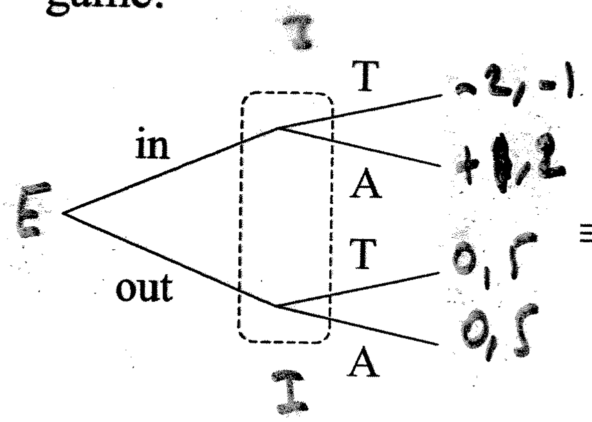
Lecture Notes 6

Observations:

1. The strategies obtained by backward induction are not necessarily unique.

> **Theorem (Khun-Zermelo):** Every finite game of perfect information admits a BI solution. Moreover, if, for every player, no two payoffs are the same, then this solution is unique.

2. Relationship between normal form and extensive form of a game:



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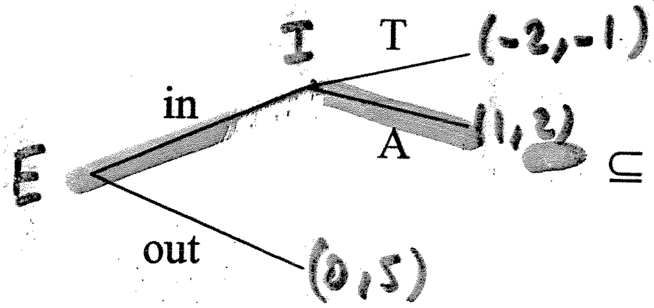
	I: T	I: A
E: in	-2, -1	1, 2
E: out	0, 5	0, 5



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	I: T	I: A
E: in	-2, -1	1, 2
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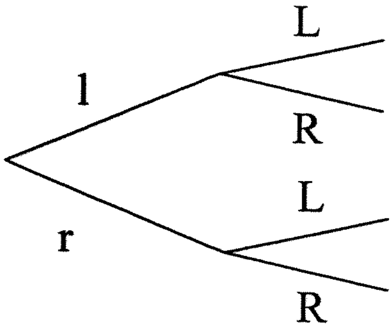


