Urban Labor Economics

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June 27, 2006

Part 1: Urban Search-Matching

Chapter 1: Simple Models of Urban Search Matching

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1. Introduction

The search-matching model is now the standard approach in labor economics (Pissarides, 2000). In this chapter, we first develop a canonical model of urban search matching, that is we introduce a land market in a standard search matching model with search intensity. The link between the land and the labor market is realized through the average search intensity of unemployed workers. Indeed, the latter depends on the location of the unemployed workers in the city, which is endogenously determined in the land-use equilibrium. The location of workers, in turn, depends on the outcomes in the labor market. In order to understand the way the two markets operate, we first develop a simple model where search intensity is exogenous (section 2). Because of this assumption, only one urban pattern emerges in which the employed reside close to jobs and the unemployed at the periphery of the city. In section 3, we extend this benchmark model by assuming that workers' search intensity negatively depends on their residential distance to jobs. This leads to two urban-land use patterns where the unemployed workers either reside close to or far away from jobs. In section 4, unemployed workers endogenously choose their search intensity and we show that they search less, the further away they reside from jobs. Apart of the two previous urban patterns, there is a third urban equilibrium that emerges: the core-periphery equilibrium where the unemployed workers reside both close to (short-run unemployed workers) and far away (long-run unemployed workers) from jobs while the employed workers live in between them. In each model we explore the labor market consequences (job creation, unemployment and wages) of the urban land use pattern. Finally, in section 5, we discuss some robustness results.

2. The benchmark model

There is a continuum of ex ante identical workers whose mass is N and a continuum of M identical firms. Among the N workers, there are L employed and U unemployed so that $N = L + U$. The workers are uniformly distributed along a *linear*, *closed* and *monocentric* city. Their density at each location is taken to be 1. All land is owned by absentee landlords and all firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers are assumed to be infinitely lived, risk neutral and decide their optimal place of residence between the CBD and the city fringe. There are *no relocation costs*, either in terms of time or money. We will relax this assumption in chapter 2.

Each individual is identified with one unit of labor. If employed, workers commute to the CBD to work. As in chapter 1, the *instantaneous* (indirect) utility of an employed worker located at a distance x from the CBD is given by:

$$W_L(x) = w_L - \tau\, x - R(x) \tag{2.1}$$

Concerning the unemployed, they commute less often to the CBD since they mainly go there to search for jobs. So, we assume that they incur a commuting cost $s\,\tau$ per unit of distance, where $0 < s \leq 1$ is a measure of search intensity or search efficiency; s is assumed to be exogenous. For example $s = 1$ would mean that the unemployed workers go everyday to the CBD (as often as the employed workers) to search for jobs. Observe that here we assume that unemployed workers need to go to the CBD to obtain information about jobs, that is why they commute there. If for example $s = 0$, which we exclude here, then they would never find a job. We would discuss alternative ways of gathering information about jobs through search in sections below and in chapter 8.

As in chapter 1, the *instantaneous* (indirect) utility of an unemployed worker residing at a distance x from the CBD is equal to:

$$W_U(x) = w_U - s\,\tau\, x - R(x) \tag{2.2}$$

Let us describe the labor market. A firm is a unit of production that can either be filled by a worker whose production is y units of output or be unfilled and thus unproductive. In order to find a worker, a firm posts a vacancy. A vacancy can be filled according to a random Poisson process. Similarly, workers searching for a job will find one according to a random Poisson process. In aggregate, these processes imply that there is a number of contacts per unit of time between the two sides of the market that are determined by the following matching function:[1]

$$d(\overline{s}U, V)$$

where $\overline{s}$ is the average search efficiency of the unemployed workers, U and V the total number of unemployed and vacancies respectively. In this simple model, it is assumed that $s = \overline{s}$, so each worker provides the same search effort.[2] As in

[1] This matching function is written under the assumption that the city is monocentric, i.e. all firms are located in one fixed location.

[2] The search intensity s will be endogeneized below.

the standard search-matching model (see e.g. Mortensen and Pissarides, 1999, and Pissarides, 2000), we assume that $d(.)$ is increasing both in its arguments, concave and homogeneous of degree 1 (or equivalently has constant return to scale). Thus, the rate at which vacancies are filled is $d(sU, V)/V$. By constant return to scale, it can be rewritten as

$$d(1/\theta, 1) \equiv q(\theta)$$

where $\theta = V/(sU)$ is a measure of *labor market tightness* in efficiency units and $q(\theta)$ is a Poisson intensity. By using the properties of $d(.)$, it is easily verified that $q'(\theta) \leq 0$: the higher the labor market tightness, the lower the rate at which firm fill their vacancy. Similarly, the rate at which an unemployed worker with search intensity s leaves unemployment is

$$a = \frac{s}{\overline{s}} \frac{d(\overline{s}U, V)}{U} \equiv s\theta q(\theta)$$

Again, by using the properties of $d(.)$, it is easily verified that $[\theta q(\theta)]' \geq 0$: the higher the labor market tightness, the higher the rate at which workers leave unemployment since there are relatively more jobs than unemployed workers. So here the higher the search intensity s (unemployed search more actively for jobs), the higher is this rate $s\theta q(\theta)$. Finally, the rate at which jobs are destroyed is exogenous and denoted by δ.

If, in this model, there are no frictions, then unemployment and vacancies disappear, and jobs are found and filled instantaneously. Indeed,

$$\lim_{\theta \to 0} [\theta q(\theta)] = \lim_{\theta \to +\infty} q(\theta) = 0 \tag{2.3}$$

and

$$\lim_{\theta \to +\infty} [\theta q(\theta)] = \lim_{\theta \to 0} q(\theta) = +\infty \tag{2.4}$$

That is, if $\theta \to 0$, then the number of unemployed is infinite and thus firms filled their job instantaneously (no frictions on the firm's side), whereas if $\theta \to 0$, then the number of vacancies is infinite and thus workers find a job instantaneously (no frictions on the worker's side).

As stated in Part 1, a steady-state equilibrium requires solving *simultaneously* an urban land use equilibrium and a labor market equilibrium. It is convenient to present first the former and then the latter.

2.1. Urban land use equilibrium

Since there are no relocation costs, the urban equilibrium is such that all the employed enjoy the same level of utility W_L while all the unemployed obtain

W_U. Bid rents are respectively given by:

$$\Psi_L(x, W_L) = w_L - \tau\, x - W_L \tag{2.5}$$

$$\Psi_U(x, W_U) = w_U - s\,\tau\, x - W_U \tag{2.6}$$

They are both linear and decreasing in x. We have the following straightforward result:

Proposition 1. *With workers' risk neutrality, fixed housing consumption and fixed search intensity, the employed reside close to jobs whereas the unemployed live at the periphery of the city.*

Let us now define the urban-land use equilibrium. We denote the agricultural land rent (the rent outside the city or opportunity rent) by R_A and, without loss of generality, we normalize it to zero. We have:

Definition 1. *An urban-land use equilibrium with no relocation costs, fixed-housing consumption and fixed search intensity is a 5-tuple* $(W_L^*, W_U^*, x_b^*, x_f^*, R^*(x))$ *such that:*

$$\Psi_L(x_b^*, W_L^*) = \Psi_U(x_b^*, W_U^*) \tag{2.7}$$

$$\Psi_U(x_f^*, W_U^*) = R_A = 0 \tag{2.8}$$

$$\int_0^{x_b^*} \frac{1}{h_L} dx = L \tag{2.9}$$

$$\int_{x_b^*}^{x_f^*} \frac{1}{h_U} dx = N - L \tag{2.10}$$

$$R^*(x) = \max\{\Psi_L(x, W_L^*), \Psi_U(x, W_U^*), 0\} \quad \textit{at each } x \in (0, x_f] \tag{2.11}$$

By solving (2.7) and (2.8), we easily obtain the equilibrium values of the instantaneous utilities of the employed and the unemployed. They are given by:

$$\begin{aligned} W_L^* &= w_L - \tau\, x_b^* - s\tau\, \left(x_f^* - x_b^*\right) \\ &= w_L - \tau\, L - s\tau\, (N - L) \end{aligned} \tag{2.12}$$

$$W_U^* = w_U - s\tau\, x_f^* = w_U - s\,\tau\, N \tag{2.13}$$

The employment zone (i.e. the residential zone for the employed workers) is thus $(0, L]$ and the unemployment zone (i.e. the residential zone for the unemployed workers) is thus $[L, N]$. By plugging (2.12) and (2.13) into (2.7) and (2.8), we easily obtain the land rent equilibrium $R(x)$. It is given by:

$$R^*(x) = \begin{cases} \tau\, (L - x) + s\tau\, (N - L) & \text{for} \quad 0 \le x \le L \\ s\,\tau\, (N - x) & \text{for} \quad L < x \le N \\ 0 & \text{for} \quad x > N \end{cases} \tag{2.14}$$

2.2. Steady-state equilibrium

We are now able to solve the labor market equilibrium and thus the steady-state equilibrium. The unemployment rate is defined by $u = U/N$. We have:

Definition 2. *A (steady-state) labor market equilibrium* (w_L^*, θ^*, u^*) *is such that, given the matching technology* $d(\cdot)$*, all agents (workers and firms) maximize their respective objective function, i.e. this triple is determined by a steady-state condition, a free-entry condition for firms and a wage-setting mechanism.*

In steady-state, the Bellman equations for the employed and unemployed are respectively given by:[3]

$$rI_L = w_L - \tau\, L - s\tau\ (N - L) - \delta\left[I_L - I_U(s)\right] \tag{2.15}$$

$$rI_U(s) = w_U - s\,\tau\, N + s\theta q(\theta)\left[I_L - I_U(s)\right] \tag{2.16}$$

where r is the exogenous discount rate. The interpretation of the Bellman equations is similar to that of Part 1. This implies that:

$$I_L - I_U(s) = \frac{w_L - w_U - (1-s)\,\tau L}{r + \delta + s\theta q(\theta)} \tag{2.17}$$

We denote respectively by I_F and I_V the intertemporal profit of a job and of a vacancy. If c is the search cost for the firm per unit of time and y is the product of the match, then, at the steady-state, I_F and I_V can be written as:

$$rI_F = y - w_L - \delta(I_F - I_V) \tag{2.18}$$

$$rI_V = -c + q(\theta)(I_F - I_V) \tag{2.19}$$

We assume that firms post vacancies up to a point where:

$$I_V = 0 \tag{2.20}$$

which is a free entry condition. From (2.20) and using (2.19), the value of a job is now equal to:

$$I_F = \frac{c}{q(\theta)} \tag{2.21}$$

Finally, plugging (2.21) into (2.18) and using (2.20), we obtain the following decreasing relation between labor market tightness and wages in equilibrium:

$$\frac{c}{q(\theta)} = \frac{y - w_L}{r + \delta} \tag{2.22}$$

[3] For the derivations of the Bellman equations, see Appendix 2 at the end of this book.

In words, the value of a job is equal to the expected search cost, i.e. the cost per unit of time multiplied by the average duration of search for the firm. So, firms' job creation is endogenous and is determined by (2.22).

Let us now determine the wage. At each period, the total intertemporal surplus is shared through a generalized Nash-bargaining process between the firm and the worker. The total surplus is the sum of the surplus of the workers, $I_L - I_U$, and the surplus of the firms $I_F - I_V$. At each period, the wage is determined by:

$$w_L = \arg\max_{w_L} (I_L - I_U)^{\beta} (I_F - I_V)^{1-\beta} \tag{2.23}$$

where $0 \leq \beta \leq 1$ is the bargaining power of workers. Observe that I_U, the threat point for the worker does not depend on the current location of the worker, who will relocate if there is a transition in his/her employment status. First order condition yields:

$$\frac{\beta}{1-\beta} \left(\frac{\partial I_L}{\partial w_L} - \frac{\partial I_U}{\partial w_L} \right) I_F + (I_L - I_u) \frac{\partial I_F}{\partial w_L} = 0 \tag{2.24}$$

Since the wage is negotiated at each period, I_U does not depend on the current wage w_L and so $\frac{\partial I_U}{\partial w_L} = 0$. Since by (2.15), $\frac{\partial I_L}{\partial w_L} = 1/(r+\delta)$, by (2.21), $I_F = c/q(\theta)$ and by (2.18), $\frac{\partial I_F}{\partial w_L} = -1/(r+\delta)$, equation (2.24) can be written as:

$$I_L - I_U = \frac{\beta}{1-\beta} \frac{c}{q(\theta)} \tag{2.25}$$

Then, using (2.17) and (2.22), we finally obtain:

$$w_L = (1-\beta) \left[w_U + (1-s)\, \tau L \right] + \beta \left(y + cs\theta \right) \tag{2.26}$$

In search-matching models, the wage-setting curve WS (a relation between wages and the state of the labor market, here θ) replaces the traditional labor-supply curve. It is here given by (2.26). Let us interpret this equation. The first part $(1-\beta)\left[w_U + (1-s)\,\tau L\right]$, is what firms must pay to induce workers to accept the job offer: firms must exactly compensate the transportation cost difference (between the employed and the unemployed) of the employed worker who is the furthest away from the CBD, i.e. located at $x = x_b = L$. This is referred to as the 'compensation effect' This is exactly the same effect that we obtained with the efficiency wage (chapter 1). The second part $\beta\,(y + cs\theta)$ is the bargaining effect where workers obtain a part of the surplus. This is referred to as the 'outside option effect'. The first effect is a pure spatial cost since $(1-s)\,\tau L$ represents the space cost differential between employed and unemployed workers paying the same bid rent whereas the second effect is a mixed labor-spatial cost one. Furthermore, observe that θ has a positive impact on w_L, implying in particular that unemployment negatively affects wages. Finally, it is interesting to note that the search intensity s has an ambiguous effect on wages. On the one hand, when s increases, unemployed workers go more to the CBD, so the difference in commuting costs between the employed and the unemployed is lower and thus firms need to compensate less the employed workers; thus wages decrease. On the other hand, when s increases, unemployed workers find jobs at a higher rate

Let us close the model. Since each job is destroyed according to a Poisson process with arrival rate δ, the number of workers who enter unemployment is $\delta(1-u)$ and the number who leave unemployment is $\theta q(\theta) s\, u$. The evolution of the unemployment rate is thus given by the difference between these two flows,

$$\overset{\bullet}{u} = \delta(1-u) - \theta q(\theta) s\, u \tag{2.27}$$

where $\overset{\bullet}{u}$ is the variation of unemployment with respect to time. In steady state, the rate of unemployment is constant and therefore these two flows are equal (flows out of unemployment equal flows into unemployment). We thus have:[4]

$$u^* = \frac{\delta}{\delta + s\theta q(\theta)} \tag{2.28}$$

It is easy to verify that $\frac{\partial u^*}{\partial \theta} < 0$. Indeed, a higher labor market tightness implies that workers leave unemployment at a faster rate and thus unemployment has to decrease for (2.28) to hold. Furthermore, by using (2.3) and (2.4), we easily obtain that:

$$\lim_{\theta \to 0} u^* = 1 \text{ and } \lim_{\theta \to +\infty} u^* = 0$$

When there are no frictions for workers then nobody is unemployed ($u^* = 0$) while all workers have to be unemployed ($u^* = 1$) for firms to have no frictions.

