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Transmission Capacity Constraints and Transmission Costs on Electricity Market Auctions

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Abstract

Capacity constraints on transmissions of electricity are raising an increasing policy concern as electricity markets are integrated around the world. But our understanding of the workings of such markets is still limited. The purpose of this paper is to highlight the impact of transmission capacity constraints and transmission costs on electricity market auctions. In the presence of transmission capacity constraints, the equilibrium is asymmetric even when the suppliers are symmetric in generation capacity and costs. An increase in transmission capacity induces non-monotonic changes in firms' profits. In the presence of transmission constraints and zero transmission costs, an increase in transmission capacity is pro-competitive; in contrast, then the transmission costs are positive, an increase in transmission capacity could be anti-competitive.

KEYWORDS: electricity auctions, transmission capacity constraints, transmission costs.
JEL codes: D43, D44, L13, L94

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1 Introduction

The capacity of electricity grids to transmit power between markets is gaining in importance, as electricity markets are becoming increasingly integrated at the national and international level. Traditionally, transmission lines have mainly been used by public utilities to exploit regional differences in consumption patterns and generation costs. But with the electricity generation industry becoming increasingly deregulated, transmission facilities become important also from the point of view of *enhancing* competition between firms in different regions.

In light of very substantial costs for installing transmission capacity, it is highly desirable to focus investments on points in the grids where the gains in terms of enhanced market performance will be the largest. But to achieve this, it is necessary to understand the functioning of electricity markets that are only partially integrated due to capacity constraints for transmissions. The purpose of this paper is to contribute to the study of such markets by characterizing the outcome of an electricity market auction in the presence of transmission capacity constraints and transmission costs.

The analysis employs a simple duopoly model similar to that in Fabra et al. (2006). In the basic set up, the two suppliers have symmetric capacities and marginal costs, but are located in two different markets ("North" and "South") that are connected through a transmission line with a limited transmission capacity.[1] The firms face a monetary charge when transmitting power through the grid.[2] Each firm faces a perfectly inelastic demand in each market that is known with certainty when suppliers submit their offer prices. Each supplier must submit a single price offer for its entire capacity[3] in a discriminatory price auction. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices set in the wholesale market, at least in the short run.

The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets where offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day. It seems reasonable to assume that in such markets, suppliers know the total demand that they face with a high degree of certainty. On the other hand, when offer prices remain fixed for longer periods, it is more appropriate to assume that suppliers face some degree of demand uncertainty, or volatility, at the time they submit their offers. Examples of such instances might be the Australian market, and the former markets in England and Wales, where prices remained constant for a full day.

When there are constraints on the possibility to deliver electricity to a market, the effective size of the market differs for the suppliers. The supplier located in the high-

[1]The term "transmission capacity constraint" is used throughout this article in the electrical engineering sense: a transmission line is constrained when the flow of power is equal to the capacity of the line, as determined by engineering standards.

[2]Electricity suppliers pay a linear tariff that depends on the location and the season/period-of-day. The locational component of the tariff penalizes the injection of electricity in points of the grid that generate high flows of electricity. The seasonal/period-of-day component of the tariff penalizes the transmission of electricity when the losses are larger.

[3]Fabra et al. (2006) show that the equilibrium outcome allocation does not change when the firms submit single price offers for their entire capacity and when they submit a set of price-quantity offers.

demand market faces a higher residual demand, while the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market has incentives to submit larger bids than the one located in the low-demand market (size effect). Hence, due to the limited transmission capacity, the equilibrium is asymmetric despite the fact that suppliers have a symmetric generation capacity and costs.

When suppliers face simultaneously transmission constraints and transmission costs. The supplier located in the high-demand market faces lower transmission costs than the one located in the low-demand market and to exploit its efficiency rents, it has incentives to submit lower bids than the one located in the low-demand market (cost effect). Hence, in the presence of transmission constraints and transmission costs, the size and the cost effects work in the opposite direction and the equilibrium outcome is determined by the one that dominates.

The consequences of transmission capacity constraints largely depend on whether there are any transmission costs. With *zero* transmission costs, an increase in transmission capacity increases the competition between suppliers and the expected bids for both firms decrease. Therefore, when the transmission costs are zero, an increase in transmission capacity is pro-competitive. But with *strictly positive* transmission costs, matters are more complex. When the transmission capacity is low, an increase in capacity increases the competition between suppliers and the expected bids decrease, as with zero transmission costs. But when the transmission capacity is higher, although still binding, an increase in capacity might induce a large increase in transmission costs and to compensate the increase in costs there is an increase in the expected bids. Therefore, when the transmission costs are positive, an increase in transmission capacity could be anti-competitive.

To understand the difference in outcome, note that with *zero* transmission costs, an increase in transmission capacity reduces the expected bid and the residual demand of the supplier located in the high-demand market and thus, also its expected profit. An increase in transmission capacity reduces the expected bid and increases the total demand that the supplier located in the low-demand market can satisfy. When the transmission capacity is low, the increase in demand dominates the decrease in the expected bid, and the expected profit thereby increases. However, when the transmission capacity is large enough, the pro-competitive effect dominates, and the expected profit decreases. In the alternative scenario of *positive* transmission costs, an increase in transmission capacity induces non-monotonic changes on both suppliers' profit functions.

This paper is not the first to study transmission constraints. Borenstein et al. (2000) characterize the equilibrium when suppliers compete in quantities. They work out the transmission capacity at which the suppliers are indifferent between satisfying the residual demand or competing in the entire market. However, they do not characterize the equilibrium when the transmission line is congested. Holmberg and Philpott (2012) solve for symmetric supply function equilibria when firms face different demand ex-ante and they do not consider any transmission costs. Escobar and Jofré (2010) analyze the effect of transmission losses on equilibrium outcome allocations; however, they assume that the transmission line is not congested. In contrast to the previous literature, this paper characterizes a setting where the transmission line is congested and where the suppliers face transmission costs.

This paper also contributes to the literature of competition under capacity constraints. Kreps and Scheinkman (1983) and Osborne and Pitchik (1986) characterize the equilibrium in a duopoly with capacity constraints. Deneckere and Kovenock (1996) and Fabra et al. (2006) extend the analysis to include asymmetries in generation capacity and production costs. However, none of these models characterize the equilibrium when the firms face transmission constraints and transmission costs. The effect of transmission constraints cannot be modeled as an asymmetry in generation capacity. Therefore, this paper presents new results for models of competition with capacity constraints. Moreover, in the presence of transmission constraints and transmission costs, the equilibrium is determined by two effects (the size and the cost effect) that work in the opposite direction. The introduction of these two effects can help us best understand the effects of the asymmetries in generation capacity and generation costs[4] presented in Kreps and Scheinkman (1983); Osborne and Pitchik (1986); Deneckere and Kovenock (1996) and Fabra et al. (2006) on equilibrium market allocations.

The results of this paper could also be of relevance for the trade literature. For instance, Krugman (1980), Flam and Helpman (1987), Brezis et al. (1993) and Motta et al. (1997) explain differences in prices and profits in international trade models based on product differentiation or product cost advantages. In contrast, in this paper, due to transmission capacity constraints and transmission costs, even when the product is homogeneous and the firms face the same production costs, there exist differences in equilibrium prices and profits.

The article proceeds as follows. Section 2 describes the model and characterizes the equilibrium in the presence of transmission capacity constraints. Section 3 characterizes the equilibrium in the presence of transmission capacity constraints and transmission costs. Section 4 concludes the paper. The proofs are found in the Appendix.

2 The model

Set up of the model. There exist two electricity markets, market North and market South, that are connected by a transmission line with capacity T. When firms transmit electricity through the grid from one market to the other, they face a symmetric linear[5] transmission tariff t. In order to reduce transmission losses,[6] the transmission tariffs in the majority of European countries have a locational and a seasonal component.[7]

[4]For the sake of the argument, suppose that two suppliers are competing in a single node electricity market; supplier one is larger and more efficient than supplier two. Supplier one faces higher residual demand and submits larger bids than supplier two (size effect). At the same time, supplier one is more efficient than supplier two and submits lower bids to extract the efficiency rents (cost effect).

[5]In European electricity markets, the transmission costs are linear. However, the model can be modified to assume convex costs. When the transmission costs are convex, the existence of the equilibrium is guaranteed by Dixon (1984).

[6]For a complete analysis of losses in Europe and a complete description of the algorithm implemented to work out power losses, consult the document "ENTSO-E ITC Transit Losses Data Report 2013".

[7]The locational and seasonal component implies that suppliers face asymmetric linear tariffs. However, the model can easily be modified to introduce this type of asymmetries. For a comparison of European tariff systems, check out the document "ENTSO-E Overview of transmission tariffs in Europe: Synthesis 2014".

There exist two duopolists with capacities k_n and k_s, where subscript n means that the supplier is located in market North and subscript s means that the supplier is located in market South. The suppliers' marginal costs of production are c_n and c_s. In this paper, I analyze the effect of transmission capacity constraints and transmission costs on equilibrium. In order to focus on this effect, I assume that suppliers are symmetric in capacity $k_n = k_s = k > 0$ and symmetric in costs $c_n = c_s = c = 0$. The level of demand in any period, θ_n in market North and θ_s in market South, is a random variable uniformly distributed that is independent across markets[8] and independent of market price, i.e., perfectly inelastic. In particular, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$ is distributed according to some known distribution function $G(\theta_i)$, $i = n, s, i \neq j$.

The capacity of the transmission line can be lower than the installed capacity in each market $T \leq k$, i.e. the transmission line could be congested for some realization of demands (θ_s, θ_n).

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation.[9] Let $b \equiv (b_s, b_n)$ denote a bid profile. On basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the capacity of the lower-bidding supplier is dispatched first. Without lost of generality, assume that $b_n < b_s$. If the capacity of supplier n is not sufficient to satisfy total demand $(\theta_s + \theta_n)$ in the case of the transmission line not being congested, or $(\theta_n + T)$ in the case of the transmission line being congested,[10] the higher-bidding supplier's capacity, supplier s, is then dispatched to serve residual demand, $(\theta_s + \theta_n - k)$ if the transmission line is not congested, or $(\theta_s - T)$ if the transmission line is congested. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$. The tie breaking rule implemented is such that if the bids of both suppliers are equal and the demand in market i is larger than the demand in region j, the auctioneer first dispatches the supplier located in market i.

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

[8] In the majority of electricity markets, demand in one market is higher than in the other market. Moreover, there exists the possibility of some type of correlation between demands across markets. In this paper, I assume uniform distribution and independence of demand. However, the model can **easily** be modified to introduce different distributions of demand and correlation between demands across markets.

[9] P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

[10] When the demand in market South is larger than the transmission line capacity $\theta_s > T$, supplier n can only satisfy the demand in its own region and T units of demand in region South $(\theta_n + T)$. Below in this section, I explain in detail the total demand and the residual demand that can be satisfied by each firm.

$$
q_i(b;\theta,T) = \begin{cases} min\{\theta_i+\theta_j, \theta_i+T, k_i\} & \text{if } b_i < b_j \\ \rho_i min\{\theta_i+\theta_j, \theta_i+T, k_i\} + & \\ \qquad [1-\rho_i] max\{0, \theta_i - T, \theta_i+\theta_j-k_j\} & \text{if } b_i = b_j \\ max\{0, \theta_i - T, \theta_i+\theta_j-k_j\} & \text{if } b_i > b_j \end{cases} \tag{1}
$$

The output function has an important role in determining the equilibrium and thus, I explain it in detail. Below, I describe the construction of supplier n's output function; the one for supplier s is symmetric.

The total demand that can be satisfied by supplier n when it submits the lower bid ($b_n < b_s$) is defined by $min\{\theta_n+\theta_s, \theta_n+T, k\}$. The realization of (θ_s, θ_n) determines three different areas (left-hand panel in figure 1).

$$
min\{\theta_n+\theta_s, \theta_n+T, k\} = \begin{cases} \theta_s+\theta_n & \text{if } \theta_n \leq k-\theta_s \text{ and } \theta_s < T \\ \theta_n+T & \text{if } \theta_n < k-T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k-\theta_s;\ \theta_s \in [0,T] \\ & \text{or if } \theta_n > k-T;\ \theta_s \in [T, k+T] \end{cases}
$$

When demand in both markets is low, supplier n can satisfy total demand ($\theta_s+\theta_n$). If the demand in market South is larger than the transmission capacity $\theta_s > T$, supplier n cannot satisfy the demand in market South, even when it has enough generation capacity for this; therefore, the total demand that supplier n can satisfy is (θ_n+T). Finally, if the demand is large enough, the total demand that supplier n can satisfy is its own generation capacity.

The residual demand that supplier n satisfies when it submits the higher bid ($b_n > b_s$) is defined by $max\{0, \theta_n-T, \theta_s+\theta_n-k\}$. The realization of (θ_s, θ_n) determines three different cases (right-hand panel in figure 1).

