeper/Penalty Kicker (left or right?)
Offense/Defense in American Football (Run or Pass?)

Poker: how often to bluff?
Case Study: Random Drug Testing (Dutta p.114-115)
Auditing tax evation
Monitoring workers/borrowers…

Interpretation of *mixed strategies*:

1. mixed strategies as objects of choice (e.g. H,T, randomise).
2. probability as a frequency of certain acts.
3. mixed strategies as one player's uncertainty about what another player will do.

Note: A given pure strategy s'_i may be dominated by a mixed strategy, even though no other pure strategy is able to dominate s'_i.

Example 1:

1/2	L	R
T	(3, _)	(0, _)
M	(0, _)	(3, _)
B	(1, _)	(1, _)

For any belief (q, 1- q) player 1 could hold about 2' s play, 1's best response is never B. It is T if q ≥ 1/2, M if q ≤ 1/2. But B not strictly dominated by either T or M…it is indeed strictly dominated by (1/2,1/2,0).

Example 2: No-name game (Dutta pp.107-110)