Urban Labor Economics

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Appendix 3: The Harris-Todaro model

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In two seminal papers, Todaro (1969) and Harris and Todaro (1970) have developed a canonical model of rural-urban migration. These papers have been so influential that they are referred in the literature to as the Harris-Todaro model. Even though these papers were written for developing countries, the general mechanism put forward can be applied to developed countries. In this Appendix, I expose in a synthetic way the same model with different extensions.

1. A simple model with exogenous wages

There are two regions: Rural and urban. The crucial assumption of the Harris-Todaro model is that workers base their migration decision on their *expected* incomes. Because the basic model is static, the expected income is just the weighted average of the urban wage and the unemployment benefit, the weights being the probabilities to find and not to find an urban job. It is assumed that the rural wage is flexible enough to guarantee that there is no rural unemployment; this wage is denoted by w_L^R. The urban wage is exogenous and set at a high-enough level, so that urban unemployment prevails in equilibrium; this wage is denoted by w_L^C and we assume that $w_L^C > w_L^R$ (superscript C is used for cities and superscript R for rural areas). There is a continuum of ex ante identical workers whose mass is N. Among the N workers, N^C and N^R live respectively in cities and rural areas, i.e. $N = N^C + N^R$, and

$$N^C = L^C + U^C$$

$$N^R = L^R$$

where L^g and U^g are respectively the total employment and unemployment levels in region $g = C, R$. As stated above, there is no unemployment in rural areas. Thus, by combining these two equations, we obtain:

$$U^C = N - L^C - L^R \tag{1.1}$$

The unemployment rate is then given by:

$$u^C = \frac{U^C}{U^C + L^C} = \frac{N - L^C - L^R}{N - L^R} \tag{1.2}$$

As a result, the probability to find an urban job is:

$$a^C \equiv \frac{L^C}{L^C + U^C} = \frac{L^C}{N - L^R} \tag{1.3}$$

What is crucial here is that the probability to find a job in cities depends on the number of jobs available in cities, L^C, but also on the number of employed workers in rural areas, L^R. Observe that L^C, the number of urban jobs, is here exogenously fixed. This means that rural workers will decide to migrate if

$$w_L^C \frac{L^C}{N - L^R} > w_L^R$$

The equilibrium is reached when

$$w_L^C \frac{L^C}{N - L^R} = w_L^R \tag{1.4}$$

which is equivalent to

$$L^R = N - \frac{w_L^C}{w_L^R} L^C \tag{1.5}$$

This is the fundamental equation of the Harris-Todaro model. We have the following definition:

Definition 1. *A Harris-Todaro equilibrium with exogenous wages is a couple (U^C, L^R) such that (1.1) and (1.5) are satisfied.*

In this simple model, an equilibrium is easy to calculate. Indeed, given that the wages w_L^C, w_L^R, the total population N and the number of urban jobs L^C are exogenous, one can calculate the number of rural employed workers L^R by solving the equilibrium migration condition (1.5). Then, using this value and the exogenous values of L^C and N, one can determine the level of urban unemployment U^C by using (1.1).

A key question that has been studied is the effect of increasing urban jobs on urban unemployment. In developing countries, it has been showed that increasing urban jobs can lead to an increase rather than a decrease in urban unemployment because of the induced effect of migration. This is referred to as the *Todaro paradox*. We have the following result.

Proposition 1. *In a Harris-Todaro model with exogenous wages, a Todaro paradox exists if and only if*

$$-\frac{\partial L^R}{\partial L^C} > 1 + \frac{U^C}{L^C} \tag{1.6}$$

To understand this result, we have to differentiate between a Todaro paradox in unemployment level, i.e. $\frac{\partial U^C}{\partial L^C} > 0$, and a Todaro paradox in unemployment rate, i.e. $\frac{\partial u^C}{\partial L^C} > 0$. By totally differentiating (1.1) and (1.2) and by observing that L^R is a function of L^C (because of the migration equilibrium condition), we obtain (1.6). To be more precise, a Todaro paradox in unemployment level exists if and only if:

$$-\frac{\partial L^R}{\partial L^C} > 1 \tag{1.7}$$

while a Todaro paradox in unemployment rate exists if and only if

$$-\frac{\partial L^R}{\partial L^C} > 1 + \frac{U^C}{L^C} \tag{1.8}$$

It is clear that if a Todaro paradox in unemployment rate exists, then a Todaro paradox in unemployment level also exists, but the reverse in *not* true. The intuition of (1.7) is as follows. If this condition holds, then the initial increase in urban jobs (L^C increases) induces rural workers to migrate to cities (since the probability a^C to obtain an urban job is higher and thus rural workers' expected income is also higher), which implies that the reduction in L^R is higher than the increase in L^C. Since $U^C = N - L^C - L^R$, urban unemployment increases.

In our model, by totally differentiating (1.5), we easily obtain:

$$\frac{\partial L^R}{\partial L^C} = -\frac{w_L^C}{w_L^R} < -1 \tag{1.9}$$

This means that a *Todaro paradox in unemployment level always exists in this model.* Indeed, if the number of urban jobs is raised by one unit, rural employment falls by w_L^C/w_L^R units. This is because creating one additional job in the urban sector induces w_L^C/w_L^R people to migrate into the urban sector and since $w_L^C > w_L^R$ and $U^C = N - L^C - L^R$, it has to be that urban unemployment U^C increases for N to stay constant. Observe that the higher the wage gap between urban and rural areas, the higher this effect. Concerning the unemployment rate, using (1.8), we have:

$$\frac{\partial u^C}{\partial L^C} > 0 \Leftrightarrow \frac{w_L^C}{w_L^R} < 1 + \frac{U^C}{L^C}$$

which is not always true and thus, in this model with exogenous wages, it may well be that increasing urban jobs increases the urban unemployment level but decreases or does not affect the urban unemployment rate. This is the case if

$$1 < \frac{w_L^C}{w_L^R} \leq 1 + \frac{U^C}{L^C} \tag{1.10}$$

Example A3.1 Let us illustrate this last result with a numerical example. Let $N = 1100$, $L^C = 800$, $w_L^C = 90$, $w_L^R = 80$, so that $w_L^C / w_L^R = 9/8$. Let us calculate the Harris-Todaro equilibrium of this economy. Using (1.5) and (1.1) we obtain:

$$L^R = 1100 - \frac{9}{8}800 = 200$$

and

$$U^C = 1100 - 800 - 200 = 100$$

The unemployment rate is given by

$$u^C = \frac{1100 - 800 - 200}{1100 - 200} = 11.11\%$$

To summarize the equilibrium is characterized by:

$$\begin{aligned} N &= 1100\ ,\ w_L^C = 90,\ w_L^R = 80\ ,\ L^C = 800\ ,\ L^R = 200 \\ U^C &= 100 \text{ and } u^C = 11.11\% \end{aligned}$$

