# Urban Labor Economics

Yves Zenou*
Research Institute of Industrial Economics

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## Part 2: Urban Efficiency Wages

## Chapter 6: Non-Monocentric Cities and Efficiency Wages

*Also affiliated with GAINS, CEPR and IZA. Address of correspondence: Research Institute of Industrial Economics, Box 55665, 102 15 Stockholm, Sweden. E-mail: yves.zenou@industrialeconomics.se

# 1. Introduction

So far (in chapters 4 and 5), we have assumed that there were only one job-center located in the city-center (the CBD) and that all firms were located there. This is obviously a restrictive assumption since quite a few cities, especially in the US, have the tendency to spread and to create new job centers in the suburbs (urban sprawl). Obviously, the creation of new job centers located in the suburbs affects the labor market both in terms of wage setting and unemployment formation. We investigate this issue in this chapter. In Section 2, we extend the standard non-spatial Harris-Todaro model in the case of efficiency wages and cities. We analyze the rural-urban migration and study its consequences in the labor market. We study the migration between cities of different sizes in section 3 and we show that there exists a fundamental trade-off between high wages and high land rents in big cities and low wages and low spatial costs in small cities. We then study in section 4 the migration of workers within a city when workers decide between the Central Business District (CBD) and the Suburban Business District (SBD). We analyze the case when the SBD is exogenous and when it is endogenously formed. We also consider the case of high-relocation costs. Because jobs in the CBD and the SBD are of different nature, efficiency wages and unemployment rates will differ (urban dual labor markets). Finally, in Section 5, in a model a la Fujita-Ogawa, we analyze the endogenous formation of the CBD using firms' externalities in production as the main force of agglomeration.

# 2. Rural-urban migration: The Harris-Todaro model with a land market

In this section, we analyze a model in which rural workers can migrate to urban areas, so that two job centers (urban and rural) will prevail in equilibrium. We use a framework similar to that of the Harris-Todaro model (Todaro, 1969, Harris and Todaro, 1970).[1] In the standard Harris-Todaro model, a city differs from a rural area only because of the specificity of its labor market. Indeed, the main difference between rural and urban areas is that unemployment and high wages (that can be due to either a minimum wage or efficiency wages or search frictions) prevail in cities and not in rural areas. The main innovation of this section is to define in a more satisfactory way a city by explicitly

[1]See Appendix 3 for a detailed description of the Harris-Todaro model, especially the efficiency wage case.

modelling the land/housing market and the location of all workers in cities. Using the developments of chapters 4 and 5, we will analyze a rural-urban migration model a la Harris-Todaro with a land market and study its policy implications.

Let us study the standard Harris-Todaro model with efficiency wages (see Appendix 3) with an explicit land market in the city. There are two regions: An urban area (the city, denoted by the superscript $C$) and a rural area (denoted by the superscript $R$). As in the standard Harris-Todaro model, it is assumed that rural wages are very flexible, and thus there is no unemployment in rural areas. So, if $N$ denotes the total population in the economy, then the total population in rural areas is $L^R = N^R$, where $L^R$ is the employment level. As a result, the total population in cities is equal to: $N^C = L^C + U^C$ (where $L^C$ and $U^C$ are respectively the employment and unemployment levels in cities), with $N = N^C + N^R$. In this context, the unemployment level in cities is given by:

$$U^C = N - L^C - L^R \tag{2.1}$$

Both regions produce the same good but use different techniques. In region $g = C, R$, $y^g$ units of output are produced and $L^g$ workers are employed. This is a short-run model where capital is fixed and the production function in region $g = C, R$ is given by

$$y^g = F^g(L^g) \quad , \; F'^g(L^g) > 0 \text{ and } F''^g(L^g) \leq 0 \tag{2.2}$$

We also assume that the Inada conditions hold, that is

$$\lim_{L^g \to 0} F^{g\prime}(L^g) = +\infty \; , \; \lim_{L^g \to +\infty} F^{g\prime}(L^g) = 0 \tag{2.3}$$

The price of the good is taken as a numeraire and, without loss of generality, normalized to 1.[2] As stated above, in rural areas, we assume that jobs are mainly menial and wages are flexible and equal to marginal product, so that there is no rural unemployment. We thus have:

$$w_L^R = F'^R(L^R) \tag{2.4}$$

In cities, jobs are more complex and thus more difficult to monitor (one way to justify this is to assume that, in cities, firms are of larger size; we investigate this issue in section 3 below). So, because shirking is more costly,

[2] which means that the city and the rural area produce an export good that is sold on world markets at a fixed price, normalized to 1.

firms set efficiency wages. Let us first solve the equilibrium in the land and labor markets in cities and then link rural and urban areas through a migration equilibrium condition. As in chapters 4 and 5, a steady-state equilibrium in cities requires solving *simultaneously* an urban land use equilibrium and a labor market equilibrium. For presentation convenience, we first present the former and then the latter.

### 2.1. The urban-land use equilibrium

In region $C$, all workers are uniformly distributed along a *linear*, *closed* and *monocentric* city. Even if there is migration, the city is still considered as closed because urban (employed as well as unemployed) workers will never consider a move to rural areas.[3] Their density at each location is taken to be 1. All land is owned by absentee landlords and all firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers decide their optimal place of residence between the CBD and the city fringe. There are *no relocation costs*, either in terms of time or money.

The urban land use equilibrium is exactly as in Chapter 4, which means that, in cities, the employed workers reside close to jobs while the unemployed workers live at the outskirts of the city (see Figure 6.1).

$[Insert\ Figure$ 6.1 $here]$

As a result, the equilibrium instantaneous utilities of the employed and unemployed workers are given by:

$$W_L^C = w_L^C - e - \tau\, L^C - s\tau\left(N - L^R - L^C\right) \tag{2.5}$$

$$W_U^* = w_U^C - s\,\tau\,\left(N - L^R\right) \tag{2.6}$$

where $w_L^C, w_U$ are the urban wage and the unemployment benefit respectively, $e$ is the effort level, $\tau$ and $s$, the pecuniary commuting cost per unit of distance and the fraction of search trips to the CBD for the unemployed, respectively. The equilibrium land rent is equal to:

$$R^C(x) = \begin{cases} \tau\,\left(L^C - x\right) + s\tau\,\left(N - L^R - L^C\right) & \text{for}\quad 0 \le x \le L^C \\ s\,\tau\,\left(N - L^R - x\right) & \text{for}\quad L^C < x \le N^C \\ 0 & \text{for}\quad x > N^C \end{cases} \tag{2.7}$$

where, for simplicity, the land rent in rural areas is normalized to zero. The main difference with the benchmark model without migration (Chapter 4) is

[3] This will be showed formally below.

that the utility of urban workers as well as the equilibrium land rent depends on $L^R$, the level of employment of rural workers because of rural-urban migration that affects land prices. This implies that now $N^C$ is endogenous (whereas it was exogenous in chapter 4) and is given by $N^C = N - L^R$, where $L^R$ will be determined by the equilibrium migration condition below.

### 2.2. The labor equilibrium in cities

The lifetime expected utilities of non-shirker, shirker and unemployed workers are, as in Chapter 4, respectively given by the following Bellman equations:

$$r\, I_L^{NS} = w_L^C - e - \tau\, L^C - s\tau\, \left(N^C - L^C\right) - \delta\, (I_L^{NS} - I_U) \tag{2.8}$$

$$r\, I_L^S = w_L^C - \tau\, L^C - s\tau\, \left(N^C - L^C\right) - (\delta + m)\, (I_L^S - I_U) \tag{2.9}$$

$$r\, I_U = w_U^C - s\tau\, N^C + a^C (I_L - I_U) \tag{2.10}$$

where $\delta, m$ and $a^C$ denote the job-destruction rate, the monitoring rate and the job-acquisition rate in cities, respectively. Firms set the efficiency wage such that $I_L^{NS} = I_L^S = I_L$ and we easily obtain that:

$$I_L - I_U = \frac{e}{m} \tag{2.11}$$

In steady state, flows out of unemployment equal flows into unemployment, so the job acquisition rate in cities is given by:

$$a^C = \frac{\delta\, L^C}{N - L^C - L^R} \tag{2.12}$$

By combining these equations and using the fact that $N_C = N - L^R$, we easily obtain the following urban efficiency wage:

$$w_L^C = w_U^C + e + \frac{e}{m}\left[\frac{\delta\, \left(N - L^R\right)}{N - L^C - L^R} + r\right] + (1 - s)\, \tau\, L^C \tag{2.13}$$

It is in particular interesting to observe that the higher rural employment $L^R$, the higher the urban efficiency wage $w_L^C$. Indeed, higher $L^R$ implies that fewer rural workers move to the city, so the urban job acquisition rate $a^C$ increases and thus urban firms raise their wages $w_L^C$. Urban firms decide their employment level by maximizing their profit. We thus have:

$$w_L^C = F'^C(L^C) \tag{2.14}$$

## 2.3. The rural-urban migration

Concerning rural-urban migration, we assume that a rural worker cannot search from home but must first be unemployed in the city (to gather information about jobs) and then search for a job. As a result, as described in Figure 6.1, a rural worker who migrates to the city will reside in the unemployment area anywhere between $x = L^C$ and $x = N^C = N - L^R$. Thus, the equilibrium migration condition can be written as (observing that the rural land rent has been normalized to zero):

$$r\, I_U = \int_0^{+\infty} w_L^R\, e^{-rt} = \frac{w_L^R}{r}$$

that is rural workers will migrate to the city up to the point that their expected lifetime utility is equal to the expected utility they will obtain in cities. Indeed, the left-hand side of this equation, $r\, I_U$, is the intertemporal utility of moving to the city (remember that a migrant must first be unemployed) while the right-hand side, $\int_0^{+\infty} w_L^R\, e^{-rt} = w_L^R/r$, corresponds to the intertemporal utility of staying in rural areas. Using (2.10), (2.11), (2.12), $N^C = N - L^R$, and (2.4, we can write this condition as:

$$w_U^C - s\tau\, \left(N - L^R\right) + \frac{e}{m} \frac{\delta\, L^C}{N - L^C - L^R} = \frac{F'^R(L^R)}{r} \tag{2.15}$$

Observe that we can now check that, in equilibrium, urban workers will never want to migrate to rural areas. Since the utility to stay in rural areas is equal to $w_L^R/r$ and, in equilibrium, is given by (2.15), it suffices to show that $r\, I_L > r\, I_U$. In equilibrium, the intertemporal utility of being unemployed and employed are respectively given by:

$$r\, I_U = w_U^C - s\tau\, \left(N - L^R\right) + \frac{e}{m} \frac{\delta\, L^C}{N - L^C - L^R} \tag{2.16}$$

$$r\, I_L = w_U^C - s\tau\, \left(N - L^R\right) + \frac{e}{m} \left[\frac{\delta L^C}{N - L^C - L^R} + r\right] \tag{2.17}$$

and it is easy to verify that $r\, I_L > r\, I_U$.

## 2.4. The equilibrium

We have the following definition.

**Definition 1.** *An Harris-Todaro equilibrium with efficiency wages and a land market is a 6-tuple* $(w_L^{C*}, L^{C*}, w_L^{R*}, U^{C*}, L^{R*}, R^{C*}(x))$ *such that (2.13), (2.14), (2.4), (2.1), (2.15) and (2.7) are satisfied.*

We have the following result (which is proved in the Appendix of this chapter).

**Proposition 1.** *There exists a unique Harris-Todaro equilibrium with efficiency wages and a land market with* $0 < L^{C*} < N$ *and* $0 < L^{R*} < N$.

### 2.5. Comparative statics and the Todaro paradox

We would like now to study the Todaro paradox in this model (see Appendix 3 for a precise definition of the Todaro paradox). This paradox says that increasing urban jobs may increase rather than decrease the level and/or the rate of urban unemployment because of the induced effect of rural-urban migration. As in Appendix 3, we will study the impact of a decrease of $w_U^C$ on urban employment and unemployment. Observe that here we have a new effect compared to the non-spatial Harris-Todaro model with efficiency wages since more urban jobs will imply more rural migrants, who will exert an upward pressure on urban land rent, which in turn may discourage rural migrants.

Let us first study the comparative statics of our equilibrium. The equilibrium is basically determined by two equations (2.15) and (A.1). If we first totally differentiate (2.15), the equilibrium migration condition, we obtain

$$L^R = L^R \left( \underset{-}{w_U^C}, \underset{-}{e}, \underset{+}{m}, \underset{-}{\delta}, \underset{+}{N}, \underset{-}{r}, \underset{-}{L^C}, \underset{+}{s}, \underset{+}{\tau} \right) \tag{2.18}$$

First, the impact of the non-spatial variables ($w_U^C$, $e$, $\delta$, $r$, $m$, $N$) are as in the non-spatial case (see Appendix 3) and are due to the fact that they affect either positively or negatively the intertemporal utility of being unemployed in the city, $I_U$. Indeed, when, it affects positively (negatively) $I_U$, then more (less) workers leave the rural area, which reduces (increases) $L^R$. Second, when $L^C$ increases, the urban job acquisition rate $a^C$ increases and more rural workers migrate to the city, thus reducing $L^R$. Finally, the two spatial variables, $s$ and $\tau$, affect positively the number of people who stay in rural areas, $L^R$, but the effect goes through the land market. Indeed, when $s$ or $\tau$ increases, the capacity to bid for land is augmented (see (2.7)). The competition in the land market becomes fiercer and, as a result, land rents paid by all workers increase. Thus, unemployed workers are worse off as their expected utility $I_U$ decreases. This discourages rural workers to migrate to the city, which in turn increases $L^R$.

If we now totally differentiate the labor demand equation (A.1), we get:

$$L^C = L^C\left(\underset{-}{w_U^C}, \underset{-}{e}, \underset{+}{m}, \underset{-}{\delta}, \underset{+}{N}, \underset{-}{r}, \underset{-}{L^R}, \underset{+}{s}, \underset{-}{\tau}\right) \tag{2.19}$$

First, the impact of the non-spatial variables ($w_U^C$, $e$, $\delta$, $r$, $m$, $N$) are as in the non-spatial case (see Appendix 3) and are due to the fact that they affect either positively or negatively the Urban Non-Shirking Condition (2.13). Indeed, when it affects positively (negatively) the UNSC, firms have to pay a higher (lower) efficiency wage to prevent shirking. This, in turn, reduces (increases) employment $L^C$ since, because of higher (lower) wage costs, maximizing-profit firms have to reduce (increase) the number of employed. Second, for $L^R$, the effect is through the job-acquisition rate $a^C$. Indeed, a higher rural employment $L^R$ increases $a^C$, which obliges firms to increase their urban efficiency wages, which in turn reduces urban labor demand $L^C$ because firms maximize their profit. Finally, concerning the spatial variables $s$ and $\tau$, they also affect the UNSC; however, not the incentive part but the spatial compensation part. Indeed, when $\tau$ increases or $s$ decreases, firms have to raise the spatial compensation of their employed workers to induce their to stay in the city. This increases the efficiency wage, which in turn reduces labor demand $L^C$.

As stated above, in order to directly increase $L^C$ and thus to create urban jobs, the government can for example reduce the unemployment benefit $w_U^C$. We have the following result:

**Proposition 2.** *In an Harris-Todaro model with urban efficiency wages and a land market, decreasing unemployment benefit leads to*

(*i*) *an increase in urban employment $L^C$, i.e. $\partial L^{C*}/\partial w_U^C < 0$;*

(*ii*) *an increase in rural employment $L^R$, i.e. $\partial L^{R*}/\partial w_U^C < 0$;*

(*iii*) *a decrease in urban unemployment (both in level and rate) $U^C$ and $u^C$, i.e. $\partial U^{C*}/\partial w_U^C > 0$ and $\partial u^{C*}/\partial w_U^C > 0$.*

*As a result, there is no Todaro paradox.*

If we compare with the non-spatial case (Appendix 3), we obtain exactly the same results. Indeed, when the government decreases the unemployment benefit, this has a direct negative effect on urban efficiency wages and thus more urban jobs are created. This acts as an attraction force to the city. But there are now three (instead of two) repulsion forces. As in the non-spatial case, the two first repulsion forces are as follows. More job creation indirectly implies that both rural wages increase and urban wages become lower. There is however an additional repulsion force due to *land-rent escalation* in cities since the initial job creation that triggers migration increases land rents because of a fiercer competition in the land market. As a result, the results are qualitatively the same but quantitatively different because of this new force triggered by land rent.

### 2.6. The specificity of a Harris-Todaro equilibrium with a land market

Because a land market has been explicitly introduced in this model, it would be interesting to see the effects of a decrease in $w_U^C$ on the equilibrium land rent $R^{C*}(x)$, which is given by (2.7). We have:

**Proposition 3.** *Decreasing the unemployment benefit,*

$(i)$ *reduces the equilibrium land rent in the unemployment zone, which is between* $x = L^{C*}$ *and* $x = N$*,*

$(ii)$ *but has an ambiguous effect on the equilibrium land rent in the employment zone, which is between* $x = 0$ *and* $x = L^{C*}$*, since*

$$\frac{\partial R^C(x)}{\partial w_U^C} = (1-s)\,\tau\,\frac{\partial L^{C*}}{\partial w_U^C} - s\tau\frac{\partial L^{R*}}{\partial w_U^C} \gtreqless 0$$

This proposition is quite intuitive. If the unemployment benefit $w_U^C$ decreases, then less rural workers migrate to the city ($L^R$ increases) and thus the land rent decreases in the unemployment zone (remember that rural migrants reside first in the unemployment zone) since there is less competition in the land market. This is not true anymore in the employment zone because the land rent is also affected by $L^C$. Indeed, when $w_U^C$ decreases, both $L^{R*}$ and $L^{C*}$ are increased (Proposition 2) so, one the one hand, there is less competition in the land market because of a higher $L^R$, but, on the other, there is more competition because of a higher $L^C$. The net effect is thus ambiguous.

