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A Nonwalrasian Model of the Business Cycle

by

J.-P. Benassy

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Jean-Pascal BENASSY

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1. INTRODUCTION [*]

The object of this paper is to show how the dynamic evolution of a short-run non Walrasian equilibrium [1] can generate cycles. Intuitively, the cycle will result from the combination of "destabilizing" quantity dynamics, and of "stabilizing" price dynamics. The destabilizing element comes from a traditional investment accelerator and from the dynamic adjustment of demand expectations [2] while the stabilizing element comes from the effects of prices on aggregate demand, and from the wage movements via a traditional Phillips curve. The resulting system may be unstable around its long-run equilibrium if the accelerator is sufficiently strong. Since we can also show that the system always remains within a bounded set, we can apply Poincaré-Bendixson type techniques ans show the existence of limit cycles [3].

The model we present here has a number of important novel elements with respect to the previous literature : (i) we are able to obtain cycles using the traditional shapes for the various functions. In the contrary most previous contributions to non linear cycle theory had to

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(1) The non-Walrasian method is developed extensively in Benassy (1982).

(2) That a strong enough accelerator effect can lead to unstable or cyclic behavior is known since the classic studies of Kalecki (1935), Samuelson (1939), Kaldor (1940), Hicks (1950), Goodwin (1951).

(3) The Poincaré-Bendixson technique has already been applied to various models of cycles by Rose (1967, 1969), Chang and Smyth (1971), Dana and Malgrange (1981), Schinasi (1982).

make fairly ad hoc assumptions such as sigmoid shapes for the investment function or the Phillips curve ; (ii) the use of a non-Walrasian equilibrium structure in the short run allows to bridge the gap between cycle theory and traditional short run Keynesian analyses, which are made in terms of such non-Walrasian equilibria ; (iii) more importantly this structure allows to give a consistent determination of prices and quantities in the short run. In the contrary, previous contributions to cycle theory using the Poincaré-Bendixson technique had no such short run structure ;as a result, in particular, discrepancies between investment and savings could occur in the short run, which could be made sense of only by invoking inventory adjustments not formalized in the models. Such discrepancies do not occur in our model.

2 . THE MODEL

The short-run

In the short-run, the structure of the model is that of a fairly traditional IS-LM model with fixed wage and flexible price [1]. We shall assume that there is an inelastic supply of labor ℓ_0 and that the productive capacities of the economy are represented by a production function with decreasing returns $F(\ell)$. The "Demand block" is given by the two following simple IS-LM type equations [2] :

$$\begin{cases} y = C(y,p) + I(x,r) \\ L(y,r,p) = \bar{M} \end{cases}$$

where y is current output and income, p current price, x expected demand, r the interest rate and $\bar{M}$ the (fixed) quantity of money in the economy. We may note that the above system is very close to the usual Keynesian system, except for one point : Here we introduce explicitly expected demand in the investment function, while in the traditional system this expected demand is usually proxied by some function of past and current levels of income. We assume that all the above functions are continuously differentiable, with the following partial derivatives [3] :

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(4) For more developments on the relation between the IS-LM model and non Walrasian theory, see Benassy (1983).

(5) More arguments could be easily added in the consumption and investment function , but we want to keep the formulas as simple as possible. Note in particular that the results below would still hold if investment depended negatively upon p and w .

(6) A subscript to a function denotes a partial derivative with respect to the corresponding variable, e.g. $C_y = \partial C/\partial y$.

$$0 < C_y < 1 \qquad C_p < 0$$

$$I_x > 0 \qquad I_r < 0$$

$$L_y > 0 \qquad L_r < 0 \qquad L_p > 0$$

The negative effect of the price level on consumption comes from some type of "real balance effect" (even though the consumption function needs not be homogeneous in price and money holdings). We shall assume that because of this effect the consumption is strictly positive for any finite price level, even with zero income.

Dynamics

We shall now study the dynamical equations governing the system. Since the quantity of money is fixed, we have only to describe the evolution of the wage w and of expected demand x . We assume that the wage evolves according to a traditional Phillips curve, i.e. that wage increases are a decreasing function of the level of unemployment u . This will be described mathematically as :

$$\dot{w} = H(u) \qquad H'(u) < 0$$

We assume for the function H the traditional shape pictured on Figure 1. In particular we assume that H(u) tends to infinity when u goes to zero. We denote by $\bar{u}$ the non inflationary rate of unemployment, i.e. the level of unemployment such that $H(\bar{u}) = 0$. Note that since output and employment are related rigidly via the production function, we can also write the Phillips curve as :

$$\dot{w} = G(y)$$

$$G(y) = H[\ell_0 - F^{-1}(y)] \qquad G'(y) > 0$$

The function G(y) tends to infinity when y tends to $y_0 = F(\ell_0)$, and is equal to zero for $\bar{y} = F(\ell_0 - \bar{u})$. As we shall see below, $\bar{y}$ will be the long run equilibrium value of output.