Now, the government decides to create 50 urban jobs, i.e. $L^{C'} = 850$ and thus $\Delta L^C \equiv L^{C'} - L^C = 50$. Now because of (1.9), we have

$$\frac{\partial L^R}{\partial L^C} = -\frac{w_L^C}{w_L^R} = -\frac{9}{8}$$

so that (in discrete units):

$$\Delta L^R \equiv L^{R'} - L^R = -\frac{9}{8}\Delta L^C = -56.25$$

As a result,

$$L^{R'} = -56.25 + L^R = 143.75$$

This means that the creation of 50 urban jobs has triggered a migration to the city of $\frac{9}{8}\Delta L^C = 56.25$ rural workers, so that the rural employment has decreased from $L^R = 200$ to $L^{R'} = 143.75$. Because $\Delta L^C = 50 < 56.25 = \left|\Delta L^R\right|$, urban unemployment has increased by 6.25 workers since

$$U^{C'} = N - L^{C'} - L^{R'} = 106.25$$

This is the fundamental idea of Harris and Todaro and this is why it is referred to as the *Todaro paradox*. Indeed, creating urban jobs may lead to an increase rather than a decrease in urban unemployment because of the induced rural-urban migration that may outweight the initial increase in urban jobs. In this example, there are 50 more urban jobs but 56.25 more migrants. So the net result is an increase in urban unemployment. Observe that what is crucial in an Harris-Todaro model is that $N^C = N - L^R$ is endogenous (since it is a function of L^R) and determined by (1.5) whereas in a model without migration (autarky) it would be exogenous.

So, an increase of $(\Delta L^C / L^C =)$ 6.25% of urban jobs has increased the urban unemployment by $(\Delta U^C / U^C =)$ 6.25%. This means that the unemployment rate has not been affected. Indeed,

$$u^{C'} = \frac{U^{C'}}{N - L^{R'}} = \frac{1100 - 850 - 143.75}{1100 - 143.75} = 11.11\%$$

so that *there is no Todaro paradox in unemployment rate*. Indeed, using (1.8), we have:

$$1 < -\frac{\partial L^R}{\partial L^C} = \frac{w_L^C}{w_L^R} = \frac{9}{8} = 1 + \frac{U^C}{L^C}$$

which means that we are in the case when (1.10) holds. To summarize after the policy, the equilibrium is characterized by:

$$\begin{aligned} N &= 1100 \text{ , } w_L^{C'} = 90, \; w_L^{R'} = 80 \text{ , } L^{C'} = 850 \text{ , } L^{R'} = 143.75 \\ U^{C'} &= 106.25 \text{ and } u^{C'} = 11.11\% \end{aligned}$$

2. The Harris-Todaro model with minimum wages

Of course, the previous analysis, though useful, was quite limited (and mechanical) since, apart of the migration decision, all variables were exogenous. Let us now describe in more details the Harris-Todaro model where L^C and w_L^R are endogenously determined but w_L^C is still exogenous (as in the original formulation). There are still two regions: Rural and urban. Both regions produce the same good but use different techniques. In region $g = C, R$ (C for cities and R for rural areas) y^g units of output are produced and L^g workers are employed. This is a short-run model where capital is fixed and the production function in region $g = C, R$ is given by

$$y^g = F^g(L^g) \quad , F'^g(L^g) > 0 \text{ and } F''^g(L^g) \leq 0 \tag{2.1}$$

We also assume that the Inada conditions hold, that is $\lim_{L^g \to 0} F^{g\prime}(L^g) = +\infty$ and $\lim_{L^g \to +\infty} F^{g\prime}(L^g) = 0$. As before, we have:

$$U^C = N - L^C - L^R \tag{2.2}$$

and, in the rural area, wages are flexible so that there is no rural unemployment. The price of the good is taken as a numeraire and, without loss of generality, normalized to 1. In the urban area, it is assumed that the government imposes a minimum wage w_m^C (which is downward rigid) that is above $w_L^C = F^{C\prime}(L^C)$, i.e. w_m^C is above the wage that would prevail if wages were set at their competitive level. The profit maximization condition in region $g = C, R$ can be written as:

$$w_L^g = F^{g\prime}(L^g) \tag{2.3}$$

As before, the equilibrium migration condition is given by (1.5). However, using the fact that $w_L^C = w_m^C$ and (2.3), this condition can be written as:

$$F^{R\prime}(L^R) = w_m^C \frac{L^C}{N - L^R} \tag{2.4}$$

Definition 2. *A Harris-Todaro equilibrium with a minimum wage is a 4-tuple* (L^C, w_L^R, U^C, L^R) *such that*

$$w_m^C = F^{C\prime}\left(L^C\right) \tag{2.5}$$

$$w_L^R = F^{R\prime}\left(L^R\right) \tag{2.6}$$

(2.2), (2.4) are satisfied.

In this model, an equilibrium is calculated as follows. First, one can calculate the number of urban employed workers L^C by solving (2.5). Second, using this value L^C and given that both w_m^C and N are exogenous, one can determine the number of rural employed workers L^R by solving the equilibrium migration condition (2.4). Then, by plugging this value L^R in (2.6), we obtain w_L^R. Finally, given that N is exogenously given and using the values of L^C and L^R calculated above, one obtains U^C by solving (2.2). It is easy to verify that the Inada conditions guarantee that (2.4) has a unique solution in L^R.

We can now totally differentiate (2.4) and we easily obtain

$$\frac{\partial L^R}{\partial L^C} = \frac{w_m^C}{\left(N - L^R\right) F^{R\prime\prime}(L^R) - w_m^C L^C / \left(N - L^R\right)} < 0 \tag{2.7}$$

Contrary to the previous model, neither a Todaro paradox in unemployment rate nor in unemployment level can be guaranteed in this model. The Todaro paradox is not always true because there are two opposite effects. Indeed, an increase in the number of urban jobs raises the number of rural migrants (attraction force to the city) since it increases their probability to find a job in cities but, because there are less workers in rural areas (due to migration), the rural wage w_L^R, which is a decreasing function of rural employment L^R, i.e. $w_L^R = F^{R\prime}(L^R)$, increases and this tends to reduce migration (repulsion force to the city). The net effect is thus ambiguous.