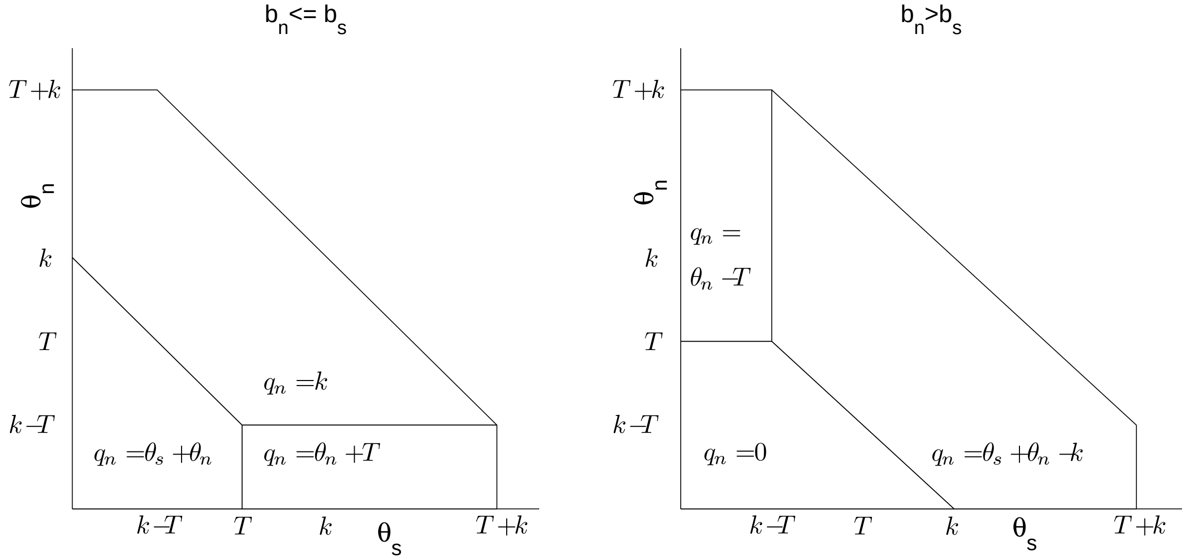
$$
max\{0, \theta_n-T, \theta_s+\theta_n-k\} = \begin{cases} 0 & \text{if } \theta_n < T;\ \theta_s \in [0, k-T] \\ & \text{or } \theta_n < k-\theta_s;\ \theta_s \in [k-T, k] \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s \in [0, k-T] \\ \theta_s+\theta_n-k & \text{if } \theta_n > k-\theta_s;\ \theta_s \in [k-T, T+k] \end{cases}
$$

When demand in both markets is low, supplier s satisfies total demand and therefore, the residual demand that remains for supplier n is zero. When total demand is large enough, supplier s cannot satisfy total demand and some residual demand ($\theta_s+\theta_n-k$) remains for supplier n. Due to the transmission constraint, the total demand that supplier s can satisfy diminishes. As soon as demand in market North is larger than the transmission capacity ($\theta_n > T$), it cannot be satisfied by supplier s and thus, some residual demand ($\theta_n - T$) remains for supplier n.

Finally, the payments are worked out by the auctioneer. When the auctioneer runs a discriminatory price auction,[11] the price received by a supplier for any positive quantity

[11]The aim of this paper is to characterize the equilibrium in an electricity auction in the presence of transmission constraints and transmission costs. I have decided to focus on discriminatory auctions because the equilibrium is unique and therefore, it is easier to make a comparative static analysis. However, using the approach presented in Fabra et al. (2006), it is simple to characterize the equilibrium when the

Figure 1: Output function for supplier n. ($k_n = k_s = 60, T = 40$)



dispatched by the auctioneer is equal to its own bid. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

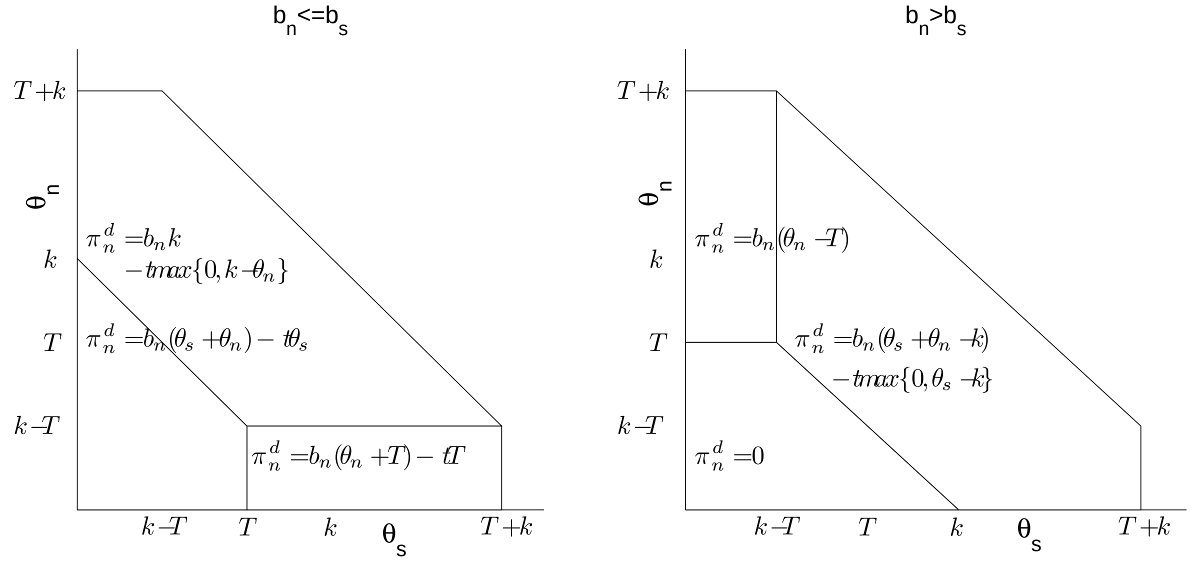
$$\pi_i^d(b; \theta, T, t) = \begin{cases} (b_i - c_i) min \{\theta_i + \theta_j, \theta_i + T, k\} - \\ \quad tmax \{0, min \{\theta_j, T, k - \theta_i\}\} & \text{if } b_i \leq b_j \text{ and } \theta_i > \theta_j \\ (b_i - c_i) max \{0, \theta_i - T, \theta_i + \theta_j - k\} - \\ \quad tmax \{0, \theta_j - k\} & \text{otherwise} \end{cases}$$

If $b_n \leq b_s$ and $\theta_n \geq \theta_s$, supplier n's payoff function is $\pi_n^d(b; \theta, T) = (b_n - c_n)$ $min \{\theta_n + \theta_s, \theta_n + T, k\}$. In addition to this expression, due to the transmission costs, supplier n is charged a transmission cost t for the power sold in market South. This cost is tT when the realization of demand in market North is low or $t(k - \theta_n)$, when the realization of demand in market North is high but lower than its generation capacity. When demand in market North is larger than the generation capacity k, supplier n cannot sell any electricity in market South and the transmission cost is zero. Hence, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b; \theta, T, t) = (b_s - c_n) min \{\theta_n + T, k\} - tmax \{0, min \{T, k - \theta_n\}\}$ (left-hand panel, figure 2).

In the rest of the cases, supplier n is dispatched last and satisfies the residual demand. Supplier n's payoff function is $\pi_n^d(b; \theta, T, t) = (b_n - c_n) min \{\theta_s + \theta_n, \theta_n + T, k\}$. In addition to this expression, due to the transmission costs, supplier n is charged a transmission cost t for the residual demand satisfied in market South. Therefore, after adding the transmission costs, supplier n's payoff is equal to $\pi_n^d(b; \theta, T) = (b_n - c_n) max \{0, \theta_n - T, \theta_s + \theta_n - k\} - tmax \{0, \theta_s - k\}$ (right-hand panel, figure 2).

auction is uniform.

Figure 2: Profit function for supplier n. ($k_n = k_s = 60, T = 40, t > 0$)



3 Effect of transmission capacity constraints

In the presence of transmission capacity constraints, the size of the market differs for both suppliers. The supplier located in the high-demand market faces higher residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. In this section, I characterize characterize the equilibrium in the presence of transmission capacity constraints and *zero* transmission costs and then I analyze the effect of an increase in transmission capacity.

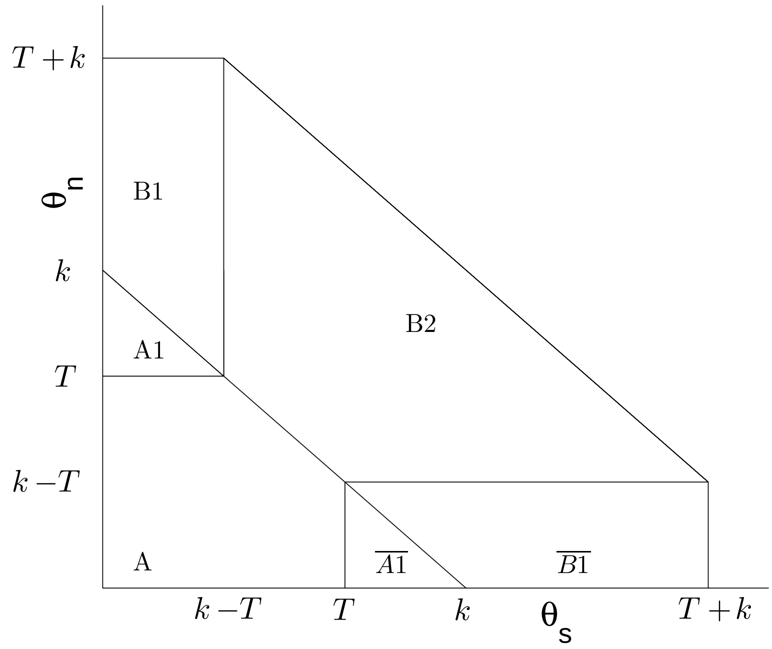
Lemma 1. When the realization of demands (θ_s, θ_n) is low (area A), the equilibrium is in pure strategies. When the realization of demands (θ_s, θ_n) is intermediate (areas $A1, B1$) or high (area $B2$), a pure strategy equilibrium does not exist (figure 3).

Proof. When the realization of demands (θ_s, θ_n) is low (area A), both suppliers have enough capacity to satisfy total demand in both markets and the transmission line is not congested. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium where both suppliers submit bids equal to their marginal cost.

When the realization of demands (θ_s, θ_n) is intermediate (areas $A1$, $B1$) or high (area $B2$), at least one of the suppliers faces a positive residual demand. Therefore, a pure strategy equilibrium does not exist. First, an equilibrium such that $b_i = b_j = c$ does not exist because at least one supplier has an incentive to deviate and satisfy the residual demand. Second, an equilibrium such that $b_i = b_j > c$ does not exist because at least one supplier has the incentive to undercut the other to be dispatched first. Finally, an equilibrium such that $b_j > b_i > c$ does not exist because supplier i has the incentive to shade the bid submitted by supplier j. □

When the realization of demands (θ_s, θ_n) is intermediate or high, a pure strategy

Figure 3: Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0$)



equilibrium does not exist. However, the model satisfies the properties[12] established by Dasgupta and Maskin (1986) which guarantee that a mixed strategies equilibrium exists.

Lemma 2. In the presence of transmission constraints. In a mixed strategy equilibrium, no supplier submits a bid lower than bid ($\underline{b}_i$) such that $\underline{b}_i min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} = Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$. Moreover, the support of the mixed strategy equilibrium for both suppliers is $S = \left[max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Proof. Each supplier can guarantee for itself the payoff $Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$, because each supplier can always submit the highest bid and satisfy the residual demand. Therefore, in a mixed strategy equilibrium, no supplier submits a bid that generates a payoff equilibrium lower than $Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$. Hence, no supplier submits a bid lower than $\underline{b}_i$, where $\underline{b}_i$ solves $\underline{b}_i min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} = Pmax\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}$.

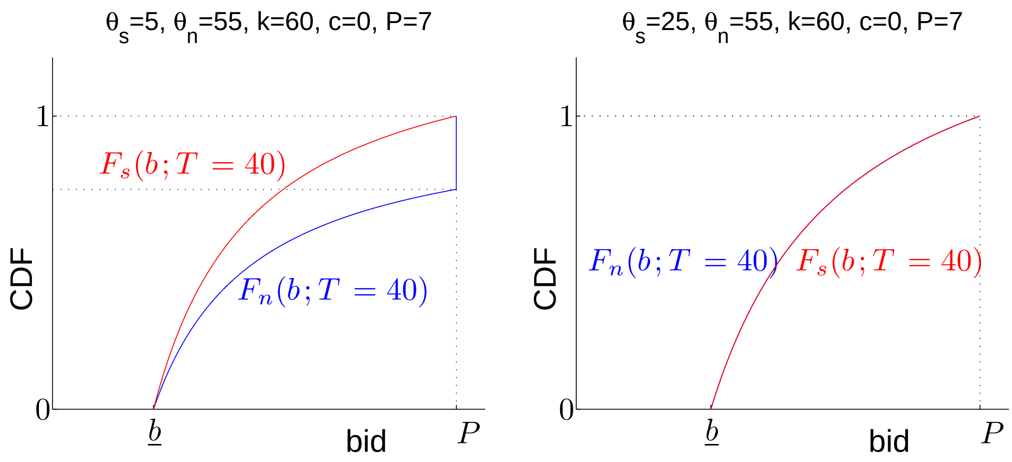
No supplier can rationalize submit a bid lower than $\underline{b}_i, i = n, s$. In the case when $\underline{b}_i = \underline{b}_j$, the mixed strategy equilibrium and the support are symmetric. In the case when $\underline{b}_i < \underline{b}_j$, supplier i knows that supplier j never submits a bid lower than $\underline{b}_j$. Therefore, in a mixed strategy equilibrium, supplier i never submits a bid b_i such that $b_i \in \left(\underline{b}_i, \underline{b}_j\right)$, because supplier i can increase its expected payoff choosing a bid b_i such that $b_i \in \left[\underline{b}_j, P\right]$. Hence, the equilibrium strategy support for both suppliers is $S = \left[max\left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$ □

Using Lemmas one and two, I characterize the equilibrium.