Figure 1

As for demand expectations, we shall assume that they adaptatively adjust towards the value of current demand y , i.e. :

$$\dot{x} = \mu(y-x) \qquad \mu > 0$$

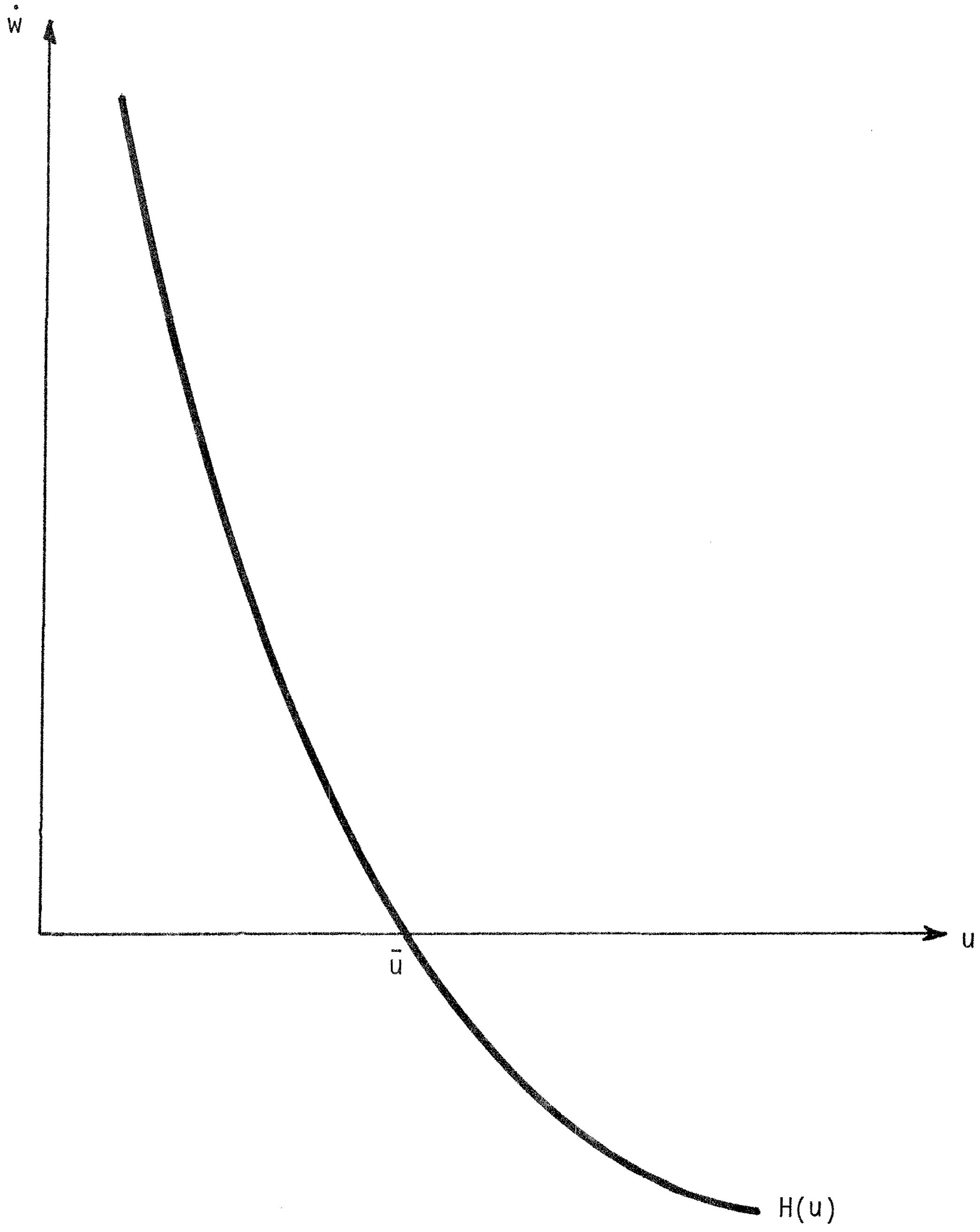

Figure 1

3. TEMPORARY KEYNESIAN EQUILIBRIUM AND DYNAMICS

Short-run equilibrium

We shall now see the equations which allow to determine the short run equilibrium for given w and x . We already saw the IS-LM equations forming the "demand block" :