Example A3.2 Let us illustrate this result with an example that we calibrate so that the equilibrium is identical to that of Example A3.1. Let $N = 1100$, $F^C(L^C) = 5091.2\sqrt{L^C}$, $F^R(L^R) = 2262.7\sqrt{L^R}$, and the government imposes a minimum wage of $w_m^C = 90$. Since $w_m^C = F^{C\prime}(L^C) = 2545.6/\sqrt{L^C}$, this implies that $L^C = 800$. Using the migration equilibrium condition (2.4), we can calculate L^R, which by observing that $F^{R\prime}(L^R) = 1131.4/\sqrt{L^R}$, is equal to $L^R = 200$. We can calculate the rural wage by solving (2.6). We easily obtain

$$w_L^R = F^{R\prime}\left(L^R\right) = \frac{1131.4}{\sqrt{200}} = 80$$

Finally, one obtains U^C by solving (2.2). We get: $U^C = 100$ and the unemployment rate is: $u^C = 11.11\%$. So we study an equilibrium that has exactly the same values (both exogenous and endogenous) as in Example A3.1. Indeed, the equilibrium is characterized by:

$$\begin{aligned} N &= 1100 \text{ , } w_m^C = 90, \, w_L^R = 80 \text{ , } L^C = 800 \text{ , } L^R = 200 \\ U^C &= 100 \text{ and } u^C = 11.11\% \end{aligned}$$

Let us implement exactly the same policy, which consists in creating 50 urban jobs, i.e. $L^{C'} = 850$ and thus $\Delta L^C \equiv L^{C'} - L^C = 50$. In this model, because L^C is endogenous, it is easy to verify that to create 50 urban jobs the government has to reduce the minimum wage by nearly 3%, i.e. from $w_m^C = 90$ to $w_m^{C'} = 87.313$. Using (2.4), one can calculate $L^{R'}$ by solving the following equation:

$$\frac{1131.4}{\sqrt{L^{R'}}} = 87.313 \frac{850}{1100 - L^{R'}}$$

We obtain: $L^{R'} = 191.72$, which implies that $\Delta L^R \equiv L^{R'} - L^R = -8.28$. By using (2.6), we obtain

$$w_L^R = \frac{1131.4}{\sqrt{191.72}} = 81.711$$

Finally, the unemployment level and the unemployment rate are respectively given by:

$$\begin{aligned} U^{C'} &= 1100 - 850 - 191.72 = 58.28 \\ u^{C'} &= \frac{58.28}{58.28 + 850} = 6.42\% \end{aligned}$$

To summarize, here are the new equilibrium values after the government policy:

$$\begin{aligned} N &= 1100 \text{ , } w_m^{C'} = 87.31, \, w_L^{R'} = 81.71 \text{ , } L^{C'} = 850 \text{ , } L^{R'} = 191.72 \\ U^{C'} &= 58.28 \text{ and } u^{C'} = 6.42\% \end{aligned}$$

If we compare these equilibrium values with the ones before the policy, it is easy to see that the unemployment level as well as the unemployment rate have been divided by two. This is mainly because the reduction in the urban wage has created fifty new urban jobs but also increased the rural wage because of the migration and the induced higher productivity. As a result, the wage gap has narrowed from $9/8 = 1.125$ to $87.31/81.71 = 1.069$. In this example, *there is no Todaro paradox* (both in unemployment level and rate) and the policy is efficient in reducing unemployment.

3. The Harris-Todaro model with efficiency wages

We want now to go further by endogeneizing both urban wages and urban unemployment. We use the standard efficiency wage model, as proposed by Shapiro and Stiglitz (1984), and developed extensively in Chapter 1 of this book. The model is now dynamic and we assume that, if rural workers want to get an urban job, they have first to move to the city, be unemployed and gather information about jobs, and then can eventually obtain an urban job. In the two previous models, which were static, the implicit assumption was that rural workers could find directly an urban job by searching from rural areas. We still have

$$U^C = N - L^C - L^R \tag{3.1}$$

The steady-state Bellman equations for non-shirkers, shirkers and unemployed workers are respectively given by:[1]

$$r\, I_L^{NS} = w_L^C - e - \delta\,(I_L^{NS} - I_U) \tag{3.2}$$

$$r\, I_L^S = w_L^C - (\delta + m)\,(I_L^S - I_U) \tag{3.3}$$

$$r\, I_U = w_U + a^C (I_L - I_U) \tag{3.4}$$

where w_L^C, w_U are the urban wage and the unemployment benefit respectively, e is the effort level, r the discount rate, δ, m and a denote the job-destruction, monitoring and job-acquisition rates, respectively. Firms set the efficiency wage such that $I_L^{NS} = I_L^S = I_L$ and we obtain that $I_L - I_U = e/m$. By combining these equations (as in Chapter 1), and observing that in steady state, flows out of unemployment equal flows into unemployment, i.e.

$$a^C = \frac{\delta\, L^C}{N - L^C - L^R} \tag{3.5}$$

[1] See Appendix 2 for the derivation of the Bellman equations.

we easily obtain the following urban efficiency wage:

$$w_L^C = w_U^C + e + \frac{e}{m}\left[\frac{\delta\left(N - L^R\right)}{N - L^C - L^R} + r\right] \tag{3.6}$$

Observe that L^R positively affects w_L^C since more employment in rural areas implies a higher urban job acquisition rate a^C (indeed higher L^R leads to a decrease in urban unemployment since there are less competition for urban jobs) and thus urban firms have to increase their wages to meet the Non-Shirking Condition (3.6). In cities, firms decide their employment level by maximizing their profit. We thus have:

$$w_L^C = F'^C(L^C) \tag{3.7}$$

In rural areas, we assume that jobs are mainly menial and wages are flexible and equal to marginal product, so that there is no rural unemployment. We thus have:

$$w_L^R = F'^R(L^R) \tag{3.8}$$

We assume that the Inada conditions on both production functions hold. Concerning rural-urban migration,[2] as stated above, we assume that a rural worker cannot search from home but must first be unemployed in the city and then search for a job. Thus, the equilibrium migration condition can be written as:

$$r\, I_U = \int_0^{+\infty} w_L^R\, e^{-rt} = \frac{w_L^R}{r} \tag{3.9}$$

The left-hand side is the intertemporal utility of moving to the city (remember that a migrant must first be unemployed) while the right-hand side corresponds to the intertemporal utility of staying in rural areas. Using (3.2)$-$(3.5), $I_L^{NS} = I_L^S = I_L$ and (3.8), we can write condition (3.9) as:

$$w_U^C + \frac{e}{m}\frac{\delta\, L^C}{N - L^C - L^R} = \frac{F'^R(L^R)}{r} \tag{3.10}$$

where L^C is determined by (3.7).

