Professor Frank A. Wolak **June 2010**

Empirical Methods in Industrial Organization and Energy Economics

Final Exam

Please send your completed exam to Pär Holmberg [Par.Holmberg@ifn.se] no later than 5 pm (Stockholm time) on 6/30/2010 . You can either type your exam answers using your favorite word processor and create a .pdf file or write your answers out by hand and scan them into a .pdf file. In either case, please make sure that the .pdf file you create is readable and has you name and e-mail address written on the first page. Use any written materials you like, such as articles, lecture notes, and textbooks, but do not consult any other person on any topic related to the exam. This is a 200 point exam. Please show your work in other to receive partial credit on questions.

1. (35 points) This problem deals with the analysis of time-series of observations on nine variables describing a monopolist's behavior. These variables are P_K, P_L, P, Q, TC, L, K, Z_1, and Z_2, where P_K is the price of capital, P_L the price of labor, and P is the price of the firm's output Q. TC denotes the total cost of producing this output, and K and L are the amounts of capital and labor used to produce Q, and finally Z_1 and Z_2 are variables not under the control of the firm which enter its demand function.

This monopolist faces an uncertain demand for its output which takes the form

$$P = AQ^{\delta}Z_1^{b_1}Z_2^{b_2}\eta,$$

$\ln(\eta)$ is $N(\mu_\eta,\sigma_\eta^2)$ (independently distributed over time) and A, δ, b_1, b_2 are the parameters of the non-stochastic portion of the demand function. The monopolist's production function is

$$Q = \Gamma K^{\alpha}L^{\beta}\varepsilon$$

where $\ln(\varepsilon)$ is $N(\mu_\varepsilon,\sigma_\varepsilon^2)$ (independently distributed over time). Suppose that ε is known to the monopolist, but unknown to the econometrician. Assume the monopolist sets Q, K, and L to maximize expected profits, where the expectation is taken with respect to η. Assume that the monopolist is a price-taker in the markets for K and L. Assume that Cov(ln(η) , ln(ε)) is zero.

(a) Set up the monopolist's objective function and compute the firm's cost function C(P_K,P_L, Q) implied by this objective function. [Be sure to account for the stochastic portions of the demand and production functions.]

(b) Compute the firm's optimal (expected profit maximizing) output schedule as a function of the input prices it faces and the disturbances to the model.

(c) Can consistent estimates of the parameters α and β be recovered from applying OLS to the equation obtained from taking the log of both sides of the production function? How about OLS applied to the log of the cost function derived in (a)? Justify your answer in both cases.

(d) Suppose OLS is applied to the log of the demand function. Will this lead to consistent estimates of the parameters of the demand function? Justify your answer.

(e) In light of your answers to (c) and (d) outline an estimation procedure that will yield consistent estimates of the parameters of the firm's demand function and production function. In the process be sure to clarify which parameters of the two functions can or cannot be uniquely identified from the data.

(f) Derive a Durbin-Wu-Hausman test of the null hypothesis that the OLS estimates of the parameters of the demand and production function parameters are in fact consistent relative to the estimates you develop in (e). Make sure to state the assumptions necessary for the validity of your hypothesis test.

2. (25 points) Recall the Robert H. Porter (1983) paper, "A study in cartel stability: the Joint Executive Committee, 1880-1886." Suppose market level demand function and firm-level cost functions are:

$$\ln(Q_t) = \alpha_0 + \alpha_1 \ln(p_t) + \alpha_2 L_t + U_{1t} \qquad \text{(D)}$$

$$C_i(q_{it}) = \exp(Z_t'\gamma + U_{2t}) a_i q_{it}^{\delta} + F_i \text{ for } (i=1,...,N) \qquad \text{(C)}$$

where N is the number of firms in the industry, Z_t is a vector of cost shifters at time t, γ is a vector of parameters to be estimated, Q_t is the sum over all i of q_{it}, and all other notation is defined in Porter (1983). Suppose that $s_{it} = q_{it}/Q_t$, the quantity-share of firm i at time t, is also observed for all firms.

(a) Derive an implicit expression for the industry supply curve assuming the Cournot quantity-setting equilibrium involving s_{it} using the first-order condition $p_t(1 + \theta_{it}/\alpha_1) = MC_i(q_{it})$ (i=1,...,N), with θ_{it} set to the appropriate value.

(b) What additional parameters in the model of equations (D) and (C) are identified as a result of the s_{it} being observed.