## 3. Migration between cities[4]

We would like to develop a model similar to that of section 2 but by focusing on urban-urban migration rather than rural-urban migration. More precisely, we

[4] This section is partly based on Zenou and Smith (1995).

would like to model the migration decision of workers between cities of different sizes. As in the benchmark model of efficiency wages (chapter 4), each firm is always motivated to discourage shirking behavior because shirkers add nothing to production. Also, each firm is assumed to have only limited ability to monitor the productivity of its workers and the monitoring capability is taken to be characterized by a *detection rate*, $m$, which represents the rapidity at which the shirking behavior of any worker can be detected. Since the behavior of single individuals is less conspicuous in large firms, this detection rate now depends of the size of the firm and it is assumed that the larger the size of the firm, the more difficult it is to detect shirking behavior. Empirical support for this dependency on firm size is given in Brown and Medoff (1989), Rebitzer and Robinson (1991), Zenger (1994) and Oi and Idson (1999). Hence detection is here taken to be a system variable that varies with average firm size, but which is beyond the control of individual firms. Since there are $M$ identical firms in the economy, the average firm size is $L/M$ (where $L$ is the level of employment in the economy) and thus our assumption implies that $m\left(L/M\right)$, with $m'\left(L/M\right) < 0$ and

$$\lim_{L \to 0} m\left(L/M\right) = +\infty \tag{3.1}$$

This is a boundary condition, which says that, as total employment $L$ approaches zero, the detection rate increases without bound so that the detection of shirkers occurs almost immediately. We can now solve this model of a monocentric city as in the benchmark model of chapter 4 since the prevailing detection rates are exogenous to the firm and the only way to prevent shirking is to offer the efficiency wage level consistent with the current level of total employment $L$.

Let us now open the city and allow for the possibility of *between-city* migration. We will consider a closed two-city system with free labor mobility between cities as described by Figure 6.2. This is a bounded one dimensional continuum with a uniform density of residential land parcels, and with CBDs at each end representing the two cities, designated respectively as city 1 (the big city) and city 2 (the small city). Each city $k = 1, 2$ has a CBD$_i$ that contains the same fixed number of firms $M$.[5] All firms are assumed identical within a CBD and all firms are assumed immobile. Each firm is assumed to produce the same good at the same market price (normalized to 1), but firms in the big city (i.e. city 1) have a higher productive capacity than those in the small city (i.e. city 2). This is a well established empirical fact (see e.g. Ciccone and Hall, 1996; Glaeser and Maré, 2001) and different reasons have

[5] This assumption is not necessary to obtain our results.

been advanced. In bigger cities, there is more extensive infrastructure and greater accessibility to those production factors other than labor. Also, cities have information externalities that increase the productivity of firms and the presence of greater demand in cities or inputs are cheaper when producers are close to other suppliers. Finally, there are human capital externalities for workers in cities (Duranton, 2006). All these theories predict that the marginal product of labor is higher in bigger cities. For example, using data on gross state output in the US, Ciccone and Hall (1996) found that a doubling of employment density increases average labor productivity by around 6 percent and more than half of the variance of output per worker across states can be explained by differences in the density of economic activities. Hence, if the production function for a firm $j$ in city $k = 1, 2$ is denoted by $f_i(el_j)$, then it is assumed that $f_1'(el_j) > f_2'(el_j)$ for all $l_j \geq 0$. Observe that we assume here that $e$, the effort level of workers, is the same in the two cities. This means that, if two identical workers put the same effort $e$, then their labor productivity will be higher if they work in city 1 than in city 2.

$[Insert\ Figure\ 6.2\ here]$

Turning next to the workers, it is assumed that there exists a fixed total population size $N$ in this two-city system, with

$$N = N_1 + N_2 \tag{3.2}$$

where $N_k$ is the number of workers in city $k$. Workers are freely mobile both within and between cities.

### 3.1. Urban land use equilibrium in each city

As in the benchmark model (see chapter 4), we can calculate the urban land use equilibrium in each city $k = 1, 2$, where the employed reside close to jobs (CBD$_k$) and the unemployed at the periphery of the city. We easily obtain (see chapter 4):

$$W_{L,k}^* = w_{L,k} - e - \tau\, L_k - s\tau\, (N_k - L_k) \tag{3.3}$$

$$W_{U,k}^* = w_U - s\,\tau\, N_k \tag{3.4}$$

$$R_k^*(x) = \begin{cases} (1-s)\,\tau\, L_k + s\tau\, N_k - \tau\, x & \text{for} \quad 0 \leq x \leq L_k \\ s\,\tau\, (N_k - x) & \text{for} \quad L_k < x \leq N_k \\ 0 & \text{for} \quad x > N_k \end{cases} \tag{3.5}$$

The main difference between the benchmark model and the present one is that there are now three more endogenous variables $N_1$ and $N_2$ (which will be determined by the equilibrium migration condition and $N = N_1 + N_2$), and $m$, which is now a function of $L_k/M$. Furthermore, each endogenous variable is now indexed by $k = 1, 2$. This is true because it is assumed that all exogenous variables (such as $\tau$, $s$, $e$, $w_U$, $\delta$) are the same between the two cities. Also, we assume that the detection function $m(L_k/m)$ is the same.

### 3.2. Steady-state equilibrium

Let is determine the labor market equilibrium. Again, as in the benchmark model, we can calculate the efficiency wage. For each city $k = 1, 2$, we obtain:

$$w_{L,k}^* = w_U + e + \frac{e}{m(L_k/M)} \left( \frac{\delta\, N_k}{N_k - L_k} + r \right) + (1 - s)\, \tau\, L_k \tag{3.6}$$

It is important to notice that we still have the same properties of the efficiency wage with respect to $L_k$, that is

$$\frac{\partial w_{L,k}^*}{\partial L_k} > 0 \text{ and } \lim_{L \to N_k} w_{L,k}^* = +\infty$$

This is a crucial property of the efficiency wage since it implies that higher unemployment is associated with lower efficiency wages ("unemployment acts as a worker discipline device") and that full employment is not compatible with efficiency wages. In particular, this implies that bigger cities have higher wages because firms are on average larger and effort monitoring and detection of shirking is more difficult. The positive relationship between wages and city size is well-established fact; see in particular Glaeser and Maré (2001).

The aggregate labor demand $L_k^*$ in city $k$ is the result of firms' profit maximization and is thus given by:

$$eF'(eL_k^*) = w_U + e + \frac{e}{m(L_k/M)} \left( \frac{\delta\, N_k}{N_k - L_k^*} + r \right) + (1 - s)\, \tau\, L_k^* \tag{3.7}$$

It is easy to verify the Inada conditions on the production function of each firm and (3.1) guarantee that there exists a unique labor market equilibrium in each city $k = 1, 2$ and thus a unique urban land use equilibrium $k = 1, 2$. We would like now to determine the equilibrium for the two-city system. Concerning the migration between the two cities, we assume that workers cannot search from home but must first be unemployed in the city where they want to migrate (to gather information about jobs) and then search for a job. In other words, a new comer in a city needs to be first unemployed in order to search for a job there.

In order to write the equilibrium migration condition, we need to determine the (steady-state) expected lifetime utility of an unemployed worker in city $k$. The corresponding Bellman equation is equal to:

$$\begin{aligned} r\, I_{U,k} &= w_U - s\tau\, N_k^* + a_k(I_{L,k} - I_{U,k}) \\ &= w_U - s\tau\, N_k^* + \delta\, e \frac{L_k^*}{(N_k^* - L_k^*)\, m(L_k^*/M)} \end{aligned}$$

since $a = \delta L_k^*/(N_k^* - L_k^*)$ and $I_{L,k} - I_{U,k} = e/m(L_k^*/M)$. In fact, since the *spatial costs* (costs of transportation plus costs land rent) of the unemployed workers are: $SC_{U,k} = s\,\tau\, N_k^*$ (see (3.4)) and the *wage surplus* that must be paid to deter shirking is: $SW_k = \delta\, e \frac{L_k^*}{(N_k^* - L_k^*) m(L_k^*/M)}$, we have:

$$r\, I_{U,k} = w_U + SW_k - SC_{U,k}$$

This expected utility consists of two components. The first one, $w_U$, is a *base utility* level that can be earned in any of the two cities. The second one, $SW_k - SC_{U,k}$, is of primary interest for our purpose and represents the difference between $SW_k$, the *utility gain* from the wage surplus in city $k$ and $SC_{U,k}$, the *utility loss* from the spatial costs in city $k$. It is this fundamental trade-off that is at the heart of all migration decisions in the present model. Indeed, the equilibrium migration condition between the two cities can be written as $r\, I_{U,1} = r\, I_{U,2}$, which is equivalent to:

$$SW_1 - SW_2 = SC_{U,1} - SC_{U,2}$$

$$\Leftrightarrow \delta\, e \left[ \frac{L_1^*}{(N_1^* - L_1^*)\, m(L_1^*/M)} - \frac{L_2^*}{(N - N_1^* - L_2^*)\, m(L_2^*/M)} \right] = s\tau\ (2N_1^* - N) \tag{3.8}$$

where we use (3.2) for the second equation. It is clear from (3.8) that, when deciding to migrate, workers trade off higher (in big cities) and lower (in smaller cities) wage surpluses and spatial costs. Compared to small cities, big cities are indeed characterized by higher wages since, because of higher productivity, firms are on average larger and thus monitoring is more difficult, and higher land rents and commuting costs since individuals are richer and on average commute more.

**Definition 2.** *A steady-state equilibrium* $(R_k^*(x), w_{L,k}^*, L_k^*, N_1^*, N_2^*)$ *for the two-city system described in Figure 6.2 consists of a land rent function (3.5) for each city, a wage (3.6) for each city, an employment level (3.7) for each city, a migration equilibrium condition (3.8), and a population condition (3.2).*

It is easy to show that there exists a unique steady-state equilibrium (Zenou and Smith, 1995). For that, there needs to be a unique solution to (3.8). For example, by introducing a positive migration cost, then one can always find a value of the migration cost for which the modified equation (3.8) (which takes into account this migration cost). We also have:

**Proposition 4.** *The expected migration utility function in city $k$, i.e. $I_{U,k}$, is always strictly decreasing in $L_k$.*

The proof of this result is given in Lemma 3.1 in Zenou and Smith (1995). This means that whenever the employment level increases in one city, then the expected utility of the unemployed in that city decreases because wages are lower and the chance to find a job is also lower. This result also establishes the stability of the system equilibrium since any continuous migration process in which migrants move toward higher expected utility must always converge to the unique system equilibrium.

### 3.3. Interaction between land and labor markets

The equilibrium conditions for this two-city system are basically given by (3.7), the equilibrium employment level in each city $k = 1, 2$, and the equilibrium migration condition (3.8) and the population condition (3.2). Indeed, equations (3.7) give $L_1^*$ and $L_2^*$. However, since $N_1^*$ and $N_2^*$ are now endogenous variables, we cannot, as in the benchmark model, calculate directly the unemployment level in each city $N_k^* - L_k^*$. Conditions (3.8) and (3.2) give $N_1^*$ and $N_2^*$ and the model is solved. The intuition of the equilibrium is as follows. In each city, the size of each firm affects the detection of shirking, which in turn determines the efficiency wage. Given this wage, firms decide their employment level by maximizing their profit. So employment in each city is basically determined by efficiency wage and monitoring reasons. The wage and employment levels also affect the competition in the land market and thus the land rent at each residential location. The complication here is that workers are freely mobile between the two cities and thus also affect the wage determination and the employment level. Indeed, when wages increase in a city, workers are more likely to move there but more unemployed workers (since a new migrants have to be first unemployed) imply lower wages. The net effect is not straightforward. What we have shown is that there is a trade off between space cost differential and wage surplus when deciding to move (see (3.8)). Indeed, bigger cities, where workers are assumed to be more productive, offer higher wages but have higher unemployment rates and housing and commuting costs are also higher. In equilibrium, individuals are indifferent between the two cities because space cost differential are totally compensated by wage surplus.

We would like to analyze the effect of an exogenous variable on equilibrium employment and unemployment levels in each city, for example the effect of a labor market variable such as $w_U$, the unemployment benefit, or $\delta$, the job destruction rate, on the equilibrium land rent $R_k^*(x)$ in city $k$.

By totally differentiating (3.7), as in the benchmark model, we obtain for each city that:

$$L_k^* \left( \underset{-}{w_U}, \underset{?}{e}, \underset{-}{\delta}, \underset{+}{N_k}, \underset{-}{r}, \underset{+}{s}, \underset{-}{\tau} \right) \tag{3.9}$$

The intuition of this comparative statics exercise is the same as in the benchmark model. The interesting relationship is between the two endogenous variables $N_k^*$ and $L_k^*$. Indeed, when $N_k^*$ increases, i.e. there is more people in the city, other things being equal, there is more unemployment, and thus, because of shirking reasons, wages are lower, which means that firms can higher more workers, so that $L_k^*$ increases. Let us now differentiate (3.8) to analyze the relationship between $N_1^*$ and $L_1^*$ and $L_2^*$. It is easy to verify that:

$$\frac{\partial N_1^*}{\partial L_1^*} > 0 \text{ and } \frac{\partial N_1^*}{\partial L_2^*} < 0 \tag{3.10}$$

Indeed, when $L_1^*$ increases there is two positive effects on $N_1^*$: a direct positive effect, which is mechanical since more employment implies more migration to the city sine it becomes more attractive (higher chance to obtain a job), and an indirect positive effect since higher employment implies that the average size of firms is bigger. The detection of shirking being more difficult, wages will be higher so that more people will migrate to the city and $N_1^*$ increases. For exactly the same reasons (but with the reverse argument), we can explain the negative relationship between $N_1^*$ and $L_2^*$. We can now analyze the effect of the unemployment benefit $w_U$ on $N_1^*$. It is easy to verify that it is ambiguous because there are two opposite effects. When $w_U$ increases, both $L_1^*$ and $L_2^*$ decrease (see (3.9)), because firms have to increase their efficiency wage in both cities to meet the non-shirking condition and thus reduce their labor demand. Now, because of (3.10), the decrease of $L_1^*$ will increase efficiency wages in city 1 (monitoring is easier since the firms' size is lower) and thus increases $N_1^*$ but the decrease of $L_2^*$ will also increase efficiency wages in city 1 and thus decreases $N_1^*$. We have the same kind of ambiguous effect if we analyze the impact of $\delta$ on $N_1^*$.

We are now able to determine the impact of $w_U$ or $\delta$ on the equilibrium land rent $R_k^*(x)$, which is given by (3.5). An increase in $w_U$ or $\delta$ affects negatively $L_k^*$ (see (3.9)) but has an ambiguous effect on $N_1^*$ so the net effect on the land rent is ambiguous. Again, the intuition is as follows. If, for example, $w_U$ increases then both cities will have higher wages because of better outside option and thus higher unemployment. This will increase in the short run the land rent in each city but because it also affects migration, the net final effect on the land rent is ambiguous since $N_1^*$ will tend to decrease from the decrease of $L_1^*$ and tend to increase from the decrease of $L_2^*$.

An interesting extension would be to consider different exogenous variables in the two cities. A natural variable for that is the job destruction rate. One can assume that big cities where jobs are more diversified are less sensitive to negative shocks while small cities where jobs are in general more specialized are more affected by shocks. Start with $\delta_1 = \delta_2 = \delta$. Then assume that there is a negative shock that only affects the small city (i.e. city 2) so that $\delta_1 = \delta$ and $\delta_2 > \delta$. Let us compare the two steady-states, before when $\delta_1 = \delta_2 = \delta$ and after when $\delta_2 > \delta = \delta_1$. After the shock, the equilibrium migration condition $r\, I_{U,1} = r\, I_{U,2}$ is now different and given by:

$$e\left[\frac{\delta\, L_1^*}{(N_1^* - L_1^*)\, m(L_1^*/M)} - \frac{\delta_2\, L_2^*}{(N - N_1^* - L_2^*)\, m(L_2^*/M)}\right] = s\tau\,(2N_1^* - N) \tag{3.11}$$

When the shock happens, $L_1^*$ is not affected in the short run but $L_2^*$ decreases (see (3.9)) since the job creation rate needs to increase for the steady-state condition on flows to be respected and thus wages also increase. The effect on $N_1^*$, given by (3.11), is follows. Because of (3.10), the decrease of $L_2^*$ increases $N_1^*$. However, there is a direct effect of $\delta_2$ on $N_1^*$. Indeed, an increase in $\delta_2$ raises the efficiency wage in the small city, reducing the wage difference between the two cities and thus more people migrate to the small city, reducing $N_1^*$. The net effect on $N_1^*$ is again ambiguous but for a different reason than before.

Consider now two cities located in different countries with different unemployment benefits. Let us start with the same level of unemployment benefit for the two countries, say zero, and then increase the one in city 1, i.e. $w_{U,1} > w_{U,2} = 0$. The equilibrium migration condition can now be written as:

$$w_{U,1} + \delta\, e\left[\frac{L_1^*}{(N_1^* - L_1^*)\, m(L_1^*/M)} - \frac{L_2^*}{(N - N_1^* - L_2^*)\, m(L_2^*/M)}\right] = s\tau\,(2N_1^* - N) \tag{3.12}$$

In that case, when the unemployment benefit in the big city increases, the employment level $L_1^*$ decreases because of higher efficiency wages in city 1 while $L_2^*$ is unchanged. The decrease of $L_1^*$ decreases $N_1^*$ because it is easier to monitor workers and wages are lower and thus more people want to migrate to the small city. There is no ambiguous effect here since $L_2^*$ is not affected and there is no, as in the case of $\delta$, direct effect of $w_U$ on $N_1^*$. Furthermore, $N_2^*$ will increase mechanically since $N_1^* + N_2^* = N$. As a result, because of (3.10), $L_1^*$ will finally decrease and $L_2^*$ increase. Looking at (3.5), one can see that the land rent for all workers will decrease in the big city and increase in the small city because of the increase in unemployment benefit in the big city.

# 4. Migration within cities: Dual labor markets in a duocentric city

We would like to study the mobility of workers within cities when there is more than one job center. We will consider the case of two job centers, the Central Business District (CBD) and the Suburban Business District (SBD).