Definition 3. *A Harris-Todaro equilibrium with efficiency wages is a 5-tuple $(w_L^C, L^C, w_L^R, U^C, L^R)$ such that (3.6), (3.7), (3.8), (3.1) and (3.10) are satisfied.*

In this model, given that $w_U^C, e, m, \delta, N, r$ are exogenous, an equilibrium is calculated as follows. First, from (3.6), one can calculate the urban efficiency

[2] In this simple model, urban-rural migration will not occur in equilibrium.

wage as a function of L^C and L^R, that is $w_L^C(L^C, L^R)$. Second, by plugging this value $w_L^C(L^C, L^R)$ in (3.7), one obtains a relationship between L^C and L^R, that we write $L_w^C(L^R)$ and is given by

$$w_U^C + e + \frac{e}{m}\left[\frac{\delta\left(N - L^R\right)}{N - L^C - L^R} + r\right] = F'^C(L^C) \tag{3.11}$$

By totally differentiating (3.11) and using the Inada conditions, we easily obtain:

$$\frac{\partial L_w^C}{\partial L^R} < 0 \ , \ \lim_{L^R \to 0} L_w^C = L_0^C \ , \ \lim_{L_w^C \to 0} L^R = N$$

where $0 < L_w^C(L^R) < L_0^C < N$ is the unique solution of the following equation

$$w_U^C + e + \frac{e}{m}\left[\frac{\delta N}{N - L_0^C} + r\right] = F'^C(L_0^C)$$

Third, the equilibrium-migration condition (3.10) gives another relationship between L^C and L^R, that we denote by $L_h^C(L^R)$ and has the following properties:

$$\frac{\partial L_h^C}{\partial L^R} < 0 \ , \ \lim_{L^R \to 0} L_h^C = N \ , \ \lim_{L_h^C \to 0} L^R = L_0^R = F'^{-1}\left(r w_U^C\right)$$

where $0 < L_h^C(L^R) < L_0^R < N$. Figure A3.1 describes the two curves (3.11) (labor demand equation) and (3.10) (migration equilibrium condition) in the plane (L^R, L^C) and it is easy to see that there exists a unique equilibrium that gives a unique value of L^C and a unique value of L^R that we denote by (L^{R*}, L^{C*}).

[*Insert Figure A*3.1 *here*]

Finally, plugging L^{R*} and L^{C*} in (3.6), (3.8) and (3.1) gives respectively the equilibrium values of $w_L^{C*}, w_L^{R*}, U^{C*}$.

Let us now study the Todaro paradox in this model. Of course, since L^C is an endogenous variable, it is difficult to study the impact of L^C on U^C or u^C. However, we can study the impact of a reduction of unemployment benefit w_U^C on urban unemployment since w_U^C has a direct impact on L^C. We have the following definition:

Definition 4. *In a model where wage w_L^C and employment L^C are endogenous, a Todaro paradox prevails if an increase or decrease in a policy variable leads to an increase in the equilibrium values of both L^C and U^C (or u^C). If one takes for example the unemployment benefit w_U^C, then a Todaro paradox prevails if by reducing w_U^C, both L^C and U^C (or u^C) increase, that is $\partial L^C/\partial w_U^C < 0$ and*

$\partial U^C/\partial w_U^C < 0$ (or $\partial u^C/\partial w_U^C < 0$). Differentiating (3.1), this implies that a Todaro paradox exists if and only if

$$\frac{\partial L^{R*}}{\partial w_U^C} > -\frac{\partial L^{C*}}{\partial w_U^C} > 0 \tag{3.12}$$

Using this definition, let us now study the Todaro paradox in this model. As stated above and described in Figure A3.1, the equilibrium is determined by two equations (3.10) and (3.11). If we differentiate (3.10), we obtain

$$L^R = L^R\left(\underset{-}{w_U^C}, \underset{-}{e}, \underset{+}{m}, \underset{-}{\delta}, \underset{+}{N}, \underset{-}{r}, \underset{-}{L^C}\right) \tag{3.13}$$

Indeed, a higher unemployment benefit, w_U^C, or effort level, e, or job-destruction rate, δ, or discount rate, r, or a lower monitoring rate, m, or total population, N, makes the city more attractive because of higher intertemporal utility of being unemployed in the city, I_U (remember that $I_L - I_U = e/m$). Thus more workers leave the rural area, which reduces L^R. When L^C increases, the urban job acquisition rate a^C increases and again more rural workers migrate to the city, thus reducing L^R.

If we now differentiate (3.11), we get:

$$L^C = L^C\left(\underset{-}{w_U^C}, \underset{-}{e}, \underset{+}{m}, \underset{-}{\delta}, \underset{+}{N}, \underset{-}{r}, \underset{-}{L^R}\right) \tag{3.14}$$

where

$$\frac{\partial L^C}{\partial L^R} = -\frac{\frac{e}{m}\delta L^C}{\frac{e}{m}\delta\left(N - L^R\right) - \left(N - L^C - L^R\right)^2 F''^C(L^C)} < 0 \tag{3.15}$$

Indeed, a higher w_U^C, or e, or δ, or r, or a lower m, or N, shifts upward the Non-Shirking Condition (3.6), so firms have to pay a higher efficiency wage to prevent shirking. This, in turn, reduces employment since, because of higher wage costs, maximizing-profit firms have to reduce the number of employed. For L^R, the effect is through the job-acquisition rate a^C. Indeed, a higher rural employment L^R increases a^C, which obliges firms to increase their urban efficiency wages, which in turn reduces urban labor demand L^C because firms maximize their profit. We obtain the following result:

Proposition 2. *In an Harris-Todaro model with urban efficiency wages, decreasing unemployment benefit leads to*

(*i*) *an increase in urban employment L^C, i.e. $\partial L^{C*}/\partial w_U^C < 0$;*

(ii) *an increase in rural employment* L^R*, i.e.* $\partial L^{C*}/\partial w_U^C < 0$*;*

(iii) *a decrease in urban unemployment (both in level and rate)* U^C *and* u^C*, i.e.* $\partial U^{C*}/\partial w_U^C > 0$ *and* $\partial u^{C*}/\partial w_U^C > 0$*.*

As a result, there is no Todaro paradox.

The proof of this proposition is tedious and can be found at the end of this appendix. There is thus no Todaro paradox in this model. The intuition is as follows. When the government decreases the unemployment benefit, this has a direct negative effect on urban wages and thus more urban jobs are created. This is the attraction force to the city. But there are two repulsion forces. As before, this implies that rural wages increase but since there are more jobs in cities and efficiency wages act as a worker's discipline device, urban firms reduce their wages because it becomes more difficult to find a job. Because the repulsion forces are strong enough, the net effect is that creating urban jobs via a reduction in unemployment benefit reduces urban unemployment because of the discouraging effect of efficiency wages on migration.

