Lecture Notes 5

Extensive Form Games

Insofar, *normal form* representation of complete information non-cooperative games.

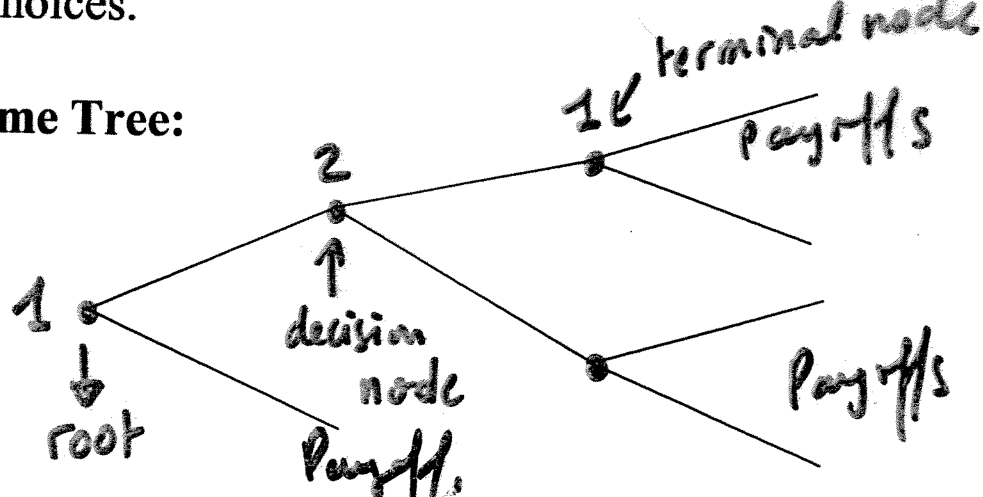
Ideal to represent situations where agents:

- make once for all choices
- move simultaneously

The *extensive form* of a game specifies:

- Physical order of play
- How many times a player gets to choose
- Actions available to players each time they get to choose
- What players know when they move
- Eventual payoffs to each player for any *sequence* of choices.

Game Tree:



- decision node: point where only one player has to make a decision

- branches: alternative choices available to that player

  - initial decision node ≡ **root**
  - **terminal nodes** (no branches emanate from them)

Restrictions are placed on the precedence between decision nodes to ensure a well-defined play for every choice of strategies (Dutta p. 158-160).

- information sets: set of nodes that are *indistinguishable* from the decision maker's standpoint.

  → If they contain:
  - 1 decision node: perfect information
  - multiple nodes: imperfect information

- In a game tree, a **strategy** s_i for player i prescribes a choice (or a probability distribution over possible choices) **for each contingency he might be called on to make a decision.**

- Solution concepts:

1. **Backward induction** (BI) - perfect information
2. **Subgame Perfect Nash Equilibrium** (SPNE) - perfect/imperfect information (generalisation of BI)

Note: A normal form game can be represented using an extensive form (e.g. Prisoner's Dilemma).

Games of Perfect Information

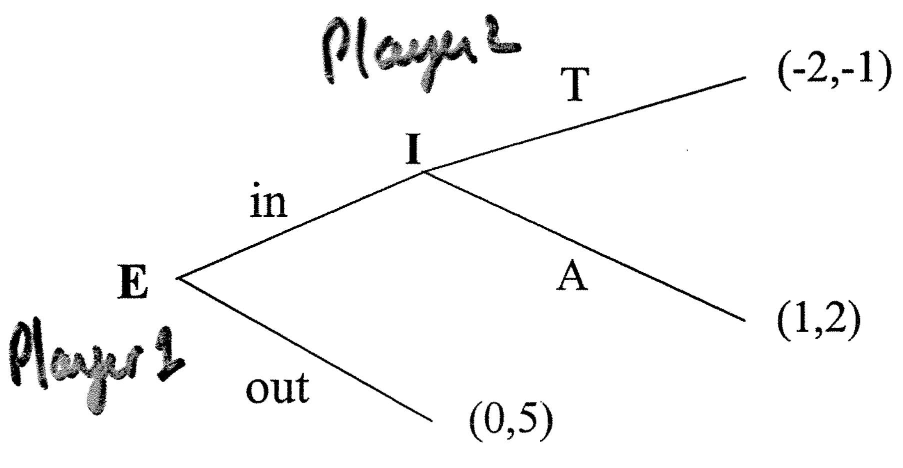
Def.: A **game of perfect information** is one in which there is no information set with multiple nodes.

⇒ at any node each player knows the entire *history* of play.

Example: **Entry Game**

- Entry decision by a firm (entrant) and subsequent post-entry decisions by firm(s) currently supplying the market.
- Entry decision depends on potential profitability of the new market.
- Potential profitability depends on how incumbent firms *react* to entry in the post-entry stage: *fight* or *accommodate* entry?

Entry game I (perfect information):



- Players: $I=\{E,I\}$
- Strategy Spaces: $S_E=\{in, out\}$; $S_I=\{T,A\}$
- Strategy profiles: e.g. $(s_E,s_I)=(in,T)$
- Payoffs at terminal nodes: e.g. $u(in,T) = (-2,-1)$

What is a reasonable prediction of play?

In dynamic games, the central issue is <u>credibility</u>:
Is the threat of playing "tough" by the incumbent if E enters the market a credible one?

Solution Concept: Backward Induction

<u>Key idea</u>: principle of sequential rationality: at each decision node, a player picks the best action given what he thinks is going to be the future play of the game.

Step 1: consider firm I decision node (final decision node):

If firm E enters, what action maximises firm I's payoff?

Step 2: consider firm E decision node (penultimate node and root at the same time):

Given firm I would ……..enter if firm E enters, what action maximises firm E's payoff? Action IN

The backward induction (sequentially rational) outcome is (IN, A)

In general:

1. Final decision nodes (any decision terminates the game):
   ⇒ decision maker picks action that max his payoff

2. Penultimate decision nodes: decision maker knows the exact consequence (final payoff) of his moves.
   ⇒ decision maker picks action that max his payoff

3. Fold the game tree back 1 step at a time till the root of the game.