A Test for Subadditivity of the Cost Function with an Application to the Bell System

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The recent literature on multiproduct firms demonstrates that when all firms in an industry (actual or potential) have access to a common technology, properties of the firm cost functions reveal the most efficient industry structure (see, for example, Elizabeth Bailey and Ann Friedlaender, 1982). An important issue addressed in this literature is the derivation of conditions which guarantee that an industry is a natural monopoly. Despite the relevance of this issue to discussions concerning the desirability of competition in regulated industries, few empirical studies offer reliable evidence on this question. The reason for this is that the theory of multiproduct industries (see, for example, William Baumol, John Panzar, and Robert Willig, 1982) suggests that global information about cost functions is required to determine the presence of natural monopoly. Such information is seldom available.

This article proposes a new test of necessary conditions for natural monopoly that does not require global information on firm cost functions. Our test does not require the extrapolation of estimated cost functions well outside the range of the available data that are required in tests currently proposed in the literature.

To see why global information is required to test for necessary *and* sufficient conditions for natural monopoly, consider a firm which produces an output vector $q=(q_1,\dots,q_n)$ with cost function $C(q)$. A firm is a natural monopoly if and only if $C(q)$ is subadditive over the relevant range of output levels.[1,2] The function $C(q)$ is subadditive at the output level $\tilde{q}$ if and only if

$$C(\tilde{q}) < \sum_{i=1}^{n} C(\tilde{q}^i), \tag{1}$$

where for nonnegative $\tilde{q}^i$

$$\sum_{i=1}^{n} \tilde{q}^i = \tilde{q}, \tag{2}$$

with at least two vectors $\tilde{q}^i$ nonzero, for all $\tilde{q}^i$ satisfying (2). Thus an industry is a natural monopoly if a single firm can produce all relevant output vectors more cheaply than two or more firms. To test this condition requires knowing the cost of all output vectors $\tilde{q}^i$ smaller than $\tilde{q}$ and thus requires global information about the cost function. Such knowledge about $C(q)$ is required for all possible industry equilibrium output values.

Less empirically demanding necessary and sufficient conditions for subadditivity remain to be developed. Baumol et al. derive separate necessary and sufficient conditions which require less information than the joint necessary and sufficient conditions for subadditivity.[3] For example, the presence of economies of scope is a necessary condition

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[1]Subadditivity of the firm cost function is a necessary and sufficient condition for natural monopoly only when all firms have access to the same technology and when market coordination between separate firms is unable to achieve the same economies (say by networking or pooling arrangements) as internal coordination within a single firm. These assumptions may not characterize many real-world industries. See our (1983a) article for further discussion.

[2]The relevant range of outputs depends on the demand and cost conditions prevailing over the period of interest to the analyst or policymaker.

[3]See Baumol et al. or Bailey and Friedlaender for formal statements of these conditions.

for subadditivity. Economies of scope exist if the costs of producing each product separately exceed the costs of producing all products jointly. Economies of scope and declining average incremental cost for each product are sufficient conditions for subadditivity. The average incremental cost of output i, when the total output vector is $\tilde{q}$, is the cost of producing output vector $\tilde{q}$ less the cost of producing all of $\tilde{q}$ excluding product i divided by the output of $i, \tilde{q}_i$. The average incremental cost for product i declines if the average incremental cost of producing i decreases with increases in the level of $\tilde{q}_i$.[4]

Baumol et al. recommend testing the necessary and sufficient conditions for subadditivity separately. If the necessary condition is rejected, subadditivity is rejected. If the sufficient conditions are accepted, subadditivity is accepted. Unfortunately, even this restrictive procedure for testing subadditivity requires more information than is usually available for most industries. Their test requires observations on the stand-alone costs of production for each product in the output vector and observations on the costs of production for all output vectors containing positive quantities of $n-1$ outputs and a zero quantity of the other output.

To illustrate the problems that arise in implementing their test, consider testing for subadditivity in the postal industry. This industry provides several types of mail service, some of which have been opened to competition in recent years. It would be interesting to determine whether a single firm could provide all postal service more cheaply than could two or more firms. But data on the stand-alone costs of providing first-class mail service are not available. Consequently, calculating economies of scope between first-class and other postal services requires extrapolation of estimated cost functions far outside the range of observations over which they are estimated.[5] Similar problems beset the calculation of average incremental costs. Moreover, the Baumol et al. test may prove inconclusive even when data are available for calculating reliable estimates of the relevant quantities. Acceptance of their necessary condition but rejection of their sufficient conditions may occur.