## 4.1. The model with a fixed SBD

The city has two centers the CBD and the SBD. It is assumed that in the CBD jobs are complex and detection of shirking is not instantaneous so that an efficiency wage policy prevails there. This referred to as sector 1 (or primary sector to use the terminology of the dual labor market literature). On the contrary, jobs are assumed to be menial in the SBD so that production tasks are easily monitored and shirking is instantaneously detected so that efficiency wages are not needed. This is referred to as sector 2 (or secondary sector).[6] It is also assumed that the SBD is fixed and located at $x = N$. We will relax this assumption below. So basically the CBD is as before (in the benchmark model), even though the efficiency wage will have a different value (see below) because of the competition with the SBD. It is also assumed that all non-work activities (such as shopping, interacting with people, etc.) take place in the CBD. Hence, the SBD is only a workplace whereas the CBD is both a workplace and a place of non-work activities.

[6] Subscript 2 refers to sector 2 (SBD) while subscript 1 refers to sector 1 (CBD).

### 4.2. Urban land use equilibrium

There is a continuum of ex ante identical workers whose mass is $N$ and a continuum of $M$ identical firms ($M_1$firms in the CBD and $M_2$ firms in the SBD, with $M_1 > M_2$ and $M = M_1 + M_2$). Among the $N$ workers, there are $L_1$ employed in the CBD, $L_2$ employed in the SBD and $U$ unemployed, so that

$$N = L_1 + L_2 + U \tag{4.1}$$

The density of workers at each location is taken to be 1 (since housing consumption is normalized to 1). The city is *linear*, *closed* and *duocentric*. The CBD is at the origin of the city ($x = 0$) while the SBD is located at the other end of the city ($x = N$). All land is owned by absentee landlords and $M_1$ firms are exogenously located in the Central Business District while $M_2$ firms are exogenously located in the SBD. Firms consume no space. Workers are assumed to be infinitely lived, *risk neutral* and decide their optimal place of residence between the CBD and the SBD. There are *no relocation costs*, either in terms of time or money.

Let us now determine the *instantaneous* indirect utilities of an employed non-shirker and shirker working in the CBD and residing at a distance $x$ from the CBD. They are respectively given by:

$$W_L^{NS}(x) = w_L - e_1 - \tau\, x - R(x) \tag{4.2}$$

$$W_L^{S}(x) = w_L - \tau\, x - R(x) \tag{4.3}$$

As in the benchmark model, we assume work trips to be separated from shopping trips, so that non-workers commute to the CBD only for non-work activities and it is assumed that they commute less than the employed, i.e. $0 < s < 1$. Thus, for an unemployed worker, we have:

$$W_U(x) = w_U - s\,\tau\, x - R(x) \tag{4.4}$$

For the workers employed in the SBD, their instantaneous utility is equal to:

$$W_{L,2}(x) = \overline{w}_2 - e_2 - s\,\tau\, x - \tau\,(N - x) - R(x) \tag{4.5}$$

Here, it should be clear why we assume that work trips are separated from shopping trips. Indeed, SBD-workers located in $x$ need to commute to the CBD for non-work activities and the costs amount to $s\,\tau\, x$ (if for example $s = 1/2$, then they will go there every other day) but they also need to go to the SBD to work, and the costs are equal to $\tau(N-x)$ since the SBD is located at $x = N$.

In equilibrium, as in the benchmark model, none of the employed workers in the CBD will shirk. Since there are no relocation costs, the urban equilibrium is such that, whatever their location $x$, all the employed workers working in the CBD and in the SBD enjoy the same level of utility $W_L^{NS}(x) \equiv W_{L,1}$ and $W_{L,2}(x) \equiv W_{L,2}$, respectively, while all the unemployed obtain $W_U$. The bid rents of the CBD-employed, SBD-employed and unemployed workers are respectively given by:

$$\Psi_{L,1}(x, W_{L,1}) = w_L - e_1 - \tau\, x - W_{L,1} \tag{4.6}$$

$$\Psi_{L,2}(x, W_{L,2}) = \overline{w}_2 - e_2 + (1-s)\,\tau\, x - \tau N - W_{L,2} \tag{4.7}$$

$$\Psi_U(x, W_U) = w_U - s\,\tau\, x - W_U \tag{4.8}$$

Observe that the CBD-employed workers' and unemployed workers' bid rents are linearly *decreasing* with $x$ while the CBD-employed workers' workers bid rent is linearly *increasing* with $x$. Observe also that the bid rent of the CBD-employed workers is steeper than that of the unemployed, so that the employed will be able to outbid the unemployed to occupy the core of the city. As a result, since each worker consumes one unit of land, the CBD-employed workers reside between $x = 0$ and $x = L_1$, the unemployed between $x = L_1$ and $x = L_1 + U$ and the SBD-employed workers live at the periphery of the city between $x = L_1 + U$ and $x = N$ (see Figure 6.3).

[*Insert Figure* 6.3 *here*]

Let us now define the urban-land use equilibrium. We denote the agricultural land rent (the rent outside the city or opportunity rent) by $R_A$ and, without loss of generality, we normalize it to zero. We have:

**Definition 3.** *An urban-land use equilibrium with no relocation costs and fixed-housing consumption in a duocentric city is a 4-tuple* $(W_{L,1}^*, W_{L,2}^*, W_U^*, R^*(x))$ *such that:*

$$\Psi_{L,1}(L_1, W_{L,1}^*) = \Psi_U(L_1, W_U^*) \tag{4.9}$$

$$\Psi_U(L_1 + U, W_U^*) = 0 \tag{4.10}$$

$$\Psi_{L,2}(L_1 + U, W_{L,2}^*) = 0 \tag{4.11}$$

$$R^*(x) = \max\left\{\Psi_{L,1}(x, W_{L,1}^*), \Psi_{L,2}(x, W_{L,2}^*), \Psi_U(x, W_U^*), 0\right\} \quad \textit{at each } x \in \left(0, x_f^*\right] \tag{4.12}$$

By solving (4.9)−(4.11) and using (4.1), we easily obtain the equilibrium values of the instantaneous utilities of all workers in the city. They are given by:

$$W_{L,1}^{*} = w_L - e_1 - \tau\, L_1 - s\,\tau\,(N - L_1 - L_2) \tag{4.13}$$

$$W_{L,2}^{*} = \overline{w}_2 - e_2 + (1-s)\,\tau\,(N - L_2) - \tau N \tag{4.14}$$

$$W_{U}^{*} = w_U - s\,\tau\,(N - L_2) \tag{4.15}$$

By plugging (4.13) into (4.6), (4.14) into (4.7), and (4.15) into (4.8), we easily obtain the land rent equilibrium $R^{*}(x)$. It is given by:

$$R^{*}(x) = \begin{cases} \tau\,(L_1 - x) + s\tau\,(N - L_1 - L_2) & \text{for} \quad 0 \leq x \leq L_1 \\ s\,\tau\,(N - L_2 - x) & \text{for} \quad L_1 < x \leq N - L_2 \\ (1-s)\,\tau\,[x - (N - L_2)] & \text{for} \quad N - L_2 < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{4.16}$$

### 4.3. Labor market equilibrium

Let us now determine the labor market equilibrium first in the CBD, then in the SBD and then the general one in the economy.

**Efficiency wages and employment in the CBD (primary sector or sector 1)** We use here the benchmark framework, that is the detection rate is exogenous and denoted by $m$. The main difference compared to the benchmark model is that CBD-workers have now two outside options if they lose their job. They can either be unemployed (as before) but they can eventually find a job in the secondary sector. In this sector, it is assumed that jobs are relatively simple and thus no shirking behavior is possible. Also, a minimum wage prevails in sector 2, which is imposed by the government, i.e. $w_2 = \overline{w}_2$. This minimum wage is assumed to be higher than the unemployment benefit, i.e. $\overline{w}_2 > w_U$.

In equilibrium, when there is no shirking, the flows in the labor market is described in Figure 6.4.

[*Insert Figure* 6.4 *here*]

It is assumed that the CBD-job acquisition rate is $a_1$, independently of the employment status of workers. In other words, SBD-employed workers and unemployed workers have exactly the same chance to find an CBD job in the primary sector. Hence, $a_1$ represents the *primary-sector job acquisition rate* for all individuals and it is the common exit rate from both the unemployed pool and sector 2 into sector 1. Observe that, even though it is not explicitly modelled, we have here on-the-job search behavior since SBD-employed workers can find directly a CBD-job (at rate $a_1$). Furthermore, the rate at which the unemployed workers obtain a CBD job is not equal to that of a SBD job. We have indeed $a_1 \neq a_2$ and thus $a_2$ represents the *secondary-sector job acquisition rate* for nonworkers. Otherwise as in the benchmark model, workers lose their job at exogenous rate $\delta$.

The state of the economy $\sigma_t$ evolves following a *three-state* Markov process with states: employed in the CBD, employed in the SBD, unemployed. We gather the Markov transitions into a matrix $P$, where its elements $p_{ij} = \Pr\{\sigma_{t+1} = i \mid \sigma_t = j\}$, where $i, j \in \{U, CBD, SBD\}$, that is, rows correspond to $t$ while columns correspond to $t+1$ (the rows sum up to one). The Markov transition matrix $P$ is given by:

$$\begin{array}{c} \\ U \\ CBD \\ SBD \end{array} \begin{array}{c} \begin{array}{ccc} U & CBD & SBD \end{array} \\ \begin{pmatrix} 1-(a_1+a_2) & a_1 & a_2 \\ \delta & 1-\delta & 0 \\ \delta & a_1 & 1-(a_1+\delta) \end{pmatrix} \end{array} = P \tag{4.17}$$

We discuss how we compute transitions. For instance, to jump from $U$ to $U$, it has to be that the unemployed worker did not find neither a CBD nor a SBD job. Therefore, $\Pr\{\sigma_{t+1} = U \mid \sigma_t = U\} = 1 - (a_1 + a_2)$. Also, to jump from $SBD$ to $CBD$, it has to be that the SBD-employed worker did find a CBD job, and this occurs at rate $a_1$. Hence, $\Pr\{\sigma_{t+1} = CBD \mid \sigma_t = SBD\} = a_1$. And so on.

We need to calculate the steady-states values of each state, that is the level of employment in the CBD, the SBD and the level of unemployment. We denote these values by $L_1^*, L_2^*$ and $U^*$, respectively. In Appendix B, we show that they are equal to:

$$U^* = \frac{\delta N}{a_1 + a_2 + \delta} \tag{4.18}$$

$$L_1^* = \frac{a_1 N}{\delta + a_1} \tag{4.19}$$

$$L_2^* = \frac{a_2\, \delta\, N}{(a_1+\delta)\,(a_1+a_2+\delta)} \tag{4.20}$$

Let us now determine the efficiency wage in the CBD. The steady-state Bellman equations for the CBD non-shirker, the CBD shirker, the unemployed and the SBD-employed workers are respectively given by:

$$r\, I_{L,1}^{NS} = w_L - e_1 - \tau\, L_1 - s\,\tau\,(N - L_1 - L_2) - \delta\,(I_{L,1}^{NS} - I_U) \tag{4.21}$$

$$r\, I^{S}_{L,1} = w_L - \tau\, L_1 - s\,\tau\,(N - L_1 - L_2) - (\delta + m)\,(I^{S}_{L,1} - I_U) \tag{4.22}$$

$$r\, I_U = w_U - s\,\tau\,(N - L_2) + a_1\,(I_{L,1} - I_U) + a_2\,(I_{L,2} - I_U) \tag{4.23}$$

$$r\, I_{L,2} = \overline{w}_2 - e_2 + (1 - s)\,\tau\,(N - L_2) - \tau N - \delta\,(I_{L,2} - I_U) + a_1\,(I_{L,1} - I_{L,2}) \tag{4.24}$$

where $r$ is the discount rate, $I^{NS}_{L,1}, I^{S}_{L,1}$, $I_U$ and $I_{L,2}$ respectively represent the expected lifetime utility of a non-shirker CBD worker, a shirker CBD worker, an unemployed worker, and a SBD worker. The main difference with the benchmark model are: (*i*) the lifetime expected utilities of CBD workers (both nonshirkers and shirkers) $I^{NS}_{L,1}$ and $I^{S}_{L,1}$, are now a function of $L_2$, (*ii*) the lifetime expected utility of the unemployed workers $I_U$, which is different now since they can find a job both in the CBD and in the SBD, (*iii*) the lifetime expected utility of a SBD-employed worker $I_{L,2}$, which did not exist before. Because of shirking behavior, firms will choose a wage $w_L$ so that $I^{NS}_{L,1} = I^{S}_{L,2} = I_{L,1}$, i.e. the efficiency wage must be set to make workers indifferent between shirking and not shirking.

By using (4.21) and (4.22), the condition $I^{NS}_{L} = I^{S}_{L} = I_L$ can be written as:

$$I_{L,1} - I_U = \frac{e_1}{m} \tag{4.25}$$

This is the same incentive condition as in the benchmark model, which says the surplus of being employed in the primary sector is strictly positive and a positive function of effort $e_1$ and a negative function of the detection rate $m$.

Also, by substracting (4.24) from (4.23), we obtain:

$$I_{L,2} - I_U = \frac{\overline{w}_2 - e_2 - w_U - \tau L_2}{r + a_1 + a_2 + \delta} \tag{4.26}$$

This is the positive surplus of working in the SBD compared to unemployment. It depends on the difference between the net wage in sector 2 and the unemployment benefit but also on the difference in spatial cost, as captured by $-\tau L_2$ since unemployed workers pay higher land rents than workers in the SBD because of the competition with CBD-workers. We can now calculate the lifetime expected utility of the unemployed, which by using (4.25) and (4.26) in (4.23), is equal to:

$$r\, I_U = w_U - s\,\tau\,(N - L_2) + a_1\,\frac{e_1}{m} + a_2\left(\frac{\overline{w}_2 - e_2 - w_U - \tau L_2}{r + a_1 + a_2 + \delta}\right) \tag{4.27}$$

Now, using (4.25) and (4.27), (4.21) can be written as:

$$\begin{aligned} w_L &= e_1 + rI_U + (r + \delta)\,(I_{L,1} - I_U) + \tau\, L_1 + s\,\tau\,(N - L_1 - L_2) \\ &= w_U + e_1 + \frac{e_1}{m}\,(r + a_1 + \delta) + a_2\left(\frac{\overline{w}_2 - e_2 - w_U - \tau L_2}{r + a_1 + a_2 + \delta}\right) + (1 - s)\,\tau\, L_1 \end{aligned}$$

By rearranging this equation, we easily obtain:

$$w_L = \Upsilon(a_1,a_2)\,w_U + [1-\Upsilon(a_1,a_2)]\,(\overline{w}_2 - e_2 - \tau L_2) + e_1 + SW_1 + \Delta SC_1 \quad (4.28)$$

where the weighting factor $0 < \Upsilon(a_1,a_2) < 1$ is defined as:

$$\Upsilon(a_1,a_2) = \frac{r+a_1+\delta}{r+a_1+a_2+\delta}\ ,$$

$SW_1 \equiv \frac{e_1}{m}\,(r+a_1+\delta)$ is as before the wage surplus that must be paid to deter shirking and $\Delta SC_1 \equiv SC_{L,1} - SC_U = (1-s)\,\tau\,L_1$ is as before the spatial-cost differential between the CBD-employed workers and the unemployed workers.

This efficiency wage is quite similar to that of the benchmark model. The main difference is that the base wage was before equal to $w_U + e$ whereas now it is equal to a convex combination of $w_U$ and the net wage in the secondary sector (i.e. $\overline{w}_2 - e_2 - \tau L_2$) plus $e_1$. The other elements of the efficiency wage are exactly as in the benchmark model and consist of the work inducement $SW_1$ and the spatial compensation $\Delta SC_1$. Of course the fact that we have $\Upsilon(a_1,a_2)\,w_U + [1-\Upsilon(a_1,a_2)]\,(\overline{w}_2 - e_2 - \tau L_2)$ instead of $w_U$ reflects the fact that, when setting the efficiency wage, firms have to take into account the opportunities that workers have outside of the CBD-labor market. In particular, when deciding whether to shirk or not, workers take into account the fact that if they are detected and fired, they will be unemployed with a possibility for the future to find a job not only in the CBD but also in the SBD. This is why this wage is higher than in the benchmark case since $\Upsilon(a_1,a_2)\,w_U + [1-\Upsilon(a_1,a_2)]\,(\overline{w}_2 - e_2 - \tau L_2) > w_U$ (it is assumed that $\overline{w}_2 - e_2 - \tau L_2 > w_U$). In some sense, $\Upsilon(a_1,a_2)\,w_U + [1-\Upsilon(a_1,a_2)]\,(\overline{w}_2 - e_2 - \tau L_2)$ represents the *composite opportunity wage* for individuals outside the primary sector. It can also be interpreted as an *expected discounted wage* for individuals moving between unemployment and SBD jobs. Of course, when $a_2 = 0$ (no secondary sector) we are back to the benchmark case since $\Upsilon(a_1,a_2) = 1$. Thus, the efficiency wage in the benchmark case is a special case of (4.28) when $a_2 = 0$ (or equivalently $L_2 = 0$).

By using the steady state flows (4.18), (4.19) and (4.20), we obtain

$$a_1(L_1) = \delta\frac{L_1}{N-L_1} \quad (4.29)$$

$$a_2(L_1,L_2) = \delta\frac{N\,L_2}{(N-L_1-L_2)\,(N-L_1)} \quad (4.30)$$

with $a_1'(L_1) \geq 0$, $\partial a_2(L_1, L_2)/\partial L_1 \geq 0$ and $\partial a_2(L_1, L_2)/\partial L_2 \geq 0$. As a result, the efficiency wage can be expressed in terms of $L_1$ and $L_2$ and we obtain:

$$\begin{aligned} w_L^* &= \Upsilon(L_1, L_2)\, w_U + [1 - \Upsilon(L_1, L_2)]\,(\overline{w}_2 - e_2 - \tau L_2) + e_1 \\ &\quad + \frac{e_1}{m}\left(r + \frac{\delta N}{N - L_1}\right) + (1 - s)\,\tau\, L_1 \end{aligned} \tag{4.31}$$

where

$$\Upsilon(L_1, L_2) = \frac{[r\,(N - L_1) + \delta N]\,(N - L_1 - L_2)}{\delta N\, L_2 + [r\,(N - L_1) + \delta N]\,(N - L_1 - L_2)}$$

We need to see if the properties of the efficiency wage ("equilibrium unemployment acts as a worker discipline device") still hold here. We have the following result:

**Proposition 5.** *The two-sector efficiency wage $w_L^*$ is increasing in $L_1$ over the relevant range $0 < L_1 < N - L_2$ and*

$$\lim_{L_1 \to N} w_L^* = +\infty$$

*Furthermore, if the minimum wage is large enough, then the two-sector efficiency wage $w_L^*$ is increasing in $L_2$.*

Indeed, higher employment in the CBD leads to higher efficiency wage because workers find easily a CBD-job and are thus more incline to shirk. Also, the efficiency wages is not compatible with full employment otherwise there would be any worker discipline device. Finally, when the level of employment in the SBD improves, CBD-firms have to increase their efficiency wages since the prospects for CBD-workers is better and will be more likely to shirk.