Proposition 1. In the presence of transmission constraints, the characterization of the

[12] In annex one, proposition one, I prove that the model satisfies the properties established by Dasgupta and Maskin which guarantee that a mixed strategy equilibrium exists.

Figure 4: Discriminatory auction. Mixed **strategy** equilibrium



equilibrium falls into one of the next two categories.

i Low demand (area A). The equilibrium strategy pair is in pure strategies.

ii Intermediate demand (areas $A1$, $B1$) and high demand (area $B2$). The equilibrium strategy pair is in mixed strategies.

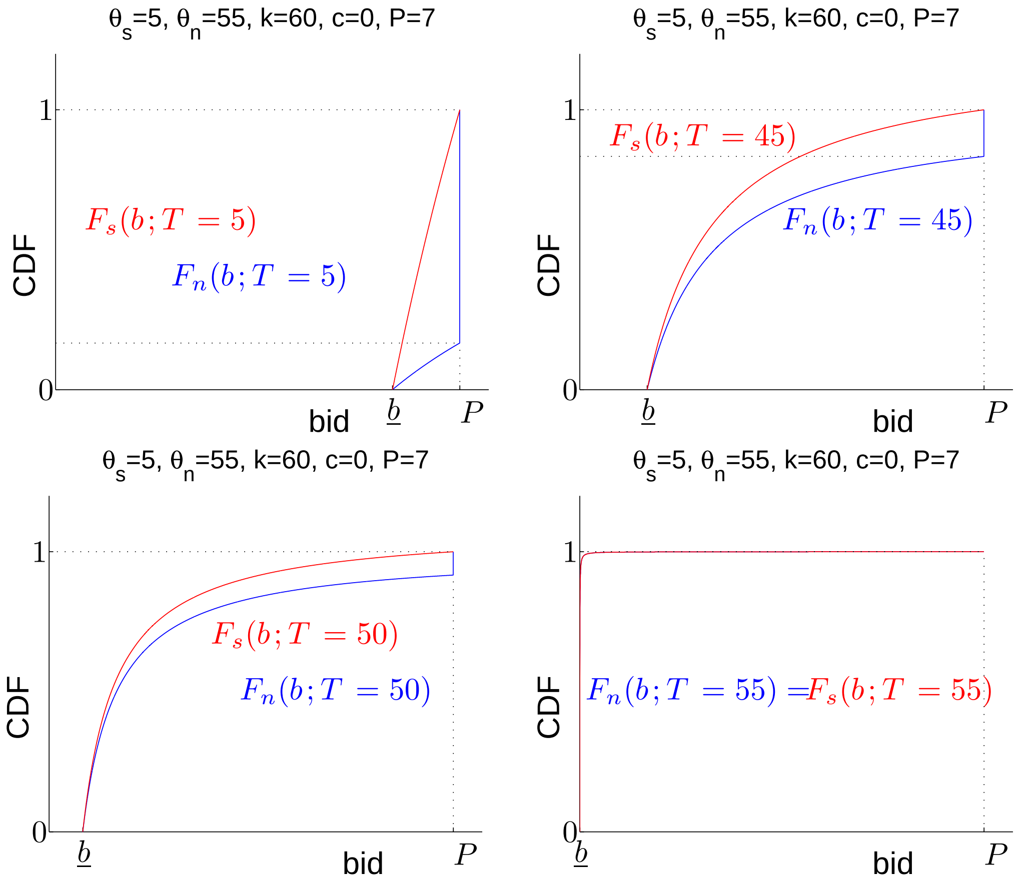
When the realization of demands (θ_s, θ_n) is low, suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium in which both suppliers submit bids equal to their marginal cost. When the realization of demands (θ_s, θ_n) is intermediate, due to the scarcity of transmission capacity, the supplier located in the high-demand market faces a higher residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the equilibrium is an asymmetric mixed strategy equilibrium where the supplier located in the high-demand market randomizes submitting higher bids with a higher probability, i.e., its cumulative distribution function stochastically dominates the cumulative distribution function of the supplier located in the low-demand market (left-hand panel, figure 4). Finally, when the realization of demands (θ_s, θ_n) is high, the transmission capacity is not binding, but the generation capacity is. Therefore, both suppliers face the same residual and total demand and the equilibrium is a symmetric mixed strategy equilibrium in which both suppliers randomize using the same cumulative distribution function (right-hand panel, figure 4).

In the presence of transmission constraints, there are two relevant constraints to explain the results. When the generation capacity is binding, even when the realization of demands is asymmetric, the equilibrium is symmetric.[13] When the transmission capacity is binding, even when the firms are symmetric in generation capacity and production costs, the equilibrium is asymmetric.

To conclude this section, I analyze the effect of an increase in transmission capacity on equilibrium outcome allocations.

[13]In the next section, I introduce transmission costs. In the presence of transmission costs, the realization of demands becomes very relevant because the transmission costs are larger for the supplier located in the low-demand market and the equilibrium is asymmetric.

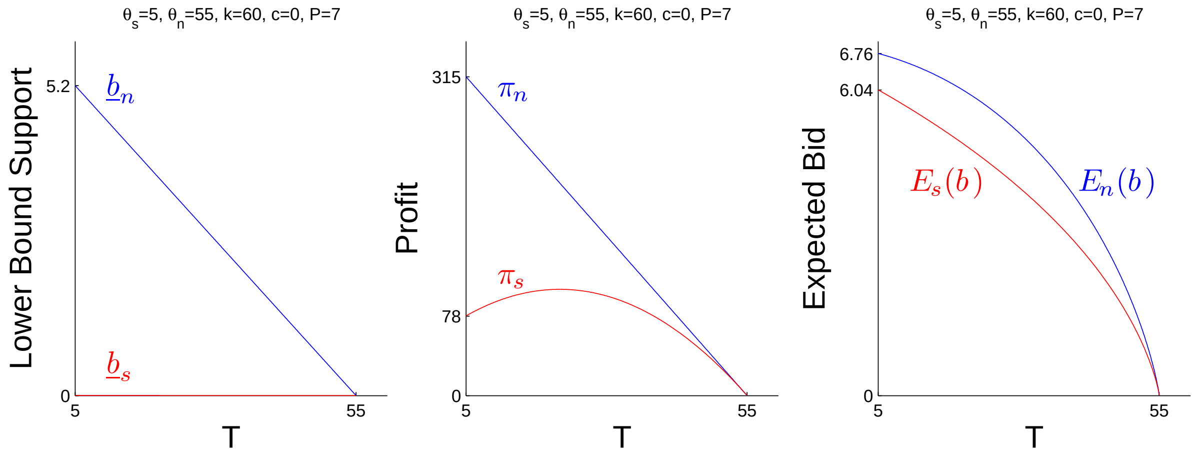
Figure 5: Increase in transmission capacity $\triangle T$. Cumulative Distribution Function



Proposition 2. In the presence of transmission constraints. An increase in transmission capacity ($\triangle T$) reduces the lower bound of support $\underline{b}$ and reduces the expected bids for both suppliers (an increase in transmission capacity is pro-competitive). Moreover, an increase in transmission capacity reduces the profit of the supplier located in the high-demand market. However, an increase in transmission capacity modifies the profit of the supplier located in the low-demand market in a non monotonic pattern (table 1 and figures 5 and 6).

An increase in transmission capacity modifies the market size as does suppliers' strategies. When the transmission capacity is very low, the supplier located in the high-demand market faces a high residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market submits higher bids than the one located in the low-demand market and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market (top-left panel, figure 5). When the transmission capacity increases, the supplier located in the high-demand market faces a reduction in its residual demand and the supplier located in the low-demand market faces an increase in the demand that it can satisfy. Therefore, the cumulative distribution function becomes more symmetric (top-right and bottom-left panels). When the transmission capacity is high enough, the residual and the total demand that both suppliers face are equal and the equilibrium is symmetric (bottom-right

Figure 6: Increase in transmission capacity $\triangle T$. Main variables



panel).[14]

The change in suppliers' strategies induced by an increase in transmission capacity modifies the main variables of the model. An increase in transmission capacity reduces the residual demand and according to lemma two, the lower bound of the support decreases (left-hand panel, figure 6). A decrease in the lower bound of the support implies that both suppliers randomize submitting lower bids and therefore, the expected bid decreases for both suppliers (right-hand panel, figure 6; columns five and seven of table 1). Finally, an increase in transmission capacity reduces the expected bid and the residual demand of the supplier located in the high-demand market and so does its expected profit. In contrast, an increase in transmission capacity reduces the expected bid and increases the total demand of the supplier located in the low-demand market. When the transmission capacity is low, the increase in demand dominates the decrease in the expected bid and the expected profit increases. However, when the transmission capacity is large enough, the decrease in bids dominates and the expected profit decreases (central panel, figure 6; columns three and four, table 1.)

An increase in transmission capacity increases the competition between markets. Moreover, an increase in transmission capacity modifies the profit of the supplier located in the low-demand market and this might increase the competition within a market. For the sake of the argument, imagine that a small hydro-power plant that faces a fixed entry cost would like to install some generation capacity in the low-demand market. When there is no transmission capacity between markets, due to the reduced size of the market, the supplier cannot cover its fixed entry cost. However, if the transmission line increases, the size of the market increases and the supplier could enter the low-demand market. This entry might increase the competition within the low-demand market.

[14]In the numerical example presented in table 1 and figures 5 and 6. When the transmission capacity is high enough ($T \leq 55$), the equilibrium is in pure strategies ($b_n = b_s = c = 0$) because both suppliers can satisfy the total demand and the transmission line is not congested.

Table 1: Increase transmission capacity $\triangle T$. Main variables. ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $P = 7$)

T	$\underline{b}$	π_n	π_s	$E_n(b)$ Ana.	$E_n(b)$ Sim.	$E_s(b)$ Ana.	$E_s(b)$ Sim.
0	–	385.07	35	7	7	7	7
5	5.835	350.1	58.35	6.8971	6.8963	6.3795	6.3830
15	4.668	280.08	93.36	6.5592	6.5587	5.6770	5.6780
25	3.501	210.06	105.03	5.9264	5.9261	4.8530	4.8532
35	2.335	140.1	93.4	4.8981	4.8981	3.8464	3.8476
45	1.168	70.08	58.4	3.2587	3.2589	2.5102	2.5109
55	0.001	0.06	0.06	0.0089	0.0093	0.0087	0.0093

$E_n(b)$ Ana. and $E_s(b)$ Ana. constitute the expected value obtained using the analytical expressions presented in proposition one and $E_n(b)$ Sim. and $E_s(b)$ Sim. constitute the expected value obtained using the simulation explained in detail in Annex 3.

I have assumed that demand in market North (θ_n) is equal to 55.01 to avoid computational problems. This is the reason why the variables in the last row are not exactly equal to zero.

4 Effect of transmission capacity constraints and transmission costs

In the presence of transmission capacity constraints, the size of the market differs for both suppliers. In the presence of transmission costs, the transmission cost differs for both suppliers depending on the realization of the demand. The supplier located in the high-demand region faces lower transmission costs than the supplier located in the low-demand region. In this section, I characterize the equilibrium in the presence of transmission capacity constraints and *positive* transmission costs.

Lemma 3. When the realization of demands (θ_s, θ_n) is low (area A) or intermediate (area $A1$), the equilibrium is in pure strategies. When the realization of demands (θ_s, θ_n) is intermediate (areas $B1a, B1b$) or high (area $B2a, B2b$), a pure strategy equilibrium does not exist (figure 7). Moreover, due to the presence of transmission costs, the pure strategy equilibria are asymmetric.

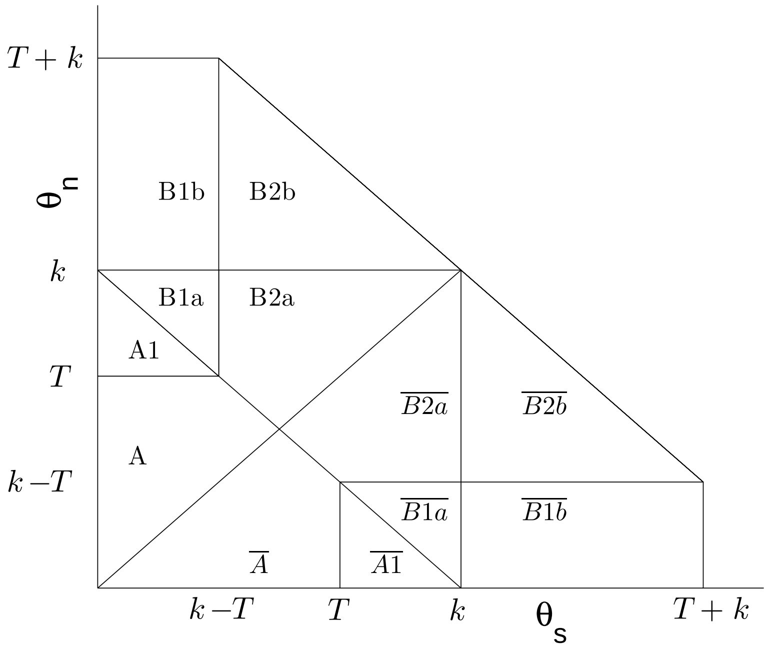
Proof. When the realization of demands (θ_s, θ_n) is low (area A), both suppliers have enough capacity to satisfy total demand and the transmission line is not congested. Therefore, the competition to be dispatched first is fierce. Moreover, the supplier located in the high-demand market (supplier j) faces lower transmission costs. Hence, the equilibrium is the typical Bertrand equilibrium with asymmetries in "costs" [15] where the supplier located in the high-demand market extracts the efficiency rents. The *pure strategies equilibrium* is $b_i = b_j = \dfrac{t\theta_j}{\theta_i + \theta_j}$.