This article proposes a new test for subadditivity within a region that avoids the need to extrapolate outside the range of the available data. The test is local and not global. It is based on the idea that if subadditivity is rejected in one region, global subadditivity must be rejected. Section I describes our test for a two-product industry. (Generalization to n-product industries is straightforward.) Section II applies the test to data from the U.S. Bell System.

I. Subadditivity Test

We begin by refining the terminology used in the introduction. Consider a two-product industry in which all firms have access to the same technology. The cost function $C(q_1, q_2)$ is subadditive at $\tilde{q} = (\tilde{q}_1, \tilde{q}_2)$ if and only if for nonnegative $\tilde{q}_1, \tilde{q}_2$

$$(3) \qquad \Sigma_i C(a_i\tilde{q}_1, b_i\tilde{q}_2) > C(\tilde{q}_1, \tilde{q}_2), \qquad i = 1, \dots, n,$$

$$(4) \quad \Sigma a_i = 1, \quad \Sigma b_i = 1, \quad a_i \geq 0, \quad b_i \geq 0,$$

for at least two a_i or b_i not equal to zero. It is superadditive at $\tilde{q}$ if and only if "$>$" is replaced with "$<$" in (3). It is additive at $\tilde{q}$ if and only if "$>$" is replaced with "$=$" in (3). A firm with superadditive costs could save money by breaking itself into two or more divisions. Unless there are firm-specific fixed factors which preclude such decentralization, we would not expect to observe profit-maximizing firms operating with superadditive costs. A firm with additive costs

[4] Economies of scope and average incremental costs can also be defined for subsets of the output vector containing multiple products. See Baumol et al.

[5] Melvyn Fuss and Leonard Waverman (1981) have used this test to determine whether Bell Canada has a natural monopoly over local, toll, and private line telephone services. They reject the hypothesis that there are economies of scope between private line service and local and toll service. But, as they readily admit, their test is unreliable since they have to extrapolate the cost function far outside their sample in order to calculate stand-alone costs for local, toll, and private line services.

may have decentralized itself into the optimal configuration. Consequently, in many situations the interesting statistical question concerns whether the cost function is additive or subadditive at observed output levels. The cost function is quasi-globally subadditive, superadditive, or additive if and only if the cost function is subadditive, superadditive, or additive, respectively, at all relevant output vectors $\tilde{q}$. The relevant output vectors are those which are consistent with industry equilibrium given demand and cost conditions for alternative possible organization patterns of the industry (for example, multifirm vs. single firm).

Our test computes (3) for an "admissible range" of outputs. For simplicity, we restrict our evaluation of (3) to $n=2$ so that we compare two-firm configurations with the monopoly configuration. We also confine our attention to the case where the analyst has time-series data on a single firm assumed to be a monopoly, although the test can be readily applied to situations where the analyst has cross section or panel data on a sample of firms. Denote each time-series observation by an output q_t, $t=1,\ldots,T$.

Denote the first hypothetical firm by A and the second hypothetical firm by B. Consider output level $\tilde{q}$ and let $\tilde{q}^A+\tilde{q}^B=\tilde{q}$. Figure 1 illustrates our test. We define an "admissible region" represented by an area on the floor of the diagram. The cost of producing $\tilde{q}^A$ is $\tilde{C}^A$, the cost of producing $\tilde{q}^B$ is $\tilde{C}^B$, and the cost of producing $\tilde{q}$ is $\tilde{C}$. If $\tilde{C}^A+\tilde{C}^B>\tilde{C}$, then the cost function is subadditive at $\tilde{q}$ with respect to the particular two-firm industry configuration described by $(\tilde{q}^A,\tilde{q}^B)$, a situation which holds true in Figure 1. We consider all two-firm configurations $(\tilde{q}^A,\tilde{q}^B)$ in the admissible region which sum to $\tilde{q}$. If $\tilde{C}^A+\tilde{C}^B>\tilde{C}$ for all such configurations, then the cost function is subadditive at $\tilde{q}$ over the admissible region, the situation depicted in Figure 1.