(c) Describe an instrumental variable estimation technique to estimate the parameters of the demand function (D) and the aggregate supply function you derived in part (a). Explain what parameters are in fact identified, what instruments you would use and what is required for them to be valid instruments.

(d) Describe a maximum likelihood approach to estimating the parameters of the demand function (D) and the aggregate supply function you derived in part (a). Explain what paramaters of the model are identified using this estimation technique. Be sure to state precisely the assumptions necessary for the consistency of your proposed estimation procedure and justify the result.

3. (20 points) For this problem assume the following the two-equation demand and supply model is the true data generation process:

$$Q_d(t) = \beta_0 + \beta_1 P(t) + \beta_2 INC(t) + \beta_3 PS(t) + \varepsilon_{dt} \qquad \text{(D)}$$

$$Q_s(t) = \alpha_0 + \alpha_1 P(t) + \alpha_2 WAGE(t) + \alpha_3 RATE(t) + \varepsilon_{st} \qquad \text{(S)}$$

for t=1,...,T, where $(\varepsilon_{dt}\ \varepsilon_{st})'$ are independent (across t) joint normally distributed with zero means and variances σ_{dd} and σ_{ss} and covariance σ_{sd}.

The other variables are defined as follows: $Q_d(t)$= quantity demanded, $Q_s(t)$ = quantity supplied, P(t) = price of output, PS(t) = price of the substitute good, INC(t) = total income, WAGE(t) = wage rate, RATE(t) = rate of return on capital. The equilibrium condition $Q_s(t)$= $Q_d(t)$ completes this system of equations, so that only the market clearing value of Q is observed. Assume that PS(t), INC(t), WAGE(t), and RATE(t) are strongly exogenous variables relative to this two-equation market equilibrium in the sense that the conditional distribution of $(\varepsilon_{dt},\varepsilon_{st})'$ given all T observations of (PS(t), INC(t), WAGE(t), RATE(t)) is bivariate normal with the mean vector and covariance matrix given above for all observations.

Prove or disprove the following statements. Justify your answers.

(a) The regression coefficient associated with PS(t) obtained from an ordinary least squares (OLS) regression of Q(t) on all of the exogenous variables is a consistent and unbiased estimate of the causal effect of a one unit change in PS(t) on the equilibrium value of Q(t).

(b) The regression coefficient associated with P(t) obtained from OLS applied to (D) yields a consistent estimate of the derivative, with respect to P(t), of the conditional expectation of Q(t) given P(t), INC(t), and PS(t), mathematically $\partial E[Q(t) \mid P(t),INC(t),PS(t)]/\partial P(t)$.

(c) The regression coefficient associated with P(t) from OLS applied to (D) yields a consistent estimate of the slope of the demand curve.

(d) The best (in a minimum mean-squared error sense) prediction of what the new equilibrium P and Q in hypothetical period s (that is not included in the estimation sample) given PS(s), INC(s), WAGE(s), and RATE(s) can be computed by the following two-step procedure. Regress both P(t) and Q(t) on all the exogenous variables. Using these OLS parameter estimates, compute the fitted values of P and Q evaluated at PS(s), INC(s), WAGE(s), and RATE(s).

4. (25 points) For this problem assume the following the two-equation demand and supply model is the true data generation process:

$$Q_d(t) = f(P(t), INC(t), PS(t),\varepsilon_{dt},\theta) \qquad (D)$$

$$Q_s(t) = g(P(t),WAGE(t),RATE(t),\varepsilon_{st},\beta) \qquad (S)$$

for t=1,...,T, where $f(P(t), INC(t), PS(t),\varepsilon_{dt},\theta)$ and $g(P(t),WAGE(t),RATE(t),\varepsilon_{st},\beta)$ are known parametric functions of θ and β, respectively, that are nonlinear in all of their arguments. These functions are also monotone in P(t) and ε_{dt} and ε_{st}, respectively. The $(\varepsilon_{dt}\ \varepsilon_{st})'$ and the exogenous variables satisfy the properties described in question 3.

Prove or disprove the following statements. Justify your answers.

(a) The regression coefficient associated with PS(t) obtained from an ordinary least squares (OLS) regression of Q(t) on all of the exogenous variables is a consistent estimate of the causal "reduced form" effect of a one unit change in PS(t) on the equilibrium value of Q(t).

(b) The regression coefficient associated with P(t) obtained from OLS applied to (D) yields a consistent estimate of the derivative, with respect to P(t), of the best (in a minimum mean-squared error sense) linear predictor of Q(t) given P(t), INC(t), and PS(t), mathematically $\partial BLP[Q(t)|P(t),INC(t),PS(t)]/\partial P(t)$.