The *equilibrium profit* is:

$$\overline{\pi}_i = (\theta_i + \theta_j)\frac{t\theta_j}{\theta_i + \theta_j} - t\theta_j = 0; \;\; \overline{\pi}_j = (\theta_i + \theta_j)\frac{t\theta_j}{\theta_i + \theta_j} - t\theta_i = t(\theta_j - \theta_i) > 0$$

[15] It is important to emphasize that the generation costs are symmetric and equal to zero. In this model, the asymmetries in costs are due to the transmission costs.

Figure 7: Equilibrium areas ($k_n = k_s = k = 60, T = 40, c = 0, t > 0$)



The *equilibrium price* is $\dfrac{t\theta_j}{\theta_i + \theta_j}$

The electricity flows from the high-demand market to the low-demand market.

When the demand belongs to area $A1$ (figure 7), the transmission constraint binds for the supplier located in the low demand market (supplier i), therefore, only the supplier located in the high demand market can satisfy the total demand. The supplier located in the high demand market prefers submit a low bid and extract the efficiency rent instead of submit a high bid and satisfy the residual demand if $(\theta_i + \theta_j)\dfrac{tT}{\theta_i + T} - t\theta_i \geq P(\theta_i - T)$. In such a case, the *pure strategies equilibrium* is $b_i = b_j = \dfrac{tT}{\theta_i + T}$.

The *equilibrium profit* is:

$$\overline{\pi}_i = (\theta_i + T)\frac{tT}{\theta_i + T} - tT = 0; \;\; \overline{\pi}_j = (\theta_i + \theta_j)\frac{tT}{\theta_i + T} - t\theta_i > 0$$

The *equilibrium price* is $\dfrac{tT}{\theta_i + T}$

The electricity flows from the high-demand market to the low-demand market.

In the rest of the cases a pure strategies equilibrium does not exist and the proof proceeds as in lemma one $\square$

When the realization of demands (θ_s, θ_n) is intermediate or high and the auction is discriminatory, a pure strategy equilibrium does not exist. However, the model satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategy equilibrium exists.

Lemma 4. In the presence of transmission constraints and *positive* transmission costs. In a mixed strategy equilibrium, no supplier submits a bid lower than bid ($\underline{b}_i$) such that

$$\underline{b}_i min \{\theta_i + \theta_j, \theta_i + T, k\} - tmax \{0, min \{\theta_j, T, k - \theta_i\}\} =$$
$$Pmax \{0, \theta_i - T, \theta_i + \theta_j - k\} - tmax \{0, \theta_j - k\}.$$

Moreover, the support for the mixed strategy equilibrium for both suppliers is $S = \left[max \left\{\underline{b}_i, \underline{b}_j\right\}, P\right]$.

Proof. The proof proceeds as in lemma two. □

Using lemmas three and four, I characterize the equilibrium.

Proposition 3. In the presence of transmission constraints and transmission costs, the characterization of the equilibrium falls into one of the next two categories.

i Low demand (area A). The equilibrium strategy pair is in pure strategies.

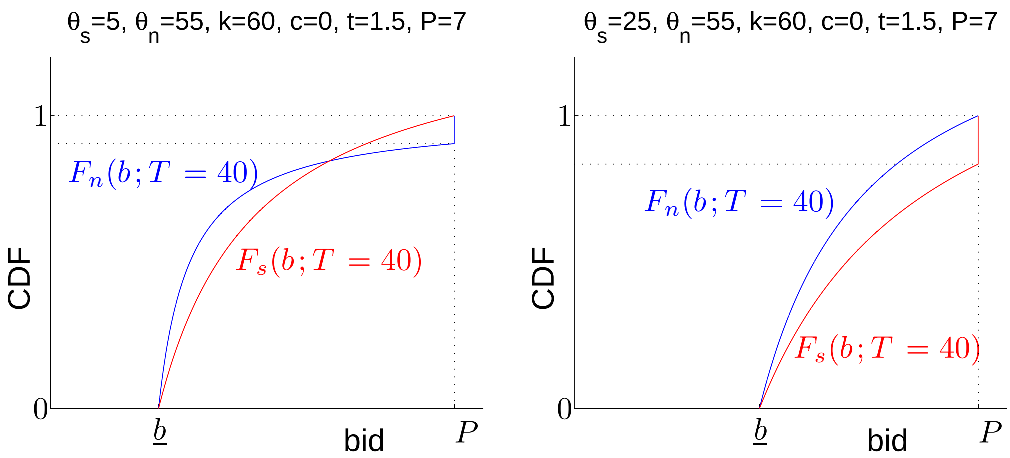
ii Intermediate demand (areas $A1$, $B1a$, $B1b$) and high demand (areas $B2a$, $B2b$). The equilibrium strategy pair is in mixed strategies.

When the realization of demands (θ_s, θ_n) is low, suppliers compete fiercely to be dispatched first in the auction and the equilibrium is the typical Bertrand equilibrium with asymmetries in costs where the supplier located in the high-demand market extracts the efficiency rents.

When the realization of demands (θ_s, θ_n) is intermediate, due to the scarcity of transmission capacity, the supplier located in the high-demand market faces a higher residual demand and the supplier located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market has higher incentives than the one located in the low-demand market to submit high bids (size effect). However, due to the presence of transmission costs, the supplier located in the high-demand market faces lower transmission costs and to exploit its efficiency rents, it has higher incentives than the supplier located in the low-demand market to submit low bids (cost effect). The cost and size effects work in the opposite direction and no stochastic dominance range can be established between the cumulative distribution functions of both suppliers (left-hand panel, figure 8). This is in contrast to the no transmission costs case where only the size effect drives the results and the cumulative distribution function of the supplier located in the high-demand region stochastically dominates the cumulative distribution function of the supplier located in the low-demand market (left-hand panel, figure 4).

When the realization of demands (θ_s, θ_n) is high, the transmission capacity is not binding, but the generation capacity is. Therefore, both suppliers face the same demand. However, due to the transmission costs, the supplier located in the high-demand market

Figure 8: Discriminatory auction. Mixed strategy equilibrium



faces lower transmission costs and submits lower bids (cost effect). Hence, the cumulative distribution function of the supplier located in the low-demand market stochastically dominates the cumulative distribution function of the supplier located in the high-demand market (right-hand panel, figure 8). This is in contrast to the no transmission costs case where both suppliers randomize using the same cumulative distribution function (right-hand panel, figure 4).

Finally, when the realization of demands (θ_s, θ_n) is in the diagonal, both suppliers face the same residual, i.e. total demand and transmission costs. Therefore, the equilibrium is a symmetric mixed strategy equilibrium.

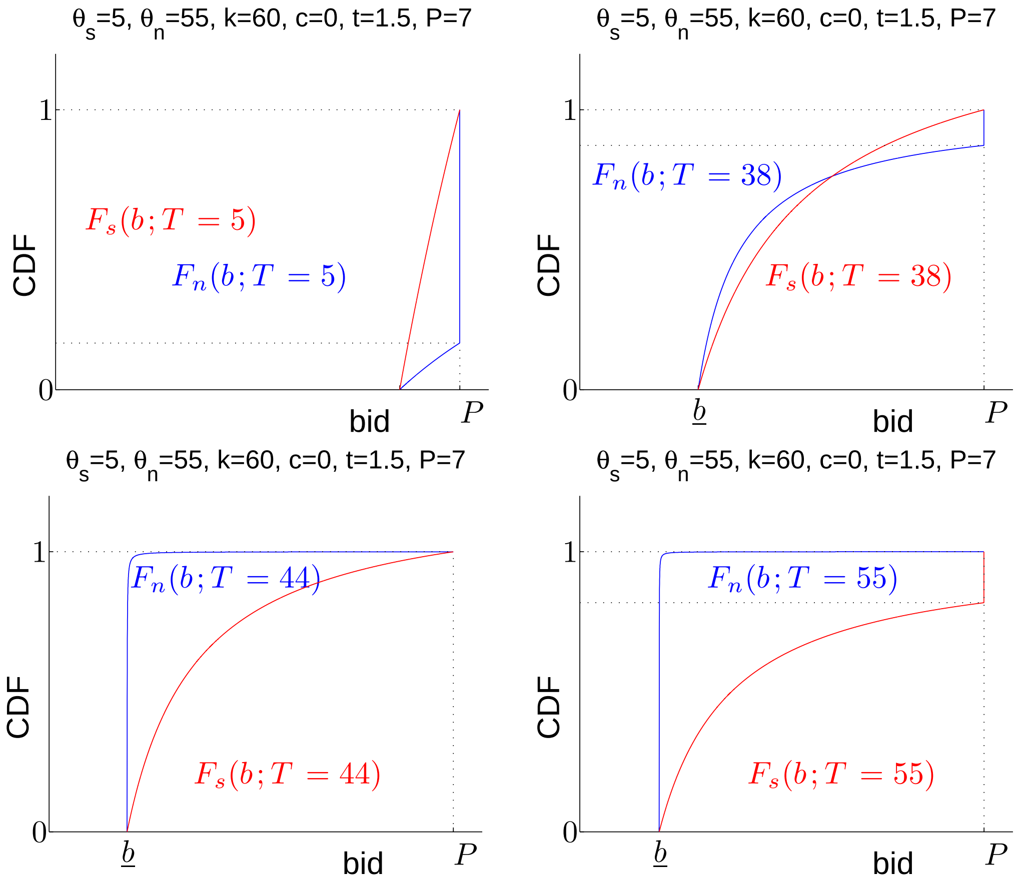
In the rest of this section, I analyze the effect of an increase in transmission capacity on the size and cost effects and thus, on equilibrium outcome allocations.[16]

Proposition 4. An increase in transmission capacity $(\triangle T)$ reduces the lower bound of the support of the supplier located in the high-demand market and increases the lower bound of the support of the supplier located in the low-demand market (left-hand panel, figure 10).

- When the lower bound of the support of the supplier located in the high-demand market is larger than the lower bound of the support of the supplier located in the low-demand region. An increase in transmission capacity reduces the expected bids of both suppliers (an increase in transmission capacity is pro-competitive), reduces the profit of the firm located in the high-demand market and modifies the profit of the supplier located in the low-demand market in a non-monotonic pattern.
- Otherwise, an increase in transmission capacity increases the expected bids of both suppliers (an increase in transmission capacity is anti-competitive), increases the expected profit of the supplier located in the high-demand market and does not modify the expected profit of the supplier located in the low-demand market (table 2; figures 9 and 10).

[16] In Annex four, I analyze the effect of an increase in transmission costs on equilibrium outcome allocations.

Figure 9: Increase in transmission capacity $\triangle T$. Cumulative Distribution Function



An increase in transmission capacity modifies the market size and the transmission costs and also supplier strategies. When the transmission capacity is very low, the size effect dominates and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market (top-left panel, figure 9). When the transmission capacity increases slightly, no cumulative distribution function stochastically dominates the other (top-right panel, figure 9). There is a considerable increase in the transmission costs when the transmission capacity is sufficiently large, mainly for the supplier located in the low-demand market. The supplier located in the high-demand market submits lower bids than the one located in the low-demand market to extract the efficiency rents and the cumulative distribution function of the supplier located in the low-demand market stochastically dominates that of the supplier located in the high-demand market (bottom-left and bottom-right panels, figure 9).

The change in suppliers' strategies induced by an increase in transmission capacity modifies the main variables of the model. When the transmission capacity is sufficiently low ($T \leq 44$ for the numerical examples in table 2 and figures 9 and 10), the size effect dominates and an increase in transmission capacity induces the same changes in the variables as when the transmission costs are null (proposition two). An increase in transmission capacity decreases the lower bound of the support and therefore, decreases the expected bid for both suppliers. Hence, an increase in transmission capacity is anti-competitive (right-hand panel, figure 10; columns five and seven, table 2); reduces the

Figure 10: Increase in transmission capacity $\triangle T$. Main variables

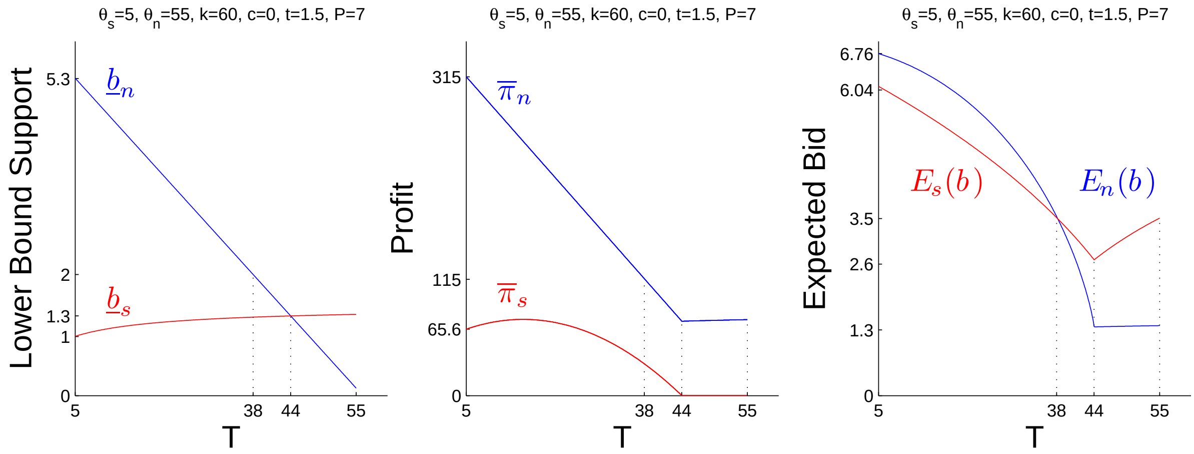


Table 2: Increase transmission capacity $\triangle T$. Main variables. ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $c = 0$, $t = 1.5$, $P = 7$)

T	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$E_n(b)$ Ana.	$E_n(b)$ Sim.	$E_s(b)$ Ana.	$E_s(b)$ Sim.
0	–	385.07	35	7	7	7	7
5	5.959	350.05	52.09	6.9079	6.9072	6.4495	6.4483
15	4.793	280.09	73.36	6.5206	6.5201	5.7472	5.7490
25	3.626	210.07	71.28	5.7253	5.7252	4.9294	4.9301
35	2.459	140.05	45.86	4.2942	4.2944	3.9306	3.9307
45	1.351	73.575	0	1.3569	1.3570	2.7299	2.7304
55	1.376	75.075	0	1.3821	1.3825	3.5073	3.5075

Here $E_n(b)$ Ana. and $E_s(b)$ Ana. constitute the expected value obtained using the analytical expressions presented in proposition one and $E_n(b)$ Sim. and $E_s(b)$ Sim. constitute the expected value obtained using the simulation explained in detail in Annex 3.

expected profit of the supplier located in the high-demand market and modifies in a non-monotonic pattern the profit of the supplier located in the low-demand market (central panel, figure 10; columns three and four, table 2).