Our choice of an admissible region for each output $\tilde{q}$ is dictated by our desire to avoid "excessive" extrapolation outside the data. It is difficult to make this notion precise without knowing in advance exactly what it is we seek to estimate. Approximation theory provides bounds for particular ap-

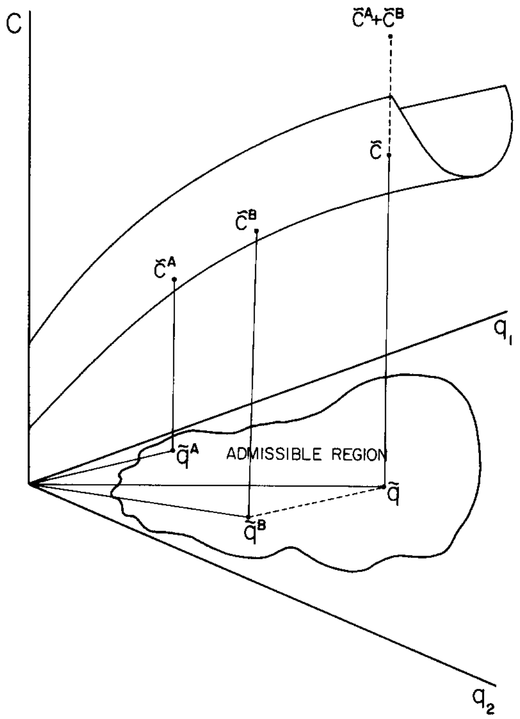

FIGURE 1. TEST FOR SUBADDITIVITY

proximations to known functions. But these bounds are of little use in applied work since, in practice, the true cost function is unknown.[6] In this paper we define the admissible region to keep hypothetical industry output configurations within the range of output configurations actually observed in the data.

Specifically, we define the admissible region so that it satisfies two constraints. The first is that no hypothetical firm in the two-firm industry is permitted to produce less of either output than the firm for which we have data. Define q_M as the vector of minimal sizes of the output of the firms permitted by this criterion:

$$q_M=(q_{1M},q_{2M})=\left(\min_t q_{1t},\ \min_t q_{2t}\right).$$

[6]See, for example, Philip Davis (1975) for a discussion of approximation theory and bounds for various functions.

Firm A produces

(5) $q_t^A = \left(\phi q_{1t}^* + q_{1M}, \omega q_{2t}^* + q_{2M}\right).$

Firm B produces

(6) $q_t^B = \left((1-\phi)q_{1t}^* + q_{1M}, (1-\omega)q_{2t}^* + q_{2M}\right).$

The parameters (ϕ, ω) satisfy

(7) $0 \leq \phi \leq 1, 0 \leq \omega \leq 1.$

Aggregating across firms A and B, we obtain

(8) $q_{1t}^* + 2q_{1M} = \tilde{q}_{1t}, \quad q_{2t}^* + 2q_{2M} = \tilde{q}_{2t},$

so that

(9) $q_{1t}^* = \tilde{q}_{1t} - 2q_{1M}, \quad q_{2t}^* = \tilde{q}_{2t} - 2q_{2M},$

for $\tilde{q}_{it} > 2q_{iM}$. We restrict the test to values of $\tilde{q}_t$ which satisfy this inequality. We test in years for which the output of both goods is at least twice the lowest output level in the sample.

The second constraint is that both firm A and B produce q_1 and q_2 in a ratio within the range of ratios observed in the data. Thus we require

(10) $$R_L < \frac{\phi q_{1t}^* + q_{1M}}{\omega q_{2t}^* + q_{2M}} < R_U,$$

$$R_L < \frac{(1-\phi)q_{1t}^* + q_{1M}}{(1-\omega)q_{2t}^* + q_{2M}} < R_U,$$

where

(11) $R_L = \min_t q_{1t}/q_{2t}, \quad R_U = \max_t q_{1t}/q_{2t}.$

This constraint precludes our hypothetical firms from specializing in either output to a greater extent than has the firm for which we have data.[7]

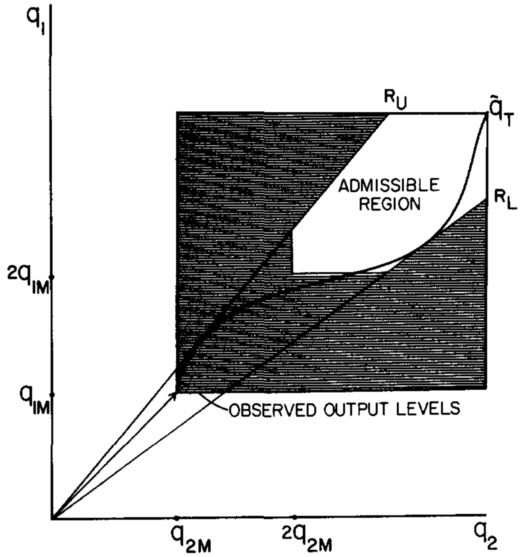

FIGURE 2. DETERMINATION OF ADMISSIBLE REGION

Figure 2 illustrates how these constraints restrict the admissible sample region. The smallest hypothetical firm size we consider is q_M. The hypothetical firm must also have output ratios between the vectors R_L and R_U, where R_L is the lowest ratio of output 1 to output 2 in the sample and R_U is the highest ratio of output 1 to output 2 in the sample. In addition, the test only applies to $\tilde{q}_t$ which satisfy (9) and therefore lie in the northeast quadrant of the box drawn in Figure 2.

[7]It is possible to define a broader admissible region by also including output configurations that are not too far removed from observed configurations. For example, we could modify the first constraint so that neither hypothetical firm is allowed to produce less than $1/\alpha$ of the minimum amount of output observed in the sample or more than α of the amount of output observed in the sample, where $\alpha \geq 1$. We could modify the second constraint so that the range of specialization for hypothetical firm lies between R_L/β and βR_U where $\beta \geq 1$. The admissible region described in the text is for the special case where $\alpha = \beta = 1$. We have chosen a conservative admissible region for two reasons. First, since this study reports the first application of our test for subadditivity, we believe it is best to be conservative and restrict the test to observed output combinations. Second, the estimates reported below deteriorate as we get further from the sample mean. Expanding the admissible region, at least for our example, does not provide much additional information. We would like to thank the referee for the suggestion for expanding the admissible region.

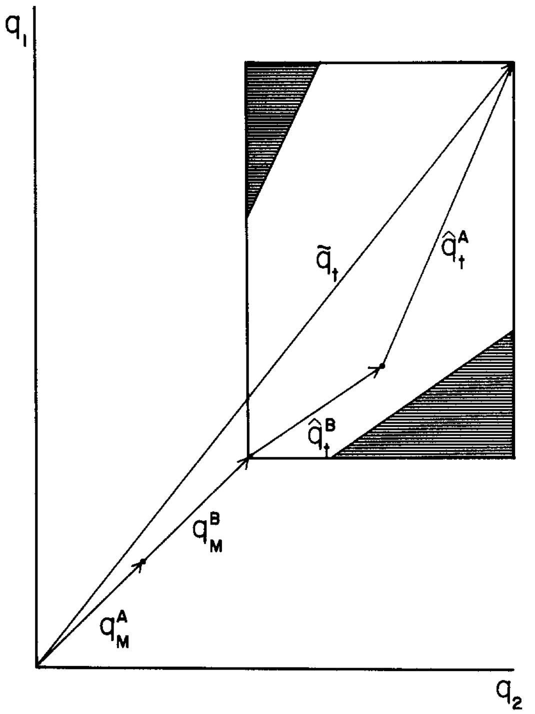

FIGURE 3. IMPLEMENTATION OF SUBADDITIVITY TEST

Figure 3 illustrates an implementation of the test for output level $\tilde{q}_t$ in the relevant output set. Let $\hat{q}_t^A = (\phi q_{1t}^*, \omega q_{2t}^*)$. Firm A produces q_M plus an increment $\hat{q}_t^A$ and firm B produces q_M plus an increment $\hat{q}_t^B$ where $\hat{q}_t^A$ and $\hat{q}_t^B$ are chosen so that $\hat{q}_t^A + \hat{q}_t^B + 2q_M$ sum to the output level $\tilde{q}_t$ at which the test is being performed. All possible vectors $\hat{q}_t^A$ and $\hat{q}_t^B$ within the admissible region, and thus all possible two-firm output configurations within the admissible region, are evaluated. In practice, the $\hat{q}_t^A$ and $\hat{q}_t^B$ are constructed by varying ϕ and ω over a grid given by $\phi = i/g$, $\omega = k/g$, $i, k = 0, \ldots, g$ where g is the grid size.

We now formally describe our test for subadditivity. Let

$$\tilde{C}_t^A(\phi, \omega) = \tilde{C}(\tilde{q}_t^A) = \tilde{C}(q_M + \hat{q}_t^A),$$

$$\tilde{C}_t^B(\phi, \omega) = \tilde{C}(\tilde{q}_t^B) = \tilde{C}(q_M + \hat{q}_t^B),$$

$$\tilde{C}_t = \tilde{C}(\tilde{q}_t^A + \tilde{q}_t^B) = \tilde{C}(\tilde{q}_t).$$

We measure the degree of subadditivity by

$$Sub_t(\phi, \omega) = \left[\tilde{C}_t - \tilde{C}_t^A(\phi, \omega) - \tilde{C}_t^B(\phi, \omega)\right] / \tilde{C}_t.$$

If $Sub_t(\phi, \omega)$ is less than zero, the cost function is subadditive with respect to the industry configuration given by (ϕ, ω). If $Sub_t(\phi, \omega)$ is equal to zero, the cost function is additive with respect to the industry configuration given by (ϕ, ω). If $Sub_t(\phi, \omega)$ is greater than zero, the cost function is superadditive over the industry configuration given by (ϕ, ω).

We calculate $\text{Max}_{(\phi, \omega)} Sub_t(\phi, \omega)$. If this quantity is negative and statistically significantly different from zero, we reject the hypothesis that the cost function is additive at $\tilde{q}_t$ over the admissible region. Values of the test statistic that are negative and statistically significantly different from zero do not contradict the hypothesis that the cost function is subadditive at $\tilde{q}_t$ relative to the admissible region. If we do not reject the composite hypothesis that the cost function is subadditive at all feasible output levels, then we do not reject the hypothesis that the cost function is quasi-globally subadditive relative to these output levels. A test of the hypothesis of subadditivity over the whole sample is based on $\text{Max}_{(\phi, \omega, T^*)} Sub_t(\phi, \omega)$, where T^* is the set of sample years for which feasible partitions exist.[8] Note that our criterion tests only necessary conditions for subadditivity as long as the admissible region is a proper subset of the possible

[8] Note that because the maximization is with respect to a continuous set of exogenous explanatory variables and because in general there is only one stationary point for the maximum problem, our procedure does not create an order-statistics problem, and so we can use conventional test statistics. We make no claim about the optimality of our test. A better test, that grew out of discussions with Kevin M. Murphy, computes $Sub_t(\phi, \omega)$ for all points in the admissible region. This is an infinite dimensional statistic but its distribution can be derived. A one-sided test of the hypothesis that this function is everywhere negative can be constructed that utilizes more information than the test proposed in this paper. While this test is no worse than the one proposed in the text, it is also more complicated to compute, and is not developed here.

region of output configurations. Failure to find subadditivity within the admissible region is informative in rejecting the hypothesis of subadditivity; evidence supporting subadditivity within the admissible region obviously does not indicate support for that hypothesis outside the admissible region.

II. Subadditivity Tests for the U.S. Bell System

We use data on the Bell System from 1947–77 to test whether the Bell System had a subadditive cost function at the output levels produced during those years. We assume the Bell System produces local and toll telephone services with capital, labor, and materials. We estimate several alternative cost function specifications under alternative assumptions concerning the structure of the disturbances. Using likelihood ratio statistics, we find that the preferred specification is a general translog cost function with first-order autoregressive disturbances in the cost and factor share equations. The estimated cost function is monotonic and concave with respect to input prices in all years. The estimated own-price elasticities of demand for capital, labor, and materials are negative in all years. Therefore our estimates satisfy the conditions required of an economically valid cost function.[9] The Appendix reports the estimated cost function.

Between 1947 and 1977, cost increased more than fourteenfold, toll service increased almost fourteenfold, and local service increased fivefold. The smallest quantities of local and toll service were both produced in 1947. Output doubled by 1958 making this year the first feasible one for our test. Over the sample period, the ratio of local to toll varies between 0.5 and 1.3.