There is another more intuitive way of representing equation (2.28). Indeed, (2.28) can be written as:

$$\delta N(1-u) - V\, q(V/Nu\overline{s}) = 0$$

and we obtain the so-called *Beveridge curve.* This is an upward sloping curve in the $U - V$ space. Now, by combining (2.22) and (2.26) and by observing

[4] Using (2.27), it is easy to show that:

$$\lim_{t \to +\infty} u_t = u^*$$

Thus, the steady-state u^* is a stable equilibrium.

that $L^* = N(1-u^*)$, the market solution that defines the equilibrium θ^* is given by:

$$y - w_U = \frac{c}{q(\theta^*)}\left[\frac{r+\delta+\beta s\theta^* q(\theta^*)}{1-\beta}\right] + (1-s)\,\tau N\,(1-u) \tag{2.29}$$

Let us study its properties in the plane (θ, u). We have:

$$\frac{\partial u}{\partial \theta} = -\frac{c}{q(\theta)^2}\frac{r+\delta}{1-\beta}q'(\theta) + \frac{c\beta s}{1-\beta} > 0$$

Indeed, when unemployment increases, it is easier for firms to find a worker and thus they create more jobs, which increases θ. Now by combining (2.28) and (2.29), we obtain:

$$\frac{c}{q(\theta^*)}\left[\frac{r+\delta+\beta s\theta^* q(\theta^*)}{1-\beta}\right] - (1-s)\,\tau N\frac{\delta}{\delta + s\theta^* q(\theta^*)} + (1-s)\,\tau N - (y - w_U) = 0 \tag{2.30}$$

Denote by $\Lambda(\theta)$, the left-hand side of this equation. Then, it is easy to verify that:

$$\Lambda'(\theta) > 0\ ,\ \lim_{\theta \to 0}\Lambda(\theta) = -\infty \text{ and } \lim_{\theta \to +\infty}\Lambda(\theta) = +\infty$$

This implies that there exists a unique θ^* that solves (2.30). Plugging this value in (2.26) and (2.28) gives a unique wage w^* and a unique unemployment rate u^*.

2.3. Interaction between land and labor markets

Let us write the four equilibrium equations that determine the four endogenous variables $R^*(x)$, W_L^*, W_U^* and θ^*. Using the value of w_L and u^* respectively given by (2.26) and (2.28), and by observing that $L^* = N(1-u^*)$, we obtain:

$$R^*(x) = \begin{cases} \tau\,(N-x) - (1-s)\,\tau N\frac{\delta}{\delta+s\theta^* q(\theta^*)} & \text{for } 0 \le x \le N\frac{s\theta^* q(\theta^*)}{\delta+s\theta^* q(\theta^*)} \\ s\,\tau\,(N-x) & \text{for } N\frac{s\theta^* q(\theta^*)}{\delta+s\theta^* q(\theta^*)} < x \le N \\ 0 & \text{for } x > N \end{cases}$$

$$W_L^* = (1-\beta)\,w_U + \beta\,(y + cs\theta^*) + (1-s)\,\beta\tau N\frac{\delta}{\delta+s\theta^* q(\theta^*)} - [1-(1-s)\,(1-\beta)]\,\tau N$$

$$W_U^* = w_U - s\,\tau\,N$$

$$y - w_U = \frac{c}{q(\theta^*)}\left[\frac{r+\delta+\beta s\theta^* q(\theta^*)}{1-\beta}\right] + (1-s)\,\tau N\frac{s\theta^* q(\theta^*)}{\delta+s\theta^* q(\theta^*)}$$

The key variables is θ^*, which appears everywhere except in the utility of the unemployed. Compared to the standard job-matching model with no space (Pissarides, 2000, chap. 1), apart from the land market, the main difference resides in equation (2.29) or (2.30) and thus in the determination of the equilibrium job creation rate θ^*. Indeed, there is here one more term, $(1-s)\,\tau N\,(1-u^*)$, which comes from the wage equation (2.26) and corresponds to the spatial compensation firms have to give to workers for them to accept a job. Interestingly, this term depends on an endogenous variable u^*, which here is a measure of distance to jobs since the worker who resides the furthest away from jobs is located at $L^* = N\,(1-u^*)$. This is why now the two equations (2.28) and (2.29) are linked together (this is not the case in Pissarides, 2000) and the comparative statics of θ^* will be affected. Indeed, by totally differentiating equation (2.30), we easily obtain:

$$\theta^* = \left(\underset{+}{y}, \underset{-}{w_U}, \underset{-}{c}, \underset{-}{r}, \underset{?}{\delta}, \underset{-}{\beta}, \underset{?}{s}, \underset{-}{\tau}, \underset{-}{N}\right) \tag{2.31}$$

where a question mark means that the sign is ambiguous. The effect of y, w_U, c and r are standard and can be found in Pissarides (2000). Indeed, an increase in productivity y or a decrease in unemployment benefit w_U, or in firms' search cost c, or in the discount rate r, or in workers' bargaining power β, leads to more job creation because more firms enter the labor market, anticipating higher expected profit. The new effects concern parameters that enter in the term $(1-s)\,\tau N\,(1-u^*)$. Take for example the job destruction rate δ. There are two opposite effects, so the net effect on θ^* is ambiguous. Indeed, when δ increases, unemployment increases, which leads to higher job creation since firms need to compensate less workers in terms of wages ($(1-s)\,\tau N\,(1-u)$ decreases); the worker who is the furthest away is now closer to jobs. However, when δ increases, jobs are destroyed at a faster rate, which implies that less firms enter in the labor market and thus there is less job created. In Pissarides (2000), because the term $(1-s)\,\tau N\,(1-u)$ is absent, only the last effect prevails and thus an increase in δ always reduces job creation.

Take now the spatial variables s and τ. For the commuting cost τ it is quite easy since it enters only in $(1-s)\,\tau N\,(1-u)$. So, when τ increases, the spatial compensation in terms of wages increases and thus less jobs are created. For the search intensity s, it is more subtle because s affects both the spatial compensation and the efficiency of the labor market (the higher s, the lower the unemployment rate). The next effect is ambiguous. Indeed, an increase in search intensity reduces the unemployment rate. This has two effects. On the one hand, it decreases the spatial compensation of the wage but, on the other, it gives a better outside option for employed workers who require higher wages. The net effect on wages and thus on job creation θ^* is thus ambiguous. Once again, in Pissarides (2000, chap. 5) where search intensity is introduced, only the second effect is present so that higher s implies higher wages and thus lower θ^*.

The interaction between land and labor markets is thus captured by the comparative statics exercise of θ^* since the distance to jobs of the last employed worker is affecting wages and thus job creation θ^*. To study further the interaction between the two markets, note that θ^* also affects both the employment and unemployment zones and the land rent $R^*(x)$ in the employment zone. Indeed, for the latter, when θ^* increases, less people are unemployed and thus the employment zone is augmented while the unemployment zone is reduced. As a result, bid rents and thus $R^*(x)$ for the employed workers increase at each $x \in [0, x_b]$. We can study the comparative statics of $R^*(x)$ in the employment zone. We obtain:

$$R^* = \left(\underset{+}{y}, \underset{-}{w_U}, \underset{-}{c}, \underset{-}{r}, \underset{?}{\delta}, \underset{-}{\beta}, \underset{?}{s}, \underset{?}{\tau}, \underset{?}{N} \right)$$

The effects of each parameter on $R^*(x)$ can be derived from those of θ^* (see (2.31)). Observe however that the spatial variables s, τ, N have now an ambiguous sign because they affect both directly and indirectly (through θ^*) bid rents.

2.4. Welfare and efficiency

Let us study the welfare of this economy. The social welfare function is given by the sum of the utilities of the employed and the unemployed, the production of the firms net of search costs and the land rents received by the (absentee) landlords. The wage w_L as well as the land rent $R(x)$, being pure transfers, are thus excluded in the social welfare function. The latter is therefore given by:

$$\mathcal{W} = \int_0^{+\infty} e^{-rt} \left\{ \int_0^L (y - \tau x) dx + \int_L^N (w_U - s\,\tau\, x) dx - c\,\theta\, s\, u \right\} dt \qquad (2.32)$$

The social planner chooses θ and u that maximize (2.32) under the constraint (2.27). In this problem, the control variable is θ and the state variable is u. Let ϑ be the co-state variable. The Hamiltonian is thus given by:

$$\mathcal{H} = e^{-rt} \left\{ (1-u)(y - \tau L) + u\,(w_U - s\,\tau\, L) - c\, s\, \theta\, u \right\} + \vartheta \left[\delta(1-u) - \theta q(\theta) s\, u \right]$$

The Euler conditions are $\frac{\partial \mathcal{H}}{\partial \theta} = 0$ and $\frac{\partial \mathcal{H}}{\partial u} = -\overset{\bullet}{\vartheta}$. They are thus given by:

$$c\, e^{-rt} + \vartheta\, q(\theta) \left[1 - \eta(\theta)\right] = 0 \tag{2.33}$$

$$\left[y - w_U - (1-u)\, \tau N \,(2-s) - \tau N\, u\, s + c\, s\, \theta\right] e^{-rt} + \vartheta \left[\delta + s\, \theta q(\theta)\right] = \overset{\bullet}{\vartheta}_k \tag{2.34}$$

where $\eta(\theta) = -q'(\theta)\theta/q(\theta)$ is the elasticity of the matching function with respect to unemployment. Let us focus on the steady state equilibrium in which $\overset{\bullet}{\theta} = 0$. By differentiating (2.33), we easily obtain that $\overset{\bullet}{\vartheta} = -r\vartheta$.[5] By plugging this value and the value of ϑ from (2.33) in (2.34), we obtain:

$$y - w_U = \frac{c}{q(\theta)} \left[\frac{r + \delta + s\theta q(\theta)\eta(\theta)}{1 - \eta(\theta)}\right] + (1-s)\, \tau N \,(1-u) \tag{2.35}$$

In order to see if the private and social solutions coincide, we compare (2.29) and (2.35). We have the following result.

Proposition 2. *In a spatial search-matching model with fixed search intensity, where the employed reside close to jobs and the unemployed far away from jobs, the market equilibrium is efficient if and only if:*

$$\eta(\theta) = \beta \tag{2.36}$$

Condition (2.36) is referred to as the Hosios-Pissarides condition and is exactly the same as in the standard matching model with no space (Pissarides, 2000, chap. 8). Indeed, in this model, market failures are caused by search externalities. As it is well-known, the land market is efficient (Fujita, 1989). Let us explain the search externalities. We have seen that the job-acquisition rate is positively related to V and negatively related to U whereas the job-filling rate has exactly the opposite sign. So, for example, negative search externalities arise because of the congestion that firms and workers impose on each other during their search process. Therefore, two types of externalities must be considered: *negative intra-group externalities* (more searching workers reduces the job-acquisition rate) and *positive inter-group externalities* (more searching firms increases the job-acquisition rate). As stated above, for a class of related search-matching models, Hosios (1990) and Pissarides (2000) have

[5] The transversality condition is given by:

$$\lim_{t \to +\infty} \vartheta\, u_t = 0$$

and is obviously verified.

established that these two externalities just offset one another in the sense that search equilibrium is socially efficient if and only if the matching function is homogenous of degree one and the worker's share of surplus β is equal to $\eta(\theta)$ the elasticity of the matching function with respect to unemployment. Of course, there is no reason for β to be equal to $\eta(\theta)$ since these two variables are not related at all and, therefore, the search-matching equilibrium is in general inefficient. However, when β is larger than $\eta(\theta)$, there is too much unemployment, creating congestion in the matching process for the unemployed. When β is lower, there is too little unemployment, creating congestion for firms.

In our present model, we have exactly the same externalities (intra- and inter-group externalities). The spatial dimension does not entail any inefficiency so this is why the Hosios-Pissarides condition still holds, i.e. $\beta = \eta(\theta)$.

