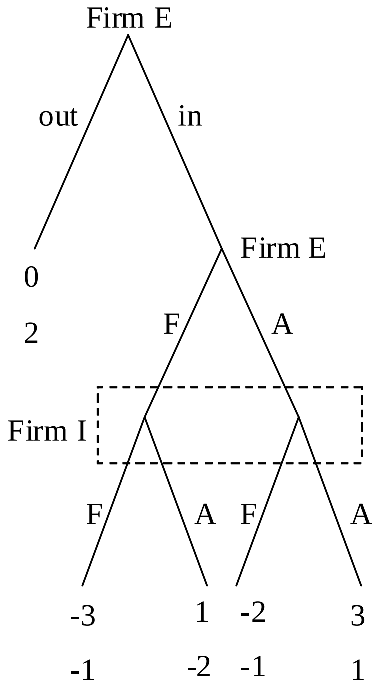
Microeconomic Theory EC104

Problem Set 3

(* is easy, ** is difficult, *** is more difficult)

1. * Consider the 2 player game depicted in Figure 1.

Figure 1

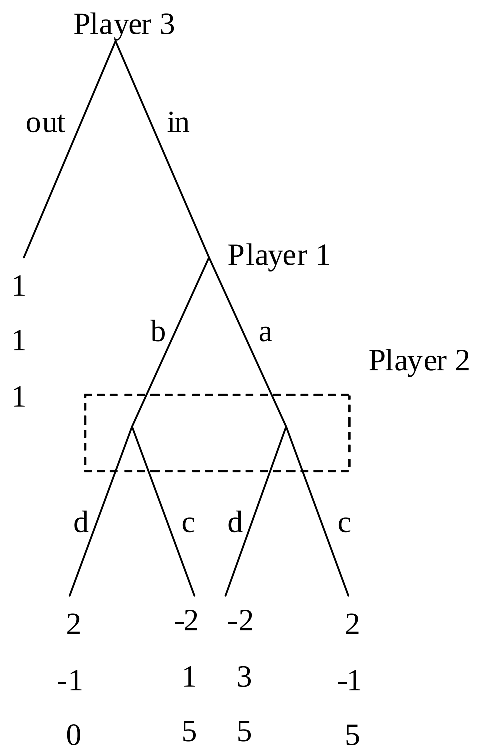


1a. Find the unique subgame perfect equilibrium of this game.

1b. Identify all other pure strategy Nash equilibria of this game. Explain why none of these other equilibria are sub-game perfect.

2. ** Consider the 3 player game depicted in Figure 2.

Figure 2



2a. Explain why there is no subgame perfect Nash equilibrium in pure strategies.

2b. Find the unique subgame perfect Nash equilibrium.

3. * Suppose three firms compete in a market for a single product with industry inverse demand curve $p = A - Q$. All three firms have constant marginal cost m. Firm 1 is a leader and selects output level q_1. Firms 2 and 3 are followers and select q_2 and q_3 after q_1. Note, q_2 and q_3 are chosen simultaneously. Total output is $Q = q_1 + q_2 + q_3$. Find the subgame perfect Nash equilibrium.

4. ** Consider three firms in an industry such that each firm $i = 1, 2, 3$ produces quantity q_i. In fact, firms produce their output sequentially, that is firm 1 produces first q_1, then, in the second stage, firm 2 produces q_2, and then, in the third stage, firm 3 produces q_3. The market price is given by:

$$p = 120 - Q$$

where $Q = q_1 + q_2 + q_3$. The marginal cost of production is assumed to be the same for all firms and constant and equals to c.

4a. Determine the subgame perfect Nash equilibrium of this game by giving the equilibrium quantities of each firm q_1^*, q_2^* and q_3^*, the equilibrium price p^* as well as the equilibrium profits π_1^*, π_2^* and π_3^*. All these equilibrium values should be expressed in terms of only c.

4b. Which firm produces the most and which firm produces the least in equilibrium? Which firm has the highest profit and which firm has the lowest profit in equilibrium? Explain why.

4c. How each firm i's ($i = 1, 2, 3$) profit varies with c? Explain.

4d. Assume now that the firm 1 is a potential entrant whereas firms 2 and 3 are incumbents. If firm 1 enters the market, the sequence of actions is as before, that is firm 1 first chooses q_1, then firm 2 chooses q_2 and finally firm 3 chooses q_3. The entry cost for firm 1 is $F > 0$. If firm 1 chooses to not enter, then its profit is zero. We assume that if firm 1 has profit equals to zero whether it enters or not enters in the market, it always prefers enter in the market. For which value of F the firm will decide to enter in this market?