In terms of comparative statics of the efficiency wage, we obtain nearly the same results as in the benchmark case. There is however one important exception and it is the effect of the commuting cost $\tau$ on $w_L^*$. In the benchmark model, an increase in $\tau$ always leads to an increase in $w_L^*$ because firms had to compensate more workers for higher space-cost differential. Here this is not true anymore. In fact, the sign is now ambiguous since

$$\frac{\partial w_L^*}{\partial \tau} = (1 - s)\, L_1 - [1 - \Upsilon(L_1, L_2)]\, L_2 \gtreqless 0$$

Indeed, $\tau$ increases not only the space-cost differential between the CBD-employed and the unemployed but also increases the commuting costs of SBD-workers. In particular, if $[1 - \Upsilon(L_1, L_2)]\, L_2 > (1 - s)\, L_1$, then the increased space costs for individuals outside the primary sector are sufficiently high to allow a decrease in efficiency wage levels. This dual effect is also present in the time-discounting parameter, $r$. In the benchmark case, an increase in $r$ always increased $w_L^*$ (since workers value more short-run benefits and are thus more likely to shirk), here this is not true anymore. While this effect is also present in the two-sector case, it can be countered by the fact that workers fired for shirking always enter unemployment before gaining work in the secondary sector. Hence, if the net wage in sector 2, $\overline{w}_2 - e_2 - \tau L_2$, is sufficiently high relative to $w_U$, then for a range of increases in $r$, this short-run loss of income reduces the prospects of individuals leaving the primary sector enough to allow a decrease in efficiency wages.

Let us now determine the labor demand $L_1$ in the primary sector. First, observe that since there is a minimum wage in the economy equals to $\overline{w}_2$ CBD-firms can not pay a wage below $\overline{w}_2$. Thus, the relevant *effective* wage in the CBD is: $w_{L,1}(L_1, L_2) = \max\{\overline{w}_2, w_L^*\}$, i.e.

$$w_{L,1}(L_1, L_2) = \max \left\{ \begin{array}{c} \overline{w}_2, \Upsilon(L_1, L_2)\, w_U + [1 - \Upsilon(L_1, L_2)]\,(\overline{w}_2 - e_2 - \tau L_2) + e_1 \\ +\frac{e_1}{m}\left(r + \frac{\delta N}{N - L_1}\right) + (1 - s)\,\tau\, L_1 \end{array} \right\} \tag{4.32}$$

There are $M_1$ identical firms in the primary sector. We normalize the price of product in this sector by 1. By maximizing their profit, each firm determines its labor demand and the aggregate labor demand is thus given by: $e_1 F'(eL_1^*) = w_L^*$, which using (4.32) yields:

$$e_1 F'(eL_1^*) = w_{L,1}(L_1, L_2) \tag{4.33}$$

Figure 6.5 describes the equilibrium in the labor market for this duocentric city. It shows that for very low levels of employment $L_1$, the CBD-firms will pay the minimum wage because it is above the Urban Non-Shirking curve $w_{L,1}(L_1, L_2)$. Then , when the two-sector efficiency wage will be above the minimum wage, it will prevail. Moreover, the dashed curve displays the efficiency wage for the one sector model (the benchmark case). Indeed, the one-sector efficiency wage is always below that the one set in the two-sector case because workers have more employment opportunities in the latter than in the former.

[*Insert Figure* 6.5 *here*]

**Employment in the SBD (secondary sector or sector 2)** Production technology in the SBD for the $M_2$ identical firms in sector 2 is taken to be representable by a production function, $F_2(L_2)$, which is assumed to be twice differentiable with $F_2(0) = 0$, $F_2'(L_2) > 0$, $F_2''(L_2) \leq 0$, and to satisfy the Inada conditions, i.e. $\lim_{L_2 \to 0} F'(L_2) = +\infty$ and $\lim_{L_2 \to +\infty} F'(L_2) = 0$. Similarly, it is assumed that all output can be sold at a fixed market price $p_2$. As stated above, production tasks are relatively simple and easy monitored in sector 2 and hence it is assumed that the effort expenditure in sector 2, $e_2$, required for production is much smaller than that of sector 1, i.e. $e_2 < e_2$, and that shirking behavior can be ignored in sector 2. The profit function of the representative firm in sector 2 can thus be written as: $\Pi_2 = p_2 F_2(L_2) - \overline{w}_2 L_2$. Hence, at the minimum wage $\overline{w}_2$, the firm needs to attract enough workers to maximize its profit. By solving the following program,

$$\max_{L_2} \Pi_2 = p_2 F_2(L_2) - \overline{w}_2 L_2$$

one obtains the equilibrium employment level in sector 2:

$$F_2'(\overline{L}_2) = \frac{\overline{w}_2}{p_2} \tag{4.34}$$

Since $\overline{w}_2$ and $p_2$ are exogenous variables so is the employment level in the secondary sector and that is why it is denoted by $\overline{L}_2$.

### 4.4. Steady-state equilibrium

We have the following definition:

**Definition 4.** *A steady-state equilibrium in the duocentric city* $(R^*(x), w_{L,1}, L_1^*)$ *consists of a land rent function (4.16), a wage for CBD-workers (4.32) and employment level in the CBD (4.33) such that the urban land use equilibrium and the labor market equilibrium are solved for simultaneously.*

Because of the Inada conditions on the production function, it is easy to verify that there exists a unique steady-state equilibrium in the duocentric city.

### 4.5. Interaction between land and labor markets

Basically equation (4.16), which gives the equilibrium land rent $R^*(x)$ and equation (4.33), which gives the equilibrium employment level in the primary sector, $L_1^*$, determine the steady-state equilibrium of this duocentric city. Observe that the equilibrium land rent is totally determined by the minimum wage $\overline{w}_2$ and this $\overline{L}_2$ between $x = L_1$ and $x = N$ (i.e. for the unemployed and the SBD-employed workers) and thus is exogenous. So the only part that is interesting and endogenous is for the CBD-employed workers, i.e. between $x = 0$ and $x = L_1$.

First, it is interesting to see if the SBD-employed workers pay a higher land rent than the unemployed workers. We have:

$$R_2^*(N) \gtreqless R_U^*(L_1) \Longleftrightarrow s \lesseqgtr \frac{L_2}{L_2 + U}$$

which means that if we compare the highest land rent that these workers pay (at $x = N$ for the SBD-employed workers and at $x = L_1$ for the unemployed workers; see Figure 6.3), then it is indeterminate. In Figure 6.3, we have represented a case where $R_2^*(N) > R_U^*(L_1)$, but this is not always true.

Second, let us analyze the effects of two key exogenous variables, $w_U$ and $\overline{w}_2$ (labor variables) and $\tau$ (spatial variable) on the endogenous variables $R^*(x)$ and $L_1^*$. By totally differentiating (4.33) and on focussing on the employment level for which $\max \{\overline{w}_2, w_L^*\} = w_L^*$, we have

$$\frac{\partial L_1^*}{\partial w_U} < 0 \ , \ \frac{\partial L_1^*}{\partial \overline{w}_2} < 0 \text{ and } \frac{\partial L_1^*}{\partial \tau} \gtreqless 0 \Longleftrightarrow (1 - s) \, L_1 \lesseqgtr [1 - \Upsilon(L_1, L_2)] \, L_2$$

All the intuition of these results goes through the efficiency wage setting. Indeed, when $w_U$ or $\overline{w}_2$ increases, then CBD-firms have to increase their wage to meet the urban non-shirking condition because workers have better outside option. As a result, since wages are higher, firms higher less. We have already discuss the ambiguous effect of $\tau$ on the efficiency wage, which translates in an ambiguous effect on the employment level in the primary sector.

Let us now analyze the impact of these variables on the land rent paid by individuals working in the CBD. Since when $L_1$ increases, at each $x \in [0, L_1]$, the equilibrium land rent $R^*(x)$ also increases (there is more competition in the land market because more workers are living between 0 and $L_1$), then we have that for $x \in [0, L_1]$:

$$\frac{\partial R^*(x)}{\partial w_U} = \frac{\partial R^*(x)}{\partial L_1^*} \frac{\partial L_1^*}{\partial w_U} < 0 \ , \ \frac{\partial R^*(x)}{\partial \overline{w}_2} = \frac{\partial R^*(x)}{\partial L_1^*} \frac{\partial L_1^*}{\partial \overline{w}_2} < 0$$

Indeed, when the unemployment benefit and/or the minimum wage in the economy increases, workers have better prospects as unemployed and/or as working in the secondary sector. Hence, CBD firms need to increase their wage to deter shirking. This reduces employment in the primary sector and thus the competition in the land market. As a result, land prices decrease at each location between $x = 0$ and $x = L_1$. Observe that land prices outside of $[0, L_1]$ are not at all affected by a change in $w_U$ but will be affected in the following way if the minimum wage varies:

$$\frac{\partial R^*(x)}{\partial \overline{w}_2} = \frac{\partial R^*(x)}{\partial L_2}\frac{\partial L_2}{\partial \overline{w}_2} > 0 \text{ for } L_1 < x \leq N - L_2$$

$$\frac{\partial R^*(x)}{\partial \overline{w}_2} = \frac{\partial R^*(x)}{\partial L_2}\frac{\partial L_2}{\partial \overline{w}_2} < 0 \text{ for } N - L_2 < x \leq N$$

Indeed, for the unemployed residing in $L_1 < x \leq N - L_2$, an increase in $\overline{w}_2$ reduces $L_2$, which in turn decreases the competition in the land market and thus the land price in this area. On the contrary, for the SBD-employed workers residing in $N - L_2 < x \leq N$, an increase in $\overline{w}_2$ reduces $L_2$, which now increases the competition in the land market and thus the land price in this area.

Finally, if we analyze the impact of the commuting cost $\tau$ on the equilibrium land rent for $x \in [0, L_1]$, we obtain:

$$\frac{\partial R^*(x)}{\partial \tau} = (1-s)\, L_1 + s\, (N - L_2) - x + (1-s)\, \tau \frac{\partial L_1^*}{\partial \tau} \gtreqless 0$$

There is indeed two effects of an increase of $\tau$ on the land rent. First, there is a *direct positive effect* since the access to the CBD is more costly and thus the competition in the land market becomes fiercer, which increases the land price at each location $x$. Second, there is an *ambiguous indirect effect* via the wage. Indeed, when $\tau$ increases, the wage decreases (increases) if the negative indirect effect on the SBD workers mentioned above outweights (is lower than) the positive direct effect on the CBD workers. In that case, labor demand in the CBD increases (decreases) and the land rent increases (decreases). As a result, if $[1 - \Upsilon(L_1, L_2)]\, L_2 > (1-s)\, L_1$, then the total effect of the commuting cost $\tau$ on the equilibrium land rent $R^*(x)$ for $x \in [0, L_1]$ is always positive. Otherwise it is ambiguous. Observe that an increases in $\tau$ have always a positive effect on the equilibrium land rent $x \in [L_1, N]$.

### 4.6. First extension: The case of high relocation costs

As in Chapter 5, we now assume that mobility costs are so high that once someone is located somewhere he/she never moves. As a result, *a worker's residential location remains fixed as he/she enters and leaves unemployment.* We also assume perfect capital markets with a zero interest rate. When there is a zero interest rate, workers have no intrinsic preference for the present, and thus they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent.

As before , we assume that changes in the employment status (employment versus unemployment) are governed by a three-state Markov process. Therefore, for the unemployment, CBD- and CBD-employment rates of non-shirkers are respectively given by:

$$u^{NS} = \frac{U^{NS}}{N} = \frac{\delta}{a_1 + a_2 + \delta} \tag{4.35}$$

$$l_1^{NS} = \frac{L_1^{NS}}{N} = \frac{a_1}{a_1 + \delta} \tag{4.36}$$

$$l_2^{NS} = \frac{L_2^{NS}}{N} = \frac{a_2\,\delta}{(a_1 + \delta)\,(a_1 + a_2 + \delta)} \tag{4.37}$$

For the case of shirkers, it is a little bit more complicated. In particular, the Markov transition matrix $P$ is now given by:

$$\begin{array}{c} \\ U \\ CBD \\ SBD \end{array} \begin{array}{c} \begin{array}{ccc} U & CBD & SBD \end{array} \\ \begin{pmatrix} 1-(a_1+a_2) & a_1 & a_2 \\ \delta+m & 1-(\delta+m) & 0 \\ \delta & a_1 & 1-(a_1+\delta) \end{pmatrix} \end{array} = P \tag{4.38}$$

So the only change compared to the non-shirking case is that CBD-workers can lose their jobs not only because of the exogenous job destruction rate $\delta$ but also because of their shirking behavior, which is detected at rate $m$. In Appendix C, we show that, in steady-state, we have:

$$u^{S} = \frac{U^{S}}{N} = \frac{(\delta + m)\,(a_1 + \delta)}{(a_1 + \delta + m)\,(a_1 + a_2 + \delta)} \tag{4.39}$$

$$l_1^{S} = \frac{L_1^{S}}{N} = \frac{a_1}{a_1 + \delta + m} \tag{4.40}$$

$$l_2^{S} = \frac{L_2^{S}}{N} = \frac{a_2\,(\delta + m)}{(a_1 + \delta + m)\,(a_1 + a_2 + \delta)} \tag{4.41}$$

It is easy to verify that:

$$l_1^{NS} - l_1^{S} = \frac{a_1 m}{(\delta + a_1)\,(a_1 + \delta + m)} > 0 \tag{4.42}$$

$$u^{S} - u^{NS} = \frac{a_1 m}{(a_1 + \delta + m)\,(a_1 + a_2 + \delta)} > 0 \tag{4.43}$$

$$\frac{u^{S} - u^{NS}}{l_1^{NS} - l_1^{S}} = \frac{a_1 + \delta}{a_1 + a_2 + \delta} < 1 \tag{4.44}$$

$$l_2^{S} - l_2^{NS} = \frac{a_1 a_2\, m}{(a_1 + \delta + m)\,(a_1 + a_2 + \delta)\,(a_1 + \delta)} > 0 \tag{4.45}$$

Indeed, if one shirks then the time he/she will spend unemployed and employed will be higher and lower, respectively, than if he/she does not work. Interestingly, $u^S - u^{NS} < l_1^{NS} - l_1^S$, which means the shirker is losing more in terms of employment than unemployment. Finally, shirking behavior also affects secondary employment since it implies that shirkers will spend more time in the secondary sector than non-shirkers because they are more likely to lose their primary sector job.

Since workers have a zero discount rate, they only care about the average net income over time. For a non shirker located at a distance $x$ from the CBD, it is equal to:

$$\begin{aligned} EW^{NS}(x) &= \left(1 - u^{NS} - l_1^{NS}\right)\left[\overline{w}_2 - e_2 + (1-s)\,\tau\,x - \tau N - R(x)\right] \\ &\quad + u^{NS}\left[w_U - s\tau x - R(x)\right] + l_1^{NS}\left[w_L - e_1 - \tau x - R(x)\right] \end{aligned} \tag{4.46}$$

whereas for a shirker residing at a distance $x$ from the CBD, it is given by:

$$\begin{aligned} EW^{S}(x) &= \left(1 - u^{S} - l_1^{S}\right)\left[\overline{w}_2 - e_2 + (1-s)\,\tau\,x - \tau N - R(x)\right] \\ &\quad + u^{S}\left[w_U - s\tau x - R(x)\right] + l_1^{S}\left[w_L - \tau x - R(x)\right] \end{aligned} \tag{4.47}$$

Let us calculate the efficiency wage. It will be determined by: $EW_L^{NS}(x) = EW_L^{NS}(x)$, which is equivalent to:

$$w_{L,1}^*(x) = \Upsilon'\,w_U + (1-\Upsilon')\left(\overline{w}_2 - e_2 - \tau N\right) + e_1 \frac{l_1^{NS}}{l_1^{NS} - l_1^S} + (2 - s - \Upsilon')\,\tau x \tag{4.48}$$

where

$$\Upsilon' = \frac{u^S - u^{NS}}{l_1^{NS} - l_1^S}$$

Using (4.44), we have that $0 < \Upsilon' < 1$.

We assume that firms do not observe the residential location of all workers. So we need to know how this efficiency wage vary with location $x$. It is easy to verify that $\partial w_L / \partial x \geq 0$ since $2 > s + \Upsilon'$ and $s < 1$ and $\Upsilon'$. Indeed, when someone shirks, he/she will spend more time unemployed and working in the SBD (see (4.42), (4.43) and (4.45)) and thus will commute less to the CBD than those who are not shirking. As a result, workers reside further away need to be paid more because they are more likely to shirk than those living closer to the CBD. Since firms know that efficiency wages are increasing with $x$, the distance to the CBD, and since they do not where people live, they will set the highest possible wage to prevent shirking, i.e. at $x = N$. Thus, the efficiency wage in the CBD will be equal for all workers and given by:

$$w_{L,1}^* \equiv w_{L,1}^*(N) = \Upsilon'\,w_U + (1-\Upsilon')\left(\overline{w}_2 - e_2\right) + e_1 \frac{l_1^{NS}}{l_1^{NS} - l_1^S} + (1-s)\,\tau N \tag{4.49}$$

This efficiency wage is in fact very close to the one without relocation costs, given in (4.31). Indeed, it is made of three parts: The base wage, $\Upsilon' w_U + (1-\Upsilon')(\overline{w}_2 - e_2)$, the wage surplus due to incentive reasons, $e_1 l_1^{NS} / \left(l_1^{NS} - l_1^S\right)$, and the spatial cost differential, $(1-s)\,\tau N$.

