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ABSTRACT

In most applications of complete systems of demand functions to national accounts data it is implicitly assumed that the demand functions are identified. For post-war Sweden this is not an altogether good assumption because due to rent control there was an excess demand for housing. In this paper the linear expenditure system is modified to include a supply function for housing. Supply is simply assumed to be a function of excess demand. The model is estimated by the maximum likelihood method and the results are compared to those obtained for the ordinary linear expenditure system.

1. INTRODUCTION

Much effort has been made within the realm of neo-classical static utility theory to find general but still useful functional forms for complete systems of demand functions, but less work is done to adapt these models to the particular features of commodities and markets. Some work has been done explicitly to explain demand for consumer durables within a complete system, see for example Phlips [1974], but to our knowledge there is almost no work on the specification and estimation of complete systems when one or more commodities are bought on a regulated market.

It is usually implicitly assumed that the demand functions in a complete system are identified. This is not always a realistic assumption. In particular, in many countries rent control and other regulations have been enforced on the housing market to keep housing costs down. This has typically created an excess demand manifested in queues for housing. In this situation when the demand model is fitted to expenditure data we will obviously not obtain estimates of a demand function for housing but rather of a supply function or a mixture between the two. The identification problem may be solved either if data on the magnitude of the excess demand is utilized or if these data are not available the supply side and the effects of rent control are adequately specified within the model. This paper is based on an idea presented in an earlier paper by one of the authors (Klevmarken [1974]). It will become clear from below that we do not aim at an analysis of the housing market *per se*. This paper is rather a modest attempt to reduce the possible bias in the estimates of a complete system of demand functions when no attention is payed to the effects of rent control. Our approach has to be rather simple because the estimation of a complete system of demand functions is not a simple matter in itself.

2. A MODEL FOR DEMAND UNDER RENT CONTROL

Rent control of appartments has been enforced in Sweden since 1942. Appartments have been allocated to households in a queing system. As the estimates in table 1 show there was a substantial excess demand until the beginning of the 1970 -ies. The queues have been particular long in the big cities. A controlled credit market, interest subsidies, goverment loan garanties and direct subsidies to households are other means of public housing policy. Under the pressure of long housing queues major resources have been allocated to house building. In addition to an increasing rent level (within the control system) this has made a successive liquidation of control possible. In 1972 only 43 cities and 13 to 14% of the total number of appartments were included.

In order to forecast future demand for housing and other commodities we thus have to estimate our model on data from a period with excess demand for housing, while in the forecasting period the housing market will most likely be close to a balance. Our prediction thus cannot be based on the assumption that the effects of the public housing policy will be the same during the forecasting period as during the sample period.

Suppose now that the linear expenditure system (Stone [1954]) is a good explanation of the average consumer demand.

$$(1) \qquad p_{it} q_{it} = p_{it} c_i + \beta_i (y_t - \sum_{k=1}^{n} p_{kt} c_k) + u_{it} \qquad i = 1, \ldots, n \quad t = 1, \ldots, T$$

where q_{it} is the demanded volume of commodity i at time t, p_{it} the price of this commodity, y_t is income (total consumption), u_{it} is a random error and β_i and c_i parameters. $p_{it} c_i$ is usually interpreted as subsistance level expenditures, while the remaining part of income, the supernumery income $(y_t - \sum_{k=1}^{n} p_{kt} c_k)$, is distributed on commodities proportionally to the marginal propensities β_i. The assumption of a constant subsistance level is sometimes unrealistic and it has for instance been suggested that it should be possible for the average consumer to adjust this minimum level to past consumption, i.e.

(2) $$c_i = \gamma_i q_{it-1} \qquad i = 1, \ldots, n$$

(See for instance Pollack 1970 and Dahlman & Klevmarken 1971).