When the transmission capacity is high enough ($T > 44$), the cost effect dominates and an increase in transmission capacity increases the lower bound of the support (left-hand panel, figure 10). An increase in the lower bound of the support entailed that both suppliers randomize submitting higher bids and therefore, the expected bid increases for both suppliers. Hence an increase in transmission capacity is anti-competitive (right-hand panel, figure 10; columns five and seven, table 2). Finally, an increase in transmission capacity increases the expected profit of the supplier located in the high-demand market because it can exploit the efficiency rents more; in contrast, the expected profit of the supplier located in the low-demand market does not change because the increase in profits derived from an increase in the expected bid is compensated by the increase in transmission costs (central panel, figure 10; columns three and four, table 2).

5 Conclusion

Electricity markets are moving through integration processes around the world. In such a context, there exists an intense debate to analyze the effect of transmission constraints and costs on suppliers' strategies. In this paper, I have characterized the equilibrium on an electricity market auction in the presence of transmission capacity constraints and transmission costs.

When there are constraints on the possibility to deliver electricity to a market, the effective size of the market differs for the suppliers. The supplier located in the high-demand market faces a higher residual demand and the one located in the low-demand market cannot sell its entire generation capacity. Therefore, the supplier located in the high-demand market has incentives to submit larger bids than the one located in the low-demand market (size effect). Hence, due to the scarcity of transmission capacity, the equilibrium is asymmetric even when the suppliers are symmetric in generation capacity and costs.

When there are constraints in delivering electricity from one market to the other, suppliers face different transmission costs depending on the realization of the demand. The supplier located in the high-demand market faces lower transmission costs than the one located in the low-demand market and to exploit its efficiency rents, it has incentives to submit lower bids than the one located in the low-demand market (cost effect). Hence, in the presence of transmission constraints and transmission costs, the size and cost effects work in the opposite direction and the equilibrium outcome is determined by the effect that dominates.

An increase in transmission capacity induces non-monotonic changes in suppliers' profits. The consequences of an increase in transmission capacity depend considerably on whether there are any transmission costs. In the presence of transmission capacity constraints and zero transmission costs, an increase in transmission capacity is always pro-competitive. In the alternative scenario of positive transmission costs, an increase in transmission capacity could be anti-competitive.

The characterization of the equilibrium in the presence of transmission constraints and transmission costs gives us the opportunity to use the toolbox of the models of competition with capacity constraints to best understand electricity markets. In particular, the model that I have used in this paper can be used to analyze mergers between suppliers located in different markets and it can be used to analyze investment decisions in generation capacity at different points of the electricity grid. Moreover, the size and cost effects described in the paper can help us to best understand the effects of the asymmetries in generation capacity and generation costs presented in Kreps and Scheinkman (1983); Osborne and Pitchik (1986); Deneckere and Kovenock (1996) Fabra et al., 2006 on equilibrium market allocations.

Annex 1. Effect of transmission capacity constraints

Proposition 1. Characterization of the equilibrium in the presence of transmission constraints.

When demand is low (area A, figure 3): $b_n = b_s = c = 0$, the *equilibrium profit* is zero for both firms. No electricity flows through the grid.

When demand is intermediate (areas $A1$ and $B1$, figure 3) or high (area $B2$, figure 3). As I have proved in lemma one, a pure strategies equilibrium does not exist; however, the model presented in section two satisfies the properties established by Dasgupta and Maskin (1986) which guarantee that a mixed strategy equilibrium exists. In particular, the discontinuities of $\pi_i, \forall i, j$ are restricted to the strategies such that $b_i = b_j$. Furthermore, it is simple to confirm that by lowering its price from a position where $b_i = b_j$, a firm discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is everywhere left lower semi-continuous in b_i, and hence weakly lower semi-continuous. Obviously $\pi_i(b_i, b_j)$ is bounded. Finally, $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is continuous, because discontinuous shifts in clientele from one firm to another only occur where both firms derive the same profit per customer. Therefore, theorem five in Dasgupta and Maskin (1986) applies and hence, a mixed strategy equilibrium exists.

The existence of the equilibrium is guaranteed by Dasgupta and Maskin (1986). However, they did not provide an algorithm to work out the equilibrium. Nevertheless, using the approach proposed by Karlin (1959); Shapley (1957); Shilony (1977); Varian (1980); Deneckere and Kovenock (1986); Osborne and Pitchik (1986) and Fabra et al. (2006), the equilibrium characterization is guaranteed by construction. I use the approach proposed by this branch of the literature to work out the mixed strategy equilibrium. In particular: first, I work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*; second, I work out the particular formulas associated with each single area[17] in figure 3.

Lower Bound of the Support. The lower bound of the support is defined according to lemma two.

Cumulative Distribution Function.

In the first step, the payoff function for any firm is:

$$\begin{aligned} \pi_i(b) &= b\left[F_j(b)max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\} + (1 - F_j(b))min\left\{\theta_i + \theta_j, \theta_i + T, k\right\}\right] = \\ &= -bF_j(b)\left[min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} - max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}\right] + \\ &\quad\; bmin\left\{\theta_i + \theta_j, \theta_i + T, k\right\} \end{aligned} \tag{2}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategies. Then,

[17] The general formulas that I will introduce below fully characterize the equilibrium. However, the equilibrium presents specific characteristics in each single area. In order to fully characterize the equilibrium, I have decided to write down the formulas for each single area.

$$\begin{aligned}
\overline{\pi}_i &= -bF_j(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]+ \\
&\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \Rightarrow \\
F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\overline{\pi}_i}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} \qquad (3)
\end{aligned}$$

The third step, at $\underline{b}$, $F_i(\underline{b})=0\forall i=n,s$. Then,

$$\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \qquad (4)$$

In the fourth step, plugging 4 into 3, I obtain the mixed strategies for both firms.

$$\begin{aligned}
F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-\underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} = \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\frac{b-\underline{b}}{b} \quad \forall i=n,s \qquad (5)
\end{aligned}$$

For further reference:
$L_i(b)=bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$ and
$H_i(b)=bmax\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}$.

It is easy to verify that equation $F_j(b)\forall i,j$ is indeed a cumulative distribution function. First, in the third step, I have established that $F_j(\underline{b})=0$. Second, $F_j(b)$ is an increasing function in b. At $\underline{b}$, $L_i(\underline{b})=H_i(b)$, for any $b>\underline{b}$, $L_i(\underline{b})<H_i(b)$; moreover, $\frac{\partial L_i(b)}{\partial b}>0$, $\frac{\partial L_i(\underline{b})}{\partial b}=0$ and $\frac{\partial H_i(b)}{\partial b}>0$, therefore, $\frac{\partial\left(L_i(b)-L_i(\underline{b})\right)}{\partial b}>\frac{\partial\left(L_i(b)-H_i(b)\right)}{\partial b}$. Third, $F_j(b)\leq 1\forall b\in S_i$. Finally, $F_j(b)$ is continuous in the support because $L_i(b)-L_i(\underline{b})$ and $L_i(b)-H_i(b)$ are continuous functions in the support.

Probability Distribution Function.

$$\begin{aligned}
f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)^2} \\
&= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)} \quad \forall i=n,s \qquad (6)
\end{aligned}$$

Expected Equilibrium Bid.

$$\begin{aligned} E_j(b) &= \int_{\underline{b}}^{P} bf_j(b)\partial b \\ &= \int_{\underline{b}}^{P} \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{b^2\left(min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right)}\partial b \\ &\quad +P\left(1-F_j(P)\right) \\ &= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}\underline{b}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}-max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\left[ln(b)\right]_{\underline{b}}^{P} \\ &\quad +P\left(1-F_j(P)\right)\ \ \forall i=n,s \qquad (7) \end{aligned}$$

where $(1-F_j(P))$ in equation 7 is the probability assigned by firm j to the maximum price allowed by the auctioneer.[18]

Expected Profit. The expected profit is defined by equation 4 and as equal to $\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$.

In the rest of the proof, I will work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* for the different possible realization of demands (θ_s,θ_n).

Area A1.

First, I work out the lower bound of the support in the border between areas $B1$ and $B2$, $\theta_s = k-T$. On the border, $\underline{b}_n$ solves $\underline{b}_n min\left\{\theta_n+\theta_s,\theta_n+T,k\right\} = Pmax\left\{0,\theta_n-T,\theta_s+\theta_n-k\right\}$, therefore $\underline{b}_n = \dfrac{P(\theta_n-T)}{k}$ and $\underline{b}_s$ solves $\underline{b}_s min\left\{\theta_n+\theta_s,\theta_s+T,k\right\} = Pmax\left\{0,\theta_s-T,\theta_s+\theta_n-k\right\}$, therefore $\underline{b}_s = \dfrac{P(\theta_n+\theta_s-k)}{\theta_s+T}$. Plugging the value of θ_s on the border between these areas into $\underline{b}_s$ formula, I obtain $\underline{b}_s = \dfrac{P(\theta_n+k-T-k)}{k-T+T} = \dfrac{P(\theta_n-T)}{k} = \underline{b}_n$. Therefore, on the border between these areas, $\underline{b}_s = \underline{b}_n = \dfrac{P(\theta_n-T)}{k}$.

In areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. In area $A1$, taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial \theta_s} = \dfrac{-P(\theta_n-T)}{(\theta_n+\theta_s)^2} < 0$ and $\dfrac{\partial \underline{b}_s}{\partial \theta_s} = \dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2} > 0$. In area $B1$, taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial \theta_s} = 0$ and $\dfrac{\partial \underline{b}_s}{\partial \theta_s} = \dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2} > 0$. Therefore, in areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. Hence, $S = \left[max\left\{\underline{b}_n,\underline{b}_s\right\},P\right] = \left[\underline{b}_n,P\right]$. In particular, in area $A1$, $S = \left[\dfrac{P(\theta_n-T)}{(\theta_n+\theta_s)},P\right]$ and

[18] When the transmission line is congested, the mixed strategy equilibrium is asymmetric. In such equilibrium, the cumulative distribution function for the firm located in the low-demand region is continuous in the upper bound of the support. By contrast, the cumulative distribution function of the firm located in the high-demand region is discontinuous, which means that the firm located in the high-demand region submits the maximum bid allowed by the auctioneer with a positive probability $(1-F_j(P))$. Hence, in order to work out the expected value, in addition to the integral, it is necessary to add the term $P\left(1-F_j(P)\right)$. Figure 4 illustrates these characteristics.

in area $B1$, $S = \left[\frac{P(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_n + \theta_s}{\theta_s + T}\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{\theta_s + T}{\theta_s + T}\frac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{\theta_s + T}\frac{P - \frac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = \frac{(\theta_s + T)}{(\theta_n + \theta_s)} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s + T}{\theta_s + T}\frac{\underline{b}}{b^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{\theta_n + \theta_s}{\theta_s + T}\frac{\underline{b}}{b}\partial b = \frac{\theta_n + \theta_s}{\theta_s + T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\underline{b}}{b^2}\partial b = \frac{\theta_s + T}{\theta_s + T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1 - F_n(P))\,P \end{aligned}$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \underline{b}(\theta_s + \theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s + T)$.

Area $B1$.

First, the lower bound of the support is $S = \left[\frac{P(\theta_n - T)}{k}, P\right]$.

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{T+k-\theta_n}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s+T}{T+k-\theta_n}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{k}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P} = 1 \\ F_n(P) &= \frac{\theta_s+T}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P} = \frac{\theta_s+T}{k} < 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b^2} \\ f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b = \frac{k}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b + \left(1-F_n(P)\right)P \\ &= \frac{\theta_s+T}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + \left(1-F_n(P)\right)P \end{aligned}$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}(\theta_s+T)$.

Area B2.

First, lower bound of the support is $S = \left[max\left\{\underline{b}_n, \underline{b}_s\right\}, P\right] = \left[\dfrac{P(\theta_s+\theta_n-k)}{k}, P\right]$.