TABLE 1—MAXIMUM PERCENT GAIN FROM MULTIFIRM PRODUCTION VS. SINGLE-FIRM PRODUCTION[a]

Year	Percent Gain	Standard Error	ϕ	ω
1958	13	15	0.0	0.0
1959	20	14	0.0	0.2
1960	25	14	0.0	0.4
1961	25	14	0.0	0.6
1962	33	14	0.0	0.5
1963	40	15	0.0	0.5
1964	44	15	0.0	0.6
1965	48	16	0.0	0.6
1966	53	23	0.5	0.9
1967	58	23	0.6	0.7
1968	51	26	0.5	0.8
1969	50	30	0.3	0.9
1970	39	22	0.5	0.7
1971	36	21	0.4	0.6
1972	39	21	0.4	0.6
1973	41	20	0.4	0.6
1974	42	21	0.4	0.6
1975	45	20	0.4	0.6
1976	59	20	0.5	0.5
1977	51	19	0.5	0.5

[a]Entries equal Max $Sub_t \times 100$. A positive number indicates that multifirm production is more efficient than single firm production.

For each year $t = 1958,\ldots,1977$, we calculate $Sub_t(\phi, \omega)$ over a grid with a $g = 10$ for (ϕ, ω) lying in the admissible region. Table 1 reports the estimates of Max $Sub_t(\phi, \omega)$. Table 2 reports the estimate of $Sub_t(\phi, \omega)$ for $t = 1961$, a year near the center of our data. We find that Max Sub_t is always greater than and often statistically significantly different from zero for output configurations produced between 1958 and 1977. Therefore we reject the hypothesis that the Bell System's cost function is subadditive at any of these output levels. We also reject the hypothesis that the Bell System's cost function is quasi-globally subadditive over these output levels. The fact that Max $Sub_t(\phi, \omega)$ is never significantly less than zero and the fact that Sub_t is frequently positive suggest that our finding that the Bell System cost function is not quasi-globally subadditive is robust.

The frequent positive and sometimes statistically significant point estimates of Sub_t

[9]Like other economists who estimate translog cost functions, we require our cost function to be linear homogeneous in input prices and to have a symmetric Hessian matrix with respect to input prices. See, for example, Laurits Christensen and William Greene (1976); Fuss and Waverman; Friedlaender, Clifford Winston, and Kung Wang (1983). Unlike these analysts, we report tests of these restrictions. We resoundingly reject them. This phenomenon occurs in all other specifications of the model that we estimate. We conjecture that these restrictions would be rejected in other translog cost function studies. In order to make our estimates consistent with the translog estimates reported by other economists, we impose homogeneity and symmetry restrictions on the cost function.

TABLE 2—PERCENT GAIN OR LOSS FROM MULTIFIRM VS. SINGLE-FIRM PRODUCTION ALTERNATIVE INDUSTRY CONFIGURATIONS, 1961 DATA[a]

	$\phi =$										
$\omega =$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	8 (21)										
0.1	8 (19)	8 (20)									
0.2	9 (18)	8 (18)	8 (19)								
0.3	12 (16)	10 (17)	9 (18)	9 (19)							
0.4	15 (15)	13 (13)	10 (17)	9 (18)	9 (18)						
0.5	20 (14)	16 (15)	13 (16)	11 (17)	9 (18)	9 (18)					
0.6	25 (14)	21 (15)	17 (16)	14 (17)	11 (17)	10 (18)	9 (18)				
0.7			23 (16)	18 (16)	15 (17)	12 (18)	10 (18)	9 (18)			
0.8					20 (17)	16 (18)	12 (18)	10 (19)	8 (19)		
0.9							17 (19)	13 (19)	10 (20)	8 (20)	
1.0										10 (21)	8 (21)

[a]Entries equal $Sub_{1961} \times 100$. A positive number indicates that multifirm production is more efficient than single-firm production.

Standard errors are reported in parentheses.