These results are quite interesting. Let us see what happens in the autarky case, i.e. the case of no mobility between rural and urban areas. Indeed, imagine now that migration was totally controlled and that workers, especially rural workers could *not* migrate to cities. In that case, the two regions (C and R) would be totally independent and we would have

$$U^C = N^C - L^C$$

$$L^R = N^R$$

so that the unemployment rate would be given by

$$u^C = \frac{U^C}{U^C + L^C} = \frac{N^C - L^C}{N^C}$$

Here, only L^C is endogenous and not L^R. Thus, the job acquisition rate and the urban efficiency wage would be given by:

$$a^C = \frac{\delta\, L^C}{N - L^C}$$

$$w_L^C = w_U + e + \frac{e}{m}\left[\frac{\delta\, N^C}{N - L^C} + r\right] \tag{3.16}$$

and the labor demand would still be given by (3.7). The urban labor equilibrium would then be defined as:

$$w_U + e + \frac{e}{m}\left[\frac{\delta\, N^C}{N - L^C} + r\right] = F'(L^C) \tag{3.17}$$

Definition 5. *An efficiency wage equilibrium with no mobility is a triple* $(w_e^{C*}, L^{C*}, w^{R*})$ *such that (3.16) (3.7) and (3.8) are satisfied.*

From this definition and by totally differentiating (3.17), we obtain the following result:

Proposition 3. *In efficiency wage equilibrium with no mobility, decreasing the unemployment benefit* w_U *always increases urban employment and decreases urban unemployment (both in level and rate), that is*

$$\frac{\partial L^C}{\partial w_U} < 0 \ , \ \frac{\partial U^C}{\partial w_U} > 0 \ , \ \frac{\partial u^C}{\partial w_U} > 0$$

This result is not surprising since when w_U decreases, firms can reduce their efficiency wages and thus hire more workers. There is no effect on rural workers. However, even when rural-urban migration is authorized, we obtain the same results because the repulsion forces are sufficiently strong to thwart the attraction force of a reduction of the unemploy,ment benefit.

4. The Harris-Todaro model with urban search externalities

We would like to endogeneize both urban wages and urban unemployment using a standard search matching model as in Mortensen and Pissarides (1999) and Pissarides (2000). This model has been extensively analyzed in Chapter 4 of this book. The starting point is the following matching function

$$d(U^C, V^C)$$

where U^C and V^C are the total number of urban unemployed and urban vacancies, respectively. This matching function captures the frictions that search behaviors of both firms and workers imply. It is assumed that $d(.)$ is increasing in its arguments, concave and homogeneous of degree 1. Thus, the rate at which vacancies are filled is $d(U^C, V^C)/V^C = d(1/\theta^C, 1) \equiv q(\theta^C)$, where

$$\theta^C = \frac{V^C}{U^C} \tag{4.1}$$

is a measure of *labor market tightness* in cities and $q(\theta^C)$ is a Poisson intensity. Similarly, the rate at which an unemployed worker leaves unemployment (job acquisition rate) is now given by

$$a^C = \frac{d(U^C, V^C)}{U^C} \equiv \theta^C q(\theta^C) \tag{4.2}$$

In steady-state, the Bellman equations for the employed and unemployed are respectively given by:[3]

$$rI_L = w_L^C - \delta\,(I_L - I_U) \tag{4.3}$$

$$rI_U = w_U^C + \theta^C q(\theta^C)\,(I_L - I_U) \tag{4.4}$$

For firms with filled and vacant jobs, we have the following Bellman equations:

$$rI_F = y^C - w_L^C - \delta(I_F - I_V) \tag{4.5}$$

$$rI_V = -c + q(\theta^C)(I_F - I_V) \tag{4.6}$$

where c is the search cost for the firm and y^C is the product of the match. Because of free entry, $I_V = 0$, and using (4.5)−(4.6), we obtain the following decreasing relation between labor market tightness and wages:

$$\frac{c}{q(\theta^C)} = \frac{y^C - w_L^C}{r + \delta} \tag{4.7}$$

Wages are negotiated between the firm and the worker. By solving the following Nash-program (where β is the bargaining power of workers):

$$w_L^C = \arg\max_{w_L^C}(I_L - I_U)^{\beta}(I_F - I_V)^{1-\beta}$$

we obtain the following wage (see Chapter 4):

$$w_L^C = (1-\beta)\,w_U^C + \beta\left(y^C + c\,\theta^C\right) \tag{4.8}$$

As before, the unemployment level in cities in equal to:

$$U^C = N - L^C - L^R \tag{4.9}$$

In steady-state, flows in and out unemployment have to be equal and we obtain the following relationship in cities:

$$L^C = \frac{\theta^C q(\theta^C)}{\delta + \theta^C q(\theta^C)}\left(N - L^R\right) \tag{4.10}$$

[3]For simplicity, it is assumed that each firm only hires one worker.

In rural areas, as before there is no unemployment and the following condition holds:

$$w_L^R = F'^R(L^R) \tag{4.11}$$

Finally, we assume that a rural worker cannot search from home but must first be unemployed in the city and then search for a job. Thus the equilibrium migration condition can be written as:

$$r\, I_U = \frac{w_L^R}{r}$$

Using (4.3) and (4.4), and observing that

$$I_L - I_U = \frac{w_L^C - w_U^C}{r + \delta + \theta^C q(\theta^C)}$$

this can be written as:

$$\frac{(r+\delta)\, w_U^C + \theta^C q(\theta^C) w_L^C}{r + \delta + \theta^C q(\theta^C)} = \frac{F'^R(L^R)}{r} \tag{4.12}$$

Definition 6. *A Harris-Todaro equilibrium with urban search externalities and bargained wages is a 5-tuple* $(w_L^C, \theta^C, w_L^R, L^C, U^C, V^C, L^R)$ *such that (4.8), (4.7), (4.11), (4.10), (4.9), (4.1) and (4.12) are satisfied.*

Here is the way the equilibrium is calculated. The system is recursive. First, by combining (4.8) and (4.7), we obtain a unique θ^{C*} that is only function of parameters and given by:

$$(1-\beta)\left(y^C - w_U^C\right) - \beta c\, \theta^C = \frac{c\,(r+\delta)}{q(\theta^C)} \tag{4.13}$$

Second, by combining (4.8) and (4.12), we obtain:

$$\frac{(r+\delta)\, w_U^C + \theta^C q(\theta^C)\left[(1-\beta)\, w_U^C + \beta\left(y^C + c\,\theta^C\right)\right]}{r + \delta + \theta^C q(\theta^C)} = \frac{F'^R(L^R)}{r} \tag{4.14}$$

which using θ^{C*} gives a unique L^{R*} as a function of parameters only. Furthermore, by plugging θ^{C*} and L^{R*} in (4.8), we obtain a unique L^{C*}. Figure A3.2 illustrates the way the equilibrium is calculated.

