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TRADE DIVERSION IN A CURRENCY UNION

by

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2. THE ANALYTICAL FRAMEWORK

Let there be three countries, the home country, the partner country and the rest of the world. The home and partner countries form a currency union, i.e. the union countries have a common currency. This common currency floats without government intervention with respect to the rest of the world currency. There are two periods, indexed by \( i = 1,2 \) and \( n \) perishable goods, \( i = 1,\ldots,n \). We may now denote period 1's prices, in the union currency, by \( p^1 = (p^1_1, p^1_2, \ldots, p^1_n) \) and expected period 2 prices (point expectations) by \( p^2 = (p^2_1, p^2_2, \ldots, p^2_n) \). Home and partner countries are small. They face exogenous world prices for all goods, \( p^1_i = (p^1_{i1}, \ldots, p^1_{in}) \), and they can not influence the income or output of the rest of the world. We assume that the law of one price holds between the union countries and the rest of the world, i.e. \( p^1_i = e p^1_i \) where \( e \) denotes the exchange rate between the two partners and the rest of the world.

We shall first study the consumer side of the home country. We assume that its labour supply is given. (For a case of variable labour supply see Rodrik (1986)). The utility function of the the representative consumer there may now be presented as
\[
U = U(c^1, c^2)
\]
where \( c^1 = (c^1_1, \ldots, c^1_n) \) is the consumption in the first period and \( c^2 = (c^2_1, \ldots, c^2_n) \) the planned consumption in the second period. The household enters the first period with a given nominal wealth, \( M \). It consists of non-interest bearing assets, cash balances (money), only. In the first period the household cannot borrow or lend but it may save a part of its money to be used in the second period. In the context of our
representative consumer model this may be interpreted as the equality of the trade balance and the current account. The revenues of the household consist of labor income and dividends. We assume that the firms distribute all its profits. These revenues are paid to households at the end of each period. Accordingly, the first period budget constraint for the household may be presented as

\[ p^1 c^1 - M = 0 \]  \hspace{1cm} (1) \]

while the overall budget constraint is

\[ p^1 c^1 + p^2 c^2 - M - w^1 L^1 + \pi^1 = y \]  \hspace{1cm} (2) \]

where \( w^1 \) denotes the nominal wage rate, \( L^1 \) the labor input and \( \pi^1 \) the profits of the firms during the first period. We shall assume that in both periods the nominal wages are rigid at a level which leads to deficient demand, i.e. there is Keynesian unemployment.

The intertemporal household optimization problem is to maximize \( U(c^1, c^2) \) subject to the constraint (2). This may be described conveniently by using the restricted expenditure function

\[ E = E(p^1, p^2, u) = \min \{ p^1 c^1 + p^2 c^2 | U(\cdot) > u \} \]

As is known, \( E \) is homogenous of degree one in prices and its partial derivatives with respect to the first and second variables are the compensated (Hicksian) demand for the first period and the expected demand for the second period, respectively. The household's equilibrium may now be presented as

\[ E(p^1, p^2, u) = y \]  \hspace{1cm} (4) \]

Especially, the period 1 overall compensated demand is
We can now solve \( u \) from eq. (4) and substitute this into (5). This yields the uncompensated demand function for period 1, denoted by \( c \),

\[ c = c(p^1, p^2, y) \]  

(6)

Eq. (6) tells us how the real demand in the first period reacts to the present prices, to expected future prices and to the overall income over the two periods. From eqs. (4) and (5) we may see that uncompensated demands are homogenous of degree zero in total income and prices together.

In the second period, the households spend their period 1 savings and the revenues earned in the first period.

For the partner country, denoted by \( * \), corresponding results apply while the consumption demand was assumed exogenous in the rest of the world.

