Lecture Notes 8bis

Some examples of Bayes-Nash Equilibria

Example 1: Relation between mixed-strategy equilibrium and Bayesian equilibrium (Gibbons)

A man (Patrick) and a woman (Lindsey) are trying to decide on an evening's entertainment. The following normal form game (Battle of Sexes) represents their payoffs.

		Patrick (Player 2)	
		Opera	Fight
Lindsey	Opera	$(2, 1)$	$(0, 0)$
(Player 1)	Fight	$(0, 0)$	$(1, 2)$

By convention, the payoff to row-player (here Lindsey, player 1) is the first payoff given, followed by the payoff of the column player (here Patrick, player 2).

1a) Find the mixed-strategy equilibria of this game.

We still suppose that Patrick and Lindsey are trying to decide on an evening's entertainment but now Lindsey is unsure whether Patrick prefers to go out with her or prefers to avoid her and Patrick is also unsure whether Lindsey prefers to go out with him or prefers to avoid him. The following nomal form of the game gives their payoffs.

		Patrick (Player 2)	
		Opera	Fight
Lindsey	Opera	$(2 + t_L, 1)$	$(0, 0)$
(Player 1)	Fight	$(0, 0)$	$(1, 2 + t_P)$

Observe that t_L is privately known by Lindsey and t_P is privately known by Patrick. We assume that t_L and t_P are independent draws from a uniform distribution on $[0, x]$.

1b) Give a formal definition of this static Bayesian game in normal form (action spaces, type spaces, beliefs).

We now have the following strategies.

Lindsey plays "Opera" if $t_L \geq c$ and plays "Fight" otherwise. Similarly, Patrick plays "Fight" if $t_P \geq p$ and plays "Opera" otherwise. Observe that $0 < c < 1$ and $0 < p < 1$.

1c) Because the distribution is uniform, what is the probability that Lindsey plays "Opera" and the probability that Patrick plays "Fight"? If the distribution was not uniform, say t_L and t_P are independent draws from a general distribution $F(.)$ on $[0, x]$.

1d) Let us assume again that t_L and t_P are independent draws from a *uniform* distribution on $[0, x]$. Then, if Lindsey and Patrick are playing the strategies describe above, what are the values of c and p such that these strategies are a Bayesian Nash equilibrium.

1e) Show that when x approaches zero, i.e. the incomplete information disappears, the players' behavior in this pure-strategy Bayesian Nash equilibrium approaches their behavior in the mixed-strategy Nash equilibrium in the original game of complete information (question 1a).