[*Insert Figure* A3.2 *here*]

Finally, by plugging L^{C*} and L^{R*} in (4.11) and (4.9), we obtain respectively w_L^{R*} and U^{C*} and by plugging θ^{C*} in (4.13), we obtain w_L^{C*}. Also, using the values of θ^{C*} and U^{C*} in (4.1), we obtain the equilibrium number of vacancies in cities, V^{C*}.

Here the migration process is more complex. If the government reduces the unemployment benefit, this will again have a direct effect by increasing urban jobs. Indeed, since the wage is reduced (see (4.8)), more firms enter the market (see (4.7)) and thus more urban jobs are created. There will still be a repulsion force because of the positive effect on rural wage. But since workers face less search frictions (more firms enter the market) more rural workers will migrate to the cities, which in turn increases workers' search frictions. We have the following result:[4]

Proposition 4. *In an Harris-Todaro model with urban search externalities and bargained wages, decreasing unemployment benefit w_U^C leads to:*

- (i) *an increase in both urban job creation θ^C and urban employment L^C,*
- (ii) *an ambiguous effect on both rural employment L^R and urban unemployment (both in level and rate) U^C and u^C.*

Furthermore, if the following condition holds,

$$w_L^C - w_L^R/r + \frac{\beta c}{\frac{\partial\left[\theta^C q(\theta^C)\right]}{\partial\theta^C}} < -F''^R(L^R)\left(N - L^R\right)\frac{\delta\left[r+\delta+\theta^C q(\theta^C)\right]}{r\left[\delta+\theta^C q(\theta^C)\right]^2}$$

then a Todaro paradox prevails, that is decreasing w_U^C increases both urban employment and unemployment.

A decrease in w_U^C has a direct negative effect on bargained wages. As a result, because it is cheaper and thus more profitable to hire a worker, more firms enter the urban labor market and more jobs are created; consequently θ^C and L^C increase. However, the effect on rural-urban migration and thus on L^R is more subtle. Indeed, when w_U^C decreases, there is a *direct negative effect* on migration since urban wages are lower and thus less rural workers migrate (thus L^R increases). There is also an *indirect positive effect* on migration since a lower w_U^C increases w_L^C and thus more firms enter the urban labor market (if the search cost c is not too large) and more jobs are created. This increases rural-urban migration and thus reduces L^R. The net effect is thus ambiguous. The same ambiguity arises when one studies the effect of w_U^C on urban unemployment. These results mean that there is a possibility for a

[4] The proof of Proposition 4 can be found at the end of this appendix.

Todaro paradox, that is a decrease in unemployment benefit can increase both urban employment and unemployment. This is true if at least the indirect positive effect on migration is larger than direct negative effect mentioned above.

As in the efficiency wage model, let us study the case with no mobility between the two regions. The wage w_L^C and the job creation rate θ^C are still be given by (4.8) and (4.13) respectively but L^C is now equal to:

$$L^C = \frac{\theta^C q(\theta^C)}{\delta + \theta^C q(\theta^C)} N^C \tag{4.15}$$

Definition 7. *A search equilibrium with no mobility is a triple* $(w^{C*}, \theta^{C*}, L^{C*}, w^{R*})$ *such that (4.8), (4.13), (4.15) and (3.8) are satisfied.*

By totally differentiating (4.13), (4.15) and (4.9), we have the following result:

Proposition 5. *In a search equilibrium with no mobility, decreasing the unemployment benefit* w_U *increases both urban job creation* θ^C *and urban employment* L^C *and decreases urban unemployment (both in level and rate), that is:*

$$\frac{\partial \theta^C}{\partial w_U} < 0 \ , \ \frac{\partial L^C}{\partial w_U} < 0 \ , \ \frac{\partial U^C}{\partial w_U} > 0 \ , \ \frac{\partial u^C}{\partial w_U} > 0$$

Here, contrary to the efficiency wage model, the results are different from the free-mobility case because of the induced effects on rural-urban migration.

5. Proof of Propositions

5.1. Proof of Proposition 2

The Harris-Todaro equilibrium is defined by equations (3.11) and (3.10). From (3.11), we obtain a $L^C\left(L^R, w_U^C\right)$, whose properties are given by (3.14). Plugging this value in (3.10), we obtain the following equation:

$$w_U^C + \frac{e}{m} \frac{\delta\, L^C(L^{R*}, h_U)}{N - L^C(L^{R*}, h_U) - L^{R*}} = \frac{F'^R(L^{R*})}{r}$$

that gives a unique L^R, which is a function of exogenous parameters only, and in particular a function of w_U^C. This is why we denote the equilibrium value that we obtain by $L^{R*} \equiv L^R(w_U^C)$. By totally differentiating this equation, we obtain:

$$\frac{\partial L^{R*}}{\partial w_U^C} = -\frac{\left[N - L^C(L^{R*}, w_U^C) - L^{R*}\right]^2 + \frac{e\delta}{m}\frac{\partial L^C(L^{R*}, w_U^C)}{\partial w_U^C}\left(N - L^{R*}\right)}{\frac{e\delta}{m}\left[\frac{\partial L^C}{\partial L^{R*}}\left(N - L^{R*}\right) + L^C(L^{R*}, w_U^C)\right] - \left[N - L^C(L^{R*}, w_U^C) - L^{R*}\right]^2 \frac{F''^R(L^{R*})}{r}} \tag{5.1}$$

where, using (3.14), we have $\frac{\partial L^C(L^{R*}, w_U^C)}{\partial w_U^C} < 0$ and $\frac{\partial L^C(L_U^{R*}, w_U^C)}{\partial L^{R*}} < 0$, so we cannot sign this derivative.

Now, plugging this value $L^{R*} \equiv L^R(w_U^C)$ in (3.11), we obtain a unique $L^{C*} \equiv L^C(w_U^C)$, which is only function of parameters and given implicitly by the following equation:

$$w_U^C + e + \frac{e}{m}\left[\frac{\delta\left(N - L^{R*}\right)}{N - L^{C*} - L^{R*}} + r\right] = F'^C(L^{C*})$$

where $L^{C*} \equiv L^C(L^{R*}, w_U^C)$. Again, by totally differentiating this equation, we obtain:

$$\frac{\partial L^{C*}}{\partial w_U^C} = -\frac{\left(N - L^{C*} - L^{R*}\right)^2 + \frac{e}{m}\delta\frac{\partial L^{R*}}{\partial w_U^C}L^{C*}}{\frac{e}{m}\delta\left(N - L^{R*}\right) - \left(N - L^{C*} - L^{R*}\right)^2 F''^C(L^{C*})} \tag{5.2}$$

where $\frac{\partial L^{R*}}{\partial w_U^C}$ is given by (5.1).