Second, I wok out the cumulative distribution function.

$$F_i(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{2k-\theta_i-\theta_j}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \quad \forall i = s, n \\ 1 & \text{if } b = P \end{cases}$$

Third, the probability distribution function is equal to:

$$f_i(b) = \frac{\partial F_i(b)}{\partial b} = \frac{k}{2k - \theta_i - \theta_j}\frac{\underline{b}}{b^2} \;\; \forall i = s, n$$

Fourth, the expected bid is determinded by:

$$E_i(b) = \int_{\underline{b}}^{P} b f_i(b_i)\partial b = \int_{\underline{b}}^{P} \frac{k}{2k - \theta_n - \theta_s}\frac{\underline{b}}{b}\partial b = \frac{k}{2k - \theta_n - \theta_s}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \;\; \forall i = s, n$$

Fifth, the expected profit is defined by equation 4 and is equal to $\overline{\pi}_n = \overline{\pi}_s = \underline{b}k$.

Proposition 2. The effect of an increase in transmission capacity.

Area A1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{(\theta_s + \theta_n)} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{(\theta_s + \theta_n)} > 0$$

$$\begin{aligned} \frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] - \frac{\partial F_n(P)}{\partial T} \\ &= \frac{\partial \underline{b}}{\partial T}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] - \frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1 \end{aligned}$$

$$\begin{aligned} \frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s + \theta_n}{\theta_s + T}\left[ln\left(\frac{P}{\underline{b}}\right)\right] - \underline{b}\frac{\theta_s + \theta_n}{(\theta_s + T)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\frac{\theta_s + \theta_n}{\theta_s + T}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] \\ &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s + \theta_n}{\theta_s + T}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] - \underline{b}\frac{\theta_s + \theta_n}{(\theta_s + T)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1 \end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{(\theta_s + \theta_n)}(\theta_s + T) + \frac{P(\theta_n - T)}{(\theta_s + \theta_n)} = \frac{P(\theta_n - 2T - \theta_s)}{(\theta_s + \theta_n)} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Area B1.

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{k} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{k} > 0$$

$$\begin{aligned}
\frac{\partial E_n(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s + T}{k + T - \theta_n}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + \underline{b}\frac{k + T - \theta_n - \theta_s - T}{(k + T - \theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\
&+\underline{b}\frac{\theta_s + T}{k + T - \theta_n}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] - \frac{\partial F_n(P)}{\partial T} \\
&= \frac{\partial \underline{b}}{\partial T}\frac{\theta_s + T}{k + T - \theta_n}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] + \underline{b}\frac{k - \theta_s - \theta_n}{(k + T - \theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\
&-\frac{\partial F_n(P)}{\partial T} < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1
\end{aligned}$$

$$\begin{aligned}
\frac{\partial E_s(b)}{\partial T} &= \frac{\partial \underline{b}}{\partial T}\frac{k}{k + T - \theta_n}\left[ln\left(\frac{P}{\underline{b}}\right)\right] - \underline{b}\frac{k}{(k + T - \theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] \\
&+\underline{b}\frac{k}{k + T - \theta_n}\left[\frac{\underline{b}}{P}\frac{-\frac{\partial \underline{b}}{\partial T}P}{\underline{b}^2}\right] \\
&= \frac{\partial \underline{b}}{\partial T}\frac{k}{k + T - \theta_n}\left[ln\left(\frac{P}{\underline{b}}\right) - 1\right] \\
&-\underline{b}\frac{k}{(k + T - \theta_n)^2}\left[ln\left(\frac{P}{\underline{b}}\right)\right] < 0 \Leftrightarrow ln\left(\frac{P}{\underline{b}}\right) > 1
\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{k}(\theta_s + T) + \frac{P(\theta_n - T)}{k} = \frac{P(\theta_n - 2T - \theta_s)}{k} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Annex 2. The effect of transmission capacity constraints and transmission losses

Proposition 3. Characterization of the equilibrium in the presence of transmission constraints and transmission costs.

For further reference:

$$\begin{aligned} H_i(\theta, P, T, t) &= max\{0, \theta_i - T, \theta_j + \theta_i - k\} \\ Ht_i(\theta, P, T, t) &= max\{0, \theta_j - k\} \\ L_i(\theta, P, T, t) &= min\{\theta_i + \theta_j, \theta_i + T, k\} \\ Lt_i(\theta, P, T, t) &= max\{0, min\{\theta_i, T, k - \theta_i\}\} \end{aligned}$$

I proceed as in proposition one: first, I work out the general formulas of the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit*; second, I work out the particular formulas associated with each single area in figure 7.

Lower Bound of the Support. The lower bound of the support is defined according to lemma four.

Cumulative Distribution Function.

In the first step, the payoff function for any firm is:

$$\begin{aligned} \pi_i(b) &= F_j(b)\left[b\left(H_i(\theta, P, T, t)\right) - t\left(Ht_i(\theta, P, T, t)\right)\right] + \\ &\quad (1 - F_j(b))\left[b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right)\right] = \\ &= -F_j(b)\left[b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) - b\left(H_i(\theta, P, T, t)\right) + t\left(Ht_i(\theta, P, T, t)\right)\right] \\ &\quad b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) \end{aligned} \tag{8}$$

In the second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategy. Then,

$$\begin{aligned} &= -F_j(b)\left[b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) - b\left(H_i(\theta, P, T, t)\right) + t\left(Ht_i(\theta, P, T, t)\right)\right] \\ &\quad b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) \Rightarrow \\ F_j(b) &= \frac{b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) - \overline{\pi}_i}{b\left[L_i(\theta, P, T, t) - H_i(\theta, P, T, t)\right] - t\left[Lt_i(\theta, P, T, t) - Ht_i(\theta, P, T, t)\right]} \end{aligned} \tag{9}$$

In the third step, at $\underline{b}$, $F_i(\underline{b}) = 0 \forall i = n, s$. Then,

$$\overline{\pi}_i = b\left(L_i(\theta, P, T, t)\right) - t\left(Lt_i(\theta, P, T, t)\right) \tag{10}$$

Fourth step, plugging 10 into 9, I obtain the mixed strategies for both firms.

$$\begin{aligned} F_j(b) &= \frac{(b - \underline{b})L_i(\theta, P, T, t)}{b\left[L_i(\theta, P, T, t) - H_i(\theta, P, T, t)\right] - t\left[Lt_i(\theta, P, T, t) - Ht_i(\theta, P, T, t)\right]} = \\ &\quad \forall i = n, s \end{aligned} \tag{11}$$

Probability Distribution Function.

$$
\begin{aligned}
f_j(b) &= \frac{\partial F_j(b)}{\partial b} \\
&= \frac{L_i(\cdot)\left[\underline{b}\left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right]-t\left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]\right]}{\left[b\left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right]-t\left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]\right]^2} \\
&\quad \forall i = n,s
\end{aligned} \tag{12}
$$

For further reference:

$$
\begin{aligned}
n(\cdot) &= L_i(\cdot)\left[\underline{b}\left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right]-t\left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]\right] \\
d_1(\cdot) &= \left[L_i(\theta,P,T,t)-H_i(\theta,P,T,t)\right] \\
d_2(\cdot) &= \left[Lt_i(\theta,P,T,t)-Ht_i(\theta,P,T,t)\right]
\end{aligned}
$$

Expected Equilibrium Bid.

$$
\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} b f_j(b)\partial b \\
&= \int_{\underline{b}}^{P} \frac{b\,(n(\cdot))}{\left[b\,(d_1(\cdot))-t\,(d_2(\cdot))\right]^2}\partial b + P(1-F_j(P)) \;\; \forall i = n,s
\end{aligned}
$$

I solve this equation by substitution of variables. In particular:

$$
\begin{aligned}
U &= \left[b\,(d_1(\cdot))-t\,(d_2(\cdot))\right] \Rightarrow b = \frac{U+t\,(d_2(\cdot))}{d_1(\cdot)} \\
\frac{\partial U}{\partial b} &= d_1 \Rightarrow \partial b = \frac{\partial U}{\partial d_1}
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
E_j(b) &= \int_{\underline{b}}^{P} \frac{\left(\dfrac{U+t\,(d_2(\cdot))}{d_1(\cdot)}\right)n(\cdot)}{U^2}\frac{\partial U}{d_1(\cdot)} + P(1-F_j(P)) \\
&= \frac{n(\cdot)}{d_1(\cdot)}\left[\int_{\underline{b}}^{P}\frac{U\partial U}{U^2} + \int_{\underline{b}}^{P}\frac{t\,(d_2(\cdot))\,\partial U}{U^2}\right] + P(1-F_j(P)) \\
&= \frac{n(\cdot)}{d_1(\cdot)^2}\left[ln(U) - \frac{t\,(d_2(\cdot))}{U}\right]_{\underline{b}}^{P} + P(1-F_j(P))
\end{aligned}
$$

Substituting again:

$$
\begin{aligned}
E_j(b) &= \frac{n(\cdot)}{d_1(\cdot)^2} \\
&\quad \left[ln\left(\frac{P\,(d_1(\cdot))-t\,(d_2(\cdot))}{\underline{b}\,(d_1(\cdot))-t\,(d_2(\cdot))}\right) - \frac{t\,(d_2(\cdot))}{P\,(d_1(\cdot))-t\,(d_2(\cdot))} + \frac{t\,(d_2(\cdot))}{\underline{b}\,(d_1(\cdot))-t\,(d_2(\cdot))}\right] \\
&\quad +P(1-F_j(P))
\end{aligned} \tag{13}
$$

In the rest of the proof, I will work out the *lower bound of the support*, the *cumulative distribution function*, the *probability distribution function*, the *expected equilibrium price* and the *expected profit* for the different possible realizations of demands (θ_s, θ_n) (figure 7).

Area A1.

First, the lower bound of the support is:

$$\begin{aligned} \underline{b}_n\theta_n + \underline{b}_n\theta_s - t\theta_s = P(\theta_n - T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n - T) + t\theta_s}{\theta_n + \theta_s} \\ \underline{b}_s\theta_s + \underline{b}_s T - tT = 0 \Rightarrow \underline{b}_s &= \frac{tT}{\theta_s + T} \end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})(\theta_n + \theta_s)}{b\left[(\theta_s + \theta_n) - (\theta_n - T)\right] - tmin\left\{\theta_s, k - \theta_n\right\}} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})(\theta_s + T)}{b(\theta_s + T) - tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} \text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(\theta_s + T) - t\theta_s)(\theta_s + T)}{(P(\theta_s + T) - tT)(\theta_s + \theta_n)} \\ \text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P(\theta_s + T) - tT)(\theta_s + \theta_n)}{(P(\theta_s + T) - t\theta_s)(\theta_s + T)} \\ F_n(P) &= 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned} f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{(b(\theta_s + T) - t\theta_s)^2} \\ f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{(\theta_s + T)(\underline{b}(\theta_s + T) - tT)}{(b(\theta_s + T) - tT)^2} \end{aligned}$$

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(b(\theta_s+T)-t\theta_s)^2} + (1-F_s(P))P \\
&= \frac{(\theta_n+\theta_s)(\underline{b}(\theta_s+T)-t\theta_s)}{(\theta_s+T)^2} \\
&\quad \left[ln\left(\frac{P(\theta_s+T)-t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s+T)-t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s+T)-t\theta_s}\right] \\
&\quad +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\theta_s+T)(\underline{b}(\theta_s+T)-tT)}{(b(\theta_s+T)-tT)^2} + (1-F_n(P))P \\
&= \frac{(\underline{b}(\theta_s+T)-tT)}{(\theta_s+T)} \\
&\quad \left[ln\left(\frac{P(\theta_s+T)-tT}{\underline{b}(\theta_s+T)-tT}\right) - \frac{tT}{P(\theta_s+T)-tT} + \frac{tT}{\underline{b}(\theta_s+T)-tT}\right] \\
&\quad +(1-F_n(P))P
\end{aligned} \tag{14}
$$

In equations 14, I have solved by substituting variables:

$$
\begin{aligned}
U &= b(\theta_s+T)-t\theta_s \Rightarrow b = \frac{U+t\theta_s}{\theta_s+T} \\
\frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T} \\
&\text{and} \\
U &= b(\theta_s+T)-tT \Rightarrow b = \frac{U+tT}{\theta_s+T} \\
\frac{\partial U}{\partial b} &= \theta_s+T \Rightarrow \partial b = \frac{\partial U}{\theta_s+T}
\end{aligned}
$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}(\theta_s+\theta_n)-t\theta_s$ and $\overline{\pi}_s = \underline{b}(\theta_s+T)-tT$.

Area B1a.