We will also assume that the volumes demanded are equal to the volumes bought except for housing. For this commodity there is an excess demand and the consumption of housing per head is jointly determined by supply and lagged demand. More specificly, except for a random deviation v_{it} the public authorities are assumed to take such policy measures that the services obtained from the stock of houses increase proportionally to excess demand. Without loss of generality let the first commodity be housing. We now obtain

(3 a) $$\bar{q}_{1t} = \bar{q}_{1t-1} + \alpha(q_{1t-1} - \bar{q}_{1t-1}) + v_{1t} \qquad t = 1, \ldots, T$$

(3 b) $$\bar{q}_{it} = q_{it} \qquad i = 2, \ldots, n, \quad t = 1, \ldots, T$$

For simplicity α is in the following treated as a constant parameter. A more realistic approach might be to look upon α as a policy parameter, the value of which the government might change depending on for instance the political pressure to satisfy demand for housing, the rate of unemployment among building workers, the tightness of the labor market and the general economic conditions in the country. It is wellknown that in Sweden an expension of investments in housing have been used in an attempted counter cyclical policy.

When the consumption of housing services at the prevailing controlled rent is restricted by supply conditions, the "supernumery income" of the average consumer is increased by the amount he would otherwise have spent on housing. This supernumery income is then allocated to commodities proportionally to the marginal propensities to consume. Thus, what is now not spent on housing is spent on other commodities. We may consider this as a result of a utility maximization on the part of the average consumer under the constraint that housing consumption is constrained to $\bar{q}_{1t}$, i.e.

$$\text{(4)} \qquad \text{Max } U(q_2, \ldots, q_n) = \log(\bar{q}_{1t} - c_1) + \sum_{i=2}^{n} \beta_i \log(q_{it} - c_i)$$

subject to

$$\text{(5)} \qquad \sum_{i=2}^{n} p_{it}\, q_{it} = y_t - p_{1t}\, \bar{q}_{1t}$$

will give the expenditure functions

$$\text{(6)} \qquad p_{it} q_{it} = p_{it} c_i + \frac{\beta_i}{1-\beta_1} \left(y_t - p_{1t} \bar{q}_{1t} - \sum_{k=2}^{n} p_{kt} c_k\right) \qquad \begin{array}{l} i = 2, \ldots, n \\ t = 1, \ldots, T \end{array}$$

Considering the possibility of a randomly shifting utility function as well as an average consumer who is not completly rational we will have to add a stochastic error at the end of eq. (6).

The expenditure function for housing is obtained if equation (3 a) is premultiplied by p_{1t} and for $i = 1$, equation (1) is substituted into the resulting expression, thus,

$$(7\,a) \qquad p_{1t}\bar{q}_{1t} = p_{1t}c_1 + \beta_1 \frac{p_{1t}}{p_{1t-1}} \left[y_{t-1} - \sum_{k=1}^{n} p_{kt}c_k \right] + $$

$$+ (1-\alpha)\, p_{1t}\bar{q}_{1t-1} + \varepsilon_{1t} \qquad t = 1, \ldots, T$$

Finally equation (3 b) substituted into (6) gives

$$(7\,b) \qquad p_{it}\bar{q}_{it} = p_{it}c_i + \frac{\beta_i}{1-\beta_1} (y_t - p_{1t}\bar{q}_{1t} - \sum_{k=2}^{n} p_{kt}c_k) + \varepsilon_{it}$$

$$i = 2, \ldots, n$$
$$t = 1, \ldots, T$$

(7 a) and (7 b) i.e. our model (LES & H) to be estimated. We will assume that the stochastic error terms have the following first and second order moments

$$(8\,a) \qquad E(\varepsilon_{it}) = 0$$

$$(8\,b) \qquad E(\varepsilon_{it}\,\varepsilon_{is}) = \begin{cases} \sigma_{ij} & \text{if} \quad s = t \\ 0 & \text{if} \quad s \neq t \end{cases}$$

The moment matrix $\{\sigma_{ij}\}$ is singular because from (7 b) summation of both sides of the equality sign over all $n - 1$ commodities gives an identity and thus it follows that

$$(8\,c) \qquad \sum_{i=2}^{n} \varepsilon_{it} = 0 \quad \text{for all} \quad t .$$

All parameters are obviously identified and it is thus possible to estimate a true demand function for housing.