4e. Assume now that $F = c^2/16$. Does firm 1 enter in the market? Discuss the results in terms of entry versus not entry for different values of c. Calculate firm 1's profit in all possible cases.

5. *** Exercise 177.2 (Osborne, only questions 5a and 5b) (The "rotten kid theorem")

A child's action a affects both her own private income $c(a)$ and her parent's income $p(a)$; for all values of a we have

$$c(a) < p(a)$$

The child is selfish: she cares only about the amount of money she has. Her loving parent cares both about how much money she has and how much her child has.

Specifically, the preferences of the child are represented by a payoff function U_c equals to the smaller of the amount of money she has and the amount of money her child has.

The parent may transfer money to the child. This transfer is denoted by $t \geq 0$. In that case, the utility U_c of the child is given by:

$$U_c(a, t) = c(a) + t$$

whereas the utility of the parent is:

$$U_p(a, t) = \min\{p(a) - t\ ,\ c(a) + t\}$$

The timing is as follows. First the child takes an action, then the parent decides how much money to transfer.

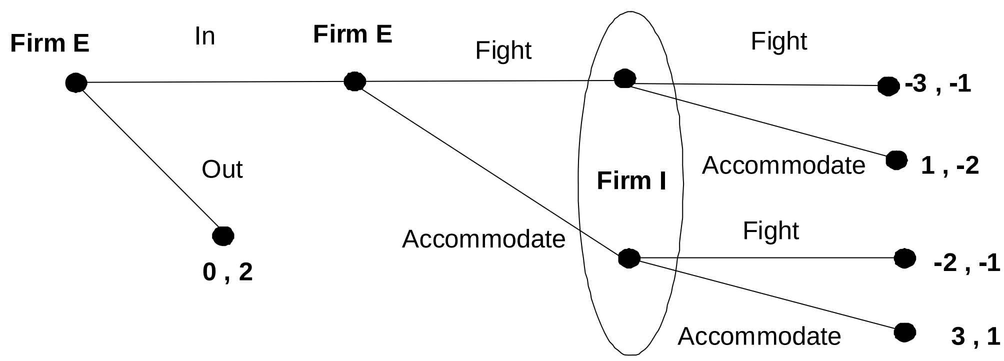
5a. Model this situation as an extensive game.

5b. Show that in a subgame perfect equilibrium the child takes an action that maximizes the sum of her private income and the parent's income. (In particular, the child's action does not maximize her own private income.)

5c. We now assume that $c(a) = a$ and $p(a) = 2a$. We also assume that a can only take values between 0 and 1, i.e. $a \in [0, 1]$. Determine the unique subgame perfect Nash equilibrium of this game. Give the equilibrium utilities of the parent and the child.

5d. Assume now assume that $c(a) = -a^2$ and $p(a) = a$. Determine the unique subgame perfect Nash equilibrium of this game. Interpret the results.

6. ** (Mas-Colell, Whinston and Green) Consider the following entry game:

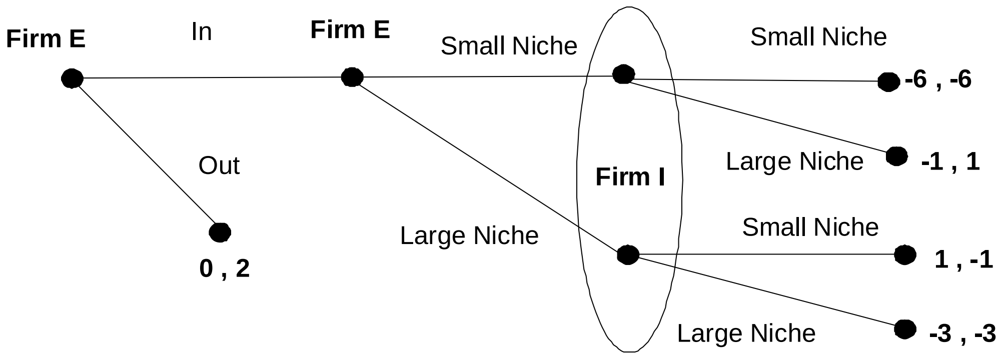


There are two stages. In the first one, firm E has to decide to enter or not. In the second stage, firms E and I play a simultaneous game which is given by:

E/I	Accommodate	Fight
Accommodate	$3, 1$	$-2, -1$
Fight	$1, -2$	$-3, -1$

6a. Calculate the pure-strategy subgame perfect Nash equilibria of this game.