3. Search effort as a function of distance to jobs[6]

So far workers' search intensity was exogenous and independent of residential location. There are strong evidence showing that distance to jobs do impact on search behavior.[7] In this section, we would like to endogeneize s by assuming that

$$s(x) = s_0 - s_a x \tag{3.1}$$

where $s_0 > 0$ and $s_a > 0$. In order for the search intensity to be always positive, we impose that $s_0 > s_a N$. The linearity assumption will be very useful in the urban land use analysis. So, with this formulation, when workers are further away from jobs, they search less intensively. Moreover, $\overline{s}$, the aggregate search efficiency in a city will now depend on the average location of the unemployed since it is given by

$$\overline{s} = s_0 - s_a \overline{x} \tag{3.2}$$

By doing so, we are integrating even more the land and the labor market because now one will need to solve both markets simultaneously to obtain outcomes.

Because the model gets quite complicated, we will slightly change the way s was viewed in the previous section. Indeed, in section 2, the search intensity s was affecting both the number of trips to the CBD unemployed workers make to gather information there and the job acquisition rate. This was mainly because, the only way workers were gathering information about jobs, was by

[6] This section is based on Wasmer and Zenou (2002, 2005).

[7] We will discuss this issue in more details in chapter 8.

going to the CBD. Here we make the other extreme assumption that unemployed workers do not need at all to go to the CBD to obtain information about jobs. They obtain information either locally through newspapers, local employment agencies, etc. or through firms' adds and it is not costly. So, basically, when someone is searching more intensively it means here that he/she is more actively searching for jobs by reading more newspapers or checking more often the adds. In this context, (3.1) states that distance to jobs negatively affects workers' search intensity because, by living further away, it becomes more difficult to obtain information about jobs and this has a discouraging effect on search intensity. In section 4 below, we will consider a more general model where s is endogenous and affects both the number of trips to the CBD and the job acquisition rate. Observe that in the present model unemployed workers still need to go to the CBD to shop (as in chapter 1).

3.1. Urban land use equilibrium

Contrary to section 2, we cannot use the instantaneous utilities to solve the urban-land use equilibrium. Indeed, now, when workers decide how much to bid for land, they have to take into account the future prospects in the labor market since their location will partly determines it. The job-acquisition rate of a worker residing at a distance x from the CBD is, as above, given by:

$$a(x) = s(x)\theta q(\theta) = (s_0 - s_a x)\,\theta q(\theta) \tag{3.3}$$

but now depends on a worker's residential location and on the average location of the unemployed since θ depends on $\overline{s}$. So, let us write the Bellman equations for the employed and unemployed workers. By denoting by τ_L and τ_U the commuting cost of the employed and unemployed workers respectively,[8] these equations are respectively given by:

$$rI_L = w_L - \tau_L x - R(x) - \delta\,(I_L - I_U)$$

$$rI_U = w_U - \tau_U x - R(x) + a(x)\,(I_L - I_U)$$

where $\tau_U < \tau_L$ because the unemployed workers go to the CBD only to shop while the employed workers go there to shop and to work. As stated above, the search intensity only affects the job acquisition rate $a(x)$ and not the commuting cost of the unemployed. The main difference is that the value of $\overline{s}$ will depend on the average location of the unemployed $\overline{x}_U$ and thus on

[8] In the previous section, we had: $\tau_L = \tau$ and $\tau_U = s\tau$.

the prevailing urban equilibrium. Since there are no relocation costs, the urban equilibrium is such that all the employed enjoy the same level of utility $rI_L = r\overline{I}_L$ as well as the unemployed $rI_U = r\overline{I}_U$. Bid rents can be written as:

$$\Psi_L(x, \overline{I}_U, \overline{I}_L) = w_L - \tau_L x + \delta \overline{I}_U - (r + \delta)\overline{I}_L \tag{3.4}$$

$$\Psi_U(x, \overline{I}_U, \overline{I}_L) = w_U - \tau_U x + a(x)\overline{I}_L - [r + a(x)]\,\overline{I}_U \tag{3.5}$$

These bid rents are linear (because $s(x)$ is linear in x) and decreasing in x. However, contrary to section 2, no clear urban pattern emerges because workers are trading off commuting costs and access to jobs. We have the following result.

Proposition 3.

(i) *If*

$$\tau_L - \tau_U < w_U \theta^1 q(\theta^1)(\overline{I}_L^1 - \overline{I}_U^1) \tag{3.6}$$

we have Equilibrium 1 in which the unemployed live close to jobs.

(ii) *If*

$$\tau_L - \tau_U > w_U \theta^2 q(\theta^2)(\overline{I}_L^2 - \overline{I}_U^2) \tag{3.7}$$

Equilibrium 2 prevails and the employed live close to jobs.

The unemployed workers would like to live close to jobs both because of their commuting costs and their unemployment duration (which is the inverse of $a(x)$). For the employed workers, it is only their commuting costs that drive their choice. Equation (3.6) states that if the differential in commuting costs per unit of distance between the employed and the unemployed is lower than the marginal expected return of search for the unemployed, then the unemployed workers bid away the employed workers and occupy the center of the city. If on the contrary (3.7) holds, then the employed live close to jobs while the unemployed workers reside at the outskirts of the city. Observe that these conditions depend on the endogenous variable θ that will be determined at the labor market equilibrium. So, at this stage, we do not know if both of them hold.

Given that conditions (3.6) and (3.7) hold, we have the following definitions:

Definition 3. *An urban land use equilibrium $k = 1, 2$ with no relocation costs, fixed-housing consumption and endogenous search intensity is a 5-tuple $(I_L^{k*}, I_U^{k*}, x_b^{k*}, x_f^{k*}, R^{k*}(x))$ such that:*

$$x_b^{1*} = N\,u^1 \qquad \textit{or} \qquad x_b^{2*} = N\left(1 - u^2\right) \tag{3.8}$$

$$x_f^{k*} = N \qquad k = 1,2 \tag{3.9}$$

$$\Psi_U(x_b^{k*}, \overline{I}_U^{k*}, \overline{I}_L^{k*}) = \Psi_L(x_b^k, \overline{I}_U^{k*}, \overline{I}_L^{k*}) \qquad k = 1,2 \tag{3.10}$$

$$\Psi_L(x_f^1, \overline{I}_U^{1*}, \overline{I}_L^{1*}) = 0 \qquad or \qquad \Psi_U(x_f^2, \overline{I}_U^{2*}, \overline{I}_L^{2*}) = 0 \tag{3.11}$$

$$R^{k*}(x) = \max\left\{\Psi_L(x, \overline{I}_U^{k*}, \overline{I}_L^{k*}), \Psi_U(x, \overline{I}_U^{k*}, \overline{I}_L^{k*}), 0\right\} \quad at\ each\ x \in \left(0, x_f^k\right] \tag{3.12}$$

These two equilibria are illustrated in Figures 1.1 and 1.2. By using (3.4), (3.5), (3.8) and replacing them in (3.10) and (3.11), we obtain:

$$I_L^k - I_U^k = \frac{w_L^k - w_U - (\tau_L - \tau_U)x_b^k}{r + \delta + (s_0 - s_a x)\,\theta^k q(\theta^k)} \qquad k = 1,2 \tag{3.13}$$

where w_L^k, u^k, θ^k will be determined at the labor market equilibrium k. The average search intensity in equilibrium k is equal to:

$$\overline{s}^k = s_0 - s_a\,\overline{x}^k \qquad k = 1,2 \tag{3.14}$$

where $\overline{x}^k$ is the average location of the unemployed in equilibrium k. It is easy to verify that $\overline{x}^1 = N\,u^1/2$ and $\overline{x}^2 = N\,(1 - u^2/2)$. Since $\overline{x}_1^1 < \overline{x}^2$ (this is always true because $u^1 + u^2 < 2$), the average search efficiency in urban equilibrium 1 is higher than in urban equilibrium 2, i.e., $\overline{s}^1 > \overline{s}^2$. This result is not surprising . Indeed, in the integrated city (equilibrium 1), the unemployed reside closer to the CBD than in the segregated city (equilibrium 2) and thus the rate at which they leave unemployment is on average higher.

[*Insert Figures* 1.1 and 1.2 *here*]

3.2. Steady-state equilibrium

We can now close the model as before. For each $k = 1,2$, we have a free-try condition that gives the following labor demand curve:

$$\frac{c}{q(\theta^k)} = \frac{y - w_L^k}{r + \delta} \tag{3.15}$$

We can also determined a bargained wage, which is given by:

$$w_L^k = (1-\beta)\left[w_U + (\tau_L - \tau_U)x_b^k\right] + \beta\left[y + \left(s_0 - s_a x_b^k\right)\theta^k c\right] \tag{3.16}$$

Equations (3.16) give the two wage-setting curves (WS_1 and WS_2) and are both positively sloped in the plan (θ_k, w_k). One can show that the curve WS_1 is steeper than WS_2 but the intercept of WS_1 is lower. The reason why WS_1

is steeper than WS_2 is the following. The slope of these curves represents the ability of workers to exert wage pressures when the labor market is tighter. Thus, for a given labor market tightness, a more efficient labor market, i.e. $\overline{s}^1 > \overline{s}^2$, leads to a higher wage pressure per unit of θ, thus a steeper WS_1 curve. The intercept is, however, higher in equilibrium 2 since when $\theta = 0$, the outside option effect vanishes and only the compensation effect remains. Since the same marginal worker is further away in equilibrium 2, he/she must have a higher compensation for his/her transportation costs.

We can finally determine a steady-state conditions on flows:

$$u^k = \frac{\delta}{\delta + \overline{s}^k \theta^k q(\theta^k)} \tag{3.17}$$

As stated in the previous section, the above equation defines in the (u^k, V^k) space a downward sloping curve referred to as the Beveridge curve. The interesting feature of this Beveridge curve is that it is indexed by $\overline{s}^k$, which depends on the spatial dispersion of the unemployed in equilibrium $k = 1, 2$. It can be shown that, in the plane (θ, w), the Beveridge curve in urban equilibrium 2 is always above the Beveridge curve in urban equilibrium 1. Indeed, a lower $\overline{s}^k$ is associated with an outward shift of Beveridge curve in the $u^k - V^k$ space because more vacancies are needed to maintain the steady-state level of unemployment. If s_a increases or s_0 decreases, the Beveridge curve is shifted away from the origin meaning that the labor market is less efficient. The same would arise if the city size increased: the unemployed would be further away.

By combining (3.15) and (3.16), we obtain the following market equilibrium:

$$y - w_U = \frac{c}{q(\theta^k)} \left[\frac{\delta + r + \theta^k q(\theta^k) \left(s_0 - s_a x_b^k\right) \beta}{1 - \beta} \right] + x_b^k \left(\tau_L - \tau_U\right) \tag{3.18}$$

If we define

$$\widehat{\theta} = \frac{1 - \beta}{\beta} \frac{(\tau_L - \tau_U)}{s_a \, c}$$

we have then the following result:

Proposition 4. *There exists a unique and stable steady-state equilibrium* $(I_L^{k*}, I_U^{k*}, x_b^{k*}, x_f^{k*}, R^{k*}(x), w_L^{k*}, \theta^{k*}, u^{k*})$, $k = 1, 2$, *and only the two following cases are possible:*

(*i*) *If* $\widehat{\theta} < \theta^{1*} < \theta^{2*}$, *urban equilibrium 1 prevails;*

(*ii*) *If* $\theta^{2*} < \theta^{1*} < \widehat{\theta}$, *urban equilibrium 2 prevails.*

These two possible cases are respectively plotted in Figures 1.3 and 1.4. Inequalities in (i) and (ii) replace (3.6) and (3.7). Observe that multiple urban equilibria can never happen (i.e. having both a segregated and an integrated city) because, according to Proposition 4, the condition for multiple equilibria is $\theta^{2*} < \widehat{\theta} < \theta^{1*}$ and it can never be verified since the intersection of the labor demand curve with the wage setting curve never satisfies this condition. This is because $\widehat{\theta}$, defined initially as the intersection point between the two wage-setting curves is also the critical value that determines which urban equilibrium prevails.

[*Insert Figures* 1.3 and 1.4. *here*]

3.3. The role of space in search-matching model

3.3.1. Interaction between land and labor markets

The interaction between land and labour markets is partly due to the dependence of search efficiency on distance. To show that, we proceed *a contrario*: we assume first that wages are exogenous and $s_a = 0$. In this case, both markets are independent. When we relax exogenous wages and keep $s_a = 0$, then we are back to section 2 and there is a one-way interaction between markets: the labor market does not depend on the land market but the land market equilibrium depends on the labor market equilibrium through labor market tightness as workers locate in one configuration or the other depending notably on θ^*.

Finally, as soon as $s_a > 0$, one has a general equilibrium interaction between the two markets. Indeed, one of the key assumption of our model is that the search efficiency s of each worker depends on the distance between residence and the job-center, i.e., $s = s(x)$ with $s'(x) < 0$. This implies that the land and labor markets are interdependent. Indeed, on the one hand, the labor market strongly depends on the land market since the equilibrium values of u^{k*}, V^{k*} and θ^{k*} are affected by the value of $\overline{s}^k$. On the other hand, the land market strongly depends on the labour market since the inequality (3.6) or (3.7) determining the land market equilibrium configuration depends on the value of θ^{k*}.