Efficiency wages are not location-dependent and land rents will only compensate for commuting costs. Let us solve the urban land use equilibrium. By plugging (4.49) in (4.46) or (4.47), we obtain the following expected utility of a (non-shirker) worker located at a distance $x$ from the CBD:

$$\begin{aligned} EW(x) &= w_U\left(\frac{l_1^{NS}u^S - u^{NS}l_1^S}{l_1^{NS} - l_1^S}\right) + (\overline{w}_2 - e_2)\left[1 - \left(\frac{l_1^{NS}u^S - u^{NS}l_1^S}{l_1^{NS} - l_1^S}\right)\right] \quad (4.50)\\ &+e_1\left(\frac{l_1^S\, l_1^{NS}}{l_1^{NS} - l_1^S}\right) + \tau N\left[(2-s)\, l_1^{NS} + u^{NS} - 1\right] - R(x)\\ &-\tau\, x\left[u^{NS} + (2-s)\, l_1^{NS} - (1-s)\right]\end{aligned}$$

If we denote that $I$ the (expected) utility reached by all workers in the city in equilibrium, then the bid rent is equal to

$$\begin{aligned} \Psi(x,I) &= w_U\left(\frac{l_1^{NS}u^S - u^{NS}l_1^S}{l_1^{NS} - l_1^S}\right) + (\overline{w}_2 - e_2)\left[1 - \left(\frac{l_1^{NS}u^S - u^{NS}l_1^S}{l_1^{NS} - l_1^S}\right)\right] \quad (4.51)\\ &+e_1\left(\frac{l_1^S\, l_1^{NS}}{l_1^{NS} - l_1^S}\right) + \tau N\left[(2-s)\, l_1^{NS} + u^{NS} - 1\right] - I\\ &-\tau\, x\left[u^{NS} + (2-s)\, l_1^{NS} - (1-s)\right]\end{aligned}$$

**Proposition 6.** *In the duocentric city with high relocation costs, the land rent is decreasing with $x$, the distance to the CBD, if $l_1^{NS} + s\left(u^{NS} + l_2^{NS}\right) > l_2^{NS}$. A sufficient condition for this to be true is $a_1 > \delta$, that is the job acquisition rate in the primary sector is greater than the job-destruction rate in the economy.*

**Proof.** By differentiating (4.51) with respect to $x$, we obtain

$$\frac{\partial \Psi(x,I)}{\partial x} = -\tau\left[u^{NS} + (2-s)\, l_1^{NS} - (1-s)\right]$$

Thus

$$\frac{\partial \Psi(x,I)}{\partial x} \lesseqgtr 0 \Leftrightarrow 1 - s \lesseqgtr u^{NS} + (2-s)\, l_1^{NS}$$

Hence,

$$\begin{aligned}\frac{\partial \Psi(x,I)}{\partial x} &< 0 \Leftrightarrow 1 - s < u^{NS} + (2-s)\, l_1^{NS}\\ &\Leftrightarrow u^{NS} + 2l_1^{NS} + s\left(1 - l_1^{NS}\right) > 1\\ &\Leftrightarrow l_1^{NS} + s\left(u^{NS} + l_2^{NS}\right) > l_2^{NS}\end{aligned}$$

Now, using (4.36) and (4.37), we obtain:

$$l_1^{NS} - l_2^{NS} = \frac{a_1 (a_1 + a_2) + \delta (a_1 - a_2)}{(a_1 + \delta) (a_1 + a_2 + \delta)}$$

A sufficient condition for $l_1^{NS} - l_2^{NS} > 0$ is $a_1 > \delta$. ▮

Indeed, there are two opposite forces that determines the marginal willingness to pay for land. On the one hand, distant workers have higher transportation costs to commute to the CBD. On the other, distant workers have lower costs to commute to the SBD. In both cases, the land rent has to compensate for that but in the former it decreases with $x$ while in the latter, it increases with $x$. If $a_1 > \delta$, or more generally if individuals spend more time unemployed and working in the primary sector than working in the secondary sector, then the first effect will dominates the second one and the land rent will be decreasing with the distance to the CBD. From now on, we assume that the land rent is always decreasing with $x$.

As in a standard land-use model, the utility $I$ is determined by the fact that the bid rent at the city-fringe is equal to zero (the agricultural land rent). We obtain:

$$\begin{aligned} I^* &= w_U \left( \frac{l_1^{NS} u^S - u^{NS} l_1^S}{l_1^{NS} - l_1^S} \right) + (\overline{w}_2 - e_2) \left[ 1 - \left( \frac{l_1^{NS} u^S - u^{NS} l_1^S}{l_1^{NS} - l_1^S} \right) \right] \\ &\quad + e_1 \left( \frac{l_1^S \, l_1^{NS}}{l_1^{NS} - l_1^S} \right) - s\tau N \end{aligned} \tag{4.52}$$

Furthermore, plugging (4.52) in (4.51), we obtain the following equilibrium land rent for $x \in [0, N]$:

$$\begin{aligned} R^*(x) &= \tau N \left[ (2 - s) \, l_1^{NS} - (1 - s) + u^{NS} \right] \\ &\quad - \tau \, x \left[ u^{NS} + (2 - s) \, l_1^{NS} - (1 - s) \right] \end{aligned} \tag{4.53}$$

We can now determine the steady-state labor market equilibrium. In the primary (CBD) and secondary sectors, each firm adjusts employment until the marginal product of an additional worker equals the efficiency wage and the minimum wage, respectively. By focussing on the part of employment for which the efficiency wage is above the minimum wage, we obtain in the CBD:

$$\Upsilon' \, w_U + (1 - \Upsilon') \, (\overline{w}_2 - e_2) + e_1 \frac{l_1^{NS}}{l_1^{NS} - l_1^S} + (1 - s) \, \tau N = e_1 F'(e L_1^*)$$

while in the SBD, we have:

$$F_2'(\overline{L}_2) = \frac{\overline{w}_2}{p_2}$$

Since employment at each center (CBD and SBD) must equal labor demand, the equilibrium unemployment and CBD-employment rates satisfy:

$$(1 - l_1^* - u^*)\, N = \overline{L}_2$$

$$l_1^* N = L_1^*$$

The labor-market equilibrium condition can thus be written as:

$$\Upsilon'\, w_U + (1 - \Upsilon')\,(\overline{w}_2 - e_2) + e_1 \frac{l_1^*}{l_1^* - l_1^S} + (1 - s)\,\tau N = e_1 F'(e l_1^* N) \tag{4.54}$$

where $l_1^* \equiv l_1^{NS}$ and $\Upsilon' = \left(u^S - u^{NS}\right) / \left(l_1^* - l_1^S\right)$.

We have the following definition:

**Definition 5.** *A steady-state equilibrium in the duocentric city with high-relocation costs* $(R^*(x), w_{L,1}, l_1^*, u^*, l_2^*, I^*)$ *consists of a land rent function (4.53), a wage for CBD-workers (4.49), an employment rate in the CBD (4.54), an unemployment rate (4.35), an employment rate in the SBD (4.37), and an (expected) utility level (4.52) such that the urban land use equilibrium and the labor market equilibrium are solved for simultaneously.*

The way the equilibrium is calculated is as follows. First, since $l_2^{NS}$ is exogenous, one obtains a relationship between $a_1$ and $a_2$ using (4.37). Then, by solving equation (4.54), one obtains a relationship between $l_1^*$ and $a_1$ (since $u^S, u^{NS}, l_1^S$ are functions of $a_1$,$a_2$ and, because of (4.37), $a_2$ is a function of $a_1$) and using equation (4.36), one obtains another relationship between $l_1^*$ and $a_1$. By combining these two last equations, one finds a unique solution for $l_1^*$. Using the fact that $1 - l_1^* - l_2 = u^*$, one obtains the value of $u^*$. By plugging these values in (4.53), (4.49), (4.52), one obtains respectively $R^*(x)$, $w_{L,1}^*$ and $I^*$.

### 4.7. Second extension: Endogenous formation of the SBD[7]

So far, the secondary employment center (SBD) was exogenously fixed. We would like now to study the case of an endogenous formation of a SBD. Consider the benchmark model and in particular the urban land-use equilibrium as described in Figure 4.1. The secondary sector is now treated as a single large firm and we would like to determine under which conditions this large firm will set up at the periphery of the city.

[7] This section is based on Smith and Zenou (1997).

In choosing a specific location $x$, the firm takes into account both location land rent, $R(x)$, and the accessibility of $x$ to the CBD, where the latter effect is represented by a CBD-interaction cost, $\kappa$, per unit of distance, reflecting the occasional need to utilize city services in the CBD and/or interact with CBD firms (see the next section, for a more elaborate model of interactions between firms). Hence the profit function of a SBD firm that decides to locate in a location $x$ is equal to:

$$\Pi(x) = p_2 F(L_2) - \overline{w}_2 L_2 - R(x) - \kappa\, x \tag{4.55}$$

Denote by $x_{SBD}$ the location of the SBD. Then, since ex ante, workers in the secondary sector do not know where the SBD will be located, their bid rent function will not be given by (4.7) but by:

$$\Psi_{L,2}(x, W_{L,2}) = \overline{w}_2 - e_2 - s\,\tau\, x - \tau\, |x_{SBD} - x| - W_{L,2} \tag{4.56}$$

Hence the slope of the bid rent function of SBD workers in all locations $x$ is given by $\Psi'_{L,2}(x, W_{L,2})$, which is seen to be $(1-s)\tau$ for all $x < x_{SBD}$, and $-(1+s)\tau$ for all $x > x_{SBD}$. In other words, it is increasing on the left of $x_{SBD}$ and decreasing on the right of $x_{SBD}$ with a steeper slope on the right of $x_{SBD}$. Indeed, we have:

$$\Psi^{-}_{L,2}(x, W_{L,2}) = \overline{w}_2 - e_2 - s\,\tau\, x - \tau\,(x_{SBD} - x) - W_{L,2} \tag{4.57}$$

$$\Psi^{+}_{L,2}(x, W_{L,2}) = \overline{w}_2 - e_2 - s\,\tau\, x - \tau\,(x - x_{SBD}) - W_{L,2} \tag{4.58}$$

where $\Psi^{-}_{L,2}(x, W_{L,2})$ and $\Psi^{+}_{L,2}(x, W_{L,2})$ are respectively the bid rent of workers located on the left and on the right of $x_{SBD}$. The SBD firm determines $x = x^{*}_{SBD}$ by maximizing its profit or equivalently by minimizing $R(x) + \kappa\, x$. We have a first result:

**Proposition 7.** *There are three different possible cases:*

(*i*) *If* $x_{SBD} > N - (1-s)L_2/2$, *the only equilibrium involves an isolated suburb (Figure 6.6a);*

(*ii*) $N - L_2/2 \leq x_{SBD} \leq N - (1-s)L_2/2$, *the only equilibrium involves an edge city (Figure 6.6b).*

(*iii*) $L_1 + L_2/2 \leq x_{SBD} < N - L_2/2$, *the only equilibrium involves a subcenter (Figure 6.6c).*

The proof of this proposition can be found in Smith and Zenou (1997). In case $(i)$, the SBD consists of an isolated suburb since there a non-empty interval between the smallest distance to the CBD of the SBD (i.e. $x = N-(1-s)L_2/2$) and the unemployed located the farthest away from the CBD (i.e. at $x = N - L_2$) so that there is no land competition between the SBD workers and the other workers in the city (see Figure 6.6a). In case $(ii)$, this interval is now equal to zero and the two distances mentioned above are equal so that we have an edge city (see Figure 6.6b). In the last case (case $(iii)$), the SBD is located within the unemployment area and thus forms a subcenter (see Figure 6.6b).

[*Insert Figures* 6.6*a*, 6.6*b*, 6.6*c here*]

Observe that we do not consider the case for which $x_{SBD} < L_1 + L_2/2$ since in that case the land rent will be much higher since SBD-workers will compete for land with employed CBD-workers. Hence for each relevant location $x_{SBD}$, the only possible equilibrium land rent, $R(x_{SBD})$, is seen from figures 6.6$a$, 6.6$b$ and 6.6$c$ to be given by:

$$R(x_{SBD}) = \begin{cases} \tau\left(\frac{1+s}{2}\right)L_2 + s\tau\left(N - \frac{L_2}{2} - x_{SBD}\right) & \text{if} \quad L_1 + \frac{L_2}{2} \leq x_{SBD} < N - \frac{L_2}{2} \\ \tau(1+s)\left(N - x_{SBD}\right) & \text{if} \quad N - \frac{L_2}{2} \leq x_{SBD} < N - (\frac{1-s}{2})L_2 \\ \tau\left(1-s\right)\left(\frac{1+s}{2}\right)L_2 & \text{if} \quad x_{SBD} > N - (\frac{1-s}{2})L_2 \end{cases}$$

We have the following result:

**Proposition 8.** *If* $0 < \varepsilon < s\tau$, *then the only equilibrium location of the SBD is given by:*

$$x^*_{SBD} = N - \left(\frac{1-s}{2}\right)L_2 \tag{4.59}$$

Indeed, as described by the profit function (4.55), there are two opposite forces that drive the SBD's location: the land rent $R(x)$ (which acts as a repulsion force to the CBD) and interaction costs $\kappa\, x$ (which act as attraction force to the CBD). So when the interactions are not too high, then the SBD firm decides the unique location that minimizes $R(x) + \kappa\, x$. This location is given by (4.59). The urban land-use equilibrium is thus described by Figure 6.7.

[*Insert Figure* 6.7 *here*]

The main difference with the case of exogenous SBD is that $x^*_{SBD}$ is given by (4.59) whereas it was before such that: $x^*_{SBD} = N$. As a result, now SBD-workers live on both sides of the location of the SBD whereas before there were always living of its left hand side (compare figures 6.3 and 6.7). Therefore, the labor market analysis is exactly as before and thus the efficiency wage for CBD-workers is still given by (4.32), the CBD-employment level by (4.33), and the SBD-employment level by (4.34). The only difference is the value of the equilibrium land rent $R^*(x)$. It is not anymore given by (4.16). Let us define the new urban land-use equilibrium:

**Definition 6.** *An urban-land use equilibrium with no relocation costs and fixed-housing consumption in a duocentric city with endogenous SBD is a 4-tuple* $(W^*_{L,1}, W^*_{L,2}, W^*_U, R^*(x))$ *such that:*

$$\Psi_{L,1}(L_1, W^*_{L,1}) = \Psi_U(L_1, W^*_U)$$

$$\Psi_U(N - L_2, W^*_U) = \Psi^-_{L,2}(N - L_2, W^*_{L,2}) = \Psi^+_{L,2}(N, W^*_{L,2}) = 0$$

$$\begin{array}{rcl} R^*(x) & = & \max\left\{\Psi_{L,1}(x, W^*_{L,1}), \Psi^-_{L,2}(x, W^*_{L,2}), \Psi^+_{L,2}(x, W^*_{L,2}), \Psi_U(x, W^*_U), 0\right\} \\ \textit{at each } x & \in & \left(0, x^*_f\right] \end{array}$$

By solving these equations using (4.59), we easily obtain:

$$W^*_{L,1} = w_L - e_1 - \tau\, L_1 - s\,\tau\,(N - L_1 - L_2)$$

$$W^*_{L,2} = \overline{w}_2 - e_2 - s\,\tau\, N - \left(\frac{1-s}{2}\right)\tau L_2$$

$$W^*_U = w_U - s\,\tau\,(N - L_2)$$

$$R^*(x) = \begin{cases} \tau\,(L_1 - x) + s\,\tau\,(N - L_1 - L_2) & \text{for} \quad 0 \le x \le L_1 \\ s\,\tau\,(N - L_2 - x) & \text{for} \quad L_1 < x \le N - L_2 \\ (1-s)\,\tau\,(x + L_2 - N) & \text{for} \quad N - L_2 < x \le N - \left(\frac{1-s}{2}\right) L_2 \\ (1+s)\,\tau\,(N - x) & \text{for} \quad N - \left(\frac{1-s}{2}\right) L_2 < x \le N \\ 0 & \text{for} \quad x > N \end{cases}$$

If we compare these equilibrium values with the ones obtained in the case of a fixed SBD (see equations (4.13)−(4.16)), we obtain the same values for $W^*_{L,1}$ and $W^*_U$ but of course a different one for $W^*_{L,2}$ since SBD workers are now located on both sides of the SBD. This is also why the equilibrium land rent $R^*(x)$ has a different value between $N - L_2$ and $N$.

## 5. Endogenous formation of monocentric cities with unemployment[8]

So far, firms' location was assumed to be fixed and the employment center was thus prespecified (in the previous section, the center was prespecified only in the CBD, not in the SBD). There is in fact a literature that deals with the endogenous location of firms and formation of cities by explaining why cities exist, why cities form where they do and why economic activities agglomerate in a small number of places. In their very complete survey and book, Fujita and Thisse (1996, 2002) give three main reasons for agglomeration economies: externalities under perfect competition (see e.g. Beckmann, 1976, Borukhov and Hochman, 1977, Fujita and Ogawa, 1982, Papageorgiou and Smith, 1983, among others), increasing returns under monopolistic competition (see e.g. Abdel-Rahman and Fujita, 1990, Krugman, 1991, Fujita and Krugman, 1995, Fujita and Mori, 1997,...) and spatial competition under strategic interaction (Hotelling types of models). In this section, we will develop a model of endogenous formation of a monocentric city where both employed and unemployed workers compete for land.

### 5.1. The model

The city is closed, linear and symmetric. The middle of the city is normalized to 0 and the length of the city is denoted by $x_f$ on its right and by $-x_f$ (symmetry) on its left. There is no vacant land and no cross-commuting (workers cannot cross each other when they go to work) in the city. All the land is owned by absentee landlords. There are two types of workers, the employed and the unemployed. As in the benchmark model of chapter 4, all workers (employed and unemployed) consume the same amount of land, which is normalized to 1 for simplicity.

We further assume that the density of workers $\psi(x)$ in each location $x$ of the city within a *residential area* is equal to 1 (a residential area is an area when only households locate). Even though workers and non-workers consume the same amount of land, they differ by their revenue and commuting costs. Let us denote by $x_c$, $w_L(x_c)$ and $w_U$, the location of firms (or equivalently workers' workplace which will be determined endogenously in equilibrium), the wage at $x_c$ and the unemployment benefit exogenously financed by the government.