$\subseteq$

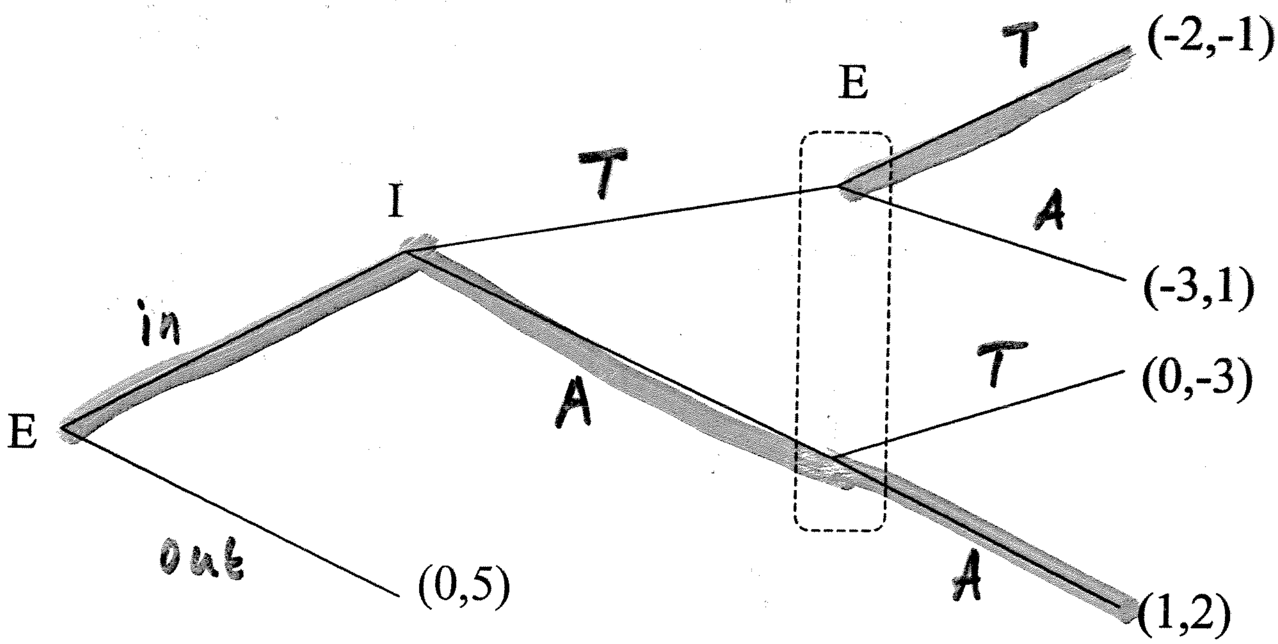
	L,L	L,R	R,L	R,R
l				
r				

3. BI allows us to rule out non-credible strategies. A BI outcome only includes strategies players can *commit* to.

4. IEDS applied to the normal form of the game gives the same solution as BI applied to the extensive form of the game.

Games of Imperfect Information

Example 2: **Entry Game III**



Players: I={E,I}
Strategy Spaces:
S_E ={(in,T), (in,A), (out,T), (out,A)}
S_I ={T,A}
Strategy profile: e.g. (s_E,s_I)=((in,T), T)
Payoffs at terminal nodes

Def.: **Successor** to node x: all of those nodes that can be reached by following some sequence of branches originating from x.

Def.: **Sub-game**: part of the extensive form of a game satisfying:
1. Starts at a single decision node.
2. It contains every successor to this node.
3. If is contains any part of an information set, then it contains all the nodes in that information set.

Def.: **Sub-game Perfect Nash Equilibrium (SPNE):**
Given a game $G = \{I, S = S_1 \times S_2 \ldots \times S_N, u_i(s), s \in S, i = 1,\ldots,N\}$, the strategy profile $s^* \in S$ forms a SPNE in G if it forms a Nash Equilibrium in every sub-game g of G.

To find SPNE of a game G, use a backward induction procedure:

1. Start from a final sub-game and find the all the NE.
2. Replace each final sub-game with one of its NE pay-offs*
3. Consider a penultimate sub-game and find all the NE.

4. Fold back the game until the sub-game that starts with the root of game G. (see next page)

* Multiple NE in sub-games lead to multiple SPNE.

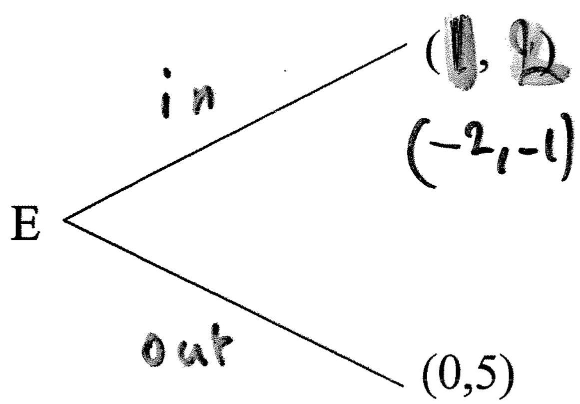
In the entry game:

1. Final sub-game: (Payoffs of the Post-entry competit

E/I	T	A
T	(-2,-1)	(0,-3)
A	(-3,-1)	(1,2)

The NE of this sub-game are: (T,T) and (A,A)

2. Penultimate sub-game:



Sub-game Perfect Nash Equilibria: (IN, A, A)

(OUT, T, T)

Normal form of the game

I

E	T	A
IT	-2,-1	0,-3
IA	-3,1	1,2
OT	0,5	0,5
OA	0,5	0,5

3 pure-strategy Nash equilibria!!!!

1. IA, A
2. OA, T
3. OT, T