Let us now calculate the exact value of $\frac{\partial L^{R*}}{\partial w_U^C}$. By plugging (3.15) and (5.2) in (5.1) and solving in $\frac{\partial L^{R*}}{\partial w_U^C}$, we obtain:

$$\frac{\partial L^{R*}}{\partial w_U^C} = \frac{\left(N - L^C - L^{R*}\right)^4 F''^C(L^{C*})}{\left(\frac{e\delta}{m}\right)^2 L^C\left(N - L^{R*}\right) - \left(N - L^{C*} - L^{R*}\right)^2 B} < 0 \tag{5.3}$$

where

$$B \equiv \frac{e}{m}\delta L^C F''^C(L^C) + \frac{F''^R(L^{R*})}{r}\left[\frac{e}{m}\delta L^C\left(N - L^{R*}\right) - \left(N - L^{C*} - L^{R*}\right)^2 F''^C(L^{C*})\right] < 0$$

We can now calculate $\frac{\partial L^{C*}}{\partial w_U^C}$. By plugging (5.3) in (5.2), we obtain:

$$\frac{\partial L^{C*}}{\partial w_U^C} = -\frac{\left(N - L^{C*} - L^{R*}\right)^2 + \frac{e}{m}\delta\frac{\partial L^{R*}}{\partial w_U^C}L^{C*}}{\frac{e}{m}\delta\left(N - L^{R*}\right) - \left(N - L^{C*} - L^{R*}\right)^2 F''^C(L^{C*})}$$

$$= -\frac{\left(N - L^{C*} - L^{R*}\right)^2 - \frac{\left(N - L^{C*} - L^{R*}\right)^4 F''^C(L^{C*})}{\left(N - L^C - L^R\right)^2 B - \left(\frac{e}{m}\delta\right)^2 L^{C*}\left(N - L^{R*}\right)}\frac{e}{m}\delta L^{C*}}{\frac{e}{m}\delta\left(N - L^{R*}\right) - \left(N - L^{C*} - L^{R*}\right)^2 F''^C(L^{C*})}$$

$$\left[\left(N - L^C - L^R\right)^2 B - \left(\frac{e}{m}\delta\right)^2 L^{C*}\left(N - L^{R*}\right)\right]\left(N - L^{C*} - L^{R*}\right)^2 < \left(N - L^{C*} - L^{R*}\right)^4 F''^C(L^{C*})$$

Let us show that $\frac{\partial L^{C*}}{\partial w_U^C} < 0$. Since the denominator is positive, we have

$$\frac{\partial L^{C*}}{\partial w_U^C} < 0$$

$$\Leftrightarrow \left(N - L^{C*} - L^{R*}\right)^2 > \frac{\left(N - L^{C*} - L^{R*}\right)^4 F''^C(L^{C*})}{\left(N - L^C - L^R\right)^2 B - \left(\frac{e}{m}\delta\right)^2 L^{C*}\left(N - L^{R*}\right)} \frac{e}{m}\delta L^{C*}$$

$$\Leftrightarrow \left(N - L^{C*} - L^{R*}\right)^2 \left[B - F''^C(L^{C*})\frac{e}{m}\delta L^{C*}\right] - \left(\frac{e}{m}\delta\right)^2 L^{C*}\left(N - L^{R*}\right) < 0$$

Since

$$\begin{aligned} & B - F''^C(L^{C*})\frac{e}{m}\delta L^{C*} \\ = & \frac{F''^R(L^{R*})}{r}\left[\frac{e}{m}\delta\left(N - L^{R*}\right) - \left(N - L^{C*} - L^{R*}\right)^2 F''^C(L^{C*})\right] < 0 \end{aligned}$$

This implies that

$$\left(N - L^C - L^R\right)^2 \left[B - F''^C(L^{C*})\frac{e}{m}\delta L^{C*}\right] - \left(\frac{e}{m}\delta\right)^2 L^{C*}\left(N - L^{R*}\right) < 0$$

is always true. Thus

$$\frac{\partial L^{C*}}{\partial w_U^C} < 0$$

Let us now calculate $\frac{\partial U^{C*}}{\partial w_U^C}$. By differentiating (3.1), we have:

$$\frac{\partial U^{C*}}{\partial w_U^C} = -\frac{\partial L^{C*}}{\partial w_U^C} - \frac{\partial L^{R*}}{\partial w_U^C} > 0$$

Moreover, since the unemployment rate is defined as

$$u^{C*} = \frac{U^{C*}}{U^{C*} + L^{C*}}$$

then

$$\frac{\partial u^{C*}}{\partial w_U^C} = \frac{\frac{\partial U^{C*}}{\partial w_U^C} L^{C*} - U^{C*}\frac{\partial L^{C*}}{\partial w_U^C}}{\left(U^{C*} + L^{C*}\right)^2} > 0$$

Finally, using Definition 4 and in particular (3.12), it is easy to verify that there is no Todaro paradox. ∎

5.2. Proof of Proposition 4

By totally differentiating (4.13), it is easy to verify that

$$\frac{\partial \theta^C}{\partial w_U^C} = \frac{1 - \beta}{-\beta c + q'(\theta^C)c\,(r + \delta)\,/\left[q(\theta^C)\right]^2} < 0 \tag{5.4}$$

By totally differentiating (4.14), we obtain:

$$\frac{\partial L^R}{\partial \theta^C} = \frac{\frac{\partial\left[\theta^C q(\theta^C)\right]}{\partial \theta^C} w_L^C + \theta^C q(\theta^C)\left[c - w_L^R/r\right]}{\left[r+\delta+\theta^C q(\theta^C)\right] F''^R(L^R)/r}$$

$$\frac{\partial L^R}{\partial w_U^C} = \frac{r+\delta+\theta^C q(\theta^C)(1-\beta) + \frac{\partial \theta^C}{\partial w_U^C}\left[\frac{\partial\left[\theta^C q(\theta^C)\right]}{\partial \theta^C}\left(w_L^C - w_L^R/r\right) + \beta c\right]}{\left[r+\delta+\theta^C q(\theta^C)\right] F''^R(L^R)/r} \tag{5.5}$$

