Advanced Microeconomic Theory EC104
Problem Set 1

1. Each of n farmers can costlessly produce as much wheat as she chooses. Suppose that the kth farmer produces W_k, so that the total amount of what produced is $W = W_1 + W_2 + ... + W_n$. The price p at which wheat sells is then determined by the demand equation $p = e^{-W}$.

1a. Show that the strategy of producing one unit of wheat strongly dominates all of a profit-maximizing farmer's other strategies. Check that the use of this strategy yields a profit of e^{-n} for a farmer.

1b. Explain why the best of all agreements that treat each farmer equally requires each to produce only $\frac{1}{n}$ units of wheat. Check that a farmer 's profit would then be $\frac{1}{en}$. Why would such an agreement need to be binding (that is, signed as a legally binding contract) for it to be honored by a profit-maximizing farmers?

1c. Confirm that xe^{-x} is largest when $x = 1$. Deduce that all the farmers would make a larger profit if they all honored the agreement rather than each producing one unit and so flooding the market.

1d. You would have realized what the exercise went through was version of the "tragedy of the commons". Why is such an n-player game a generalization of the Prisoners' Dilemma?

2. Consider the following case of a *differentiated good Cournot model.* Firm i produces type i widgets at a constant unit cost of c_i, $i = 1, 2$. If $q_1$and q_2 are the quantities of the two varieties produced, the respective prices for the two goods are determined by the demand equations $p_1 = M - 2q_1 - q_2$ and $p_2 = M - q_1 - 2q_2$. Adapt Cournot's duopoly model to this new situation, and find:

2a. The firms' reaction functions

2b. The quantities produced in equilibrium and prices at which the goods are sold and the equilibrium profits.

3. Two firms set prices in a market whose demand curve is given by $Q = D(p)$, where $D(p)$ is a downward-sloping function and p is the lower of the two prices. The lower priced firm meets all of the demand; if the two firms post the same price, then they each get half the market. Assume costs of production are zero and that prices can only be quoted in dollar *discrete* units (0, 1, 2...).

3a. Show that if the rival firm charges a price above the monopoly price p_m, then the best response is to charge the monopoly price.

3b. Show further that if the rival firm charges a price $p(> 1)$ at or below the monopoly price, then the best response is to charge a price below p.

3c. Conclude from the preceding arguments that the unique Nash Equilibrium price must be for each firm to price at 1 dollar.

3d. What would be the Nash Equilibrium if there were 3 firms in this market? More generally, if there were n firms? Explain.

4. (Osborne, Exercise 60.1) (Variant of Cournot's game, with market-share maximizing firms) Consider a standard Cournot game with two firms 1 and 2 competiting in quantities q_1 and q_2, where the marginal cost of production is constant and the same for both firms and denoted by c and the (inverse) market demand is given by:

$$P(Q) = \begin{cases} \alpha - Q & \text{if} \quad Q \leq \alpha \\ 0 & \text{if} \quad Q > \alpha \end{cases}$$

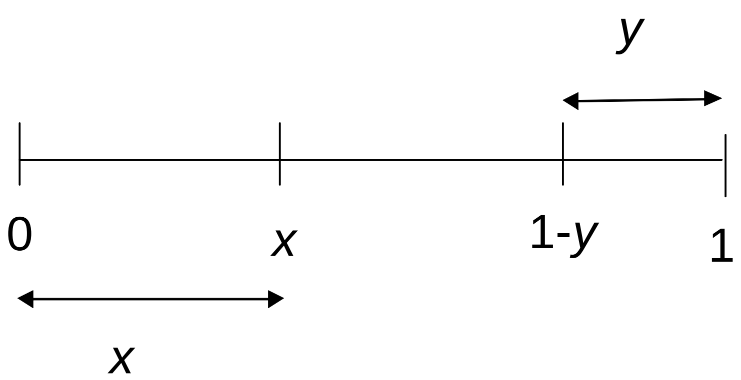
where $\alpha > 0$ and $c \geq 0$ are constant and $Q = q_1 + q_2$ is the market demand.

4a. Find the Nash equilibrium (equilibria?) of a variant of the standard Cournot's duopoly game (linear inverse demand, constant unit cost) that differs only in that one of the two firms chooses its output to maximize its market share subject to not making a loss, rather than to maximize its profit.

4b. What happens if each firm maximizes its market share?

5. (Hotelling voting game with 3 candidates) An election has 3 candidates and takes place under the plurality rule. Voters are uniformly spread along an ideological spectrum from left to right whose extreme points are 0 (extreme left) and 1 (extreme right). Each voter votes for the candidate whose declared position is closest to the voter's own position. The candidates have no ideological attachment and take up any position along the line, each seeking only to maximize her share of votes.

Suppose you are one of the three candidates. The leftmost of the other two is at point x, and the rightmost is at point $(1-y)$, where $x+y<1$, so the rightmost candidate is a distance y from 1. The following figure illustrates this situation:



5a. Show that your best response is to take up the following positions under the given conditions:

(i) just slightly to the left of x if $x>y$ and $3x+y>1$.

(ii) just slightly to the right of $(1-y)$ if $y>x$ and $x+3y>1$.

(iii) exactly halfway between the other candidates if $3x+y<1$ and $x+3y<1$.

5b. In a graph with x and y along the axes (y is on the vertical axis and x on the horizontal axis), show the areas (the combination of x and y values) where each of the response rules (i), (ii), and (iii) of question (5a), is best for you.

5c. From your analysis, what can you conclude about the Nash equilibrium of the game where the three candidates each choose positions?

6. (Osborne Exercise 80.1) (Timing product release) Two firms are developing competing products for a market of fixed size. The longer a firm spends on development, the better its product. But the first firm to release its product has an advantage: the customers it obtains will not subsequently switch to its rival. Once a person starts using a product, the cost of switching to an alternative, even one significantly better, is too high to make a switch worthwhile.

A firm that releases its product first, at time t, captures the share $h(t)$ of the market, where h is a function that increases from time 0 to time T, with $h(0) = 0$ and $h(T) = 1$. The remaining market share is left for the other firm. If the firms release their products at the same time, each obtains half of the market. Each firm wishes to obtain the highest possible market share.

6a. Model this situation as a strategic game and plot the utility function u_i of firm i as a function of t_i for different values of t_j (i.e. consider three cases: $h(t_j) < 1/2$, $h(t_j) = 1/2$, and $h(t_j) > 1/2$).

6b. Calculate the best-reponse functions of each player. Denote the time t for which $h(t_1) = h(t_2) = h(t^*) = 1/2$ by t^*. Observe that when finding firm i's best response to firm j's release time t_j, three cases must be considered:

(1) if t_j is such that $h(t_j) < 1/2$;

(2) if t_j is such that $h(t_j) = 1/2$;

(3) if t_j is such that $h(t_j) > 1/2$.

6c. Show that there is unique a Nash equilibrium in which both firms release their product at time t^* where $h(t_1) = h(t_2) = h(t^*) = 1/2$.