For simplicity, we assume that the shopping center is always located exactly

[8] This section is based on Zenou (2000).

in 0 the middle of the city. This assumption is made to capture the idea of the standard CBD developed in chapters 4 and 5 where workers go there to shop and to work. Observe that the shopping center is where consumers buy goods but not where production takes place, goods being produced by firms in the workplace. The latter will be determined endogenously in equilibrium but since we focus on a monocentric city, it will be in the city-center.

Concerning commuting costs, we have the same assumptions as in previous chapters so that the total commuting cost of an employed worker residing in $x$ and working in $x_c$ is equal to: $s\tau x+(1-s)\,\tau\,|x-x_c|$ while the total commuting cost of an unemployed worker residing in $x$ is equal to: $s\tau x$.

We are now able to write the budget constraint of an employed worker residing in $x$ and working in $x_c$. It is given by:

$$R(x)+s\tau x+(1-s)\,\tau\,|x-x_c|+z_L=w_L(x_c)$$

The unemployed located at $x$ has the following budget constraint:

$$R(x)+s\tau x+z_U=w_U$$

We assume the same preferences for all workers as in the benchmark model in chapter 4. Therefore, each employed and unemployed worker has the following bid rent function:

$$\Psi_L(x,W_L)=w_L(x_c)-s\tau x-(1-s)\,\tau\,|x-x_c|-W_L \tag{5.1}$$

$$\Psi_U(x,W_U)=w_U-s\tau x-W_U \tag{5.2}$$

There exists a continuum of identical firms, which allows us to treat their distribution in the city in terms of density. The firms' density in each point $x$ of the city is denoted by $\lambda(x)$ and the total mass of firms is equal to $M$. Each firm uses a fixed quantity of land $\overline{H}$ and a variable quantity of labor $l$ to produce $y$. The production function in each firm is thus given by: $y=f(\overline{H},el)$, with $f(\overline{H},0)=f(0)=0$, $\partial f(.)/\partial l>0$, $\frac{\partial^2 f(.)}{\partial L^2}\leq 0$ , and the Inada conditions hold, i.e., $f'(0)=+\infty$ and $f'(+\infty)=0$.

Observe that $l$ is the labor demand per firm so that the total employment level in the economy is $L=l\,M$. The labor demand in each firm, $l$, is determined by profit maximization. Since all firms are identical, we have $l=L/M$ and the aggregate production function is given by: $F(\overline{H},L)=Mf(\overline{H},eL/M)$. Moreover, since $eF'(\overline{H},eL)=ef'(\overline{H},eL/M)$, the labor demand can be determined by the profit maximization of one (representative) firm.

We have now to model agglomeration forces and this is the main original part of this section (compared to the benchmark model). In our framework, the main force of agglomeration is the fact that production needs transactions between firms (information exchanges, face to face communication...). There are different ways to model these transactions. Since we want to focus on the endogenous formation of a monocentric city, we have chosen the following one. The total transaction cost between a firm located at $x$ and all the other firms in the city is equal to:[9]

$$\kappa\Theta(x) = \kappa \int_{-x_f}^{x_f} \lambda(\xi) \left| x - \xi \right| d\xi = \kappa \left[ \int_{-x_f}^{x} \lambda(\xi)(x-\xi)d\xi + \int_{x}^{x_f} \lambda(\xi)(\xi - x)d\xi \right]$$

where $\kappa$ denotes the transaction cost per unit of distance, $\lambda(x)$, the density of firms at $x$, and $\Theta(x)$, the total distance of transaction for a firm located at $x$.

This assumption is very important for the urban equilibrium configuration since it affects both workers' and firms' bid rents. For example, with this type of function we cannot obtain a duocentric city (see Fujita, 1990, for an extensive discussion of this issue). In fact, it is essentially the second derivative of $T(x)$ that plays a fundamental role. We further assume that within a *business area* the density of firms $\lambda(x)$ is constant and equal to $1/\overline{H}$ (a business area is an area when only firms locate). We have therefore:

$$\Theta'(x) = \int_{-x_f}^{x_f} \lambda(\xi)d\xi - \int_{x}^{x_f} \lambda(\xi)d\xi = 2x\lambda(x) = \frac{2x}{\overline{H}} \tag{5.3}$$

$$\Theta''(x) = \frac{2}{\overline{H}} \tag{5.4}$$

where $\Theta(x)$ is a convex function inside an area where firms are concentrated (business area), i.e., $\lambda(x) > 0$, and is linear in residential areas, i.e., $\lambda(x) = 0$. We are now able to write the profit function of each firm located at $x$ as follows:

$$\Pi(x) = y - R(x)\overline{H} - w_L(x)l - \kappa\Theta(x) \tag{5.5}$$

where the price of product is normalized to 1, $w_L(x)$ is the wage profile that will be defined below. The objective of each firm is to chose a location $x$ that maximizes its profit (5.5). Its bid rent, which is the maximum land rent that a firm is ready to pay at location $x$ to achieve profit level $\Pi_F$, given the distribution of firms $\lambda(x)$, is therefore given by:

$$\Phi(x, \Pi_F) = \frac{1}{\overline{H}} \left[ y - w_L(x)l - \kappa\Theta(x) - \Pi_F \right] \tag{5.6}$$

[9] In the previous section, the interaction between firms was mainly a black box and was such that: $\kappa\,\Theta(x) = \kappa\, x$.

where $\Pi_F$ is the equilibrium profit level common to all firms.

Finally, by using the following definition: two firms $x$ and $x'$ are connected if $|x_c - x'_c| = 0$, we can spell out our last assumption. There are no commuting costs for workers within connected firms. This assumption is made for simplicity but does not affect the main result. It can be relaxed in two ways. First, workers can bear positive commuting costs within connected firms (as in Fujita and Ogawa, 1980). Second, all workers can have the same total commuting cost whenever they enter the interval of connected firms which is equal to a fixed cost times the average size of the interval. However, both cases complicate the analysis (the second one being easier) without altering the main results.

In equilibrium, we will focus only on a monocentric configuration so that all firms will be connected in the middle of the city. In this context, a natural interpretation of this last assumption is that this connected interval corresponds to a shopping mall so that workers have a positive commuting cost to go there but then, within the mall, no commuting cost. The idea is to open the black box of the (spaceless) CBD developed in the urban literature while keeping the same interpretation of a CBD in which individuals work and shop.

### 5.2. The urban-land use equilibrium

We want to find equilibrium conditions for the endogenous formation of a *linear* and *monocentric* city. We have assumed that the city is symmetric so that we can consider only the right side of it, that is the interval $[0, x_f]$. A monocentric city is such that (on the right of 0):

$$\begin{array}{llll} \psi(x) = 0 & \text{and} & \lambda(x) = 1/\overline{H} & \text{for} \quad x \in [0, x_e] \\ \psi(x) = 1 & \text{and} & \lambda(x) = 0 & \text{for} \quad x \in [x_e, x_f] \end{array}$$

where $\psi(x)$ is the density of workers at $x$. This means that firms locate in the CBD, i.e., in the interval $[-x_e, x_e]$, and workers reside outside of it.

Because of the assumptions of no commuting costs for workers within connected firms and no cross-commuting for workers, in a monocentric city the equilibrium wage profile is given by:

$$w_L(x_c) = w_L \tag{5.7}$$

where $w_L$ is the efficiency wage that will be determined later. Equation (5.7) means that *there is no wage gradient in the city* since wages do no depend on distance. By using (5.6), this implies that the bid rent of firms is equal to:

$$\begin{aligned} \Phi(x,\Pi_F) &= \frac{1}{\overline{H}}\left[f(\overline{H},l)-w_L l-\kappa\Theta(x)-\Pi_F\right] \\ &= \frac{1}{\overline{H}}\left[f(\overline{H},l)-w_L l-\kappa\left(\frac{x^2+x_e^2}{\overline{H}}\right)-\Pi_F\right] \end{aligned} \quad (5.8)$$

We thus have

$$\Phi'(x,\Pi_F)=\begin{cases} -2\kappa x/\overline{H}^2<0 & \text{for } x\in[0,x_e] \\ 0 & \text{for } x\in\,]x_e,x_f] \end{cases} \quad (5.9)$$

and

$$\Phi''(x,\Pi_F)=\begin{cases} -2\kappa/\overline{H}^2<0 & \text{for } x\in[0,x_e] \\ 0 & \text{for } x\in\,]x_e,x_f] \end{cases} \quad (5.10)$$

We are now able to locate all workers in the city. By using (5.1) and (5.2), the employed and unemployed workers respectively have the following bid rents:

$$\Psi_L(x,W_L)=w_L-\tau(x-x_e)-W_L \quad (5.11)$$

$$\Psi_U(x,W_U)=w_U-s\tau(x-x_e)-W_U \quad (5.12)$$

**Proposition 9.** *The unemployed reside at the outskirts of the city whereas the employed workers locate at the vicinity of the city-center.*

In the present model, Proposition 9 is derived because the housing consumption is the same for all workers and commuting trips are lower for the unemployed.

Let us denote by $x_b$ on the right of 0 (and thus $-x_b$ on the left of 0) the border between the employed and the unemployed. This means that the employed reside between $x_e$ and $x_b$ (on the right of 0) and the unemployed between $x_b$ and $x_f$ (see Figure 6.8).

[*Insert Figure* 6.8 *here*]

The monocentric urban equilibrium configuration is when firms outbid workers outside the CBD. Consequently, let us write the equilibrium conditions for a monocentric city. As stated above, all firms are located in the CBD between $-x_e$ and $x_e$ (0 being in the middle of this interval), the employed workers reside between $-x_b$ and $-x_e$ (on the left of 0) and between $x_b$ and $x_e$

(on the right of 0) and the unemployed workers reside between $-x_f$ and $-x_e$ (on the left of 0) and between $x_e$ and $x_f$ (on the right of 0), as described by Figure 6.8. Since the equilibrium is symmetric, the analysis can be performed only on the right side of the city, i.e., between 0 and $x_f$. If we denote by $R_A$ the agricultural land rent (outside the city) and normalize it to zero, the equilibrium conditions are given by:

**Definition 7.** *An urban-land use equilibrium with endogenous CBD is a* $7-$ *uple* $(W_L^*, W_U^*, \Pi_F^*, x_e^*, x_b^*, x_f^*, R^*(x))$ *such that:*

$$\Phi(x_e^*, \Pi_F^*) = \Psi_L(x_e^*, W_L^*) \tag{5.13}$$

$$\Psi_L(x_b^*, W_L^*) = \Psi_U(x_b^*, W_U^*) \tag{5.14}$$

$$\Psi_U(x_f^*, W_U^*) = R_A = 0 \tag{5.15}$$

$$R^*(x) = \max\left\{\Psi_L(x, W_L^*), \Psi_U(x, W_U^*), \Phi(x, \Pi_F^*), 0\right\} \qquad \text{for } x \in \left[0, x_f^*\right] \tag{5.16}$$

$$\int_0^{x_e^*} L\lambda(x)dx = \frac{L\,M}{2} \qquad \text{for } x \in [0, x_e^*] \tag{5.17}$$

$$\int_{x_e^*}^{x_b^*} \psi(x)dx = \frac{L\,M}{2} \qquad \text{for } x \in [x_e, x_b^*] \tag{5.18}$$

$$\int_{x_b^*}^{x_f^*} \psi(x)dx = \frac{U}{2} \qquad \text{for } x \in \left[x_b, x_f^*\right] \tag{5.19}$$

Let us comment these equilibrium conditions. The land market conditions (5.13)-(5.16) ensure that landlords offer land to the highest bid rents, that in the CBD firms outbid workers and outside the CBD the employed outbid the unemployed and that the land rent market is continuous. The last three equations are the standard population constraints. By solving (5.17)-(5.19), we easily obtain:

$$x_e^* = -x_e^* = \frac{\overline{H}M}{2} \tag{5.20}$$

$$x_b^* = -x_b^* = \frac{L + \overline{H}M}{2} \tag{5.21}$$

$$x_f^* = -x_f^* = \frac{N + \overline{H}M}{2} \tag{5.22}$$

We are now able to determine the equilibrium utility and profit levels. We easily obtain:

$$W_L^* = w_L - \frac{\tau}{2}\left[sN + (1-s)\,L^*\right] \tag{5.23}$$

$$W_U^* = w_U - s\tau\frac{N}{2} \tag{5.24}$$

$$\Pi_F^* = f(\overline{H}, eL^*/M) - w_L L^*/M - \frac{\tau}{2}\overline{H}\left[sN + (1-s)\,L^*\right] - \kappa\frac{\overline{H}M^2}{2} \tag{5.25}$$

where $L^* = l^* M$ is the equilibrium employment level in the economy and $N = L^* M + U$. Observe that the equilibrium profit $\Pi_F^*$ depends (negatively) on workers' commuting costs $\tau$ because of the competition in the land market. Indeed, firms have to bid away the employed workers to occupy the core of the city.

Moreover, it is useful to identify the *equilibrium space costs,* i.e., land rent plus travel costs plus transaction costs (the latter is only for firms) for the employed, the unemployed and firms (identified by the subscript $F$) which are respectively given by:

$$SC_L^* = \frac{\tau}{2}\left[sN + (1-s)\,L^*\right] \tag{5.26}$$

$$SC_U^* = s\tau\frac{N}{2} \tag{5.27}$$

$$SC_F^* = \frac{\tau}{2}\overline{H}\left[sN + (1-s)\,L^*\right] + \kappa\frac{\overline{H}M^2}{2} \tag{5.28}$$

This yields the following *space-cost differential* between the employed and the unemployed:

$$\Delta SC^* = SC_L^* - SC_U^* = (1-s)\,\frac{\tau}{2}L^* \tag{5.29}$$

which will have a crucial role in the model. In fact, given that commuting costs are null within the CBD, $\Delta SC^*$ corresponds to the commuting costs of the last worker employed by firms.

Finally, the equilibrium land rent is given by:

$$R^*(x) = \begin{cases} \tau\left[sN + (1-s)L^*\right]/2 + \kappa\left(\frac{M^2}{4} - \frac{x^2}{\overline{H}^2}\right) & \text{for } x \in [-x_e^*, x_e^*] \\ \tau\left[\left(sN + (1-s)L^* + \overline{H}M\right)/2 - |x|\right] & \text{for } x \in [-x_b^*, -x_e^*] \\ & \text{and } x \in [x_e^*, x_b^*] \\ s\tau\left[\left(\overline{N} + \overline{H}M\right)/2 - |x|)\right] & \text{for } x \in \left[-x_f^*, -x_b^*\right] \\ & \text{and } x \in \left[x_b^*, x_f^*\right] \\ 0 & \text{for } x \in \left]-\infty, -x_f^*\right] \\ & \text{and } x \in \left[x_f^*, +\infty\right[ \end{cases} \tag{5.30}$$

## 5.3. The steady-state equilibrium

As usual, let us solve the labor-market equilibrium using the efficiency wage framework. First, observe that $W_L^S = W_L^*$ and $W_L^{NS} = W_L^S - e = w_L - e - \frac{\tau}{2}\left[sN + (1-s)L^*\right]$. Now, to calculate the urban efficiency wage, we have to solve:

$$I_L^{NS} = I_L^S = I_L$$

where

$$r\, I_L^{NS} = w_L - e - \frac{\tau}{2}\left[sN + (1-s)L\right] - \delta\left(I_L^{NS} - I_U\right)$$

$$r\, I_L^S = w_L - \frac{\tau}{2}\left[sN + (1-s)L\right] - (\delta + m)\left(I_L^S - I_U\right)$$

$$r\, I_U = w_U - s\tau\frac{N}{2} + a(I_L - I_U)$$

We easily obtain the following *urban* efficiency wage:

$$w_L = w_U + e + \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau\frac{L}{2} \tag{5.31}$$

which is exactly the same efficiency wage as in the benchmark case ($L$ is divided by 2 because we consider the whole city instead of half of it as in the benchmark case).

The labor market equilibrium is now described. Each firm solves the following program:

$$\max_L \Pi_F^* \qquad s.t. \qquad w \geq w_L^* \tag{5.32}$$

where $\Pi_F^*$ is defined by (5.25). The solution of (5.32) is:

$$w_L + (1-s)\,\tau \overline{H} M/2 = eF'(\overline{H}, eL) \tag{5.33}$$

which defines the labor demand curve. It is important to observe that, contrary to the case where firms were exogenously located in the CBD (Chapters 4 and 5), the labor demand curve is now negatively affected by $\tau$ the (per unit of distance) commuting cost of workers. Why? Because when a firm wants to hire one additional worker, the gain is $eF'(\overline{H}, eL)$ the marginal productivity of this worker. However, hiring this worker will impose two costs to the firm: the wage $w_L^*$ as well as the one resulting from a fiercer competition in the land market. Indeed, firms have to propose higher bids to push away more employed workers in order to occupy the central part of the city. This leads to an additional cost of $(1-s)\,\tau \overline{H} M/2$, where $(1-s)\,\tau$ is the marginal increase in land rent when an additional worker is hired and $\overline{H} M/2$, the location of the firm which is the furthest away, i.e. located at $x_e^*$. This effect is new and never present in standard urban labor market models since it is generally assumed that the location of firms is exogenous and the CBD is reduced to a point. We believe that it is quite interesting since it establishes a link between land and labor markets. In fact, when a firm hires an additional worker, it anticipates the additional cost in the land market so that the total cost of hiring a new worker is $w_L + (1-s)\,\tau \overline{H} M/2$ while the gain is $eF'(\overline{H}, eL)$. It is easy to show that there exists a unique labor market equilibrium.