As a comparison, results are also given below for the ordinary linear expenditure system (LES). The stochastic structure remains the same as specified above except that for LES the sum of all error terms are identically zero.

3. DATA

Expenditure and price data are revised and updated from Dahlman & Klevmarken [1971]. In particular, the new SNA national accounts estimates of housing consumption have been used. Our series, however, differ from those in the national accounts because we have defined the expenditures for housing net of direct subsidies to households. Volumes are implicitly defines as the ratio of expenditures to a price index. All expenditures and volumes are per head. Income is defined as the sum of all expenditures per head.

Data are given on two levels of commodity aggregation. The most detailed commodity grouping includes nine commodities. These have then been aggregated to four commodities. Housing and clothing are the same two commodities on both levels of aggregation, while food and beverages and tobacco have been aggregated to one commodity and the remaining five commodities to one Remainder commodity [1]. Previous studies have shown that the level of commodity aggregation and the definitions of the commodities may seriously affect the parameter estimates (see e.g. Klevmarken [1977]). To investigate these aggregation effects in this particular case each model was estimated for both levels of aggregation.

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1) Although there are commodities including consumer durables no attempt has been made to include the particular characteristics of durables in the analysis.

4. ESTIMATION AND NUMERICAL METHODS

4.1. Estimation

In section 2 it was pointed out that the contemporaneous covariance matrix defined by equation (8 b) is singular because one of the equations is redundant. In order to avoid this singularity any equation can be excluded. The contemporaneous covariance metric of the thus reduced model is below denoted Σ . It was also assumed in section 2 that the residuals are free from autocorrelation. Thus the residual vector

$$\varepsilon' = \left\{ \varepsilon_{11}, \ldots, \varepsilon_{n-1,1}, \varepsilon_{12}, \ldots, \varepsilon_{n-1,2}, \ldots, \varepsilon_{n-1,T} \right\}$$

has the covariance matrix

$$\text{(9)} \qquad E(\varepsilon \varepsilon') = \Sigma \cdot I$$

when I is a $T \cdot T$ identity matrix. If we further add the assumption that the observations are generated from a multivariate normal distribution, the joint probability for the T observations is:

$$\text{(10)} \qquad p(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_T) = (2 \ )^{-(n-1)T/2} |\Sigma|^{-T/2} \exp.\left\{ -\frac{1}{2} \sum_{t=1} \varepsilon_t' \Sigma^{-1} \varepsilon_t \right\}$$

From this expression the likelihood for the observed endogenous variables $z_{it} = p_{it} q_{it}$ can be derived as:

$$\text{(11 a)} \qquad p(z_1, z_2, \ldots, z_t) = (2 \ )^{-(n-1)T/2} |B|^T |\Sigma|^{-T/2} \exp.\left\{ \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t' \Sigma^{-1} \varepsilon_t \right\}$$

In this likelihood B is the Jacobian matrix

$$\text{(11 b)} \qquad \left\{ b_{ij} \right\}_t = \frac{\delta \varepsilon_{ij}}{\delta z_{it}}$$

In our models estimation is simplified because $|B| = 1$.