A sufficient condition for $\frac{\partial L^R}{\partial \theta^C} < 0$ is $c > w_L^R/r$. We also have:

$$\frac{\partial L^R}{\partial w_U^C} = \frac{dL^R}{dw_U^C} + \frac{\partial L^R}{\partial \theta^C}\frac{\partial \theta^C}{\partial w_U^C} \tag{5.6}$$

with

$$\frac{dL^R}{dw_U^C} = \frac{r+\delta+\theta^C q(\theta^C)(1-\beta)}{\left[r+\delta+\theta^C q(\theta^C)\right] F''^R(L^R)/r} < 0 \tag{5.7}$$

By totally differentiating (4.8), we obtain:

$$\frac{\partial L^{C*}}{\partial \theta^C} = \left(N - L^R\right)\frac{\partial\left[\theta^C q(\theta^C)\right]}{\partial \theta^C}\frac{\delta}{\left[\delta+\theta^C q(\theta^C)\right]^2} > 0$$

$$\frac{\partial L^{C*}}{\partial L^R} = -\frac{\theta^C q(\theta^C)}{\delta+\theta^C q(\theta^C)} < 0$$

$$\frac{\partial L^{C*}}{\partial w_U^C} = \frac{\partial L^{C*}}{\partial \theta^C}\frac{\partial \theta^C}{\partial w_U^C} < 0$$

Thus, since $U^C = N - L^C - L^R$, we have

$$\frac{\partial U^{C*}}{\partial w_U^C} = -\left(\frac{\partial L^{C*}}{\partial w_U^C} + \frac{\partial L^R}{\partial w_U^C}\right)$$

Finally, a Todaro paradox exists if

$$\frac{\partial L^{R*}}{\partial w_U^C} > -\frac{\partial L^{C*}}{\partial w_U^C}$$

which using (5.6), (5.7) and (5.5) is equivalent to

$$\begin{aligned} & r+\delta+\theta^C q(\theta^C)(1-\beta) \\ > \quad & -\frac{\partial \theta^C}{\partial w_U^C}\left[\frac{\partial\left[\theta^C q(\theta^C)\right]}{\partial \theta^C}\left[w_L^C - w_L^R/r + \left(N-L^R\right)\frac{\delta\left[r+\delta+\theta^C q(\theta^C)\right]}{\left[\delta+\theta^C q(\theta^C)\right]^2}F''^R(L^R)/r\right] + \beta c\right] \end{aligned}$$

Thus if

$$\frac{\partial\left[\theta^C q(\theta^C)\right]}{\partial \theta^C}\left[w_L^C - w_L^R/r + \left(N-L^R\right)\frac{\delta\left[r+\delta+\theta^C q(\theta^C)\right]}{\left[\delta+\theta^C q(\theta^C)\right]^2}F''^R(L^R)/r\right] + \beta c < 0$$

a Todaro paradox always exists. This is equivalent to:

$$-\left(N-L^{R}\right)\frac{\delta\left[r+\delta+\theta^{C}q(\theta^{C})\right]}{\left[\delta+\theta^{C}q(\theta^{C})\right]^{2}}F''^{R}(L^{R})/r>\left(w_{L}^{C}-w_{L}^{R}/r\right)+\frac{\beta c}{\frac{\partial\left[\theta^{C}q(\theta^{C})\right]}{\partial\theta^{C}}}$$

which is the condition displayed in the proposition. ▮

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Figure A3.1: Harris-Todaro equilibrium with efficiency wages

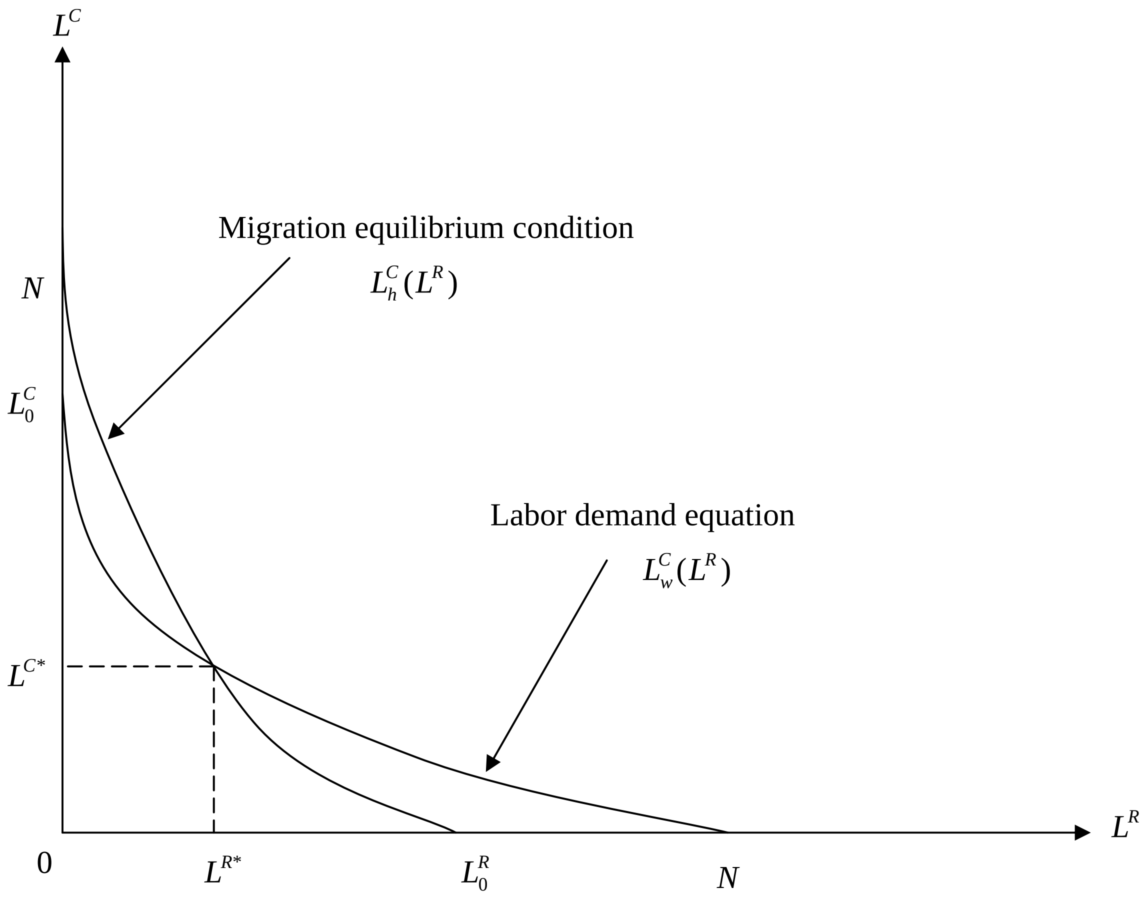

Figure A3.2: Harris-Todaro equilibrium with search externalities