Let us modify this game in the following way. Instead of having the two firms to choose whether to fight or accommodate each other, we suppose that there are actually two niches in the market, one large and one small. After entry, the two firms E and I decide simultaneously which niche they will be in. For example, the niches might correspond to two types of customers, and the firms may be to which type they are targeting their product design. Both firms lose money if they choose the same niche, with more lost if it is the small niche. If they choose different niches, the firm that targets the large niche earns a profit, and the firm with the small niche incurs a loss, but a smaller loss than if the two firms targeted the same niche. The extensive form of the game is depicted in the following figure:



As before, this means that there are two stages. In the first one, firm E has to decide to enter or not. In the second stage, firms E and I play a simultaneous game which is given by:

E/I	Small Niche	Large Niche
Small Niche	$-6, -6$	$-1, 1$
Large Niche	$1, -1$	$-3, -3$

6b. Calculate the pure-strategy subgame perfect Nash equilibria of this new game.

6c. Solve the mixed strategy equilibrium involving actual randomization in the post-entry subgame. Is there an SPNE that induces that behavior in the post-entry subgame? What are the SPNE strategies?

7. ** (Exercise 192.1 Osborne) (Sequential variant of Bertrand's duopoly game)

Consider the variant of Bertrand's duopoly game in which first firm 1 chooses a price (first stage), then firm 2 chooses a price (second stage). Assume that each firm is restricted to choose a price that is an integral number of cents, that each firm's unit cost is constant, equal to c (an integral number of cents), and that the monopoly profit is positive. The payoff function of firm $i = 1, 2$ $(i \neq j)$ is given by:

$$\pi_i\left(p_i, x_j\right) = \begin{cases} (p_i - c)\, D(p) & \text{if} \quad p_i < p_j \\ \frac{1}{2}\left(p_i - c\right) D(p) & \text{if} \quad p_i = p_j \\ 0 & \text{if} \quad p_i > p_j \end{cases}$$

where $D(p)$ is the market demand.

7a. Specify an extensive game with perfect information that models this situation.

7b. Give an example of a strategy of firm 1 and an example of a strategy of firm 2.

7c. Find the subgame perfect equilibria of the game.

8. *** (Exercise 211.1 Osborne) (Timing claims on an investment)

An amount of money accumulates; in period t ($t = 1, 2, ..., T$), its size is \$$2t$. In each period two people simultaneously decide whether to claim the money. If only one person does so, she gets all the money; if both people do so, they split the money equally; and if neither person does so, both people have the opportunity to do so in the next period. If neither person claims the money in period T, each person obtains \$$T$. Each person cares only about the amount of money she obtains.

8a. Formulate this situation as an extensive game with perfect information and simultaneous moves.

8b. Find the subgame perfect equilibrium (equilibria?) of this game. (Start by considering the cases $T = 1$ and $T = 2$.)

9. *** There are two players, a seller and a buyer, and two dates. At date 1, the seller chooses her investment level $I \geq 0$ at cost I. At date 2, the seller may sell *one unit* of a good and the seller has cost $c(I)$ of supplying it, where $\lim_{I \to 0} c'(I) = -\infty$, $c'(I) < 0$, $c''(I) > 0$, and $c(0)$ is less than the buyer's valuation of one unit of the good, which is $V > 0$, so that $c(0) < V$. Suppose that at date 2, the buyer observes the investment I and makes a take-it-or-leave-it offer in terms of price of the good (denoted by P) to the seller.

9a. What is the unique subgame-perfect equilibrium of this game.

9b. Determine the socially efficient outcome of this game, which you denote by I^*. Show that this is a unique maximum and that it is strictly positive, i.e. $I^* > 0$. Explain why the subgame-perfect equilibrium and the socially efficient outcome are different.

9c. Assume now that

$$C(I) = \frac{1}{\left(I + \frac{1}{V} + \frac{1}{2}\right)}$$

What are the conditions on $C(I)$ that are now satisfied? Determine the socially efficient outcome of this game. What is the condition on V that is needed to be assumed for I^* to be strictly positive?

9d. Can you think of a contractual way of avoiding the inefficient outcome of the subgame perfect equilibrium derived in question 9a? In particular, can you find a contract that leads to the socially optimal outcome?