The following table summarizes our discussion:

Table 1.1: Interaction between land and labor markets

	Exogenous wages	Endogenous wages
$s_a = 0$	No Interaction	Partial Interaction ($LME \rightarrow LE$)
$s_a > 0$	Complete Interaction	Complete Interaction

($LME \rightarrow LE$ means that the interaction is from the land market equilibrium to the labour equilibrium)

3.3.2. Decomposition of unemployment

We pursue our analysis by determining the part of unemployment only due to spatial frictions. For simplicity, we only focus on equilibrium 2 where the unemployed live far away from jobs. We also normalize the total population so that $N = 1$. The analysis for the other equilibrium is straightforward since nearly identical. Let us start with exogenous wages. In this case, θ^2 is constant and determined by (3.15). By using (3.17), the unemployment rate is given by:

$$u^2 = \frac{\delta}{\delta + \theta^2 q(\theta^2)\left[s_0 - s_a(1 - u^2/2)\right]} \tag{3.19}$$

Let us further define:

$$u_0^2 = \frac{\delta}{\delta + \theta^2 q(\theta^2) s_0} \tag{3.20}$$

the part of *unemployment that is independent of spatial frictions*, i.e. when $s_a = 0$. By a Taylor first-order expansion for small s_a/s_0, we easily obtain:

$$u^{2*} = u_0^2 \left[1 + \frac{s_a}{s_0}\left(1 - u_0^2\right)\left(1 - u_0^2/2\right)\right] = u_0^2 + u_\sigma^2 \tag{3.21}$$

where $u_\sigma^2 \equiv u_0^2 \left[s_a \left(1 - u_0^2\right)\left(1 - u_0^2/2\right)/s_0\right]$ is the *unemployment that is only due to spatial frictions* and u_0^2 is defined by (3.20). Observe that u_σ^2 is increasing in s_a/s_0, the parameter representing the loss of information through distance to jobs and null when $s_a = 0$. Observe also that the pure frictional unemployment u_0^2 affects u_σ^2 in the following way:

$$\text{If} \quad u_0^2 < 1 - \frac{\sqrt{3}}{3} \approx 0.42 \ , \quad \text{then} \quad \frac{\partial u_\sigma^2}{\partial u_0^2} > 0$$

In general $u_0^2 < 0.42$ so that u_0^2 affects positively u_σ^2, showing the full interaction between land and labour markets. This is quite natural: higher 'spaceless' unemployment u_0^2 affects positively frictions due to spatial heterogeneity (this is a side-effect of the dispersion of space on the unemployed themselves, which increases the average distance to jobs).

Overall, compared to the non-spatial case, unemployment increases because of the loss of information due to spatial dispersion of agents and also because of the wage compensation of commuting costs. However, it also tends to decrease because of the outside option effect that reduces wages.

3.4. Welfare

Let us now proceed to the welfare analysis. The first issue we address is the respective efficiency of the two types of land market configurations. Since there are no multiple equilibria in the land market, we cannot Pareto-rank the equilibria and it is difficult to compare them. Nevertheless, we are able to investigate what happens to the welfare difference in the range of parameters around the frontier separating the two types of land market equilibria. We show in fact that the welfare difference between the two cities is very marginal so that a social planner does not necessarily want to impose the integrated city. We then focus on the efficiency and the welfare analysis of each equilibrium. We show that each equilibrium is in general not efficient and that a social planner can restore the efficiency using transportation policies.

3.4.1. Welfare comparison between the two cities

We first investigate the welfare of the city, and study how it varies in each urban configuration. For each city, we use the welfare function defined by (2.32).

To see how this quantity varies and if it can be compared across cities, let us proceed to a simple numerical resolution of the model. We normalize the total population to 1, i.e. $N = 1$. We use the following Cobb-Douglas function for the matching function: $d(\overline{s}^k u^k, V^k) = \sqrt{\overline{s}^k u^k V^k}$. This implies that $q(\theta^k) = 1/\sqrt{\theta^k}$, $\theta^k q(\theta^k) = \sqrt{\theta^k}$ and, whatever the prevailing urban equilibrium, the elasticity of the matching rate (defined as $\eta(\theta^k) = -q'(\theta^k)\theta^k/q(\theta^k)$) is equal to 0.5. The values of the parameters (in yearly terms) are the following: the output y is normalized to unity. The relative bargaining power of workers is equal to $\eta(\theta^k)$, i.e. $\beta = \eta(\theta^k) = 0.5$. Unemployment benefits w_U have a value of 0.3 and the costs of maintaining a vacancy c are equal to 0.3 per unit of time. Commuting costs τ_L are equal to 0.4 for the employed, and $\tau_U = 0.1$ for the unemployed. The discount rate $r = 0.05$, whereas the job destruction rate $\delta = 0.1$, which means that jobs last on average ten years. Finally, s_0 is normalized to 1, implying that $0 \leq s_a \leq 1$.

We have the following results:

Table 1.2: Comparison between cities

s_a	City	$u^{k*}(\%)$	$u_0^{k*}(\%)$	$u_\sigma^{k*}(\%)$	u_σ^{k*}/u^{k*}	θ^{k*}	x_b^{k*}	$\overline{s}^{k*}$	Welfare
1	1	6.86	6.64	0.22	0.03	1.98	0.069	0.966	0.715
0.75	1	6.85	6.69	0.16	0.02	1.95	0.069	0.974	0.714
0.6	1	6.85	6.72	0.13	0.02	1.93	0.069	0.979	0.714
0.55	1	6.85	6.73	0.12	0.02	1.92	0.069	0.981	0.714
0.525	1	6.85	6.73	0.15	0.02	1.92	0.069	0.982	0.714
0.522+	1	6.85	6.73	0.12	0.02	1.92	0.069	0.982	0.714
0.522−	2	12.4	6.73	5.65	0.46	1.92	0.876	0.511	0.712
0.52	2	12.4	6.74	5.63	0.45	1.91	0.876	0.512	0.712
0.5	2	12.1	6.82	5.31	0.44	1.86	0.879	0.530	0.713
0.25	2	10.0	7.81	2.20	0.22	1.39	0.900	0.763	0.727
0.1	2	9.15	8.35	0.80	0.09	1.20	0.909	0.905	0.732

In this table, we have chosen to vary a key parameter s_a, the loss of information per unit of distance (remember that workers' search intensity is defined by $s(x) = s_0 - s_a x$). This parameter s_a varies from a very large value 1 (where city 1 is the prevailing equilibrium) to a very small value 0.1 (where city 2 is the prevailing equilibrium). The cut-off point is equal to $s_a = 0.522$. The sign '−' indicates the 'limit to the left', whereas the sign '+' indicates the 'limit to the right'.

The first interesting result of this table is that, when we switch from an integrated city (equilibrium 1) to a segregated city (equilibrium 2), for values very close to the cut-off point $s_a = 0.522$, the unemployment rate u^{k*} nearly doubles (from 6.85% to 12.4%). However, it is clear that this result is due to the spatial part of unemployment u_σ^k since the non-spatial one u_0^k is not at all affected by this increase. Indeed, when we switch from equilibrium 1, where the unemployed are close to jobs and are very efficient in their job search ($\overline{s}^1 = 0.982$), to equilibrium 2, where the unemployed reside far away from jobs and are on average not very active in their search activity ($\overline{s}^2 = 0.511$), the spatial part of unemployment changes values from 0.12 to 5.65. Another way to see this is to consider column 6 (u_σ^k/u^{k*}): the part of unemployment due to space varies from 2% to 46%. So the main effect from switching from one equilibrium to another is that search frictions are amplified by space and consequently unemployment rates sharply increase. So here the spatial access to jobs is crucial to understanding the formation of unemployment.

The last column of the table shows the value of the welfare $\mathcal{W}^k$ when s_a varies. The result is very striking: even though unemployment rates are higher in equilibrium 1 than in equilibrium 2, this does not imply that the general welfare of the economy is higher in the first equilibrium. Indeed, even though the unemployed are better off in equilibrium 1 (lower unemployment spells and lower commuting costs), the employed can in fact be worse off because of much higher commuting costs in equilibrium 1. In the above table, it is interesting to see that at the vicinity of $s_a = 0.522$, switching from equilibrium 1 to equilibrium 2 does not involve much change in the welfare level (from 0.714 to 0.712).

Welfare within each city The shape of the city has thus little impact on welfare, since in the segregated city, what is lost from lower search efficiency is gained through lower commuting costs. We now investigate the issue of the optimality of the decentralized equilibrium *within each land market equilibrium.*

In our present model, we have exactly the same externalities (intra- and inter-group externalities) as in the previous section. The spatial dimension does not entail any inefficiency so that it is easily verified that, for each equilibrium $k = 1, 2$, Proposition 2.2 still holds.

4. Endogenous search intensity and housing consumption[9]

So far, we have assumed that search intensity was either exogenous or depending on distance to jobs. In the present section, we would like to derive endogenously the search behavior of workers and show under which condition their search intensity depends on distance to jobs.

In this section, workers' search intensity s affects both the number of trips to the CBD and the job acquisition rate. So, when an unemployed worker wants to obtain information about jobs, he/she needs to go to the CBD. Also, here workers' search intensity s is interpreted as the fraction of commutes to the CBD and thus s is between $s_0 > 0$, its minimum value and 1. If $s = 1$, then the unemployed go to the CBD as often as the employed workers, that is every day.

[9] This section is based on Smith and Zenou (2003a).

4.1. The model

We assume that all workers have identical preferences among consumptions bundles (h, z) of *land* (*housing*), h, and *composite good*, z, representable by a log-linear utility

$$\Gamma(q, z) = q^{\alpha} z^{\omega} \tag{4.1}$$

with $\alpha, \omega > 0$, where it is also assumed that $\alpha + \omega < 1$. However the budget constraints for employed and unemployed workers are different. Each *employed* worker living at location, x, has the standard budget constraint

$$h_L R(x) + \tau x + z_L = w_L \tag{4.2}$$

where z is taken as the numeraire good with unit price. For an *unemployed* worker at x, we have the following budget constraint:

$$h_U R(x) + s\tau x + z_U = w_U \tag{4.3}$$

Maximizing utility (4.1) subject to (4.2) yields the following *land demand for employed workers* at x:

$$h_L(x) = \frac{\alpha}{\alpha + \omega} \cdot \frac{w_L - \tau x}{R(x)} \tag{4.4}$$

Similarly, maximizing (4.1) subject to (4.3) yields the following *land demand for unemployed workers* at x:

$$h_U(x) = \frac{\alpha}{\alpha + \omega} \cdot \frac{w_U - s(x)\tau x}{R(x)} \tag{4.5}$$

As one can see, s is now a function of x, distance to jobs. We can now derive the following indirect utility

$$\Omega_L(x) = \chi (w_L - \tau x)^{\alpha+\omega} R(x)^{-\alpha} \tag{4.6}$$

for each *employed* worker at x, where $\chi = [\alpha/(\alpha + \omega)]^{\alpha} [\omega/(\alpha + \omega)]^{\omega}$ and the following indirect utility

$$\Omega_U(s, x) = \chi (w_U - s\tau x)^{\alpha+\omega} R(x)^{-\alpha} \tag{4.7}$$

for each *unemployed* worker at x, where in this case s is now included as a relevant choice variable.

We can now write the different Bellman equations:

$$rI_L = \Omega_L(x) - \delta \left[I_L - I_U(s, x) \right] \tag{4.8}$$

$$rI_U(s,x) = \Omega_U(s,x) + s\theta q(\theta)\left[I_L - I_U(s,x)\right] \tag{4.9}$$

where $\theta = V/(\bar{s}U)$ and where $\Omega_L(x)$ and $\Omega_U(s,x)$ are respectively given by (4.6) and (4.7). Using (4.8) and (4.9), we obtain:

$$I_L - I_U(s,x) = \frac{\Omega_L(x) - \Omega_U(s,x)}{r + \delta + s\theta q(\theta)} > 0 \tag{4.10}$$

Since there are no relocation costs, at any urban equilibrium it has to be that $rI_L = r\bar{I}_L$ and $rI_U(s,x) = r\bar{I}_U$. Unemployed workers optimally choose s^* by maximizing (4.9). We obtain:

$$r\frac{\partial I_U(s,x)}{\partial s} = -\chi\tau x\left(\alpha+\omega\right)\left(w_U - s\tau x\right)^{\alpha+\omega-1}R(x)^{-\alpha} + \theta q(\theta)\left(\bar{I}_L - \bar{I}_U\right) = 0 \tag{4.11}$$

There is a fundamental trade-off between short-run and long-run benefits for an unemployed worker. On the one hand, increasing search effort s is costly in the short run (more commuting costs) but, on the other, it increases the long-run prospects of employment. Using (4.6), the first order condition can be written as:

$$\Omega_U(s,x)\frac{\left(\alpha+\omega\right)\tau x}{w_U - s\tau x} = \theta q(\theta)\left(\bar{I}_L - \bar{I}_U\right) \tag{4.12}$$

Now, observe that by plugging (4.10) in (4.8), we obtain:

$$\Omega_U(s,x) = r\bar{I}_U - s\theta q(\theta)\left(\bar{I}_L - \bar{I}_U\right) \tag{4.13}$$

As a result, by plugging the value of $\Omega_U(s,x)$ from (4.13) in (4.12), we finally obtain:

$$s^*(x) = \frac{\left(\alpha+\omega\right)}{\left[1-\left(\alpha+\omega\right)\right]}\left[\frac{w_U}{\left(\alpha+\omega\right)\tau x} - \frac{r\bar{I}_U}{\theta q(\theta)\left(\bar{I}_L - \bar{I}_U\right)}\right] \tag{4.14}$$

We have the following result:

Proposition 5.

(*i*) *At each location x, there is a unique search intensity s that maximizes (4.9).*

(*ii*) *For any prevailing job acquisition rate, $\theta q(\theta)$, and constant lifetime values, $\bar{I}_L, \bar{I}_U$, the optimal search intensity function, $s(x)$, for unemployed workers is given for each location, $x \in [0, \frac{w_U}{s_0\tau})$, by*

$$s(x) = \begin{cases} 1 & \textit{for} \quad x \leq x(1) \\ \frac{(\alpha+\omega)}{[1-(\alpha+\omega)]}\left[\frac{w_U}{(\alpha+\omega)\tau x} - \frac{r\bar{I}_U}{\theta q(\theta)\left(\bar{I}_L-\bar{I}_U\right)}\right] & \textit{for} \quad x(1) < x < x(s_0) \\ s_0 & \textit{for} \quad x \geq x(s_0) \end{cases} \tag{4.15}$$

where

$$x(s) = \frac{w_U}{\tau} \cdot \frac{\theta q(\theta)\left(\overline{I}_L - \overline{I}_U\right)}{s\left[1-(\alpha+\omega)\right]\theta q(\theta)\left(\overline{I}_L - \overline{I}_U\right) + (\alpha+\omega)\, r\overline{I}_U} \tag{4.16}$$

which is the unique inverse function of (4.15).

The proof of Proposition 5 is given in Appendix A.4.1 at the end of this chapter and shows why it is optimal for workers to have constant search intensities close and far away from the CBD. Figure 1.5 describes $s(x)$. There is a *non-linear decreasing* relationship between the residential distance to jobs of the unemployed and their search intensity s. In fact, individuals living sufficiently close to jobs search every day, $s = 1$, whereas those residing far away provide a minimum search intensity, $s = s_0$. Workers living in between these two areas see a decrease in their search intensity from $s = 1$ to $s = s_0$. The intuition runs as follows. As stated above, there is a fundamental trade-off between short-run and long-run benefits of various location choices for the unemployed. Indeed, locations near jobs are costly in the short run (both in terms of high rents and low housing consumption), but allow higher search intensities which in turn increase the long-run prospects of reemployment. Conversely, locations far from jobs are more desirable in the short run (low rents and high housing consumption) but allow only infrequent trips to jobs and hence reduce the long-run prospects of reemployment. Therefore, for workers residing further away from the CBD, it is optimal to spend the minimal search effort whereas workers residing close to jobs provide high search effort. Compared to the previous section, we have endogeneized the relationship between s and x and given a mechanism that explains why search intensity is a decreasing function of distance to jobs.

$[Insert\ Figure\ 1.5\ here]$

For interior $s^*(x)$, that is search intensity for workers living between $x(1)$ and $x(s_0)$, we can perform a comparative statics exercise. An interesting and non-standard result is that, when the unemployment benefit w_U increases, workers' search intensity s^* also increases. Indeed, since

$$\frac{\partial \Omega_U(s,x)}{\partial s} = -\chi\tau x\,(\alpha+\omega)\,(w_U - s\tau x)^{\alpha+\omega-1} R(x)^{-\alpha} < 0$$

then $\partial^2\Omega_U(s,x)/\partial s\partial w_U \geq 0$ since $\alpha + \omega < 1$, which means that an increase in w_U reduces the marginal cost of searching and thus workers put more effort in their search activities. For the same reason, we have the contrary result for the commuting cost per unit of distance τ since $\partial^2\Omega_U(s,x)/\partial s\partial w_U \leq 0$. So, when transportation becomes cheaper (for example because of transportation subsidies or better public transportation), then unemployed workers search more intensively. Another interesting result is about land rent. For a given x, an increase in land rent $R(x)$ increases search intensity since $\partial^2\Omega_U(s,x)/\partial s\partial R \geq 0$.[10] Finally, not surprisingly, an increase in the labor market tightness increases search intensity.

4.2. The different urban land use equilibria

We have determined the optimal search intensity of the unemployed at each location in the city (Proposition 5). Knowing this function $s(x)$, one may ask where do the unemployed and the employed locate in the city? The basic trade-off for the employed is between commuting costs and housing consumption whereas for the unemployed, it is between commuting/search costs, housing consumption and search intensity (and thus duration of unemployment). As usual, in order to determine the urban land use equilibrium, we have to define the bid rent function of each group of workers.

Given the utilities and lifetime values above, we now define the equilibrium bid-rents. It follows from (4.8), that the bid rent function for employed workers at each location, $x \in [0, w_L/\tau)$, is equal to:

$$\Psi_L(x, \overline{I}_U, \overline{I}_L) = \left[\frac{\chi(w_L - \tau x)^{\alpha+\omega}}{r\overline{I}_L + \delta\left(\overline{I}_L - \overline{I}_U\right)}\right]^{1/\alpha} \tag{4.17}$$

As usual, this bid rent is decreasing in x but is not anymore linear, which implies that more than two urban configurations can emerge. The bid rent function for unemployed workers is considerably more complex, in that it depends on the optimal search intensity level at each location. Using (4.9), we obtain the following bid rent function for unemployed workers at each location, $x \in [0, \frac{w_U}{s_0\tau})$:

$$\Psi_U(x, \overline{I}_U, \overline{I}_L) = \left[\frac{\chi\left[w_U - s(x)\tau x\right]^{\alpha+\omega}}{r\overline{I}_U - s(x)\theta q(\theta)\left(\overline{I}_L - \overline{I}_U\right)}\right]^{1/\alpha} \tag{4.18}$$

where $s(x)$ is given by (4.15). An instance of this (piecewise continuously differentiable) bid rent function is shown in the bottom half of Figure 1.5,

[10] Using a panel of English sub-regional data, Patacchini and Zenou (2005) have shown that optimal search intensity is higher in areas characterized by larger cost of living (i.e. higher land rent) R and/or higher labor market tightness θ.

where the curve represents a typical 'slice' through the two-dimensional rent surface.

It should be clear that the bid rents are calculated such that the lifetime utilities of both the employed and the unemployed workers, respectively, $\overline{I}_U$ and $\overline{I}_L$, are spatially invariant. Compare for example an unemployed worker residing close to jobs and another unemployed worker living far away from jobs. The former has a lower search (commuting) cost and a higher chance to find a job but consume less land whereas the latter has a higher search (commuting) cost and a lower chance to find a job but consume more land. The bid rent defined by (4.18) exactly compensates these differences by ensuring that these two workers obtain the same lifetime utility $\overline{I}_U$. This is not true for the current utility of the unemployed $\Omega_U(s,x)$ because, as can be seen in (4.13), the land rent does not compensate for $s(x)$. In fact, the unemployed residing close to jobs have a lower current utility than the ones living far away from jobs because they provide more search intensity (indeed, using (4.13), it is easy to see that $\Omega_U'(x) > 0$). However, because they provide more search intensity, they have a higher chance to find a job, and thus in the long-run they compensate the short-run disadvantage so that all unemployed workers obtain the same $\overline{I}_U$.

Definition 4. *An urban-land use equilibrium is such that:*

$$R(x) = \max\{\Psi_L(x,\overline{I}_U,\overline{I}_L), \Psi_U(x,\overline{I}_U,\overline{I}_L), R_A\} \tag{4.19}$$

$$\rho_{es}(x) > 0 \Rightarrow R_{es}(x) = R(x)\ ,\ es = U, L \tag{4.20}$$

$$h_U(x)\rho_U(x) + h_L(x)\rho_L(x) \leq \Xi(x) \tag{4.21}$$

$$R(x) > R_A \Rightarrow h_U(x)\rho_U(x) + h_L(x)\rho_L(x) = \Xi(x) \tag{4.22}$$

where R_A is the agricultural land rent, $\rho_L(x)$ and $\rho_U(x)$ denote respectively the population densities of employed and unemployed workers at x, and $\Xi(x)$ is the land distribution in the city.

This is a more general definition than before since we do not know the way the urban equilibrium looks like. Equations (4.19) and (4.20) state that land at each x is assigned to the highest bidder and that there is no vacant land in the city. Equation (4.21) is the *land capacity condition* that no more land be consumed than is available, equation (4.22) is the *land filling condition* that all land with rents higher than agricultural rent must be occupied by workers. Observe that $\Xi(x)dx$ is the amount of land available for housing between distance x and $x+dx$. For a circular city, $\Xi(x) = 2\pi x$ while for a linear city, $\Xi(x) = 1$. Observe also that, by definition, $\rho_{es}(x) = \Xi(x)/h_{es}(x)$.

4.3. Classification of equilibrium land use patterns

With the non-linear bid rents defined by (4.17) and (4.18), different urban configurations can emerge. Indeed, the land market being perfectly competitive, all workers propose different bid rents at different locations and (absentee) landlords allocate land to the highest bids. So depending on the different steepness of the bid rents (as captured by their slopes), at each location, the employed can outbid the unemployed or can be outbidden by the unemployed. An example of the equilibrium rent function defined by (4.19) is shown in Figure 1.6. In particular, this figure illustrates a case where unemployed workers occupy both a central core of locations and a peripheral ring of locations about the CBD, separated by an intermediate ring of employed workers. Other urban configurations may also emerge. For example, the unemployed can occupy the core of the city and the employed the suburbs. The reverse pattern may also prevail.

[*Insert Figure* 1.6 *here*]

Since we want to focus on interesting urban configurations in which the unemployed workers can outbid the employed workers for peripheral land in equilibrium, we shall assume

$$w_L < \frac{w_U}{s_0} \tag{4.23}$$

Proposition 6. *In equilibrium there are exactly three possible locational patterns:*

(i) *A central core of unemployed surrounded by a peripheral ring of employed,*

(ii) *A central core of employed surrounded by a peripheral ring of unemployed,*

(iii) *Both a central core and peripheral ring of unemployed separated by an intermediate ring of employed.*

This proposition shows that, in a framework where workers' search intensity is location dependent, different urban equilibrium configurations can emerge. In the first one (i), referred to as the *Integrated Equilibrium*, the unemployed reside close to the CBD, have high search intensities and experience short unemployment spells. In the second one (ii), referred to as the *Segregated Equilibrium*, the employed occupy the core of the city and bid away the unemployed in the suburbs. In this case, the latter tend to stay unemployed for a longer time since their search intensity is quite low. Finally, the third case (*iii*), referred to as the *Core-Periphery Equilibrium*, is when there are two categories of unemployed: the short-run ones who reside close to jobs and the long-term ones who live at the periphery of the city. To be consistent with section 3, the integrated equilibrium is referred to as Equilibrium 1 (Figure 1.1), the segregated equilibrium as Equilibrium 2 (Figure 1.2) and the core-periphery equilibrium as Equilibrium 3 (Figure 1.6).

4.4. The steady-state labor equilibrium

For each equilibrium $k = 1, 2, 3$, we can calculate the steady-state labor equilibrium. To define the latter, we can still use definition 2.

Definition 5. *A labor-market equilibrium* $k = 1, 2, 3$ *with endogenous search intensity is a triple* (θ^k, w_L^k, u^k) *such that the following equations hold:*

$$\frac{c}{q(\theta^k)} = \frac{y - w_L^k}{r + \delta} \tag{4.24}$$

$$w_L^k = \arg\max_{w_L^k} (\overline{I}_L^k - \overline{I}_U^k)^\beta (I_F^k - I_V^k)^{1-\beta} \tag{4.25}$$

$$u^k = \frac{\delta}{\delta + \overline{s}^k \theta^k q(\theta^k)} \tag{4.26}$$

The key variable that differs between equilibria is clearly $\overline{s}^k$ since its value depends on the average location of the unemployed. However, we have seen (Proposition 5 and Figure 1.5) that two *constant* search intensity levels for unemployed workers can emerge: all unemployed workers in the central core search with *full intensity*, $s = 1$, and all in the peripheral ring search with *minimum intensity*, $s = s_0$. These constant-search-intensity patterns are particularly easy to analyze. Moreover, Proposition 6 shows that essentially all equilibrium properties of the system can be studied in terms of these simple cases. For in the other two possible locational patterns, it is clear that so long as the equilibrium bid-rent curves, Ψ_U and Ψ_L, do not cross in the region $[x(1), x(s_0)]$, only maximal and minimal search intensities will be involved. In fact, the region $[x(1), x(s_0)]$ can be shown to be relatively small. This assertion is supported by the following result, which shows that if utility is 'almost linearly homogeneous' in the sense that $\alpha + \omega$ is close to one, then the interval $[x(1), x(s_0)]$ is necessarily very small:

Proposition 7. *If* $\alpha + \omega \approx 1$, *then in equilibrium* $|x(1) - x(s_0)| \approx 0$.

Proof: It is enough to observe from (4.16) that for any given lifetime values and hiring probability $(\overline{I}_L^k, \overline{I}_U^k, \theta^k q(\theta^k))$, the locations $x(1)$ and $x(s_0)$ have a common limiting value, $\frac{w_U}{\tau}\frac{\theta^k q(\theta^k)\left(\overline{I}_L^k - \overline{I}_U^k\right)}{r\overline{I}_U^k}$, as $\alpha + \omega \to 1$. ■

Hence if diminishing marginal utility (along rays) is sufficiently small, then equilibrium can be safely assumed to involve only maximal and/or minimal search intensities for unemployed workers. With these observations, we now restrict attention to the constant-search-intensity case. To complete the analysis, we have to calculate $\overline{s}^k$ for each equilibrium k. Observing that, contrary to the two previous sections and since workers consume different amounts of land, $\overline{s}^k$ is not anymore the search intensity of the unemployed worker located at the average location.

For any given *lifetime values*, $\overline{I}_L^k, \overline{I}_U^k$, with $\overline{I}_L^k > \overline{I}_U^k$, and *hiring probability*, $\theta^k q(\theta^k) \in (0, +\infty)$, we define the following set of functions. First, let the function s be defined by (4.15) with ranges, $x(1)$ and $x(s_0)$, given by (4.16). In terms of s and $(\overline{I}_L^k, \overline{I}_U^k, \theta^k q(\theta^k))$, we may then define the additional functions, Ω_U^k, Ψ_L^k, Ψ_U^k,and R^k, respectively by (4.13), (4.17), (4.18), (4.19). Using Ψ_L^k, Ψ_U^k, and R^k, we next define the *indicator functions*, φ_{es}^k, $es = U, L$, specifying the relevant regions occupied by unemployed and employed workers, respectively:

$$\varphi_{es}^k(x) = \begin{cases} 1 & , \ \Psi_{es}^k(x, \overline{I}_U^k, \overline{I}_L^k) = R^k(x) \\ 0 & , \ \text{otherwise} \end{cases} \quad , \quad es = U, L \tag{4.27}$$

It can be shown (Smith and Zenou, 2003b) that these indicator functions are ambiguous only on a set of measure zero [i.e., the equality $\Psi_L^k(x) = \Psi_U^k(x)$ holds only on a set of measure zero in the interval of relevant distances, x]. Hence one can now sharpen the general set of locational equilibrium conditions [(4.20),(4.21),(4.22)] above by noting in the present case that at almost every distance, x, at most one of the population densities, $\rho_L^k(x)$ and $\rho_U^k(x)$, can be positive. Hence, by substituting (4.4) and (4.5) into (4.22), and observing that by definition, $\Psi_{es}^k(x) = R^k(x)$ iff $\varphi_{es}^k(x) = 1$, it follows that the appropriate *population densities*, must have the form (in a linear city where $\Xi(x) = 1$):

$$\rho_L^k(x) = \frac{\Xi(x)}{h_L^k(x)} = \left(\frac{\alpha + \omega}{\alpha}\right) \frac{\Psi_L(x, \overline{I}_U^k, \overline{I}_L^k)}{w_L^k - \tau x} \tag{4.28}$$

$$\rho_U^k(x) = \frac{\Xi(x)}{h_U^k(x)} = \left(\frac{\alpha + \omega}{\alpha}\right) \frac{\Psi_U(x, \overline{I}_U^k, \overline{I}_L^k)}{w_U - s^k(x)\tau x} \tag{4.29}$$

where $s^k(x)$ can only take constant values 1 or s_0. Since $N = U^k + L^k$, with these functions, we can now give a formal general definition of $\overline{s}^k, U^k, L^k$ as follows:

$$\overline{s}^k = \frac{1}{U^k}\int_0^N s^k(x)\varphi_U^k(x)\rho_U^k(x)dx \tag{4.30}$$

$$U^k = \int_0^N \varphi_U^k(x)\rho_U^k(x)dx \tag{4.31}$$

$$L^k = \int_0^N \varphi_L^k(x)\rho_L^k(x)dx \tag{4.32}$$

Equation (4.30) defines $\overline{s}^k$ in terms of the search intensities, $s^k(x)$, and population densities, $\rho_U^k(x)$, at each location x occupied by unemployed workers [i.e., with $\varphi_U(x) = 1$]. Equation (4.31) defines the population totals for employed and unemployed workers, together with the accounting condition ($N = U^k + L^k$) that all workers are either employed or unemployed.

Since equilibria 1 and 2 are particular cases of equilibrium 3, let us now characterize the latter.

4.5. The core-periphery steady-state equilibrium

In this equilibrium, the unemployed workers who reside close to jobs are short-run unemployed (when we refer to them we use the superscript sr and we denote them by U^{sr}) since their search intensity is $s = 1$ while those who live far away from jobs are long-run unemployed (when we refer to them we use the superscript lr and we denote them by U^{lr}). Denote by x_{b_1} the border between short-run unemployed and employed workers, and by and x_{b_2} the border between long-run unemployed and employed workers. We have the following general definition.[11]

Definition 6. *Assume $x_{b_1}^{3*} \leq x(1)$ and $x_{b_2}^{*} \geq x(s_0)$, where $x(.)$ is defined by (4.16). Then, a core-periphery steady-state equilibrium (Equilibrium 3) with endogenous search intensity is a 11-tuple $(\overline{I}_U^{3*}, \overline{I}_L^{3*}, x_{b_1}^{3*}, x_{b_2}^{3*}, x_f^{3*}, U^{sr*}, U^{lr*}, \overline{s}^{3*}, \theta^{3*}, w_L^{3*}, R^*(x))$ such that:*

$$\Psi_L^3(x_{b_1}^{3*}, \overline{I}_U^{3*}, \overline{I}_L^{3*}) = \Psi_U^3(x_{b_1}^{3*}, \overline{I}_U^{3*}, \overline{I}_L^{3*}) \tag{4.33}$$

$$\Psi_L^3(x_{b_2}^{3*}, \overline{I}_U^{3}, \overline{I}_L^{3}) = \Psi_U^3(x_{b_2}^{*}, \overline{I}_U^{3}, \overline{I}_L^{3}) \tag{4.34}$$

$$\Psi_U^3(N, \overline{I}_U^{3}, \overline{I}_L^{3}) = R_A = 1 \tag{4.35}$$

$$\int_0^{x_{b_1}^{3*}} \rho_U^k(x)dx = U^{sr*} \tag{4.36}$$

[11] For analytical simplicity and without loss of generality, the agricultural land rent R_A is now normalized to 1 and not to zero.

$$\int_{x_{b_1}^{3*}}^{x_{b2}^{3*}} \rho_L^k(x) = N - U^{sr*} - U^{lr*} \tag{4.37}$$

$$\int_{x_{b_1}^{3*}}^{x_f^{3*}} \rho_U^k(x) = U^{sr*} \tag{4.38}$$

$$R^*(x) = \max\left\{\Psi_L(x, \overline{I}_U^{3*}, \overline{I}_L^{3*}), \Psi_U(x, \overline{I}_U^{3*}, \overline{I}_L^{3*}), 0\right\} \quad \text{at each } x \in (0, x_f] \tag{4.39}$$

$$\overline{s}^{3*} = \frac{U^{sr*} + s_0 U^{lr*}}{U^{sr*} + U^{lr*}} \tag{4.40}$$

$$u^{3*} = \frac{\delta}{\delta + \overline{s}^{3*}\theta^{3*}q(\theta^{3*})} = \frac{U^{sr*} + U^{lr*}}{N} \tag{4.41}$$

$$\frac{c}{q(\theta^{3*})} = \frac{y - w_L^{3*}}{r + \delta} \tag{4.42}$$

$$w_L^{3*} = \arg\max_{w_L^{3*}} (\overline{I}_L^{3*} - \overline{I}_U^{3*})^{\beta} (I_F^{3*} - I_V^{3*})^{1-\beta} \tag{4.43}$$

where $\rho_L^k(x)$ and $\rho_U^k(x)$ are respectively given by (4.28) and (4.29).

This definition of equilibrium encompasses both urban and labor markets. First, it follows by hypothesis that full search intensity, $s = 1$, is optimal for core unemployed workers, and thus, in equilibrium, from (4.15), it has to be true that the core boundary point, $x_{b_1}^{3*}$, must satisfy $x_{b_1}^{3*} \leq x(1)$. Similarly, minimal search intensity, $s = s_0$, is assumed to be optimal for peripheral unemployed workers, so that the peripheral boundary point, $x_{b_2}^{3*}$, must satisfy $x_{b_2}^{3*} \geq x(s_0)$. Second, for the core-periphery land use to be an equilibrium, it has to be that the bid rents cross twice (equations (4.33) and (4.34)), the bid rent of the long-run unemployed equal R_A at the city fringe (equation (4.35)), the population constraints have to be satisfied (equations (4.36)−(4.38)), and the equilibrium land rent has to be the upper envelope of all bid rents in the city (equation (4.39)). Finally, the equilibrium land market conditions are given by (4.40)−(4.43).

This is obviously quite messy but one can show that there is a unique equilibrium. In this equilibrium, it is really $\overline{s}^{3*}$ that makes the link between the labor and land market. To give some intuition of the way the labor market operates, let us show the way the wage is calculated. The Bellman equations for the employed, short-run and long-run unemployed workers are respectively given by:

$$r\overline{I}_L^3 = \Omega_L(x) - \delta\left(\overline{I}_L^3 - \overline{I}_U^3\right) \tag{4.44}$$

$$r\overline{I}_U^{sr} = \Omega_U^{sr} + \theta^3 q(\theta^3)\left(\overline{I}_L^3 - \overline{I}_U^3\right) \tag{4.45}$$

$$r\overline{I}_U^{lr} = \Omega_U^{lr} + s_0\theta^3 q(\theta^3)\left(\overline{I}_L^3 - \overline{I}_U^3\right) \tag{4.46}$$

Since in equilibrium it has to be that $rI_U^{lr} = rI_U^{sr} = r\overline{I}_U^3$, by combining (4.45) and (4.46), we have

$$\Omega_U^{lr} - \Omega_U^{sr} = (1 - s_0)\,\theta q(\theta)\left(\overline{I}_L^3 - \overline{I}_U^3\right) > 0$$

This again underscores the essential difference between unemployed workers in the central core (the short-run unemployed workers) and those in the periphery (the long-run unemployed workers). Those in the central core are giving up short-run utility for long-run utility gains. Hence, if the lifetime value, $\overline{I}_U^3$, of all unemployed workers is the same, then the short-run utility of those in the periphery must be greater than for those in the central core.

Let us determine the wage through a bargaining between the firm and the worker. First, observe that, since workers are not attached to location (they freely relocate each time they change employment status), the wage will be the same for all employed workers. Then, by solving (4.33)−(4.35), and using (4.17) and (4.18), one obtains:

$$\overline{I}_L^{3*} - \overline{I}_U^{3*} = \frac{\chi\left(w_U - s_0\tau x_f^{3*}\right)^{\alpha+\omega}}{r + \delta + s_0\theta^{3*}q(\theta^{3*})}\left[\left(\frac{w_L^3 - \tau x_{b_2}^{3*}}{w_U - s_0\tau x_{b_2}^{3*}}\right)^{\alpha+\omega} - 1\right]$$

where $x_{b_2}^{3*}$ and x_f^{3*} are determined by (4.36)−(4.38). Since the first order condition of the Nash-bargaining is given by:

$$\frac{\beta}{1-\beta}\left(\frac{\partial\overline{I}_L^{3*}}{\partial w_L^3} - \frac{\partial\overline{I}_U^{3*}}{\partial w_L^3}\right) I_F + \left(\overline{I}_L^{3*} - \overline{I}_U^{3*}\right)\frac{\partial I_F}{\partial w_L} = 0$$

and since $I_F = c/q(\theta^{3*})$ and $\frac{\partial I_F}{\partial w_L^3} = -1/(r+\delta)$, one can obtain a non-linear relationship that defines the bargain wage w_L^3.

5. Discussion

5.1. Fixed-housing consumption

This model is admittedly quite complicated, even though we were able to completely characterize it. In fact, one of the complication came from the fact that housing was endogenous. Imagine now a similar model but with fixed housing consumption.

Each individual's search efficiency s now depends only on his/her job search effort denoted by e. We assume decreasing returns to scale to effort, i.e.,

$s'(e) > 0$ and $s''(e) \leq 0$. As above, each interview is carried out in the employment center and thus involves transport costs. We denote by $e\,\tau\,x$ the search costs associated with a level of effort e for a worker living at a distance x from the employment center. Let us write the Bellman equation of the unemployed workers. It is given by:

$$rI_U = w_U - e\,\tau\,x - R(x) + \theta q(\theta)\,s(e)\,(I_L - I_U) \tag{5.1}$$

while the value of employment is equal to:

$$rI_L = w_L - \tau_L x - R(x) - \delta\,(I_L - I_U)$$

The unemployed worker located at a distance x from the employment center chooses e^* that maximizes his/her intertemporal utility (5.1). The first order condition on effort yields:

$$s'(e^*)\theta q(\theta)(I_L - I_U) = \tau\,x \tag{5.2}$$

Once again, when choosing the optimal search effort e^*, workers trade off short-run costs of commuting with long-run gains. By totally differentiating (5.2), we obtain:

$$\frac{\partial e^*}{\partial x} = \frac{\tau}{\theta q(\theta)s''(e^*)(I_L - I_U)} < 0 \tag{5.3}$$

and thus

$$\frac{\partial s}{\partial x} = s'(e)\frac{\partial e}{\partial x} < 0 \tag{5.4}$$

The bid rent functions are given by (3.4) and (3.5) with $\tau_L = \tau$, $\tau_U = e^*(x)\tau\,x$, and $a(x) = s\left[e^*(x)\right]\theta q(\theta)$, where $e^*(x)$ is given by (5.2), The bid rent of the employed workers is linear and such that

$$\frac{\partial \Psi_L(x, \overline{I}_U, \overline{I}_L)}{\partial x} = -\tau < 0$$

while the bid rent of the unemployed workers is non-linear and its slope is given by:

$$\frac{\partial \Psi_U(x, \overline{I}_U, \overline{I}_L)}{\partial x} = -\tau\left[\frac{\partial e^*}{\partial x}\,x + e^*(x)\right] + s'(e^*)\frac{\partial e^*}{\partial x}\theta q(\theta)\left(\overline{I}_L - \overline{I}_U\right)$$

which using (5.2) yields

$$\begin{aligned}\frac{\partial \Psi_U(x, \overline{I}_U, \overline{I}_L)}{\partial x} &= \frac{\partial e^*}{\partial x}\left[s'(e^*)\theta q(\theta)\left(\overline{I}_L - \overline{I}_U\right) - \tau\,x\right] - \tau e^*(x) \\ &= -\tau e^*(x) < 0\end{aligned}$$

One can also verify that

$$\frac{\partial^2 \Psi_U(x, \overline{I}_U, \overline{I}_L)}{\partial x^2} = -\tau \frac{\partial e^*}{\partial x} = -\frac{\tau^2}{\theta q(\theta) s''(e^*)(I_L - I_U)} > 0$$

We can have here also three urban configurations. If the slope of $\Psi_U(x, \overline{I}_U, \overline{I}_L)$ is steep enough close to the CBD, i.e. $e^*(0) > 1$, then the integrated equilibrium (Equilibrium 1) emerges while if we have the contrary, i.e. $e^*(0) < 1$, then the segregated equilibrium (Equilibrium 2) prevails. Finally, if $e^*(N) < 1 < e^*(0)$, then a core-periphery equilibrium (Equilibrium 3) can emerge. Of the course, the latter analysis will be easier since population densities are equal to 1, which implies in particular that $x_{b_1}^3 = U^{sr}$, $x_{b_2} = N - U^{lr}$, $x_f^3 = N$. However, now search intensities are not anymore constant close or far away from the CBD, which makes the analysis more complicated.

5.2. Gathering information about jobs

So far, the way workers gathered information about jobs was mainly by commuting to the CBD. Imagine now the following. Each unemployed individual commutes to the center to gather information about jobs. This is not the only way to obtain information since one can also obtain job information by buying newspapers or calling friends. However, each return trip from the residential location to the employment center allows the worker to have some additional information that is not accessible without going to the center (for example, looking at some ads that are locally posted or having interviews with employment agencies that are located in the center).

If τ and ι denote respectively the *pecuniary* and *time* cost per unit of distance to commute to the employment center, then for the unemployed workers the total cost per return trip of gathering information about jobs in the employment center is given by:

$$\tau x + \iota x \tag{5.5}$$

We can now determine the *total* cost of gathering information about jobs at a distance x from the employment center. It is given by:

$$(\tau + \iota) e x \tag{5.6}$$

where $0 \leq e \leq 1$ is the search-effort rate provided by each unemployed worker. For example, $e = 1$ would be searching every day while $e = 1/2$ would be searching every other day. Obviously the higher e the more often the unemployed worker has to travel to the employment center to gather information about jobs. In this formulation, x is a measure of job access (how "well" the unemployed worker is connected to jobs) while e is a measure of search intensity (how many hours per day the unemployed worker spends in searching for a job).

In this context, the budget constraint of an unemployed worker living at a distance x from the employment center is equal to:

$$w_U = z + R(x) + C(e) + (\tau + \iota)\, e\, x \tag{5.7}$$

where $C(e)$ denotes all searching costs that are not distance-related. The latter encompasses the costs of buying newspapers, making phone calls, etc. We assume that $C(0) = 0$, $C'(e) > 0$ and $C''(e) > 0$. In this formulation, the total cost of searching is thus $C(e) + (\tau + \iota)\, e\, x$, which encompasses both search costs that are not distance-related and costs that involve commuting to the employment center.

Let us now focus on employed workers. He/she has the following budget constraint:

$$w_L = z + R(x) + (\tau + \iota\, w_L)\, x \tag{5.8}$$

The time cost of commuting for an employed worker residing at a distance x from the CBD is $\iota\, w\, x$; in accordance with empirical observation, it increases with income. As usual, w_L is the opportunity cost of leisure (even though this is not explicitly modelled here). The Bellman equations for unemployed and employed workers can be written as:

$$rI_U = w_U - C(e) - (\tau + \iota)\, e\, x - R(x) + \theta q(\theta)\, s(e)\, (I_L - I_U) \tag{5.9}$$

$$rI_L = w_L\, (1 - \iota\, x) - \tau x - R(x) - \delta\, (I_L - I_U) \tag{5.10}$$

The unemployed worker located at a distance x chooses e^* that maximizes his/her intertemporal utility (5.9). The first order condition yields:

$$\theta q(\theta)\, s'(e^*)\, (I_L - I_U) = C'(e^*) + (\tau + \iota)\, x \tag{5.11}$$

By totally differentiating (5.11), we obtain:

$$\frac{\partial e^*}{\partial x} = \frac{\tau + \iota}{\theta q(\theta) s''(e^*)(I_L - I_U) - C'(e^*)} < 0$$

and thus

$$\frac{\partial s}{\partial x} = s'(e^*)\frac{\partial e^*}{\partial x} < 0$$

Our previous results are thus robust to different search costs. It is easy to check that the employed workers' bid rent is linearly decreasing in x while the unemployed workers' one will be decreasing and convex in x. We will have again the three above urban configurations and the labor-market analysis can be carried through.

6. Conclusion

We have exposed the basic urban search matching model with different variations. This model will constitute the core of part two of this book and will be used to explained urban ghettos and labor-market outcomes in part three of this book.

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A. Appendix A.4.1. Proof of Proposition 5

Let us first establish the uniqueness of solutions to (4.12). For that, we partially differentiate (4.11) once more to obtain:

$$r\frac{\partial^2 I_U(s,x)}{\partial s^2} = -\chi\tau x\,(\alpha+\omega)\,[1-(\alpha+\omega)](w_U - s\tau x)^{\alpha+\omega-2}R(x)^{-\alpha} - \theta q(\theta)\frac{\partial I_U(s,x)}{\partial s} \tag{A.1}$$

Observing that the first term in the numerator is negative, we may conclude that

$$\frac{\partial I_U(s,x)}{\partial s} \geq 0 \;\Rightarrow\; \frac{\partial^2 I_U(s,x)}{\partial s^2} < 0 \tag{A.2}$$

In particular this implies that stationary points of (4.9) can only be local maxima, and thus [by continuity of (4.11)] that there is at most one stationary point. Thus, at each location x there is *at most one solution to* (4.12).

Let us now prove the second part of the proposition.

First, note that in equilibrium this optimal lifetime value must agree with the prevailing lifetime value, I_U, for unemployed workers, i.e., that $I_U(s,x) = I_U$ in (4.12). Note also that in equilibrium we must have (4.13). Hence, by substituting these results into (4.12) and solving for s, we obtain

$$s(x) = \frac{(\alpha+\omega)}{[1-(\alpha+\omega)]}\left[\frac{w_U}{(\alpha+\omega)\,\tau x} - \frac{r\overline{I}_U}{\theta q(\theta)\left(\overline{I}_L - \overline{I}_U\right)}\right] \tag{A.3}$$

with unique inverse function, $x(s)$, given by (4.16).

In terms of this inverse function (4.16), it follows at once from (4.11) that

$$\frac{\partial I_U(s,x)}{\partial s} \gtrless 0 \Leftrightarrow x \lesseqgtr x(s) \tag{A.4}$$

Let us now prove parts (*i*), (*ii*), and (*iii*) of (4.15). They are established respectively as follows:

(*i*) [$x < x(1)$] Observe from (A.4) and (A.2) that $x < x(1) \Rightarrow \partial I_U(1,x)/\partial s > 0 \Rightarrow \partial^2 I_U(1,x)/\partial s^2 < 0$, so that $I_U(\cdot,x)$ must be increasing near $s = 1$. Hence if there is some $s_1 \in [s_0, 1)$ with $I_U(s_1,x) > I_U(1,x)$, then it follows from the continuity of (4.11) that $I_U(\cdot,x)$ must achieve a differentiable minimum at some point interior to $[s_1, 1]$. But since this contradicts (A.2), it follows that no such s_1 can exist, and hence that $I_U(1,x)$ is maximal.

(*ii*) $[x > x(s_0)]$ Again by (A.4), $x > x(s_0) \Rightarrow \partial I_U(s_0, x)/\partial s < 0$, so that $I_U(\cdot, x)$ must be decreasing near $s = s_0$. Hence if there is some $s_1 \in (s_0, 1]$ with $I_U(s_1, x) > I_U(s_0, x)$, then it again follows from the continuity of (4.11) that $I_U(\cdot, x)$ must achieve a differentiable minimum interior to $[s_0, 1]$, which contradicts (A.2). Thus $I_U(s_0, x)$ must be maximal.

(*iii*) $[x(1) \leq x \leq x(s_0)]$ Finally, it also follows from (A.4) that $x(1) \leq x \Rightarrow \partial I_U(1, x)/\partial s \geq 0$, and $x \leq x(s_0) \Rightarrow \partial I_U(s_0, x)/\partial s \leq 0$, so that by continuity there is some $s \in [s_0, 1]$ with $\partial I_U(s, x)/\partial s = 0$. Hence $s = s(x)$ in (A.3), and we may conclude from the uniqueness of differentiable maxima observed above that $I_U[s(x), x]$ must be maximal. ■

Figure 1.1: Urban equilibrium 1 (The integrated city)

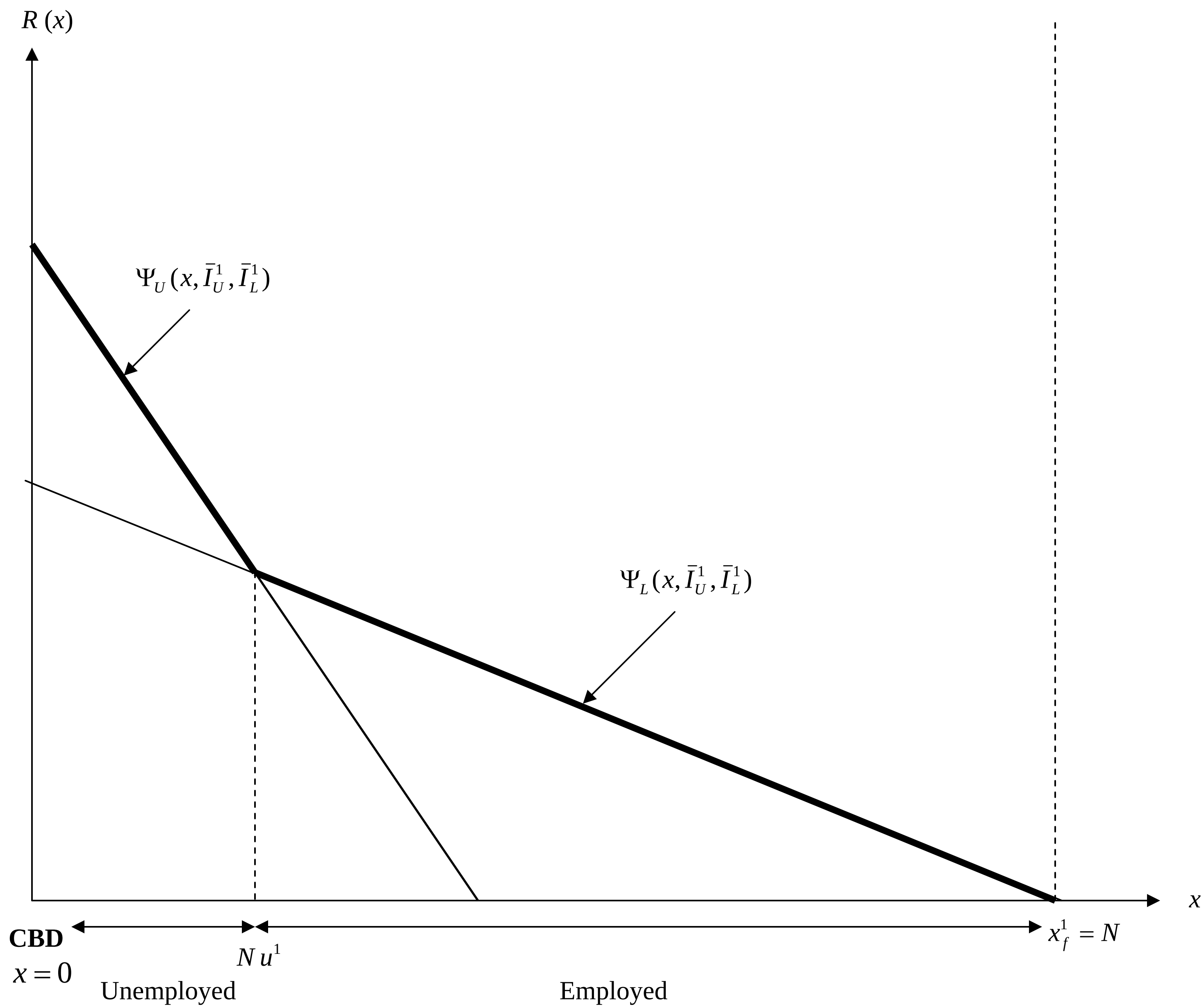

Figure 1.2: Urban equilibrium 2 (The segregated city)

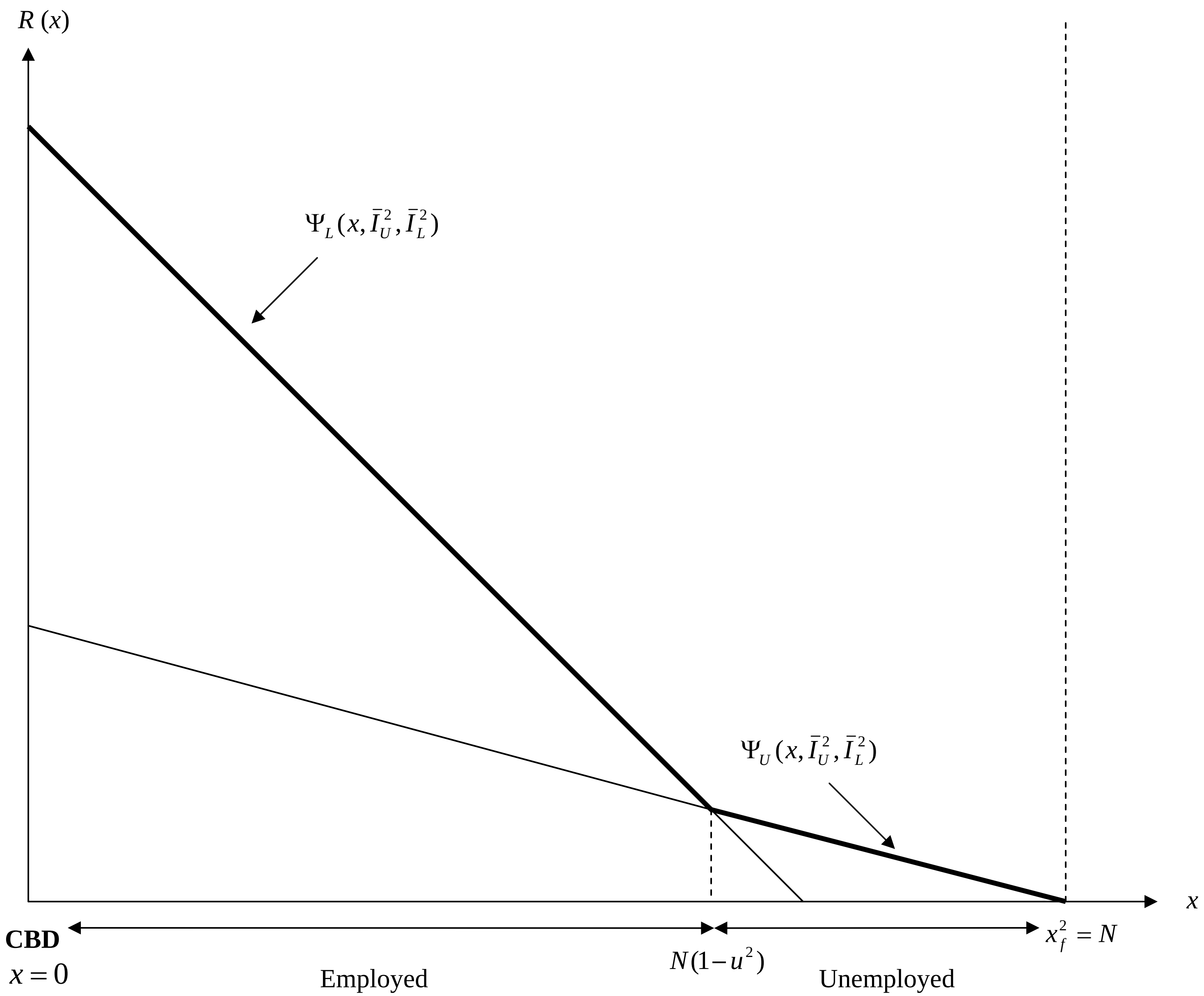

Figure 1.3: Steady-state equilibrium 1

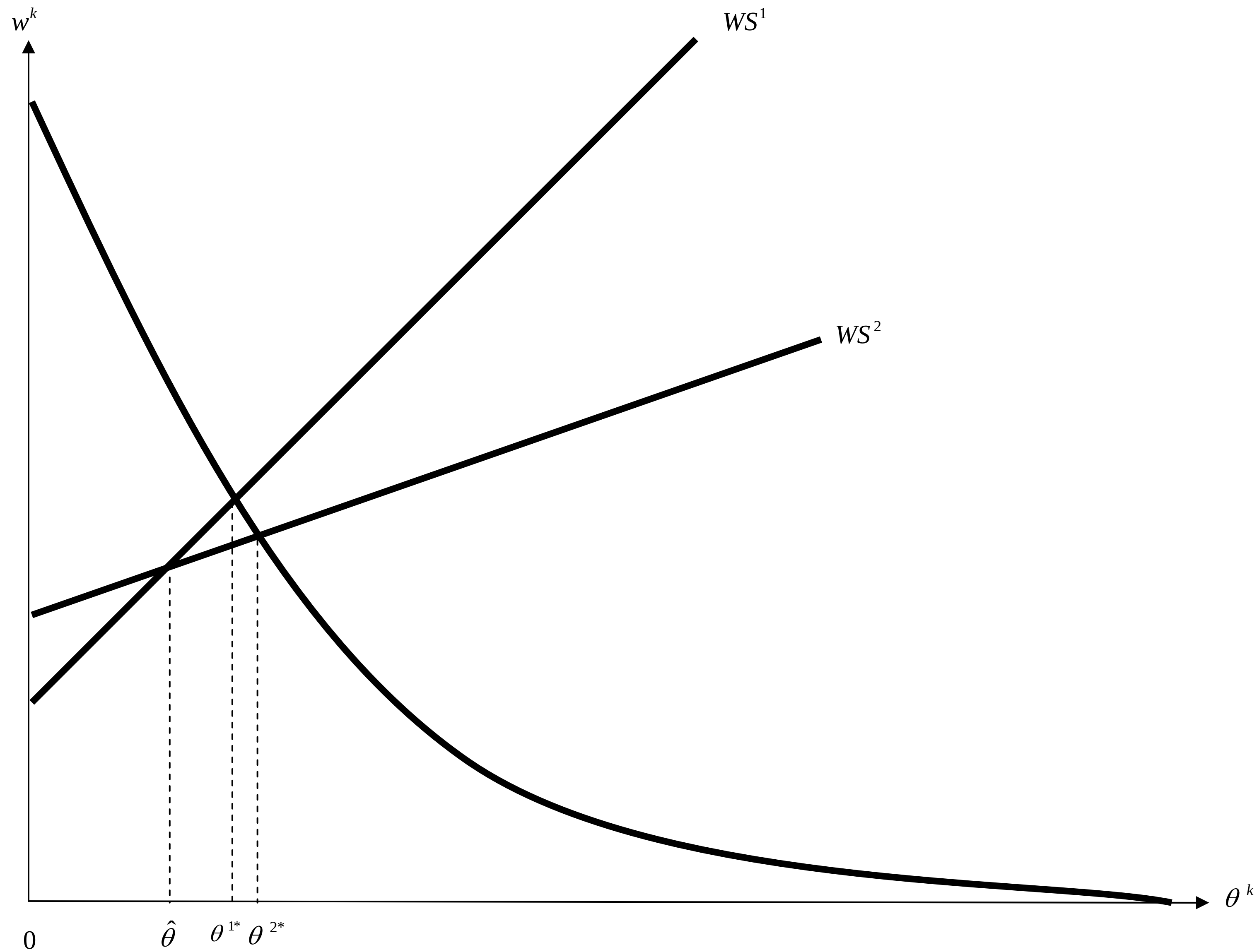

Figure 1.4: Steady-state equilibrium 2

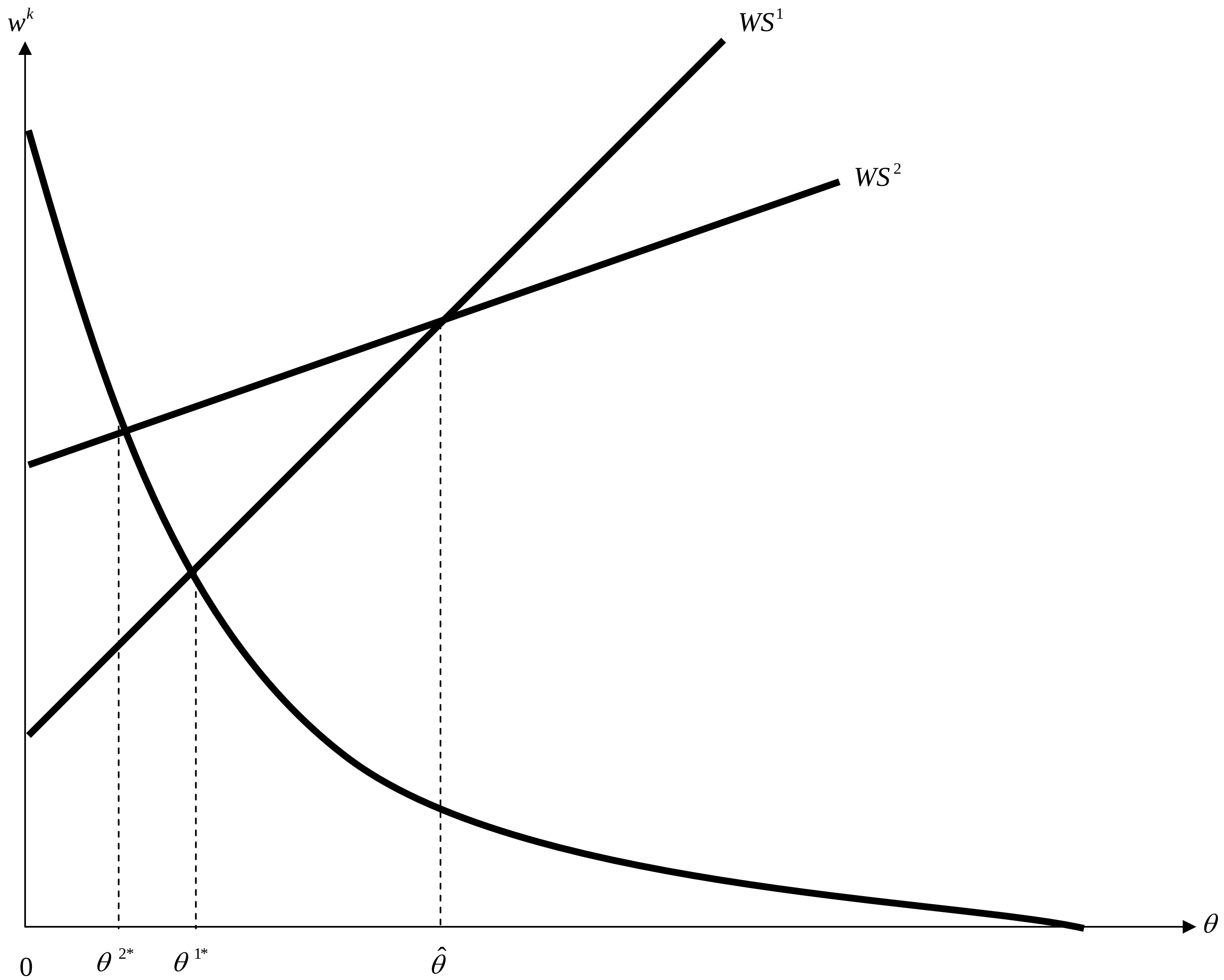

Figure 1.5: Optimal search intensities

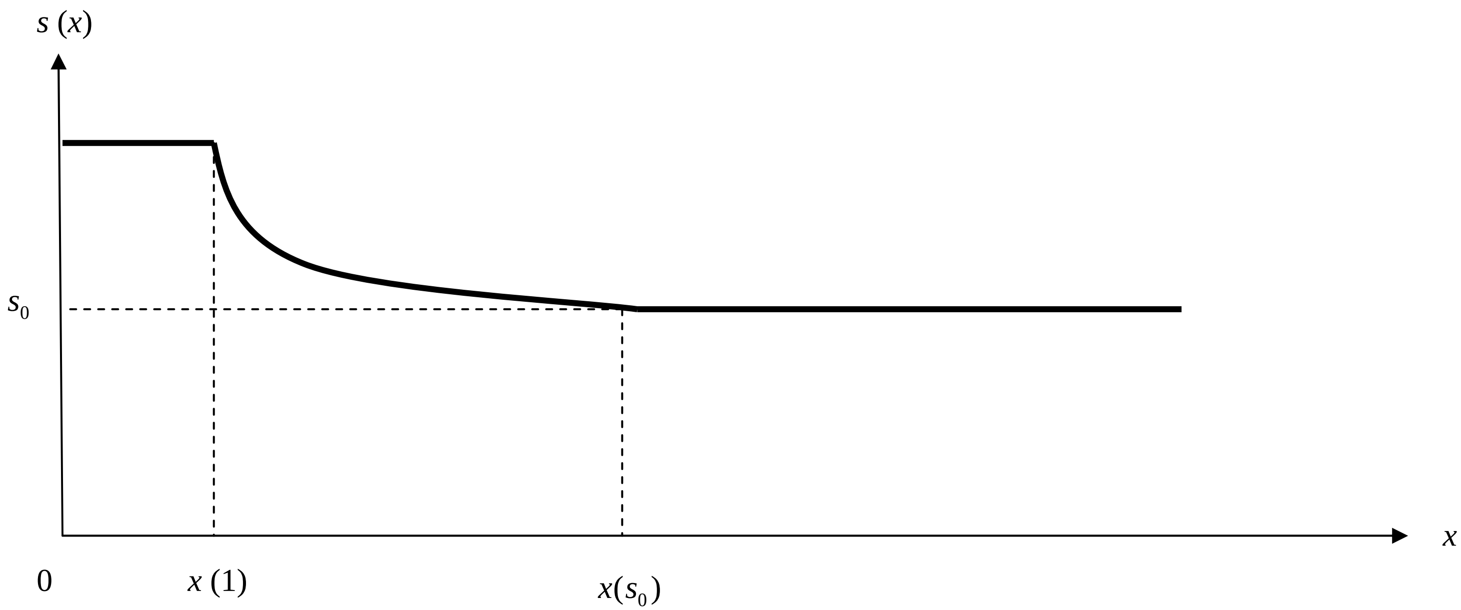

Figure 1.6: The core-periphery equilibrium