The logarithm of the likelihood is then obtained as:

$$(12) \qquad L = -\frac{(n-1)T}{2} \ell n(2\pi) - \frac{T}{2} \ell n |\Sigma| - \frac{1}{2} \sum_{t=1}^{T} \varepsilon_t' \Sigma^{-1} \varepsilon_t$$

Following Eisenpress and Greenstadt (1966) the necessary conditions for the maximum of L^* are

$$(13\,a) \qquad \frac{\delta L^*}{\delta \sigma_{ij}} = \frac{T}{2} \sigma^{ji} - \frac{1}{2} \sum_t \varepsilon_{it} \varepsilon_{jt} = 0$$

$$(13\,b) \qquad \frac{\delta L^*}{\delta \alpha} = 0$$ (for the LES & H model).

$$(13\,c) \qquad \frac{\delta L^*}{\delta \beta_i} = 0$$

$$(13\,d) \qquad \frac{\delta L^*}{\delta c_i} = 0$$

Condition (13 a) makes the last term in (12) a constant, and the non-constant part of the log-likelihood can therefore be written as

$$(14) \qquad L = -\frac{1}{2} \ell n |\Sigma| = \frac{1}{2} \ell n |\Sigma^{-1}|$$

The elements in Σ can be expressed as functions of the parameters α, β_i and c_i and by means of (13 b-d) the maximum of (14) can be evaluated. The expressions are, however, non-linear in the parameters and therefore demand an iterativ approach.

3.2. Numerical techniques

In order to find the maximum of (14) two different subroutines VA06AD and VA09AD from the Harwell Subroutine Library (1973) were tested. The second routine is based on a quasi-Newton method described by Fletcher (1970). As convergence criterion this routine use a user specified accuracy for each parameter to be estimated. Our experience was that if the specified accuracy was set to about $0.1‰$ of the actual parameter value the process "converged" but the conditions (13 b-d) were far from fulfilled. If higher accuracy was specified the routine failed to converge, at least within reasonable time. The subroutine VA06AD [1], which we finally used, is a hybrid one, based on the steepest descent algorithm and on the generalized Newton iteration. According to the author; the Newton characteristics provide a fast final rate of convergence, and the steepest descent characteristics ensure that the convergence criterion will be satisfied. The subroutine finishes when a parameter point is found at which the first derivative vector satisfies the inequality

$$(15) \qquad \sum_{i=1}^{n} \left(\frac{\delta L}{\delta \Theta_i}\right)^2 < k$$

By means of this criterion the degree av deviation from the conditions (13 b-d) can be controlled, which we found to be a great advantage.

The first derivatives of the likelihood function were evaluated analytically while second order derivatives were evaluated numerically. The main program and all subroutines were written in FORTRAN IV and all computations were carried out in doubel precision. The number of iter-

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1) Powell M.J.D. Harwell report R.6469 , 1970 .

ations required is, of course, dependent of the initial guess and the accuracy constant k in (15). With $k = 10^{-8}$ and the best guess of the parameter value we could make from á priori information, about 700 iteration at an average CPU -time of 10 minutes were necessary in order to get the parameter estimates on an IBM 360/165 computer.

In our experience the gradient could be far from a zero--vector and the element in the information matrix computed from the Hessian matrix could be very far from reasonable values even if the elements of the estimated parameter vector were close their final values. For this reason a warning must be set out for the accuracy of the estimated standard errors. They may be afflicted with errors from the numerical computation. In the opinion of Eisenpress and Greenstad (1966) the use of differences instead if higher-order derivatives is probably inefficient. It is possible that our rather computer-time consuming procedures could have been fasten up by use of analytical, instead of numerically computed second-order derivatives. It is also possible that better estimates of the covariance matrix could have been achieved in this way.

5. RESULTS

Table 2 shows the estimates of the LES & H model for both levels of aggregation. The estimates of the adjustment coefficient α indicate a very slow adjustment to changes in excess demand. There is however a large uncertainty attached to these estimates and neither coefficient is significantly different from zero on a 5% level. The marginal propensities to consume are all positive with relatively small standard errors, while some of the subsistance level parameters are negative. The fit to expenditure data is excellent as usual with linear expenditure models, while the residuals, except for housing in this particular model, show a strong autocorrelation.