We now have to check that there exists a unique urban equilibrium as described by Figure 6.8. By plugging the efficiency wage (5.31) in (5.23), (5.24) and (5.25), we obtain:

$$W_L^* = w_U + e + \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right) - s\tau \frac{N}{2} \tag{5.34}$$

$$W_U^* = w_U - s\tau \frac{N}{2} \tag{5.35}$$

$$\begin{aligned} \Pi_F^* &= f(\overline{H}, eL^*/M) - \left[w_U + e + \frac{e}{m}\left(\frac{\delta N}{N-L} + r\right)\right]\frac{L^*}{M} \\ &\quad -\frac{\tau}{2}\left[(1-s)\,L^{*2}/M - \overline{H}\left[sN + (1-s)\,L^*\right]\right] - \kappa \frac{\overline{H} M^2}{2} \end{aligned} \tag{5.36}$$

where $L^* \equiv L^*(\tau, \overline{H}, M, w_U, m, e)$ is defined by (5.33).

We have the following result.

**Proposition 10.** *The monocentric city is an equilibrium configuration if the following condition holds:*

$$\tau \leq \frac{\kappa M}{2\overline{H}} \tag{5.37}$$

The following comments are in order. First, the endogenous formation of a monocentric city is possible only if workers' commuting cost $\tau$ is low and firms' transaction cost $\kappa$ (per unit of distance) is important. This is quite intuitive since the transaction cost is the *agglomeration force* to the CBD for firms (via $\kappa\Theta(x)$), and the commuting cost is the *dispersion force* for firms (via the efficiency wage) and the *attraction force* for workers. Thus in order to have a monocentric city it must be that firms bid away workers from the CBD so that the agglomeration force dominates the dispersion force. Second, the augmentation of $\overline{H}$, firms' land consumption, has a negative impact on the city formation $\overline{H}$ since it affects negatively profits and thus firms' bid rent. Third, the endogenous monocentric city formation is more likely to occur when $M$, the number of firms, is large since transaction costs increase with $M$.

More generally, we have the following comments. First, when $\tau$ varies, it modifies the Urban Non Shirking Condition (UNSC) curve through its effect on the space-cost differential. If, for example, $\tau$ decreases, then the UNSC curve shifts downward (or rightward) so that, for any given employment level $L$, wages are lower compared to the initial situation. This is because the space-cost differential decreases and thus firms, who want to induce workers to stay employed, have to compensate less their workers in terms of commuting costs. This is what we call the *compensation effect.*

Second, when $\tau$ varies, it affects the labor demand curve since the cost of an additional worker is modified because of changes in the intensity of competition in the land market. More precisely, when commuting are lower, the attraction to the city-center is weaker since it less costly to go there and thus competition for central location is less intense so that land prices decrease. Therefore, when $\tau$ decreases, the labor demand curve shifts upward (or rightward) so that, for any given wage level, employment is higher compared to the initial case. The explanation is that, when a firm hires an additional worker, its marginal cost is lower than before because of a weaker competition in the land market. This is referred to as the *spatial effect.*

Third, the net effect of this variation is the following. When $\tau$ is reduced, the UNSC curve shifts downward and the labor demand curve shifts upward. Thus, employment unambiguously rises but wages can either increase or decrease depending of the slopes of these two curves. The main message of this analysis is that both land and labor markets interact.

## 6. Non-technical summary and notes on the literature

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# A. Appendix A.6.1. Proof of Proposition 1

In this model, given that $w_U^C, e, m, \delta, N, r$ are exogenous, an equilibrium is calculated as follows. First, from (2.13), one can calculate the urban efficiency wage as a function of $L^C$ and $L^R$, that is $w_L^C(L^C, L^R)$. Second, by plugging this value $w_L^C(L^C, L^R)$ in (2.14), one obtains an (implicit) relationship between $L^C$ and $L^R$, that we write $L_w^C(L^R)$ and is given by

$$w_U^C + e + \frac{e}{m}\left[\frac{\delta\,(N - L^R)}{N - L^C - L^R} + r\right] + (1-s)\,\tau\, L^C = F'^C(L^C) \qquad \text{(A.1)}$$

By totally differentiating (A.1) and using the Inada conditions, we easily obtain:

$$\frac{\partial L_w^C}{\partial L^R} < 0\ ,\ \lim_{L^R \to 0} L_w^C = L_0^C\ ,\ \lim_{L_w^C \to 0} L^R = N$$

where $0 < L_w^C(L^R) < L_0^C < N$ is the unique solution of the following equation

$$w_U^C + e + \frac{e}{m}\left[\frac{\delta\, N}{N - L_0^C} + r\right] + (1-s)\,\tau\, L_0^C = F'^C(L_0^C)$$

Third, the equilibrium-migration condition (2.15) gives another (implicit) relationship between $L^C$ and $L^R$, that we denote by $L_h^C(L^R)$ and has the following properties:

$$\frac{\partial L_h^C}{\partial L^R} < 0\ ,\ \lim_{L^R \to 0} L_h^C = N\ ,\ \lim_{L_h^C \to 0} L^R = L_0^R$$

where $0 < L_h^C(L^R) < L_0^R < N$ is the unique solution of the following equation

$$w_U^C - s\tau\,(N - L^R) = \frac{F'^R(L^R)}{r}$$

Figure A3.1 in Appendix 3 (we have here qualitatively the same figure than in the spaceless case) describes the two curves (A.1) (labor demand equation, $L_w^C(L^R)$) and (2.15) (migration equilibrium condition, $L_h^C(L^R)$) in the plane $(L^R, L^C)$ and it is easy to see that there exists a unique equilibrium that gives a unique value of $L^C$ and a unique value of $L^R$ that we denote by $(L^{R*}, L^{C*})$. It is also easy to verify that this equilibrium is such that $0 < L^{C*} < N$ and $0 < L^{R*} < N$.

Finally, plugging $L^{R*}$ and $L^{C*}$ in (2.13), (2.4), (2.1) and (2.7) gives respectively the unique equilibrium values of $w_L^{C*}, w_L^{R*}, U^{C*}, R^{C*}(x)$. ∎

# B. Appendix A.6.2. Proof of (4.18), (4.19), and (4.20)

Let us determine the steady-states values of $L_1^*$, $L_2^*$, and $U^*$ respectively given by (4.18), (4.19), and (4.20). There are two ways to determine these values.

In the first approach, given a transition matrix $P$, we have to solve the following system:

$$\begin{pmatrix} U & L_1 & L_2 \end{pmatrix} P = \begin{pmatrix} U \\ L_1 \\ L_2 \end{pmatrix}$$

Now, given that $N = U + L_1 + L_2$ (see (4.1)), this system can be written as:

$$\begin{pmatrix} U & L_1 & N - L_1 - U \end{pmatrix} P = \begin{pmatrix} U \\ L_1 \\ N - L_1 - U \end{pmatrix} \tag{B.1}$$

Since the transition matrix $P$ is given by (4.17), this system can be written as:

$$\begin{pmatrix} U & L_1 & N - L_1 - U \end{pmatrix} \begin{pmatrix} 1-(a_1+a_2) & a_1 & a_2 \\ \delta & 1-\delta & 0 \\ \delta & a_1 & 1-(a_1+\delta) \end{pmatrix}$$
$$= \begin{pmatrix} U \\ L_1 \\ N - L_1 - U \end{pmatrix}$$

By solving this system of equations, we easily obtain (observe that the last equation is redundant):

$$U^* = \frac{\delta N}{a_1 + a_2 + \delta}$$
$$L_1^* = \frac{a_1 N}{\delta + a_1}$$

Then, by using (4.1), we finally obtain:

$$L_2^* = \frac{a_2\, \delta\, N}{(a_1+\delta)\,(a_1+a_2+\delta)}$$

These are the values obtained in the text and corresponding to (4.18), (4.19), and (4.20), respectively.

In the second approach, which has more economic intuition, we have to write the dynamic equations that describe the different flows in the labor market. From the transition matrix $P$ given by (4.17), we have:

$$\overset{\bullet}{U} = \delta\,(L_{1,t} + L_{2,t}) - (a_1 + a_2)\,U_t$$

$$\overset{\bullet}{L}_1 = a_1 (U_t + L_{2,t}) - \delta L_{1,t}$$

$$\overset{\bullet}{L}_2 = a_2 U_t - \delta L_{2,t} - a_1 L_{2,t}$$

where $\overset{\bullet}{U} = dU/dt$, $\overset{\bullet}{L}_1 = dL_1/dt$ and $\overset{\bullet}{L}_2 = dL_2/dt$. In steady-state, $\overset{\bullet}{U} = \overset{\bullet}{L}_1 = \overset{\bullet}{L}_2 = 0$ and thus we obtain

$$\delta (L_1 + L_2) = (a_1 + a_2) U$$

$$a_1 (U + L_2) = \delta L_1$$

$$a_2 U = \delta L_2 + a_1 L_2$$

Solving these three equations using (4.1) leads to (4.18), (4.19), and (4.20). ∎

## C. Appendix A.6.3. Proof of (4.39), (4.39), and (4.41)

We proceed as in Appendix B but for the case of shirking.

In the first approach, given that $P$ is determined by (4.38), (B.1) can now be written as:

$$\begin{pmatrix} U & L_1 & N - L_1 - U \end{pmatrix} \begin{pmatrix} 1-(a_1+a_2) & a_1 & a_2 \\ \delta + m & 1-(\delta+m) & 0 \\ \delta & a_1 & 1-(a_1+\delta) \end{pmatrix}$$

$$= \begin{pmatrix} U \\ L_1 \\ N - L_1 - U \end{pmatrix}$$

By solving this system of equations and using (4.1), we easily obtain (observe that the last equation is redundant):

$$U^{S*} = \frac{(\delta+m)(a_1+\delta) N}{(a_1+\delta+m)(a_1+a_2+\delta)}$$

$$L_1^{S*} = \frac{a_1 N}{a_1+\delta+m}$$

$$L_2^{S*} = \frac{a_2 (\delta+m) N}{(a_1+\delta+m)(a_1+a_2+\delta)}$$

In the second approach, which has more economic intuition, we have to write the dynamic equations that describe the different flows in the labor market. From the transition matrix $P$ given by (4.17), we have:

$$\overset{\bullet}{U} = (\delta+m) L_{1,t} + \delta L_{2,t} - (a_1+a_2) U_t$$

$$\overset{\bullet}{L}_1 = a_1 \left(U_t + L_{2,t}\right) - \left(\delta + m\right) L_{1,t}$$

$$\overset{\bullet}{L}_2 = a_2 U_t - \delta L_{2,t} - a_1 L_{2,t}$$

where $\overset{\bullet}{U} = dU/dt$, $\overset{\bullet}{L}_1 = dL_1/dt$ and $\overset{\bullet}{L}_2 = dL_2/dt$. In steady-state, $\overset{\bullet}{U} = \overset{\bullet}{L}_1 = \overset{\bullet}{L}_2 = 0$ and thus we obtain