$$\begin{cases} y = C(y,p) + I(x,r) \\ L(y,r,p) = \bar{M} \end{cases}$$

Let us call K(x,p) the solution in y of this system, which corresponds to the traditional "aggregate demand curve" of Keynesian theory. It is easy to compute that

$$K_x = \frac{I_x L_r}{(1-C_y)L_r + L_y I_r} > 0$$

$$K_p = \frac{C_p L_r - I_r L_p}{(1-C_y)L_r + L_y I_r} < 0$$

We shall assume that this aggregate demand curve is price responsive for extreme values of p , more specifically that :

$$\lim_{p \to 0} K(0,p) > y_0 \qquad\qquad \lim_{p \to +\infty} K(y_0,p) < \bar{y}$$

The first assumption derives naturally from the existence of a "real balance" type effect. The second is also quite natural as price increases reduce both components of aggregate demand, the consumption directly and the investment via the LM equation and the induced interest rate increases.

As for the "supply side", since we assume that the goods market clears and the firm behaves competitively, output and price will be related by the following equation :

$$y = \min\{F[F'^{-1}(w/p)] \ , \ y_0\}$$

Indeed if the employment constraint is not binding, the firm is on its neoclassical supply curve. If however this neoclassical supply is higher than full employment $y_0 = F(\ell_0)$, then output is limited to y_0 .

The short run equilibrium is defined by the above supply equation, and the two IS-LM equations. Combining these last two in the function K , we find that y and p are defined by the simpler system :

$$\begin{cases} y = K(x,p) \\ y = \min\{F[F'^{-1}(w/p)], \ y_0\} \end{cases}$$

A solution will exist for all x such that :

$$\lim_{p\to\infty} K(x,p) < y_0$$

With the assumptions on the function K made above, we know that existence will be ensured at least for all $x \leq y_0$ (which is all we will need in what follows), but usually it will be the case for a much larger range of values of x . The short-run equilibrium could possibly be either one of full employment, or one with unemployment, depending on which part of the "supply curve" the system is. However the shape we assumed for the Phillips curve implies that full employment will never be reached in the dynamic evolution of the system, as wage increases become infinite when y approaches y_0 . As a result the supply side of the model is simply described by :

$$y = F[F'^{-1}(w/p)]$$

and the values of y , r , p in the short-run are determined by the following system , where we revert to the original IS-LM equations :

$$\begin{cases} y = C(y,p) + I(x,r) \\ L(y,r,p) = \bar{M} \\ y = F[F'^{-1}(w/p)] \end{cases}$$

We shall denote as Z(x,w) the solution in y of this system. Let us call :

$$S(p,w) = F[F'^{-1}(w/p)] \qquad S_p > 0 \qquad S_w < 0$$

Then the partial derivatives of the function Z are :

$$Z_x = \frac{S_p L_r I_x}{S_p L_y I_r + S_p(1-C_y)L_r + L_p I_r - C_p L_r} > 0$$

$$Z_w = \frac{(L_p I_r - C_p L_r)\ S_w}{S_p L_y I_r + S_p(1-C_y)L_r + L_p I_r - C_p L_r} < 0$$

We may note that Z_x is higher, the higher is I_x , i.e. the "acceleration" coefficient. We have drawn on Figure 2 a few typical curves Z(x,w) as a function of w for given values of x . Because consumption is strictly positive for all p , the curves Z(x,w) are strictly above the horizontal axis for all w . The assumptions made above on the function K ensure moreover that, as is depicted on Figure 2 :

$$\lim_{w \to 0} Z(0,w) > y_0 \qquad \lim_{w \to +\infty} Z(y_0,w) < \bar{y}$$

A property which will notably ensure the existence of a long run equilibrium.

<u>Figure 2</u>

The dynamic system and long-run equilibrium.

The full dynamical system can now be summarized by the following equations :

$$\begin{cases} y = Z(x,w) \\ \dot{w} = G(y) \\ \dot{x} = \mu(y-x) \end{cases}$$

The long-run equilibrium (x^*,y^*,w^*) of this system is evidently given by the following relations :

$$\begin{cases} y^* = x^* = \bar{y} \\ Z(\bar{y},w^*) = \bar{y} \end{cases}$$

It exists and is unique, given the above assumptions, as is easily seen on Figure 2. We shall now investigate the stability properties of the model and show that it is either stable around the long run equilibrium, or unstable in which case there will be at least a limit cycle.