In comparison table 3 exhibits the estimates of the ordinary LES model. For the nine-commodity-grouping they are approximately the same as for LES & H . When the particular features of the housing market are included the marginal propensity to consume housing services increases by approximately 10% . For the four-commodity-grouping the estimate of this parameter is however the same in the two models, while the estimated propensity for food, beverages and tobacco for LES & H is almost twice the estimate for LES . The two remaining propensities in LES & H are correspondingly smaller. These differences are also exhibited by the income elasticities in table 6 . All substistance parameters are positive in the LES model and they are also better determined than in LES & H . The fit is however marginally lower in LES and there is autocorrelation in all commodities.

The inequality measures in table 4 offer another comparison of the two models. For both aggregation levels and the two sample periods the housing model LES & H shows a closter fit to data. The same is also true when predictions are made from the sample period 1950-1970 to the period 1971-1972, although the difference in fit is smaller. The observed differences in information in accuracy should not be given too much importance because they only reflect small differences in predicted expenditure shares. As table 5 reveals, both models give very close predictions of the observed expenditure shares.

Attempts were also made to estimate the linear expenditure system with habit formation [1] and with a restricted supply of housing services. These attempts were however not very successful because the likelihood function had no distinctive maximum and the estimates were obtained with large standard errors. In particular it was difficult to distinguish between the marginal propensity to consume housing services and the adjustment factor α. The fit and the predictions also turned out to be poor compared to a linear expenditure system with habit formation but with housing supply neglected. For these reasons the results for the models with habit formation are not given here.

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1) C.f. section 2 eq. (2).

6. CONCLUSIONS

Our results show that although an attempt to include the effects of a regulated housing market in a complete system of demand functions gave some improvement to fit and predictions, these were only marginal. The estimates of the income elasticities were not greatly altered either.
This result may be due to a low informational content in data. They are not rich enough to discriminate between the two models. It may also be due to bad models. The high auto-correlation indicates that something might be gained by another model specification.

7. REFERENCES

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TABLE 1. Excess demand for housing

Year	Excess demand in percent of		
	stock of appartments	stock of rooms	housing expenditures
1945	2.6	5.4	7-13
1960	3.9	9.6	10-16
1965	7.6	13.9	17-23
1970	2.7	2.7	5-10
1975	-0.3	0.7	5-10

Source: IUI :s långttidsbedömning 1976, page 99, Industrins utredningsinstitut, Stockholm, ISBN 91 - 7204 - 037 - 8, Kugel Tryckeri AB, Stockholm 1976.

TABLE 2. ML estimates of the parameters in model LES & H from annual data for the period 1950-1972. Two levels of aggregation, nine and four groups of commodities.

Commodities	$\hat{\beta}_i$	C_i	R_i	DW_i
Housing	0.205 (0.039)	0.089 (0.041)	0.999	2.273
Food	0.056 (0.008)	0.156 (0.008)	0.999	0.585
Beverages and tobacco	0.110 (0.007	0.012 (0.009)	0.997	0.869
Clothing	0.069 (0.006)	0.033 (0.005)	0.981	0.691
Household equipment	0.072 (0.005)	-0.002 (0.005)	0.987	0.464
Autos and travelling	0.182 (0.010)	-0.018 (0.015)	0.996	0.916
Recreation	0.194 (0.003)	-0.020 (0.003)	0.990	0.357
Medical care, sanitation articles and services	0.042 (0.003)	0.005 (0.003)	0.989	0.386
Other goods and services	0.070	0.002 (0.007)	0.994	0.800
Housing	0.186 (0.026)	0.027 (0.029)	0.999	2.226
Food, beverages and tobacco	0.221 (0.017	0.077 (0.040)	0.999	0.791
Clothing	0.058 (0.006)	0.023 (0.006)	0.980	0.669
Remainder	0.535	-0.164 (0.058)	0.999	0.961

$\hat{\alpha}$ (9 commodites) = 0.071 (0.060)

$\hat{\alpha}$ (4 commodites) = 0.098 (0.063)

Estimated asymptotic standard errors in parenthesis.