(c) The best (in a minimum mean-squared error sense) prediction of what the new equilibrium P and Q in period s given PS(s), INC(s), WAGE(s), and RATE(s) can be computed by the following two-step process. Regress both P(t) and Q(t) on all the exogenous variables. Using these OLS parameter estimates, compute the fitted values of P and Q evaluated at PS(s), INC(s), WAGE(s), and RATE(s).

(d) It is possible to formulate an objective function whose maximum value in $\delta = (\theta', \beta', \sigma_{dd}, \sigma_{ss}, \sigma_{sd})'$ yields a consistent, asymptotically normal estimate of δ. If so, suggest a procedure to derive this objective function.

(e) It is possible to formulate a hypothesis test of the statement in part (c). If so, describe this hypothesis test.

(f) Given of your answers to parts 3(a)-3(d) and 4(a)-4(d), why should an empirical researcher ever be interested in simultaneous equations estimation techniques?

5. (30 points) Suppose that you are a large buyer participating in a wholesale electricity market with many small suppliers. Suppose that the revenues you earn from consuming electricity to produce widgets is equal to $TR(Q) = 100*Q - ½*Q^2$. Depending on the realizations of forced outages among the generators serving this market, the aggregate marginal cost curve is $MC_L(Q) = 10 + Q$ or $MC_H(Q) = 20 + Q$. These marginal cost curves occur with equal probability. Suppose that the remaining demand for electricity is 20 MWh regardless of market-clearing price. Assume that market prices are set by solving the following equation for DB(P) + 20 = S(P), where S(P) is the aggregate supply curve (the realized inverse aggregate marginal cost curve, because generators are all assumed to act as price-takers) and DB(P) is your demand bid for electricity, your announced willingness to buy electricity as a function of the price paid. Besides consuming electricity, widgets are costless to produce and sell.

(a). Suppose you simply bid your true marginal willingness to pay into the wholesale market. In other words, your demand bid curve is equal to your true marginal willingness to pay at each quantity level. Compute the market clearing price for each possible realization of the aggregate marginal cost curve. Compute your expected profits from selling widgets associated with this bidding strategy.

(b). Suppose the market rules restrict you to submitting demand bid curves of the form DB(P) = A + B*P. Compute the bid curve that maximizes your expected profits from selling widgets.

(c). Suppose the market rules allowed you to submit an arbitrary downward sloping demand bid curves. What demand bid curve would you submit?

6. (30 points). Suppose two large generator faces an uncertain but inelastic demand that has a standard exponential distribution on the interval $(0,\infty)$. The density of demand is $f(t) = \exp(-t^2)$. There a price-taking fringe of suppliers, with aggregate marginal cost curve equal to MC(Q) = 2*Q. Suppose these two large generators have a marginal curve equal to $mc(q) = ½*Q^2$.

(a). Compute the symmetric equilibrium bid function, SB(p), for each generator.

(b). Suppose each generator has a forward contract obligation of QC = 50 MWh at a contract price, PC, of \$10/MWh. Compute symmetric equilibrium bid function for this level of contract cover.

(c). Compare the optimal bid functions derived in (a) and (b). Is there any relationship between these two bid functions?

7. (35 points) A supplier offering into a short-term bid-based market faces the following set of circumstances. The hourly market demand is equal to 500 MWh with probability π, for $0 \leq \pi \leq 1$, and 400 MWh with probability $(1 - \pi)$. In both states of the world, this supplier faces a competitve fringe with a supply curve of $SO(p) = p$ up to a maximum capacity of 400 MW. This suppler owns 500 MW of generation capacity with a marginal cost of zero. Assume that p_m is the cap on price offers into the short-term market and that suppliers are required to submit non-decreasing continuous offer curves into the short term market. The market clearing price is the solution in p of $Q_d = S(p) + SO(p)$, where Q_d is the realized value of the market demand, $S(p)$ is the supplier's offer curve, and $SO(p)$ is defined above.

(a) Suppose that p_m = \$500/MWh. Derive an expression for the supplier's expected profit-maximizing offer curve for all $\pi \in [0,1]$.

(b) Suppose that p_m = \$1,000/MWh. Derive an expression for the supplier's expected profit-maximizing offer curve for all $\pi \in [0.7,1]$.

(c) Derive a general relationship between the values of p_m and π and the supplier's expected profit-maxmizing offer curve?

(d) What do the results of this problem tell you about the level of the offer cap and the unilateral incentive a supplier has to submit an offer price equal to the offer cap?