$$\left(\delta + m\right) L_1 + \delta L_2 = \left(a_1 + a_2\right) U$$

$$a_1 \left(U + L_2\right) = \left(\delta + m\right) L_1$$

$$a_2 U = \delta L_2 + a_1 L_2$$

Solving these three equations using (4.1) leads to (4.39), (4.39), and (4.41). ∎

Figure 6.1: Rural-urban equilibrium

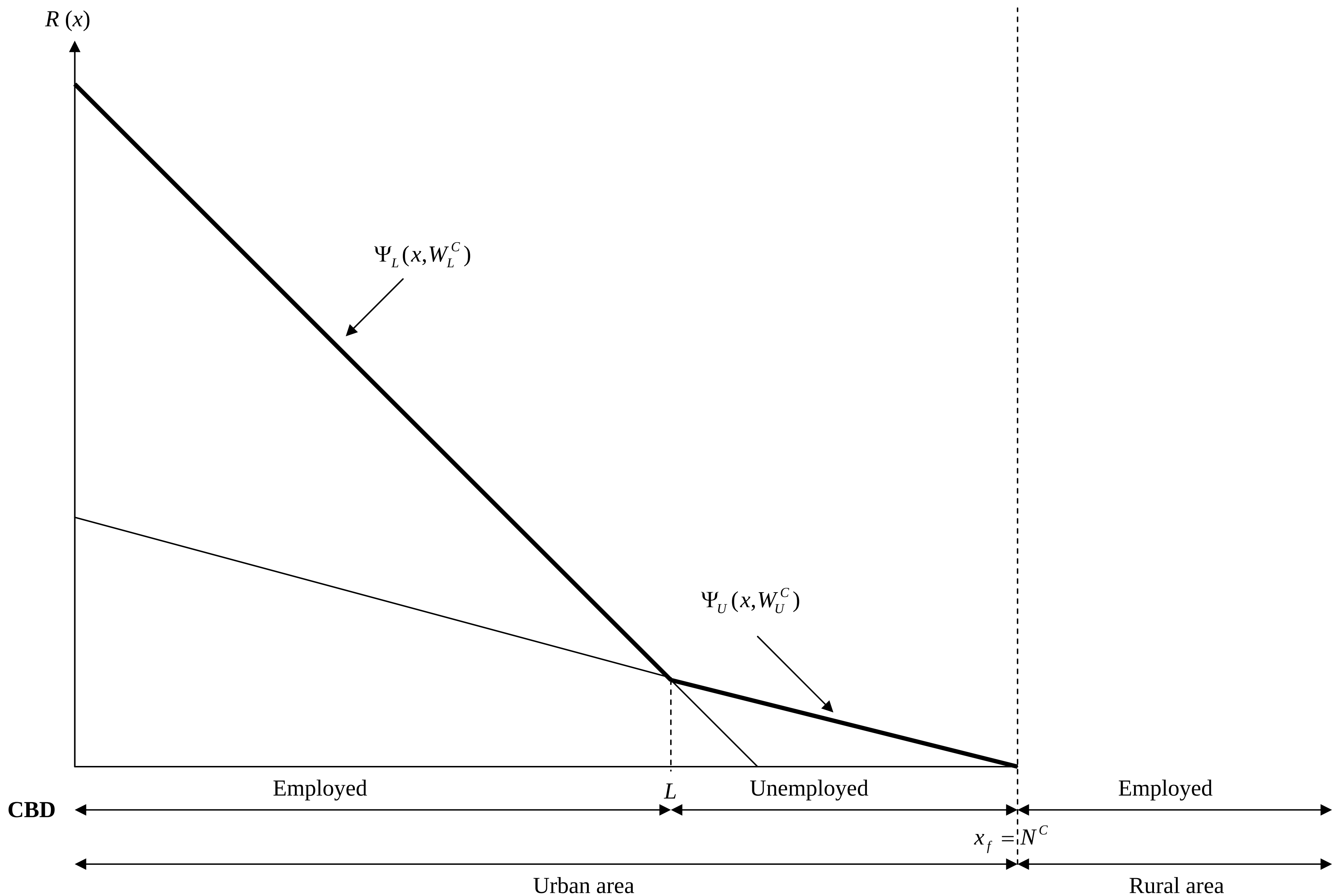

Figure 6.2. Two-city equilibrium

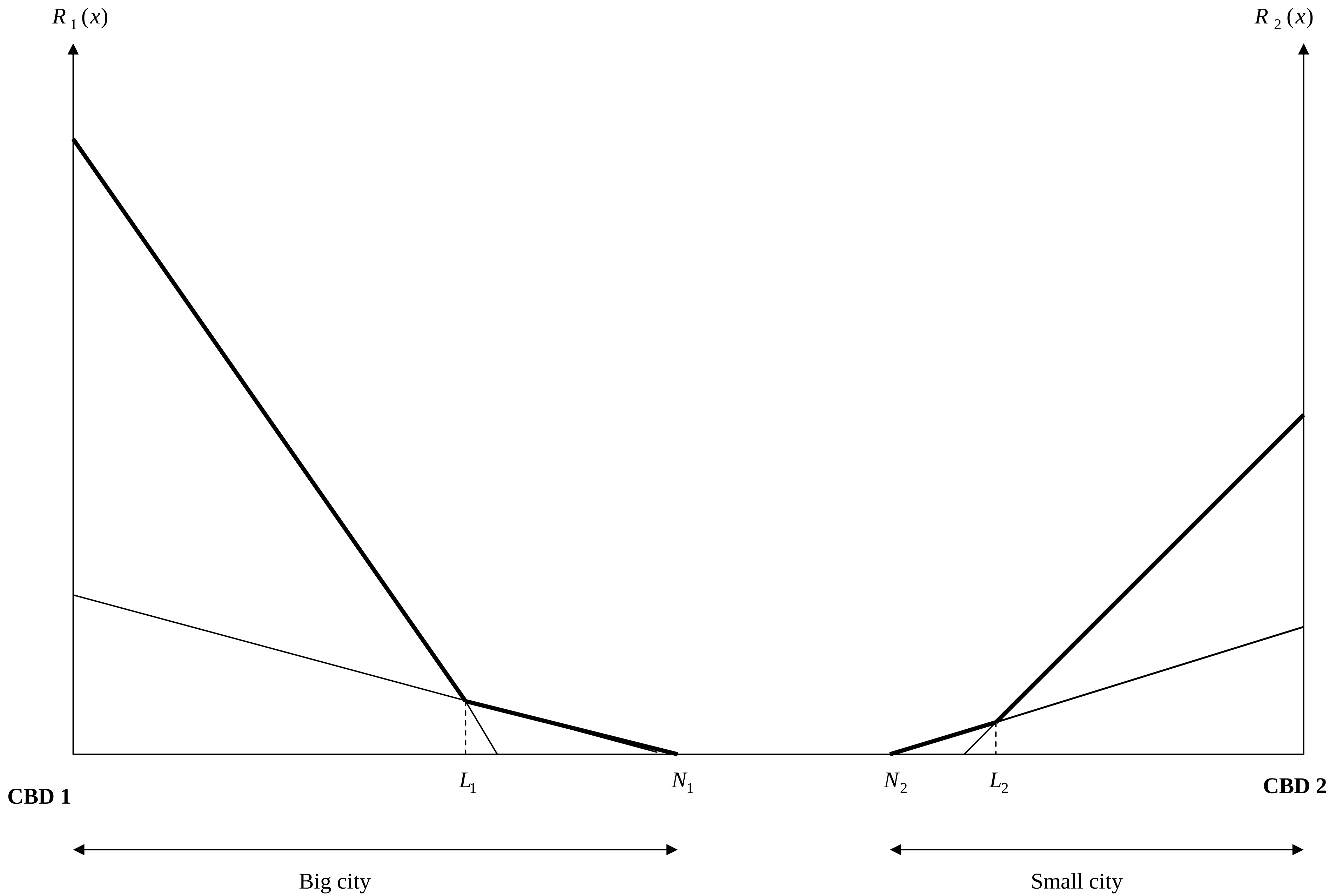

Figure 6.3. Land-use equilibrium in a duocentric city

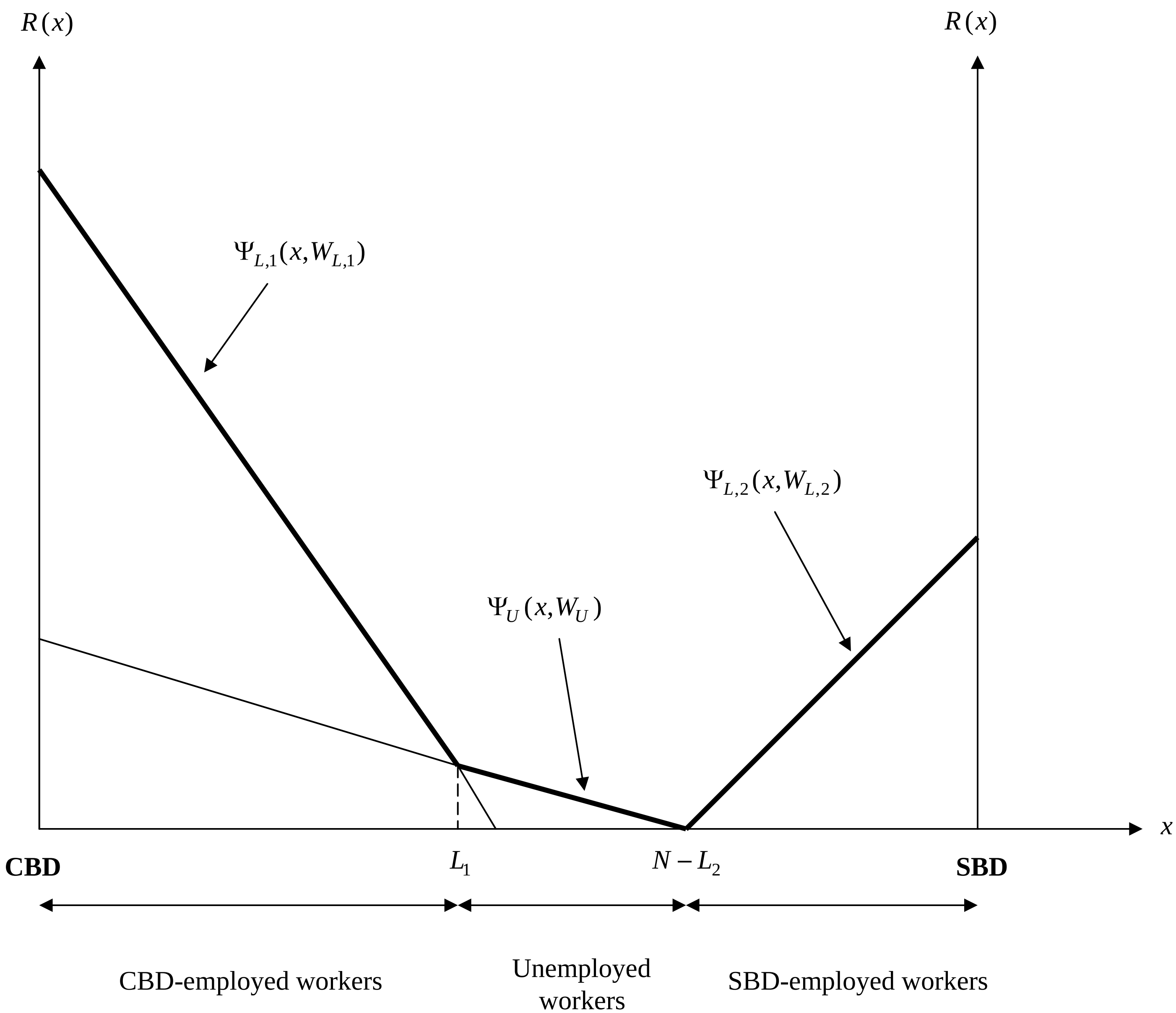

Figure 6.4. Flows in the labor market in a duocentric city

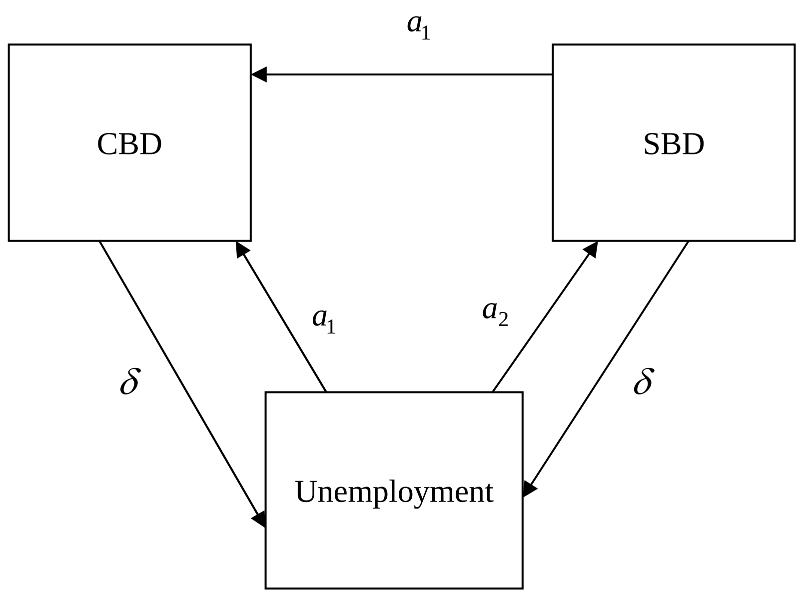

Figure 6.5: Labor equilibrium in the duocentric city

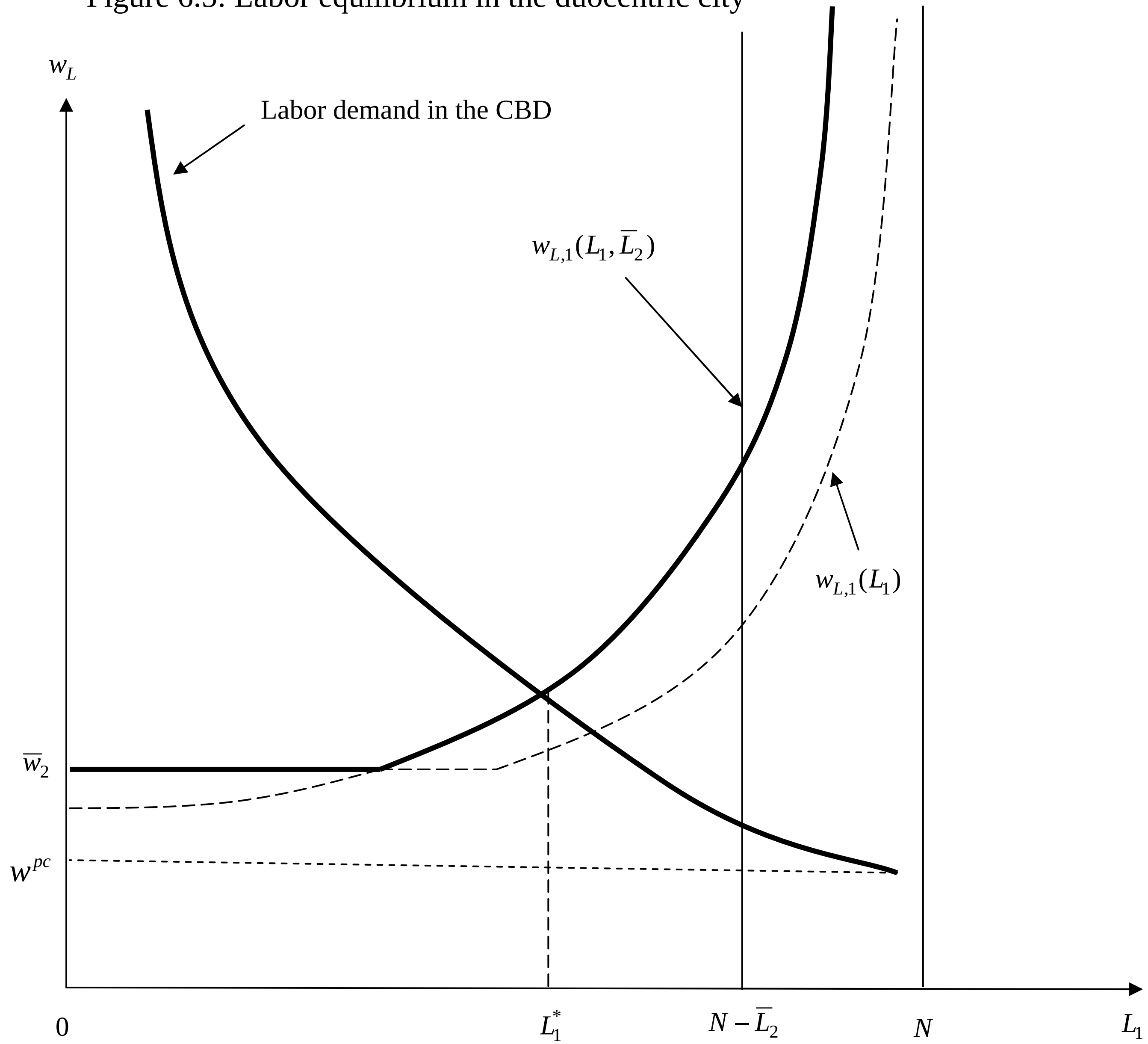

Figure 6.6a: The case of a suburb

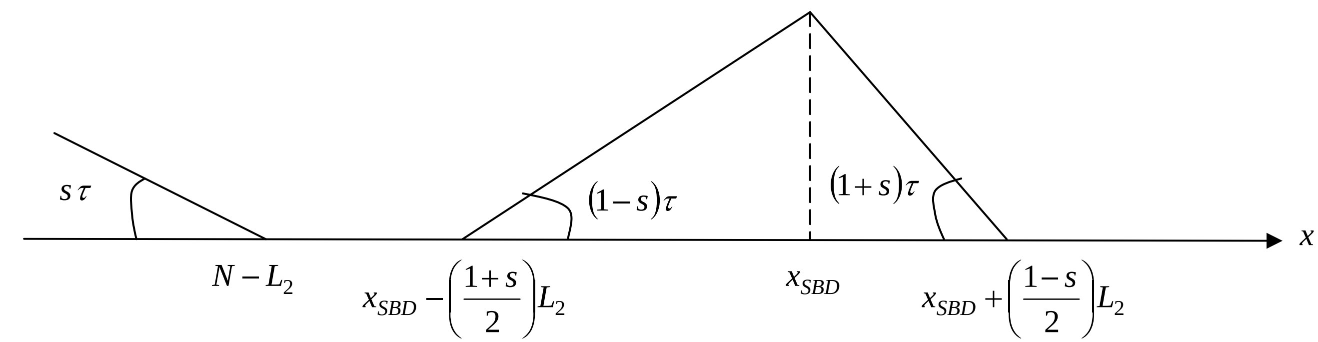


Figure 6.6b: The case of an edge city

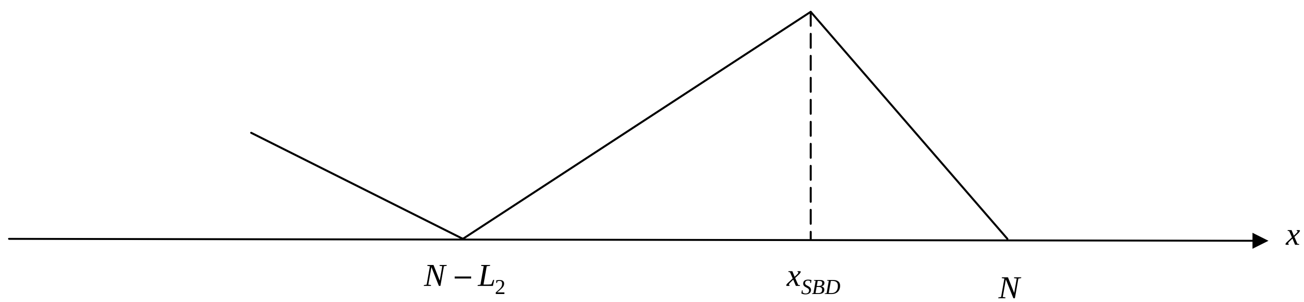


Figure 6.6c: The case of a subcenter

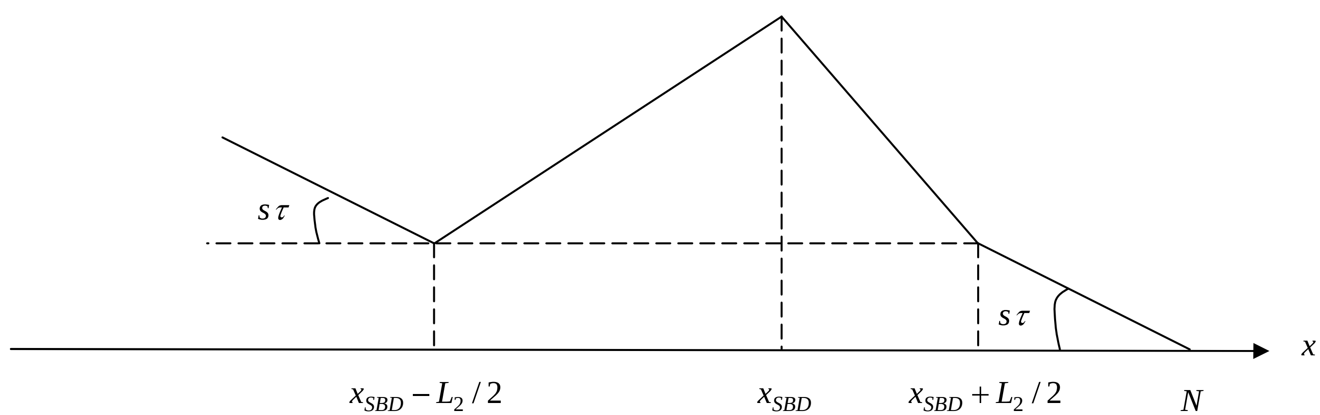

Figure 6.7. Land-use equilibrium in a duocentric city with endogenous SBD

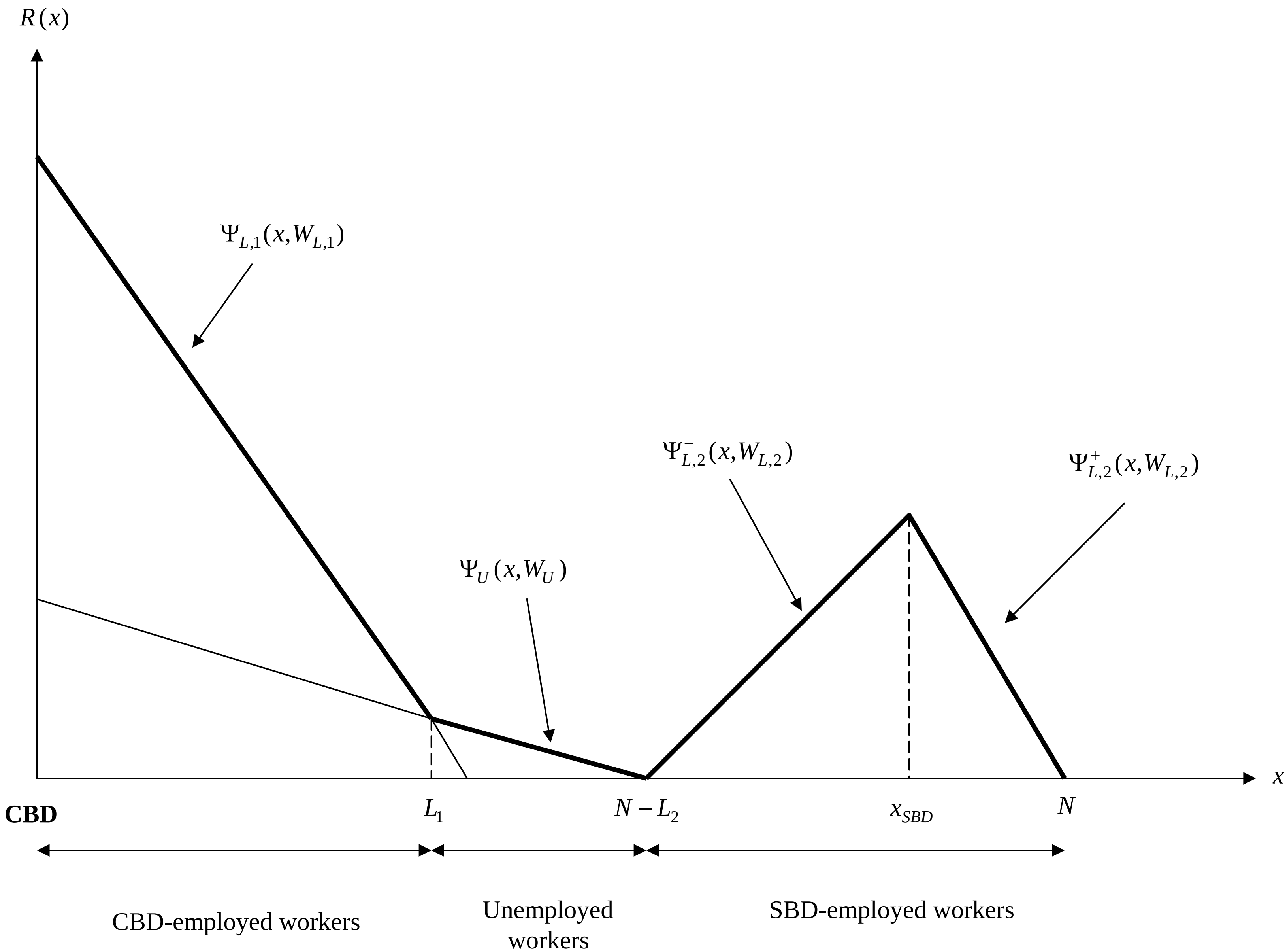

Figure 6.8. Agglomeration economies with unemployment