On the production side, according to our Keynesian framework, the production of firms is rationed by foreign and domestic demand. There is no investment. All countries produce all products. For simplicity, we assume that the home country, for example, demands a fixed proportion out of its overall consumption of every good from domestic and the two foreign sources. (For this assumption, see Dixit and Norman (1980), ch.8) Accordingly, if the home country production vector for period 1 is denoted by \( x^1 = (x^1_1, \ldots, x^1_n) \),

\[ x^1 = ac + a^*c^* + AC \]  

(7)

where \( a \) denotes the home country share, \( a^* \) the foreign country share and A
the rest of the world share of the home country good in the consumption of the respective countries. Correspondingly, for the partner country

\[ x^* = bc + b^*c^* + BC \] (8)

Because the output is equal to the income to be paid in the end of period, the equilibrium conditions for the markets of the home goods and partner goods during the first period may now be presented as the following equations

\[ p^1x^1 = y - M = ap^1c(p^1,p^2,y) + a^*p^1c^*(p^1,p^2,y^*) + Ap^1C \] (9)

\[ p^1x^* = y^* - M^* = b^*p^1c^*(p^1,p^2,y^*) + Bp^1C \] (10)

In the absence of lending and borrowing possibilities, assuming that the households in the union countries are not allowed to keep the rest of the world money (correspondingly for the rest of the world households) and with no government interference, the exchange rate balances the trade between the union and the rest of the world. Denoting \( h = 1-a-b, h^* = 1-a^*-b^* \) this can be described as

\[ hp^1c(p^1,p^2,y) + h^*p^1c^*(p^1,p^2,y^*) - (A+B)p^1C = 0 \] (11).

3. TRADE DIVERSION

To begin with we have to deal with the problem of price expectations. For simplicity, we shall assume that the price expectations for period 2 reflect period 1 prices homogenously, i.e.
where $g$ is homogenous of degree one. The partner country shares the same expectations. Let us in addition assume that the rest of the world prices remain constant in the two periods, i.e. $\Pi^1 = P$. In this situation, because of the law of one price, $p^1 = eP$. Note that the expectations for the exchange rate for period 2 may well be wrong. It does not matter, however, within our model, because the nominal balances (consisting of period 1 savings and period 1 production revenues) to be used in period 2 change correspondingly. In other words, the real balances remain unaltered. Using (12) and the homogeneity of the uncompensated demand functions, we may now rewrite the eqs. (9) - (11) as

\[
\begin{align*}
y/e &= aPc(P,g(P),y/e) + a^*Pc^*(P,g(P),y^*/e) + APC + M/e \quad (9') \\
y^*/e &= bPc(P,g(P),y/e) + b^*Pc^*(P,g(P),y^*/e) + BPC + M/e \quad (10') \\
hPc(P,g(P),y/e) + h^*Pc^*(P,g(P),y^*/e) - (A+B)PC &= 0 \quad (11')
\end{align*}
\]

The incomes measured in the currency of the rest of the world $y/e$ and $y^*/e$, which may be called the real intertemporal incomes in the respective countries, are the variables which determine the changes in the intertemporal utilities. This can be seen directly from the intertemporal equilibrium condition (4). Because of the characteristics of the expenditure function it may be written as

\[
E(P,g(P), u) = y/e \quad (12)
\]

and differentiating (12) yields the connection between changes in the utility and in the real income as
Let us now go to comparative statics and start with the simplest case. Assume that the initial money balances in the two union countries are equal, i.e. $M = M^*$ and that there is a shift in the home country demand from the products of the rest of the world to the partner goods, $dZ$. Differentiating eqs. (9') - (11') we obtain

$$
\begin{bmatrix}
1 - aPC_3 & -a^*PC_3^* & -1 \\
-bPC_3 & 1 - b^*PC_3^* & -1 \\
hPC_3 & h^*PC_3^* & 0
\end{bmatrix}
\begin{bmatrix}
\frac{d(y/e)}{1} \\
\frac{d(y^*/e)}{1} \\
\frac{d(M/e)}{1}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
1 \\
1
\end{bmatrix}
$$

(14)

where $c_3$ and $c^*_3$ denote the derivatives of $c$ and $c^*$ with respect to $y/e$ and $y^*/e$. We assume that $PC_3$ and $PC^*_3$ (the marginal propensities to consume in period 1 out of expected real overall income) are smaller than or equal to unity.

Solving (14) yields

$$
\frac{d(y/e)}{dZ} = \frac{1 + (a^* - b^* - h^*)PC_3^*}{D} > 0 \quad (15a)
$$

$$
\frac{d(y^*/e)}{dZ} = \frac{1 + (-a + b + h)PC_3}{D} > 0 \quad (15b)
$$

$$
\frac{d(M/e)}{dZ} = \frac{a^*PC_3^*(1 - PC_3)}{D} > 0 \quad (15c)
$$
from (15c)

\[
d{e}/dZ = (-M/e^2)a^*Pc^*3(1-Pc^3)/D < 0
\]  \hspace{0.5cm} (16).