First, the lower bound of the support is:

$$
\begin{aligned}
\underline{b}_n\theta_n + \underline{b}_n(k-\theta_n) - t(k-\theta_n) = P(\theta_n-T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n-T)+t(k-\theta_n)}{k} \\
\underline{b}_s\theta_s + \underline{b}_sT - tT = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+tT}{\theta_s+T}
\end{aligned}
$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(k+T-\theta_n) - t(k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s+T)}{b(k+T-\theta_n) - tT} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} \text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\ F_n(P) &= \frac{(P(k+T-\theta_n) - t(k-\theta_n))(\theta_s+T)}{(P(k+T-\theta_n) - tT)k} \\ \text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P(k+T-\theta_n) - tT)k}{(P(k+T-\theta_n) - t(k-\theta_n))(\theta_s+T)} \\ F_n(P) &= 1 \end{aligned}$$

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{k(\underline{b}(k+T-\theta_n) - t(k-\theta_n))}{(b(k+T-\theta_n) - t(k-\theta_n))^2}$$

$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\theta_s+T)(\underline{b}(\theta_s+T) - tT)}{(b(\theta_s+T) - tT)^2}$$

Fourth, the expected bid is determined by:

$$\begin{aligned} E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s) \partial b = \int_{\underline{b}}^{P} b \frac{k(\underline{b}(k+T-\theta_n) - t(k-\theta_n))}{(b(k+T-\theta_n) - t(k-\theta_n))^2} + (1 - F_s(P))P \\ &= \frac{k(\underline{b}(k+T-\theta_n) - t(k-\theta_n))}{(k+T-\theta_n)^2} \\ &\quad \left[ln \left(\frac{P(k+T-\theta_n) - t(k-\theta_n)}{\underline{b}(k+T-\theta_n) - t(k-\theta_n)} \right) \right] \\ &\quad \left[-\frac{t(k-\theta_n)}{P(k+T-\theta_n) - t(k-\theta_n)} + \frac{t(k-\theta_n)}{\underline{b}(k+T-\theta_n) - t(k-\theta_n)} \right] \\ &\quad +(1 - F_s(P))P \\ E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s) \partial b = \int_{\underline{b}}^{P} b \frac{(\theta_s+T)(\underline{b}(k+T-\theta_n) - tT)}{(b(k+T-\theta_n) - tT)^2} + (1 - F_n(P))P \\ &= \frac{(\theta_s+T)(\underline{b}(k+T-\theta_n) - tT)}{(k+T-\theta_n)^2} \\ &\quad \left[ln \left(\frac{P(k+T-\theta_n) - tT}{\underline{b}(k+T-\theta_n) - tT} \right) - \frac{tT}{P(k+T-\theta_n) - tT} + \frac{tT}{\underline{b}(k+T-\theta_n) - tT} \right] \\ &\quad +(1 - F_n(P))P \end{aligned} \tag{15}$$

In equations 15, I have solved by substituting variables:

$$\begin{aligned}
U &= b(k+T-\theta_n) - t(k-\theta_n) \Rightarrow b = \frac{U + t(k-\theta_n)}{k+T-\theta_n} \\
\frac{\partial U}{\partial b} &= k+T-\theta_n \Rightarrow \partial b = \frac{\partial U}{k+T-\theta_n} \\
\text{and} & \\
U &= b(k+T-\theta_n) - tT \Rightarrow b = \frac{U+tT}{k+T-\theta_n} \\
\frac{\partial U}{\partial b} &= k+T-\theta_n \Rightarrow \partial b = \frac{\partial U}{k+T-\theta_n}
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k - t(k-\theta_n)$ and $\overline{\pi}_s = \underline{b}(\theta_s + T) - tT$.

Area B1b.

First, the lower bound of the support is:

$$\begin{aligned}
\underline{b}_n k = P(\theta_n - T) \Rightarrow \underline{b}_n &= \frac{P(\theta_n - T)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s T - tT = P(\theta_s + \theta_n - k) - t(\theta_n - k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s + \theta_n - k) + t(k+T-\theta_n)}{\theta_s + T}
\end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(k+T-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})(\theta_s + T)}{b(k+T-\theta_n) - t(T+k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned}
\text{If } \underline{b}_n \geq \underline{b}_s \Rightarrow F_s(P) &= 1 \\
F_n(P) &= \frac{P(k+T-\theta_n)(\theta_s+T)}{(P-t)(k+T-\theta_n)k} \\
\text{If } \underline{b}_n < \underline{b}_s \Rightarrow F_s(P) &= \frac{(P-t)(k+T-\theta_n)k}{P(k+T-\theta_n)(\theta_s+T)} \\
F_n(P) &= 1
\end{aligned}$$

Third, the probability distribution function is equal to:

$$f_s(b) = \frac{\partial F_s(b)}{\partial b} = \frac{\underline{b}k}{b^2(k+T-\theta_n)}$$
$$f_n(b) = \frac{\partial F_n(b)}{\partial b} = \frac{(\underline{b}-t)(\theta_s+T)}{(b-t)^2(k+T-\theta_n)}$$

Fourth, the expected bid is determined by:

$$\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{\underline{b}k}{b^2(k+T-\theta_n)} + (1-F_s(P))P \\
&= \frac{\underline{b}k}{(k+T-\theta_n)}\left[ln\left(\frac{P}{\underline{b}}\right)\right] + (1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{(\underline{b}-t)(\theta_s+T)}{(b-t)^2(k+T-\theta_n)} + (1-F_n(P))P \\
&= \frac{(\underline{b}-t)(\theta_s+T)}{(k+T-\theta_n)}\left[ln\left(\frac{P-t}{\underline{b}-t}\right) - \frac{t}{P-t} + \frac{t}{\underline{b}-t}\right] + (1-F_n(P))P \qquad (16)
\end{aligned}$$

In equations 16, I have solved by substituting variables:

$$\begin{aligned}
U &= b-t \Rightarrow b = U+t \\
\frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U
\end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}(\theta_s+T) - tT$.

Area B2a.

First, the lower bound of the support is:

$$\begin{aligned}
\underline{b}_n\theta_n + \underline{b}_n(k-\theta_n) - t(k-\theta_n) = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_n &= \frac{P(\theta_s+\theta_n-k)+t(k-\theta_n)}{k} \\
\underline{b}_s\theta_s + \underline{b}_s(k-\theta_s) - t(k-\theta_s) = P(\theta_s+\theta_n-k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s+\theta_n-k)+t(k-\theta_s)}{k}
\end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_n)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$
$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b-\underline{b})k}{b(2k-\theta_n-\theta_s)-t(k-\theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$
\begin{aligned}
F_s(P) &= \frac{P(2k-\theta_n-\theta_s)-t(k-\theta_s)}{P(2k-\theta_n-\theta_s)-t(k-\theta_n)} \\
F_n(P) &= 1
\end{aligned}
$$

Third, the probability distribution is equal to:

$$
\begin{aligned}
f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} \\
f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2}
\end{aligned}
$$

Fourth, the expected bid is determined by:

$$
\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} + (1-F_s(P))P \\
&= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_n))^2} \\
&\quad \left[ln\left(\frac{P(2k-\theta_n-\theta_s)-t(k-\theta_n)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n)}\right)\right] \\
&\quad \left[-\frac{t(k-\theta_n)}{P(k+T-\theta_n)-t(k-\theta_n)} + \frac{t(k-\theta_n)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_n)}\right] \\
&\quad +(1-F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s)\partial b = \int_{\underline{b}}^{P} b\frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} + (1-F_n(P))P \\
&= \frac{k(\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s))}{(b(2k-\theta_n-\theta_s)-t(k-\theta_s))^2} \\
&\quad \left[ln\left(\frac{P(2k-\theta_n-\theta_s)-t(k-\theta_s)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s)}\right)\right] \\
&\quad \left[-\frac{t(k-\theta_s)}{P(k+T-\theta_n)-t(k-\theta_s)} + \frac{t(k-\theta_s)}{\underline{b}(2k-\theta_n-\theta_s)-t(k-\theta_s)}\right] \\
&\quad +(1-F_n(P))P
\end{aligned} \tag{17}
$$

Where in equations 17, I have solved by substituting variables:

$$
\begin{aligned}
U &= b(2k-\theta_n-\theta_s)-t(k-\theta_n) \Rightarrow b = \frac{U+t(k-\theta_n)}{2k-\theta_n-\theta_s} \\
\frac{\partial U}{\partial b} &= 2k-\theta_n-\theta_s \Rightarrow \partial b = \frac{\partial U}{2k-\theta_n-\theta_s} \\
&\text{and} \\
U &= b(2k-\theta_n-\theta_s)-t(k-\theta_s) \Rightarrow b = \frac{U+t(k-\theta_s)}{2k-\theta_n-\theta_s} \\
\frac{\partial U}{\partial b} &= 2k-\theta_n-\theta_s \Rightarrow \partial b = \frac{\partial U}{2k-\theta_n-\theta_s}
\end{aligned}
$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k - t(k - \theta_n)$ and $\overline{\pi}_s = \underline{b}k - t(k - \theta_s)$.

Area B2b.

First, the lower bound of the support is:

$$\begin{aligned}
\underline{b}_n k = P(\theta_s + \theta_n - k) \Rightarrow \underline{b}_n &= \frac{P(\theta_s + \theta_n - k)}{k} \\
\underline{b}_s \theta_s + \underline{b}_s (k - \theta_s) - t(k - \theta_s) = & \\
P(\theta_s + \theta_n - k) - t(\theta_n - k) \Rightarrow \underline{b}_s &= \frac{P(\theta_s + \theta_n - k) + t(2k - \theta_n - \theta_s)}{k}
\end{aligned}$$

Second, I work out the cumulative distribution function.

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})k}{b(2k - \theta_n - \theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{(b - \underline{b})k}{(b - t)(2k - \theta_n - \theta_s)} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned}
F_s(P) &= \frac{P(2k - \theta_n - \theta_s) - t(2k - \theta_n - \theta_s)}{P(2k - \theta_n - \theta_s)} \\
F_n(P) &= 1
\end{aligned}$$

Third, the probability distribution function is equal to:

$$\begin{aligned}
f_s(b) = \frac{\partial F_s(b)}{\partial b} &= \frac{\underline{b}k}{b^2(2k - \theta_n - \theta_s)} \\
f_n(b) = \frac{\partial F_n(b)}{\partial b} &= \frac{(\underline{b} - t)k}{(b - t)^2(2k - \theta_n - \theta_s)}
\end{aligned}$$

Fourth, the expected bid is determined by:

$$\begin{aligned}
E_s(b) &= \int_{\underline{b}}^{P} b f_s(b_s) \partial b = \int_{\underline{b}}^{P} b \frac{\underline{b}k}{b^2(2k - \theta_n - \theta_s)} + (1 - F_s(P))P \\
&= \frac{\underline{b}k}{(2k - \theta_n - \theta_s)} \left[ln \left(\frac{P}{\underline{b}} \right) \right] + (1 - F_s(P))P \\
E_n(b) &= \int_{\underline{b}}^{P} b f_n(b_s) \partial b = \int_{\underline{b}}^{P} b \frac{(\underline{b} - t)k}{(b - t)^2(2k - \theta_n - \theta_s)} + (1 - F_n(P))P \\
&= \frac{(\underline{b} - t)k}{(2k - \theta_n - \theta_s)} \left[ln \left(\frac{P - t}{\underline{b} - t} \right) - \frac{t}{P - t} + \frac{t}{\underline{b} - t} \right] + (1 - F_n(P))P \qquad (18)
\end{aligned}$$

Where in equations 18, I have solved by substituting variables:

$$\begin{aligned} U &= b - t \Rightarrow b = U + t \\ \frac{\partial U}{\partial b} &= 1 \Rightarrow \partial b = \partial U \end{aligned}$$

Fifth, the expected profit is defined by equation 10 and is equal to $\overline{\pi}_n = \underline{b}k$ and $\overline{\pi}_s = \underline{b}k - t(k - \theta_s)$.

Proposition 4. Effect of an increase in transmission capacity.

In the presence of transmission capacity constraints and transmission costs, the "size" and "cost" mechanisms determine the equilibrium. These two mechanisms work in opposite directions which has important implications on equilibrium outcome allocations. Hence, an increase in transmission capacity modifies the relevant variables of the model (lower bound of the support, expected bids and expected profits) in a non-monotonic pattern. Therefore, no clear conclusions can be obtained through the analysis of the partial derivatives.

In this section, I present the static comparative in order to illustrate the difficulties to obtain a formal analysis from the analytical solutions. I present the results for area $A1$, the analysis is the same for the rest of the areas.

Area $A1$.

$$\begin{aligned} \frac{\partial \underline{b}_n}{\partial T} &= \frac{-P}{(\theta_s + \theta_n)} < 0 \\ \frac{\partial \underline{b}_s}{\partial T} &= \frac{t(\theta_s + T) - tT}{(\theta_s + T)^2} = \frac{t\theta_s}{(\theta_s + T)^2} > 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial F_n(P)}{\partial T} = & \frac{(2P(\theta_s + T) - t\theta_s)\left((P(\theta_s + T) - tT)(\theta_n + \theta_s)\right)}{\left((P(\theta_s + T) - tT)(\theta_n + \theta_s)\right)^2} + \\ & \frac{t(\theta_n + \theta_s)(P(\theta_s + T) - t\theta_s)(\theta_s + T)}{\left((P(\theta_s + T) - tT)(\theta_n + \theta_s)\right)^2} > 0 \end{aligned}$$

$$\frac{\partial E_n(b)}{\partial T} = \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + (\underline{b} - t)(\theta_s + T) - \underline{b}(\theta_s + T) + tT}{(\theta_s + T)^2}$$

$$\left[ln\left(\frac{P(\theta_s + T) - tT}{\underline{b}(\theta_s + T) - tT}\right) - \frac{tT}{P(\theta_s + T) - tT} + \frac{tT}{\underline{b}(\theta_s + T) - tT}\right] +$$

$$\frac{\underline{b}(\theta_s + T) - tT}{\theta_s + T}$$

$$\left[\frac{\underline{b}(\theta_s + T) - tT}{P(\theta_s + T) - tT}\right]$$

$$\left[\frac{(P - t)(\underline{b}(\theta_s + T) - tT) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + \underline{b} - t\right)(P(\theta_s + T) - tT)}{(\underline{b}(\theta_s + T) - tT)^2}\right] +$$

$$\frac{\underline{b}(\theta_s + T) - tT}{\theta_s + T}\left[-\frac{t(P(\theta_s + T) - tT) - (P - t)tT}{(P(\theta_s + T) - tT)^2}\right] +$$

$$\frac{\underline{b}(\theta_s + T) - tT}{\theta_s + T}\left[\frac{t(\underline{b}(\theta_s + T) - tT) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + \underline{b} - t\right)tT}{(\underline{b}(\theta_s + T) - tT)^2}\right]$$