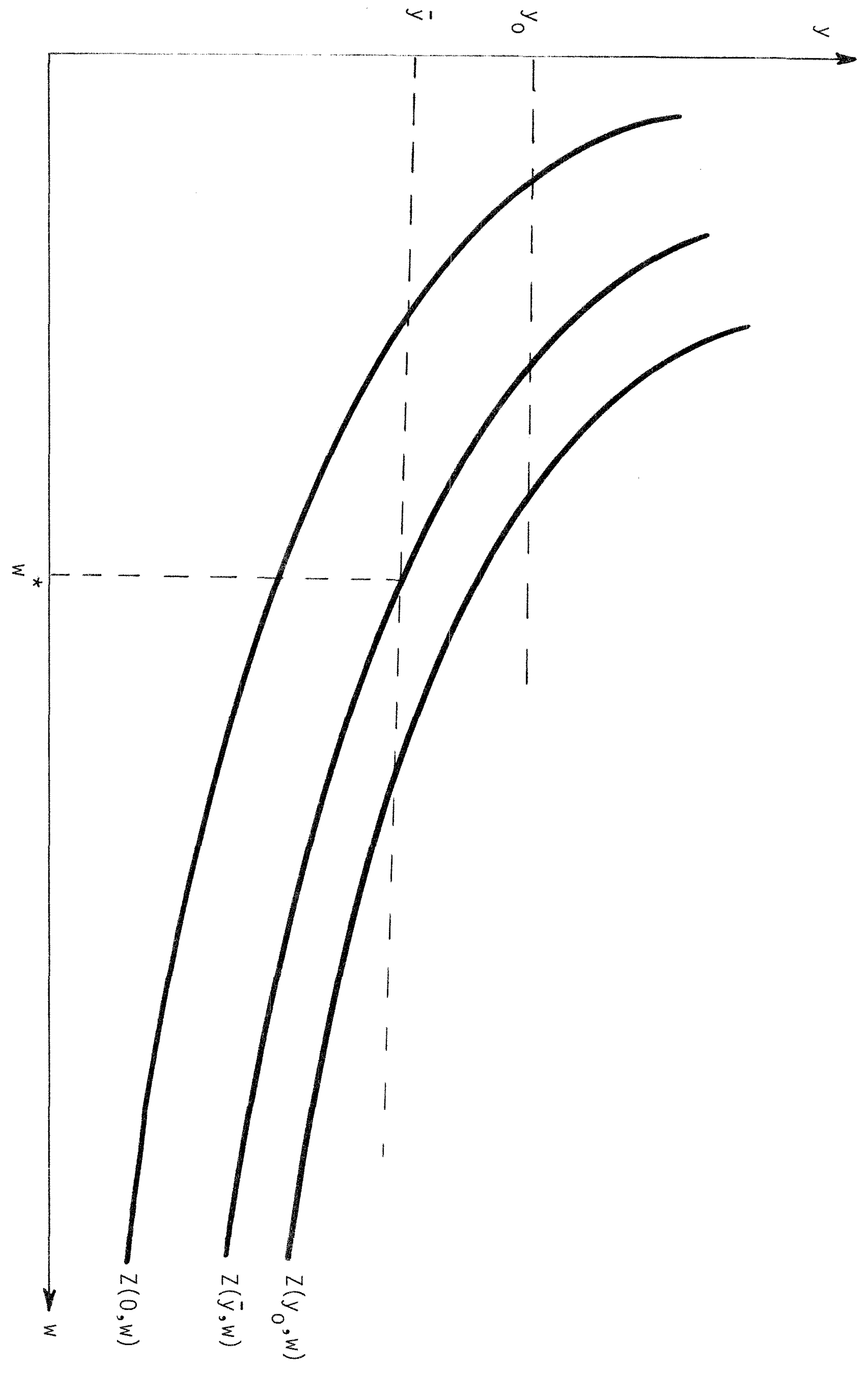

Figure 2

4. STABILITY AROUND THE LONG-RUN EQUILIBRIUM

If we linearize the above dynamical system around equilibrium, we obtain the following in matrix form :

$$\begin{bmatrix} \dot{w} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} \phi Z_w & \phi Z_x \\ \mu Z_w & \mu(Z_x - 1) \end{bmatrix} \begin{bmatrix} w - w^* \\ x - x^* \end{bmatrix}$$

where ϕ is the derivative of G evaluated at the equilibrium point. The determinant of this matrix is equal to $-\phi\mu Z_w$, which is always positive. The local stability of the system will thus be determined by the value of the trace, i.e. :

$$T = \phi Z_w + \mu(Z_x - 1)$$

We see that if $T < 0$, both roots of the characteristic polynomial of the matrix have a negative real part and the system is locally stable. However if $T > 0$, i.e. if :

$$\mu(Z_x - 1) > -\phi Z_w$$

then the system is unstable around the long run equilibrium since both roots have positive real parts. This will happen in particular if the acceleration coefficient I_x , and thus Z_x , is high. In such a case we shall show below that there exists at least a limit cycle. We have drawn on Figure 3 two typical phase diagrams . The curves $\dot{w} = 0$ and $\dot{x} = 0$ have respectively for equations :

$$\dot{w} = 0 \iff Z(x,w) = \bar{y}$$

$$\dot{x} = 0 \iff Z(x,w) = x$$

The curve $\dot{w} = 0$ has always a positive slope. The curve $\dot{x} = 0$ has a positive slope if $Z_x > 1$, a negative one if $Z_x < 1$. We thus see that in the case represented by Figure 3.a we always have local stability, while local instability implies that the locus $\dot{x} = 0$ is upward sloping at the long run equilibrium, as in Figure 3.b. We may also note that, because $Z(x,w)$ is always strictly positive, the locus $\dot{x} = 0$ is strictly above the horizontal axis.

Figure 3

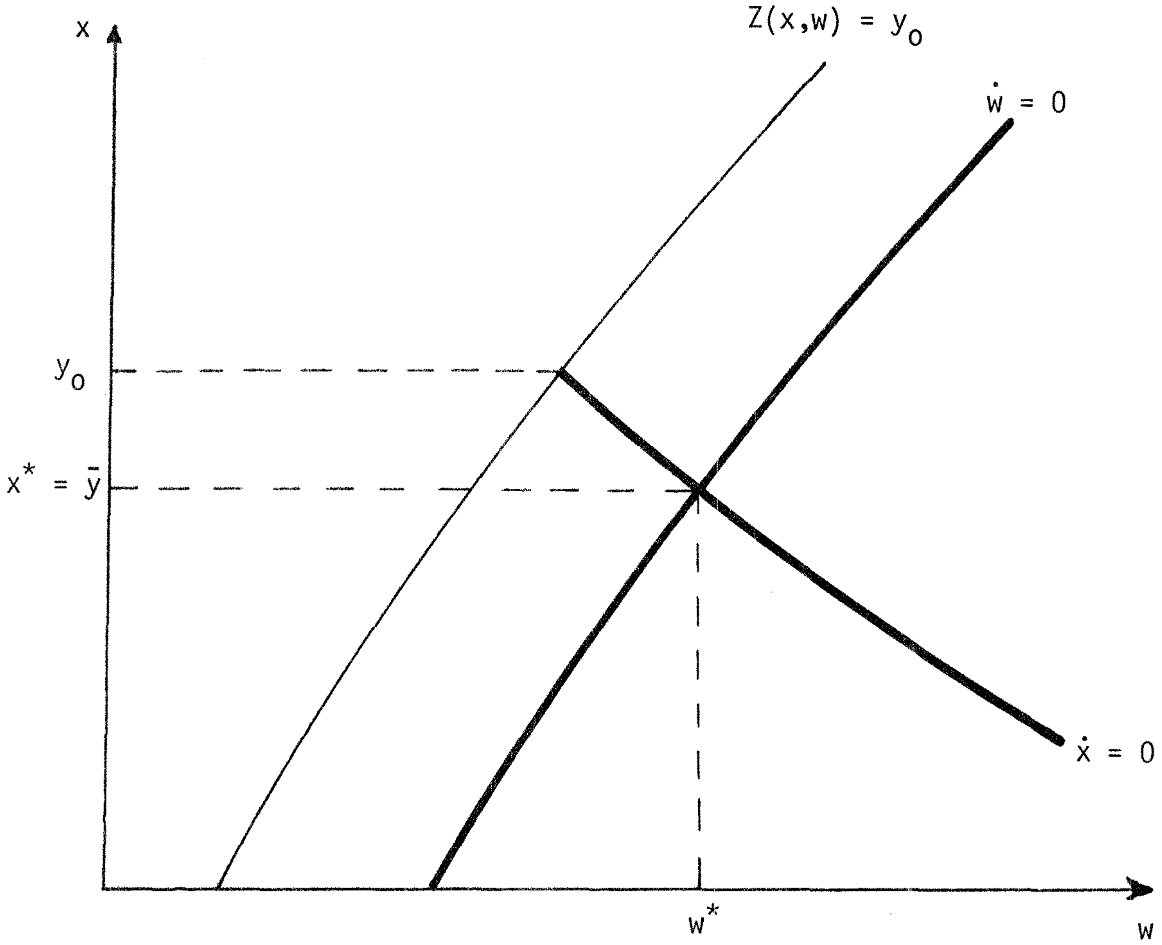

Figure 3.a

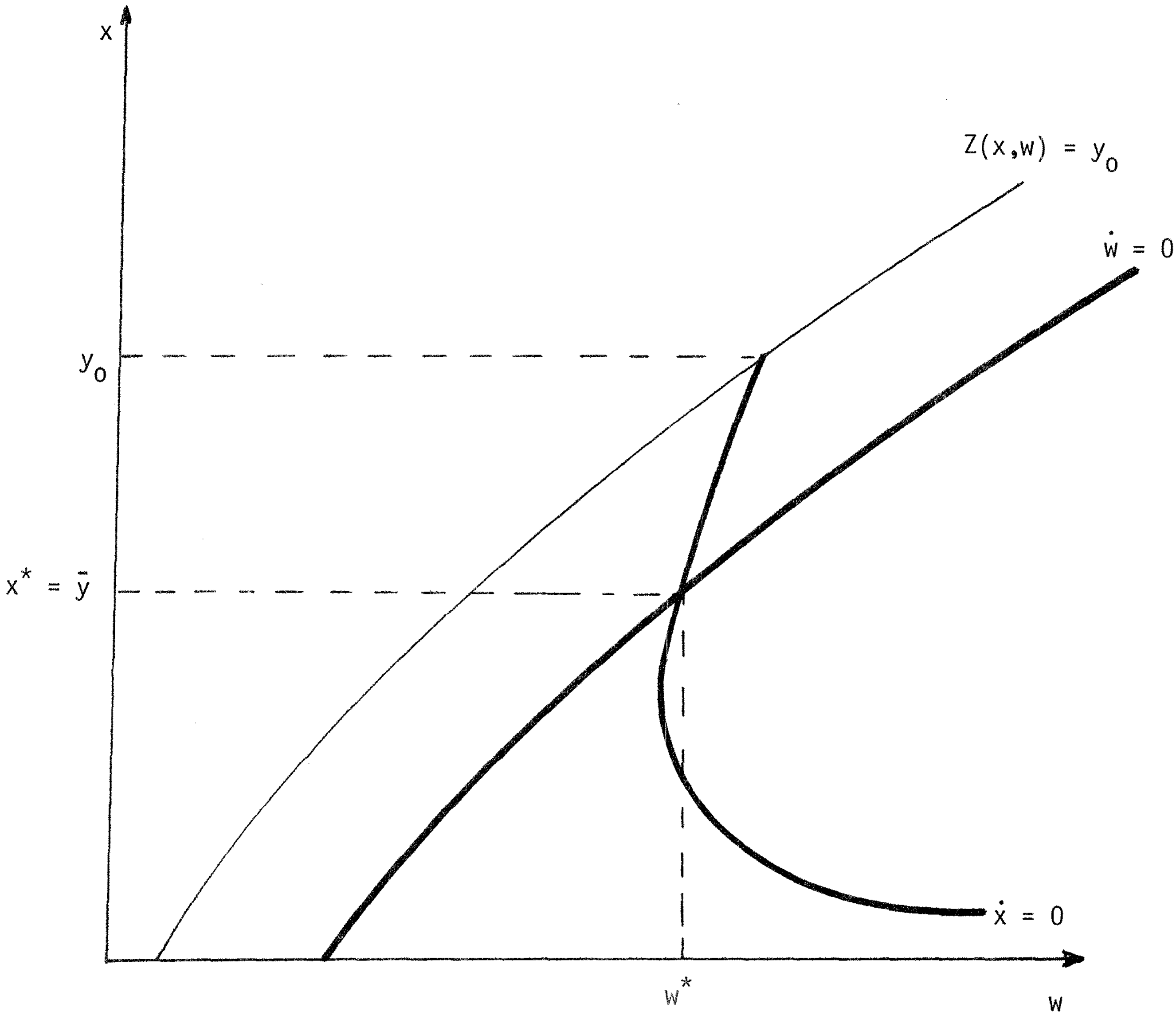

Figure 3.b

5. THE EXISTENCE OF CYCLES

We shall now show that our model can generate limit cycles by using the Poincaré-Bendixson theorem [7]. Applied to our case this theorem says that there will be at least a limit cycle if:(i) there exists a compact set in (x,w) space that the dynamic system points inwards everywhere on the boundary,(ii) the (unique) long run equilibrium is unstable. We can now prove the following proposition :

Proposition : *Assume that at the long run equilibrium* (x^*,w^*) :

$$\mu(Z_x - 1) > - \phi Z_w$$

Then there exists at least a limit cycle.

Proof : As we saw in the preceding section, the condition indicated in the proposition is sufficient for the long run equilibrium to be unstable, since in that case both roots of the characteristic equation have a positive real part. We thus only have to construct a compact set in (x,w) space such that the dynamic system points inwards on the boundary. This compact set is shown on Figure 4 as the set bounded by the line ABCDE. The line AC corresponds to the equation :

$$Z(x,w) = \hat{y}$$

where $\hat{y}$ must be high enough, in a way which will be determined below. The line CD has for equation $x = y_0$. The line DE has for equation $w = \hat{w}$

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(7) See for example Coddington and Levinson (1955) ch. 16 or Hirsch and Smale (1974) ch. 11.

where $\hat{w}$ is high enough so that $Z(y_0,\hat{w}) < \bar{y}$. Finally the segment EA is along the horizontal axis.

Figure 4

Note that, except along the segment AB, the dynamic system unambiguously points inwards the compact along its boundary. We must thus now choose $\hat{y}$ such that it points inwards along the segment AB as well. But since this segment is characterized by a constant y , all that is necessary is that $\dot{y} < 0$ along the segment AB . Since $y = Z(x,w)$, we obtain :

$$\begin{aligned}\dot{y} &= Z_x.\dot{x} + Z_w.\dot{w} \\ &= Z_x.\mu(y-x) + Z_w.G(y) \\ &< \mu.y_0\ Z_x + Z_w.G(y)\end{aligned}$$

because we are in a compact such that $x \geq 0$ and $y < y_0$. Now a sufficient condition to have $\dot{y} < 0$ along AB is to choose $\hat{y}$ such that on the segment AB :

$$\mu y_0\ Z_x + Z_w\ G(\hat{y}) < 0$$

or

$$G(\hat{y}) > - \frac{\mu y_0\ Z_x}{Z_w}$$

which is evidently possible since G(y) tends to infinity when y tends towards y_0 . Q.E.D.