From eqs. (13a) - (14) we may see that the home country trade diversion has effects via two channels. First, the increase in the demand for partner goods rises its output and income. This leads, via repercussion effects, to an increase in the income of the home country also. Secondly, the decrease in the union demand for goods produced in the rest of the world causes an appreciation in the exchange rate of the union currency. This increases the value of the initial real balances in both union countries. Both of these effects make possible a larger consumption in both periods. Accordingly, the intertemporal utility of the two union countries increases as is shown by eq. (14) and its analogy for the partner country.

We may also see from these equations that the output in period 1 measured in foreign currency increases. The change in the home country is from (2), (13a) and 13(c)

\[
d (P_x)/dZ = \{d(y/e) - d(M/e)\} > \{(1-(b^*+h^*)Pc^3)/D > 0 \}  \hspace{0.5cm} (17a)
\]

\[
d (P_x^*)/dZ = \{d(y^*/e) - d(M/e)\} > \{(1-a+b+h)Pc^3)/D > 0 \}  \hspace{0.5cm} (17b)
\]

We know, on the other hand, that the consumption in the second period is at least as large as the production revenues earned in the first period. These are equal to \(P_x^1\) and \(P_x^*1\) if measured in the currency of the rest of the world. As shown by eqs. (17) trade diversion causes an increase in these. By our assumption of constant marginal shares \(a,b,a^*\) and \(b^*\), this must increase the real demand for all union goods. Accordingly, the value of production measured
We may now turn to changes in employment. If there were only one good produced in the world we could see directly from eqs. (17a) and (17b) that employment increases. In our multiproduct case this is not quite so obvious. We shall restrict ourselves to a case where labour is the only production factor. Because the goods are perishable, there are no intertemporal connections in production decisions. Accordingly, period i production function for good j may be presented as

$$x_j^i = f_j^i(L_j^i)$$

(18)

where $L_j^i$ is the employment in sector j in period i. Differentiating (18) yields

$$dx_j^i = \frac{\partial f_j^i}{\partial L_j^i} \, dL_j^i.$$  

By adding the sectors we obtain the change in the total employment in the home country as

$$dL^i = \sum_j \frac{1}{\partial x_j^i / \partial L_j^i} \, dx_j^i$$

(19).

The firms would like to sell more than they do, i.e. \( \frac{\partial f_j^i}{\partial L_j^i} \geq \frac{w_j}{p_j} \). In case the constraint is binding, inserting this into (19) and using (18) yields

$$dL^i = \sum_j p_j \frac{dx_j^i}{w_j} = \frac{e(\sum_j p_j x_j^i)}{\bar{w}^i} = \frac{e(Px^i)}{\bar{w}^i},$$

(20).

We may now see directly from eqs. (17) that, in the special case presented above, employment increases in period both periods in both union countries. (The presentation by eqs. (18)- (20) follows closely the one in Dixit and Norman, ch.8).
The obvious extensions are to analyse a situation where both union countries divert demand from the rest of the world to each other and a situation where the initial money balances differ between these countries. Neither of these extensions affect essentially the conclusions, however, as could be easily seen from eqs. (14). For example, in the first case, with an equal trade diversion in both union countries, by symmetry the real income increase is $2dZ/D$ and the real money balance increase is $\{aP_3(1-P_3^*)+a^*P_3^*(1-P_3)\}dZ/D$ in both of them. From this we can proceed as above.

4. SUMMARY

We have analysed within a two period, three country optimizing model the consequences of trade diversion in a currency union. The union countries were small, i.e. the prices measured in the currency of the rest of the world were exogenous for these countries and they were not able to affect the output or demand outside the union. The exchange rate between the union currency and the rest of the world currency was freely floating and the economic situation in all countries was one of Keynesian unemployment.

We were able to find out rather easily that trade diversion (a shift in demand from the goods outside the union to union partner goods) causes within our model an unambiguous increase in intertemporal welfare in both union countries. This was the result of the direct demand effect and the real balance effect of the ensuing appreciation of the union currency. In a limiting case we were able to show that the unemployment decreased in both periods in both union countries.
REFERENCES:


