

## Lecture Notes 7

### Some examples of Extensive Form Games and SPNE

#### Example 1: The ultimatum game

Two people use the following procedure to split  $c$  dollars. Person 1 offers person 2 an amount of money up to  $c$  dollars. If 2 accepts this offer, then 1 receives the remainder of the  $c$  dollars. If 2 rejects the offer, then neither person receives any payoff. Each person cares only about the amount of money he receives, and prefers to receive as much as possible.

Assume that the amount person 1 offers can be any number, not necessarily an integral number of cents.

**1a)** Formulate this situation as an extensive game with perfect information.

**1b)** Find the subgame perfect equilibrium (equilibria?) of the game.

**1c)** Find the subgame perfect equilibrium (equilibria?) of the variant of the ultimatum game in which the amount of money is available only in multiple of a cent (indivisible units).

#### Example 2: Firm-union bargaining (Osborne)

A firm's output  $Q$  is given by

$$Q = \begin{cases} L(100 - L) & \text{when } L \leq 50 \\ 2500 & \text{when } L > 50 \end{cases}$$

where  $L$  is the number of units of labor that the firm uses to produce  $Q$ . The price of output is normalized to 1.

A union that represents workers presents a wage demand (a nonnegative number  $w$ ), which the firm either accepts or rejects. If the firm accepts the demand, it chooses the number  $L$  of workers to employ (that maximizes its profits). If the firm rejects the demand, no production takes place and  $L = 0$ .

The firm's preferences are represented by its profit

$$\Pi = \begin{cases} L(100 - L) - wL & \text{when } L \leq 50 \\ 2500 - wL & \text{when } L > 50 \end{cases}$$

and the union's preferences are represented by

$$U = wL$$

**2a)** Formulate this situation as an extensive game with perfect information.

**2b)** Find the subgame perfect equilibrium (equilibria?) of the game.

**2c)** Is there an outcome of the game that both parties prefer to any subgame perfect equilibrium outcome?

**2d)** Find a Nash equilibrium for which the outcome differs from any subgame perfect equilibrium outcome.