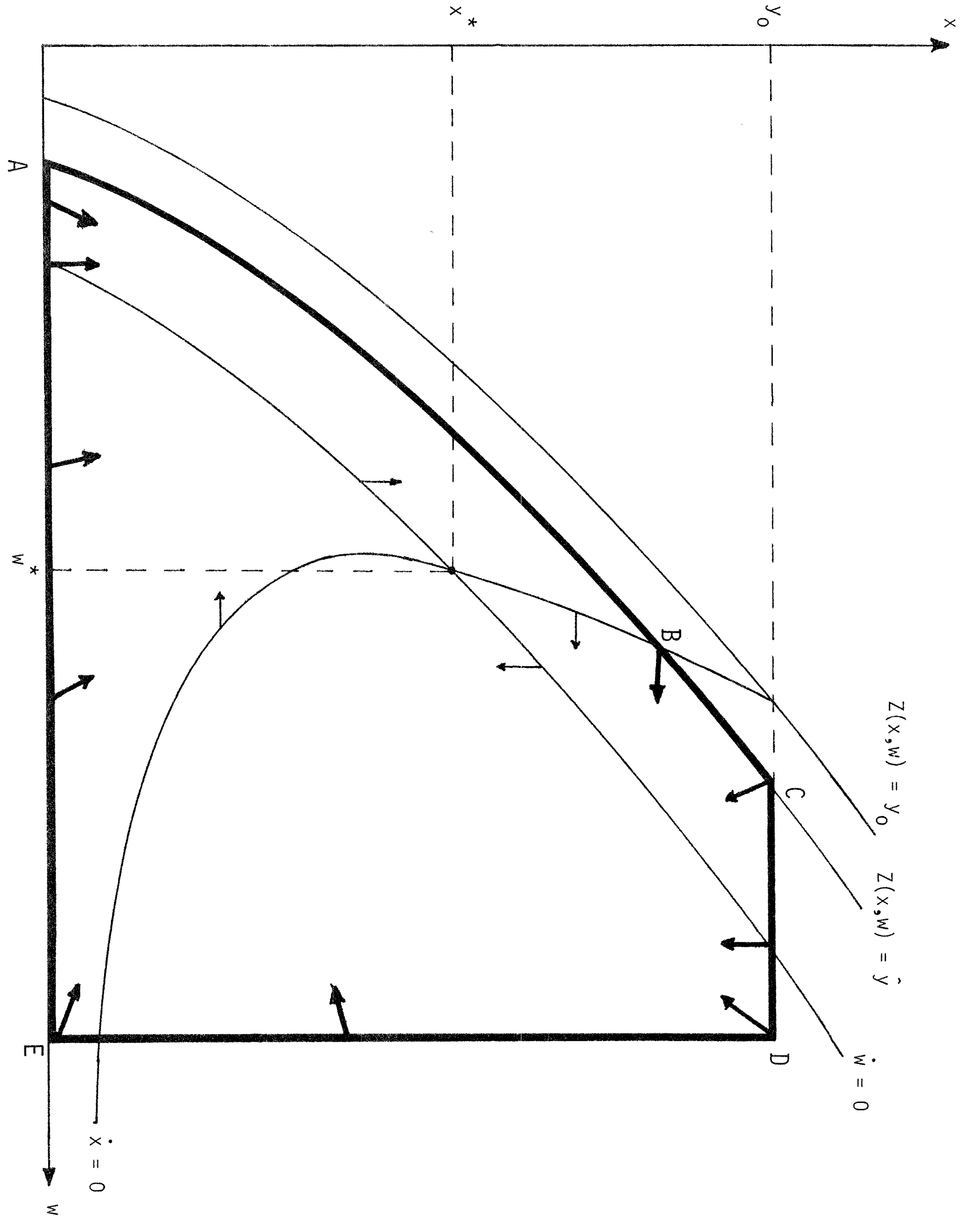

Figure 4

6. CONCLUSIONS

The model presented here allows to bridge the gap between traditional short run equilibrium analyses and cycle theory, by describing the business cycle as a succession of short run non Walrasian equilibria. The results obtained are quite intuitive : if the investment accelerator is strong enough as compared to the speed of adjustment of wages, there will be some cycles. Otherwise the model will converge towards its long run equilibrium. The model also highlighted the importance of expectations whose adjustment speed plays a role in the stability results.

The model used in this article was voluntarily simplified in order to enable us to derive simple analytical results from the Poincaré-Bendixson theorem. If not for this technical constraint, which limits the number of state variables to two, other variables could have naturally been added : first inflationary expectations could have entered in the Phillips curve, the investment and money demand functions. Secondly, we could have added an equation depicting the evolution of the capital stock, so that investment would have effects on both the demand and supply sides, and some elements of growth would be added. The inclusion of these variables, and possibly other relevant ones, would have however increased the dimensionality of our dynamic system beyond that which can be elegantly handled by available mathematical cycle theories. This should be nevertheless the subject of future research.

BIBLIOGRAPHY

BENASSY J.P. (1982), The Economics of Market Disequilibrium, Academic Press New York.

BENASSY J.P. (1983), "The Three Regimes of the IS-LM Model : A Non-Walrasian Analysis", European Economic Review, 23, pp 1-17.

CHANG W.W. and SMYTH D.J. (1971), "The Existence and Persistence of Cycles in a Nonlinear Model : Kaldor's 1940 Model Reexamined", Review of Economic Studies, 38, pp 37-44.

CODDINGTON E.A. and LEVINSON N. (1955), Theory of Ordinary Differential Equations, Mc Graw Hill, New York.

DANA R.A. and MALGRANGE P. (1981), "The Dynamics of a Discrete Version of a Growth Cycle Model", CEPREMAP, forthcoming in J.P. Ancot (Ed.), Analysing the Structure of Econometric Models, M. Nijhoff, Amsterdam.

GOODWIN R.M. (1951), "The Non-linear Accelerator and the Persistence of Business Cycles", Econometrica, 19, pp 1-17.

HICKS J.R. (1950), A Contribution to the Theory of the Trade Cycle, Oxford University Press, Oxford.

HIRSCH M.W. and SMALE S. (1974), Differential Equations, Dynamical Systems and Linear Algebra, Academic Press, New York.

KALDOR N. (1940), "A Model of the Trade Cycle", Economic Journal, 50, pp 78-92.

KALECKI M. (1935), "A Macrodynamic Theory of Business Cycles", Econometrica, 3, pp 192-226.

ROSE H. (1967), "On the Non-Linear Theory of the Employment Cycle", Review of Economic Studies, 34, pp 153-173.

ROSE H. (1969), "Real and Monetary Factors in the Business Cycle", Journal of Money, Credit and Banking, 1, pp 138-152.

SAMUELSON P.A. (1939), "Interaction between the Multiplier Analysis and the Principle of Acceleration", Review of Economic Statistics, 21, pp 75-78.

SCHINASI G. (1982), "Fluctuations in a Dynamic Intermediate-Run IS-LM Model : Applications of the Poincaré-Bendixon Theorem", Journal of Economic Theory, 28, pp 369-375.

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