TABEL 3. FIML estimates of the parameters in model LES from annual data for the period 1950-1972. Two levels of aggregation, nine and four groups of commodities.

Commodities	$\hat{\beta}_i$	C_i	R_i	DW_i
Housing	0.181 (0.006)	0.105 (0.012)	0.997	0.703
Food	0.039 (0.007	0.181 (0.003)	0.998	0.515
Beverages and tobacco	0.110 (0.004)	0.057 (0.008)	0.994	0.490
Clothing	0.082 (0.006)	0.058 (0.006)	0.981	0.781
Household equipment	0.085 (0.004)	0.026 (0.006)	0.986	0.509
Autos and travelling	0.185 (0.006)	0.053 (0.013)	0.996	1.105
Recreation	0.207 (0.008)	0.057 (0.014)	0.990	0.383
Medical care, sanitation articles and services	0.048 (0.002)	0.021 (0.003)	0.991	0.331
Other goods and services	0.063	0.031 (0.005)	0.997	1.530
Housing	0.186 (0.005)	0.126 (0.017)	0.997	0.802
Food, beverages and tobacco	0.139 (0.012)	0.258 (0.015)	0.997	0.296
Clothing	0.086 (0.007)	0.069 (0.008)	0.979	0.772
Remainder	0.589	0.260 (0.057)	0.999	1.185

Estimated asymptotic standard errors in parenthesis.

TABLE 4. Computed measure of information (I) .

Model	1950-1972	1950-1970	Prediction 1971-1972
9 commodities			
LES & H	511	461	1139
LES	714	626	1170
4 commodities			
LES & H	192	186	253
LES	396	374	257

$$I = \frac{1}{T} \sum_{1}^{T} W_i \log \left(\frac{W_i}{\hat{W}_i} \right) \cdot 10^6$$

TABLE 5. Observed and predicted expenditure shares (W_i) for 1971 and 1972.

Commedities	1971			1972		
	Ob-served $\hat{W}_i$	Model LES & H $\hat{W}_i$	Model LES $\hat{W}_i$	Ob-served $\hat{W}_i$	Model LES & H $\hat{W}_i$	Model LES $\hat{W}_i$
Housing	0.182	0.180	0.178	0.171	0.168	0.171
Food	0.237	0.239	0.239	0.241	0.243	0.244
Beverages and tobacco	0.100	0.098	0.098	0.098	0.098	0.097
Clothing	0.084	0.090	0.090	0.085	0.090	0.090
Household equipment	0.050	0.055	0.055	0.051	0.056	0.056
Autos and travelling	0.119	0.119	0.119	0.119	0.122	0.121
Recreation	0.133	0.122	0.123	0.137	0.125	0.123
Medical care, sanitation articles and services	0.037	0.039	0.039	0.036	0.039	0.039
Other goods and services	0.058	0.058	0.059	0.062	0.059	0.059
Housing	0.181	0.181	0.178	0.171	0.168	0.170
Food, beverages and tobacco	0.337	0.336	0.336	0.339	0.339	0.339
Clothing	0.084	0.090	0.090	0.085	0.090	0.090
Remainder	0.398	0.393	0.396	0.405	0.403	0.401

Note. Predictions were made with estimates obtained for the sample period 1950-1970.

TABLE 6. Estimated income elasticity of demand for 1960 .

Commodities	Model	
	LES & H	LES
Housing	1.010	0.891
Food	0.216	0.151
Beverages and tobacco	0.170	1.170
Clothing	0.683	0.812
Household equipment	1.565	1.848
Autos and travelling	1.838	1.869
Recreation	1.780	1.899
Medical care, sanitation articles and services	1.105	1.263
Other goods and services	1.373	1.235
Housing	0.916	0.916
Food, beverages and tobacco	0.628	0.395
Clothing	0.574	0.851
Remainder	1.555	1.712

Note. Sample period was 1950-1972 .