Correction of Lecture Notes 7

Some examples of Extensive Form Games and SPNE

Correction of Example 2: Firm-union bargaining (Osborne)

2a) The following extensive game models the situation.

Players: The firm and the union

Terminal histories: All sequences of the form (w, Y, L) and (w, N) for nonnegative numbers w and L, where w is the wage, Y means accept, N means reject, and L is the number of workers hired.

Player function: $P(\emptyset)$ is the union, and for any nonnegative number w, $P(w)$ and $P(w, Y)$ are the firm.

Preferences: Profit Π for the firm and U for the union.

2b) To find the subgame perfect equilibrium of this game (Nash equilibrium in each subgame), we have to use a backward induction to solve this problem.

Let us thus first consider the subgame following a history (w, Y), i.e. the union demands w and the firm has accepted the demand. In a subgame perfect equilibrium, the firm chooses L to maximize its profit, given w.

For $L \leq 50$, $\Pi = L(100 - L) - wL = (100 - w)L - L^2$. This is a quadratic function in L that is concave and that is equal to zero when $L = 0$ and when $L = 100 - w$, and reached a maximum L^* in between these two values. This maximum is

$$\frac{\partial \Pi}{\partial L} = 100 - w - 2L^* = 0$$

which implies that

$$L^* = \begin{cases} (100 - w)/2 & \text{if} \quad w \leq 100 \\ 0 & \text{if} \quad w > 100 \end{cases}$$

Given the firm's optimal action in such a subgame, consider now the subgame following a history w, in which the firm has to decide whether to accept or reject w. For any w, the firm's profit, given its subsequent optimal choice of L, is such that:

(i) if $w < 100$, this profit is positive;

(ii) if $w \geq 100$, this profit is zero.

Thus, in a subgame perfect equilibrium, the firm accepts any demand $w < 100$ and either accepts or rejects any demand $w \geq 100$.

Finally, consider the union's choice at the beginning of the game. If it chooses $w < 100$, then the firm accepts and chooses $L-(100-w)/2$, yielding the union a payoff of $w(100-w)/2$. If it chooses $w > 100$, then the firm either accepts and chooses $L = 0$ or rejects; in both cases, the union's payoff is 0.

Thus the best value of w for the union is the number that maximizes $U = w(100-w)/2$. This function U is quadratic in w and concave, is zero when $w = 0$ and $w = 100$ and reaches a maximum w^* in between. This value is

$$\frac{\partial U}{\partial w} = \frac{100 - 2w^*}{2} = 0$$

which implies that

$$w^* = 50$$

As a result, there is a unique subgame perfect equilibrium in which the union's strategy is $w^* = 50$ and the firm's strategy is to accept this demand and chooses $L^* = 25$. In this case,

$$\Pi^* = (100 - w^*)L^* - L^{*2} = 625$$

$$U^* = w^* L^* = 1250$$

2c) Yes. In any subgame perfect equilibrium $\Pi^* = 625$ and $U^* = 1250$. Thus both parties are better off at the outcome (w, L) that they are in the unique subgame perfect equilibrium if and only if

$$L \leq 50$$

and

$$U = wL > 1250$$

and

$$\Pi = L(100 - L) - wL > 625$$

or

$$L \geq 50$$

and

$$wL > 1250$$

and

$$2500 - wL > 625$$

These conditions are satisfied for a nonempty set of pairs (w, L). For example, if $L = 50$, the conditions are satisfied by $25 < w < 37.5$. If $L = 100$, they are satisfied by $12.5 < w < 18.75$.

2d) There are many Nash equilibria in which the firm "threatens" to reject high wage demands. In one such Nash equilibrium the firm threatens to reject any positive wage demand. In this equilibrium, the union's strategy is $w = 0$, and the firm's strategy rejects any demand $w > 0$ and accepts the demand $w = 0$ and chooses $L = 50$.