Correction of Lecture Notes 3bis

Some examples of Mixed Strategy Equilibria

Correction of Example 1: Battle of Sexes

1a) Let $(q, 1-q)$ be the mixed strategy in which Patrick plays "Opera" with probability q and let $(r, 1-r)$ be the mixed strategy in which Lindsey plays "Opera" with probability r.

If Patrick plays $(q, 1-q)$, then Lindsey's expected payoff are

$$2.q + 0.(1-q) = 2q$$

from playing "Opera" and

$$0.q + 1.(1-q) = 1-q$$

from playing "Fight". Thus,

(i) if $q > 1/3$, then Lindsey's best response is "Opera" (i.e. $r = 1$);

(ii) if $q < 1/3$, then Lindsey's best response is "Fight" (i.e. $r = 0$);

(iii) if $q = 1/3$, then any value of r is a best response.

Similarly, if Lindsey plays $(r, 1-r)$, then Patrick's expected payoff are

$$1.r + 0.(1-r) = r$$

from playing "Opera" and

$$0.r + 2.(1-r) = 2(1-r)$$

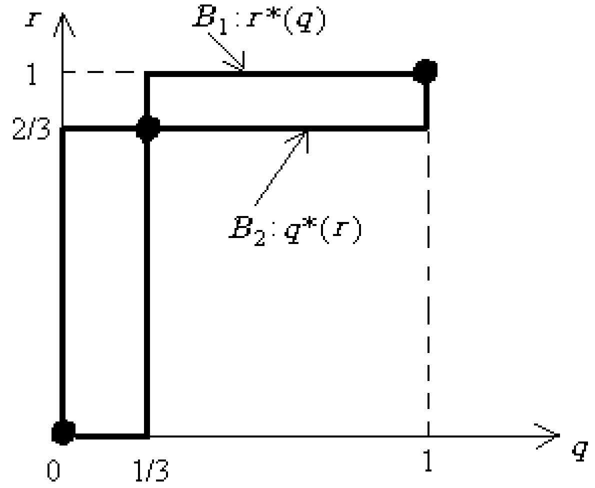
from playing "Fight". Thus,

(i) if $r > 2/3$, then Patrick's best response is "Opera" (i.e. $q = 1$);

(ii) if $r < 2/3$, then Patrick's best response is "Fight" (i.e. $q = 0$);

(iii) if $r = 2/3$, then any value of q is a best response.

We have the following figures of best responses:



From the figure, we see that the game has three mixed strategy Nash equilibria:

(i) $(q, 1-q) = (0, 1)$ for Patrick and $(r, 1-r) = (0, 1)$ for Lindsey, i.e. the pure strategy equilibrium (Fight, Fight)

(ii) $(q, 1-q) = (1, 0)$ for Patrick and $(r, 1-r) = (1, 0)$ for Lindsey, i.e. the pure strategy equilibrium (Opera, Opera),

(iii) the mixed strategies $(q, 1-q) = (1/3, 2/3)$ for Patrick and $(r, 1-r) = (2/3, 1/3)$ for Lindsey are a Nash equilibrium.