The numbers reported here are based on an autoregressive translog cost function reported in our (1983b) article.

suggest that the Bell System was not optimally decentralized during our sample period. This finding is consistent with anecdotal evidence on the Bell System. *Fortune Magazine* reported that William L. Weiss, head of the new operating company for the Midwest region, "thinks AT&T's marketing was too centralized with the result that managers became 'less creative and more dependent on the signal caller'" (June 27, 1983, p. 64). The Midwest operating company will reportedly operate as a decentralized confederation of the five companies from which it is to be constituted.

III. Conclusions

This article presents a new test for subadditivity of the cost function that requires less information on the cost function than does the test proposed by Baumol, Panzar, and Willig. The test provides an easily computed rejection criterion for the hypothesis of subadditivity. Applying this test to the Bell System using 1947–77 time-series data, we reject the hypothesis that the Bell System's cost function is subadditive at the output levels produced between 1958–77. We find limited evidence that the Bell System did not optimally decentralize itself during these years and was therefore operating inefficiently.

APPENDIX

We estimate the Bell System cost function using aggregate time-series data on the Bell System for 1947–77. We estimate a cost function rather than a production function in order to make our approach consistent with previous studies of the production characteristics of the telecommunications industry, and because the theoretical literature on natural monopoly relies on the cost function rather than the production function. We disaggregate outputs into local and long distance services and inputs into capital, labor, and materials. As a proxy for technological change, we follow previous studies of the production characteristics of the Bell System (see, for example, Laurits Christensen, Diane Cummings and Philip Schoech, 1981; M. Ishaq Nadiri and Mark Shankerman, 1981; and Hrishikesh Vinod, 1976), by using an index of lagged research and development expenditures by Bell Laboratories.

Following conventional practice (see, for example, Christensen and William Greene, 1976), we use the translog approximation to the cost function and estimate a system consisting of the cost equation, the capital share equation, and the labor share equation using maximum likelihood techniques. We find that a specification with first-order autoregressive disturbances maximizes the likelihood function and purges the residuals of serial correlation.[10,11] The estimated cost function is reported in Table 3. This cost function is monotonically increasing and concave with respect to all input prices in all years. The own-price elasticities of demand for capital, labor and materials are negative in all years. The estimates thus satisfy the sufficient conditions for an economically valid cost function.[12] We reject the hypothesis that the cost function is separable in local and long distance output. Thus it is not valid to aggregate local and long distance telephone service into a single measure of telephone service and estimate a single-product cost function as is done in the Christensen et al., Nadiri-Shankerman, and Vinod studies. Our (1983b) article provides further details on our estimation procedures and results and on our data sources.

[10]We also estimate two alternatives to the translog cost function. The first alternative applies a Box-Cox transformation to the output variables. The second alternative applies a Box-Cox transformation to all right-hand side variables. We are unable to reject the hypothesis that the correct specification is translog. See our (1983b) article.

[11]We assume that output levels and factor prices are exogenous. Although both assumptions are questionable on a priori grounds for the telephone industry, using a Durbin (1954)-Wu (1973) test, we reject the hypothesis that output levels and input prices are endogenous.

[12]But see fn. 9.

TABLE 3—ESTIMATED COST FUNCTION

Parameter	Coefficient Estimate	Standard Error
Constant	9.054	(.005)
Capital	.535	(.008)
Labor	.355	(.007)
Toll	.260	(.309)
Local	.462	(.226)
Technology	−.193	(.086)
Capital^2	.219	(.024)
Labor^2	.174	(.027)
Capital-Labor	−.180	(.019)
Toll^2	−8.018	(2.170)
Local^2	−4.241	(1.314)
Local-Toll	11.663	(3.144)
Technology^2	−.176	(1.033)
Capital-Toll	.337	(.138)
Capital-Local	−.359	(.122)
Labor-Toll	−.179	(.083)
Labor-Local	.164	(.071)
Capital-Technology	.083	(.053)
Labor-Technology	−.057	(.047)
Toll-Technology	−1.404	(1.497)
Local-Technology	1.207	(1.431)
Autocorrelation Parameter for:		
Cost Equation	.187	(.105)
Share Equations	.712	(.094)

Notes:

Summary Statistics	R^2	Degrees of Freedom	Durbin-*h*
Cost Function	.9999	15	.65
Capital Share Equation	.9756	27	1.50
Labor Share Equation	.9835	27	1.37

Generalized Variance for System = 10.568

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