$$\frac{\partial E_s(b)}{\partial T} = \frac{\frac{\partial \underline{b}}{\partial T}(\theta_s + T)^3(\theta_s + \theta_n) + \underline{b}(\theta_n + \theta_s)(\theta_s + T)^2 - 2(\theta_s + T)\left[(\theta_s + \theta_n)(\underline{b}(\theta_s + T) - t\theta_s)\right]}{(\theta_s + T)^4}$$

$$\left[ln\left(\frac{P(\theta_s + T) - t\theta_s}{\underline{b}(\theta_s + T) - t\theta_s}\right) - \frac{t\theta_s}{P(\theta_s + T) - t\theta_s} + \frac{t\theta_s}{\underline{b}(\theta_s + T) - t\theta_s}\right] +$$

$$\frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - tT)}{(\theta_s + T)^2}$$

$$\left[\frac{(\underline{b}(\theta_s + T) - t\theta_s)}{P(\theta_s + T) - t\theta_s}\right]$$

$$\left[\frac{P(\underline{b}(\theta_s + T) - t\theta_s) - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T) + \underline{b}\right)(P(\theta_s + T) - t\theta_s)}{(\underline{b}(\theta_s + T) - t\theta_s)^2}\right] +$$

$$\frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{(\theta_s + T)^2}\left[-\frac{Pt\theta_s}{(P(\theta_s + T) - t\theta_s)^2}\right] +$$

$$\frac{(\theta_n + \theta_s)(\underline{b}(\theta_s + T) - t\theta_s)}{\theta_s + T}\left[\frac{-\underline{b}t\theta_s - \left(\frac{\partial \underline{b}}{\partial T}(\theta_s + T)t\theta_s\right)}{(\underline{b}(\theta_s + T) - t\theta_s)^2}\right]$$

$$\frac{\partial \pi_n}{\partial T} = -P < 0$$

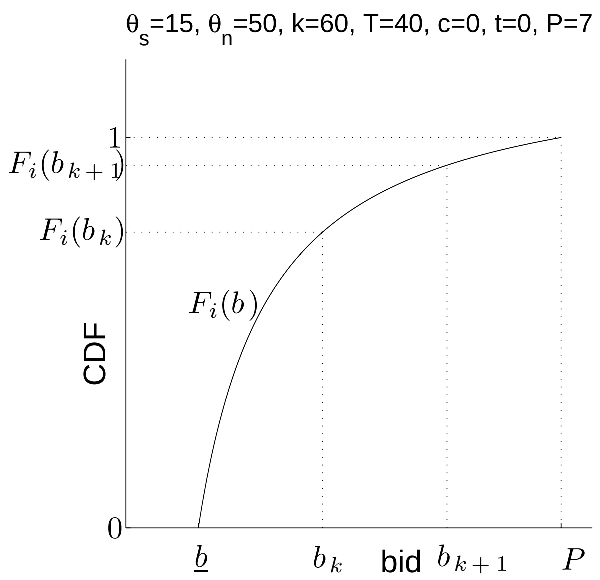
$$
\begin{aligned}
\frac{\partial \overline{\pi}_s}{\partial T} &= \frac{-P}{(\theta_s+\theta_n)}(\theta_s+T) + \frac{P(\theta_n - T) + t\theta_s}{(\theta_s+\theta_n)} - t \\
&= \frac{P(\theta_n - 2T - \theta_s) - t\theta_n}{(\theta_s+\theta_n)}
\end{aligned}
$$

Annex 3. Expected equilibrium price: Simulation

Propositions one and three fully characterize the equilibrium. However, due to the complexity of calculations and to ensure that I did not make any algebra mistake, I work out the expected bid for both firms using the algorithm presented in this annex. The algorithm is based on the cumulative distribution function that is the mixed strategies equilibrium from which the rest of variables of the model are derived.

As can be observed in tables 1, 2 and 3, the differences between the expected bid using the analytical formulas from propositions one and three and using the algorithm proposed here are almost null.[19]

Figure 11: Expected bid. Simulation.



Algorithm: (figure 11)

1. I split the support of the mixed strategies equilibrium into K grid values (where K is a large number e.g., 5000 or 10000). I call each of these values $b_i(k)$ $\forall i = s, n$.

2. For each $b_i(k)$, I work out $F_i(b_i(k))$ using the formulas obtained in propositions one and three.

[19] I have applied this algorithm to work out the expected value for any realization of demand (all areas) and I have compared this with the analytical values and the results are almost identical.

3. The probability assigned to $p_i(b_i(k))$ equals the difference in the cumulative distribution function between two consecutive values $F_i(b_i(k+1)) - F_i(b_i(k))$. Therefore, $p(b_i(k)) = F_i(b_i(k+1)) - F_i(b_i(k))$. It is important to remark that one observation is lost during the process to work out the probabilities.

4. The expected value is the sum of each single bid multiplied by its probability: $E_i(b) = \sum_{k=0}^{K-1} b_i(k)p_i(b_i(k)) \;\; \forall i = s, n$

Annex 4. Increase in transmission costs (numerical example)

In this annex, I analyze the effect of an increase in transmission costs on equilibrium market allocations.

The effects of an increase in transmission costs ($\triangle t$) can be summarized as follows: An increase in transmission costs increases the lower bound of the support of both suppliers

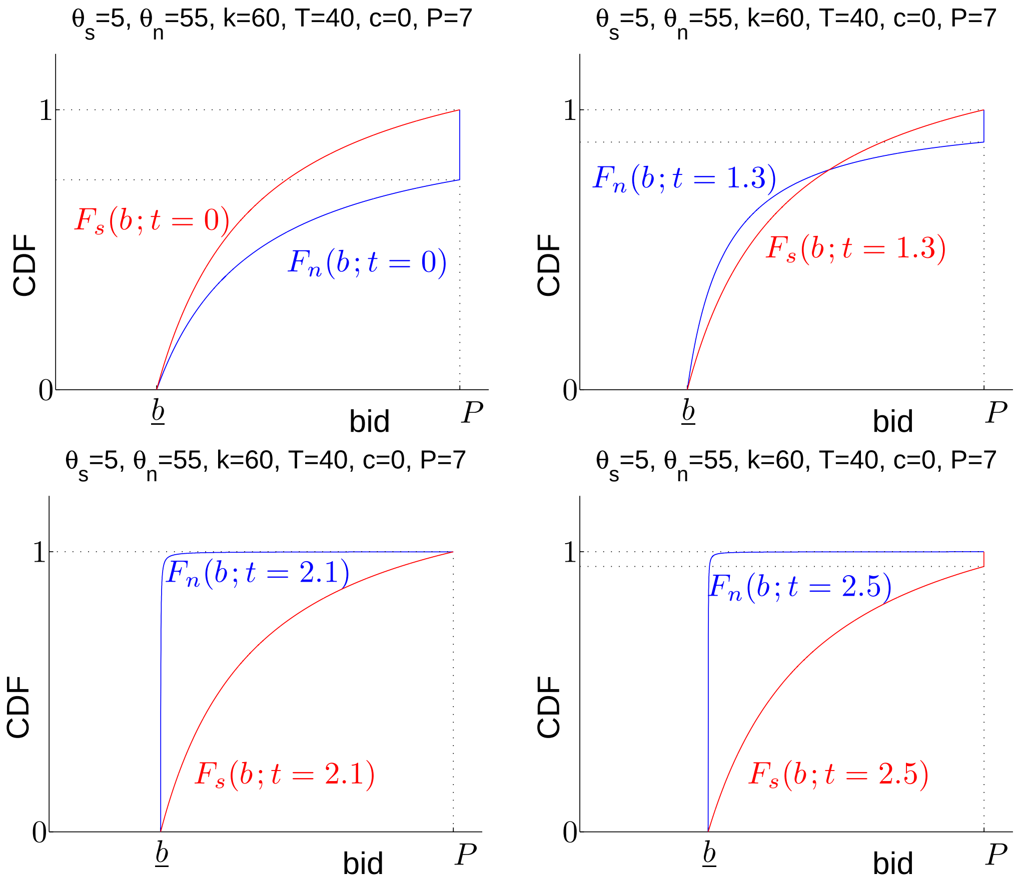
- When the lower bound of the support of the supplier located in the high-demand market is larger than the lower bound of the support of the supplier located in the low-demand market. An increase in transmission costs reduces the expected bids of the supplier located in the high-demand market and increases the expected bids of the supplier located in the low-demand market. Moreover, an increase in transmission costs does not change the profit of the supplier located in the high-demand market and reduces the profit of the supplier located in the low-demand market.

- Otherwise, an increase in transmission costs increases the expected bids of both suppliers, increases the expected profit of the supplier located in the high-demand market and does not modify the expected profit of the supplier located in the low-demand market(table 3; figures 12 and 13).

An increase in transmission costs modifies suppliers' strategies. When the transmission costs are null, the size effect dominates the cost effect and the cumulative distribution function of the supplier located in the high-demand market stochastically dominates that of the supplier located in the low-demand market (top-left panel, figure 12). When the transmission costs increase slightly, no cumulative distribution function stochastically dominates the other (top-right panel, figure 12). When the transmission costs are large enough, the firm located in the high-demand market submits low bids to extract the efficiency rents, the cost effect dominates and the cumulative distribution function of the supplier located in the low-demand market stochastically dominates that of the supplier located in the high-demand market (bottom-left and bottom-right panels, figure 12).

The change in suppliers' strategies induced by an increase in transmission costs modifies the main variables of the model. In particular, when the transmission costs are low enough ($t \leq 2.17$ for the numerical examples in table 3 and figures 12 and 13), an increase in transmission costs increases the cost of satisfying total demand and the lower bound of the support increases (left-hand panel, figure 13). There is an increase in the transmission costs of the firm located in the low-demand market and its expected bid. In contrast, the supplier located in the high-demand market submits lower bids to exploit its efficiency rents (right-hand panel, figure 13; columns five and seven, table 3). Finally, an increase in transmission costs does not change the expected profit of the supplier located

Figure 12: Increase in transmission costs $\triangle t$. Cumulative Distribution Function



in the high-demand market and decreases the expected profit of the supplier located in the low-demand market because its costs increase (central panel, figure 13; columns three and four, table 3)

When the transmission costs are high enough ($t > 2.17$), the cost effect dominates and an increase in transmission costs induces the same changes in the main variables as the one described in proposition four. In particular, an increase in transmission costs increases the expected bids of both suppliers, increases the expected profit of the supplier located in the high-demand market and does not modify the expected profit of the supplier located in the low-demand market.

Figure 13: Increase in transmission costs $\triangle t$. Main variables

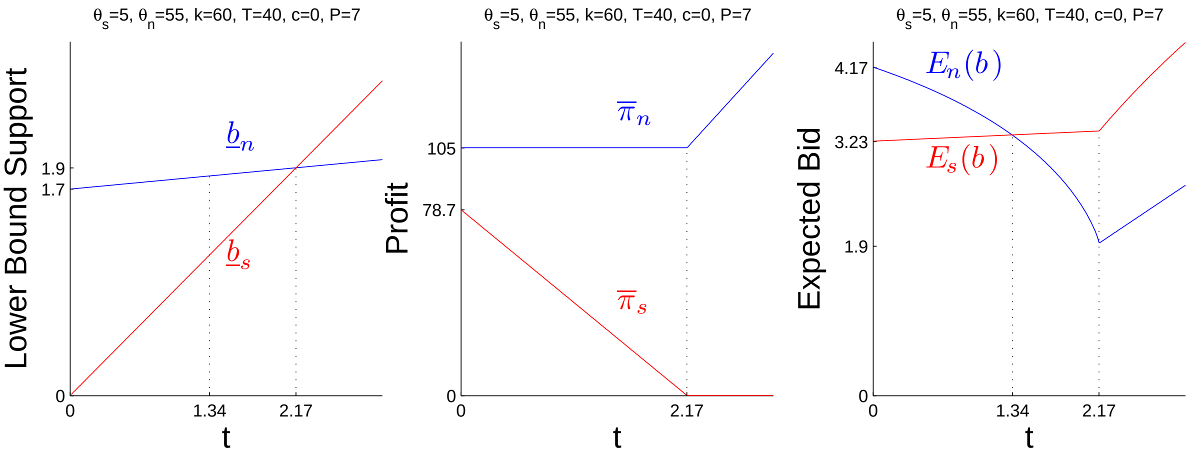


Table 3: Increase in transmission costs $\triangle t$. Main variables. ($\theta_s = 5$, $\theta_n = 55$, $k = 60$, $T = 40$, $c = 0$, $P = 7$)

t	$\underline{b}$	$\overline{\pi}_n$	$\overline{\pi}_s$	$E_n(b)$ Ana.	$E_n(b)$ Sim.	$E_s(b)$ Ana.	$E_s(b)$ Sim.
0	1.751	105.06	78.79	4.1768	4.1769	3.2359	3.2362
0.5	1.793	105.08	60.68	3.9247	3.9249	3.2660	3.2668
1	1.834	105.05	42.53	3.5971	3.5974	3.2955	3.2955
1.5	1.876	105.07	24.42	3.1477	3.1481	3.3255	3.3261
2	1.918	105.1	6.31	2.4232	2.4236	3.3555	3.3567
2.5	2.224	120.96	0	2.2332	2.2341	3.8482	3.8486

Here $E_n(b)$ Ana. and $E_s(b)$ Ana. are the expected values obtained using the analytical expressions presented in Proposition one and $E_n(b)$ Sim. and $E_s(b)$ Sim. are the expected values obtained using the simulation explained in detail in Annex 3.

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