THE RESEARCH INSTITUTE OF INDUSTRIAL ECONOMICS

Working Paper No. 549, 2001

Cooperative Product Development and Endogenous Information Sharing

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COOPERATIVE PRODUCT DEVELOPMENT AND ENDOGENOUS INFORMATION SHARING

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December 2000

Abstract

In this paper, a model of product innovation is developed that endogenizes the degree of cooperation. Two opposing forces affect firm profit in an R&D joint venture. Cooperation increases the quality of the product but it also makes the new products more similar. The increasing substitutability of the product intensifies competition in the production stage. Thus, it may not be optimal to share all of the product information. The basic model is altered to allow for a joint-selling agreement and for tariffs or transport costs. Firms are found to increase R&D cooperation if they are protected from product market competition.

Keywords: R&D cooperation, product innovation, information sharing

JEL-classifications: L00, L10

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* This paper was written while the author held a position as a visiting researcher at the Research Institute of Industrial Economics (IUI) in Stockholm. The author is grateful for helpful discussions with researchers at the IUI, in particular with Johan Stennek, Lars Persson, and Pehr-Johan Norbäck, and with James Markusen at the University of Colorado. Financial support by the Center of Economic Policy Research in form of a TMR Fellowship is gratefully acknowledged.

1. Introduction

Cooperative research and development is the topic of an extensive body of economic and business literature[1]. The existing literature provides valuable insights into welfare aspects and motives for cooperative R&D. However, despite the large number of papers, the vast majority of models are based on process innovation. Process innovation has, of course, its rightful place in the R&D agendas of many firms. Businesses undertake efforts to develop new technologies or to change their production organization in order to reduce the cost of producing their current products. However, if we look at firms' business strategies, product innovation seems to play a much more prominent role. Product development is vital for companies to remain competitive in their current market, to add to their product line, to penetrate existing or create completely new markets. Several databases such as the INSEAD[2] database or the MERIT-CATI database with over 7000 agreements attempt to document cooperative agreements between firms. Although the data are not systematic with respect to motivations for, incidence of, and implications of cooperative agreements, it appears that the majority of such agreements involve product development.[3]

Product innovation is not only important empirically. It also invites the researcher to look at cooperative R&D from a different angle. Price competition

[1] A review of this literature can be found in Veugelers (1998).

[2] European Institute of Business Administration, Fontainebleau, France

[3] Hergert and Morris (1988)

seems the more realistic form of competition after a new product was developed instead of the standard Cournot analysis mostly used for process innovation. In the context of price competition and product development, it becomes important whether or not the firms produce a homogeneous product. However, what kind of product is produced should depend on how closely the firms cooperate in its development. Previous work has studied process innovation assuming that firms either cooperate and share all of the R&D outcome or that they do not cooperate at all. Keeping in mind that real-life firms forming a research and development joint venture (RJV) can write contracts in which they specify to what extent they will cooperate in R&D, it seems natural to assume that the degree of cooperation is endogenous[4].

In this paper, a model of product innovation is developed that allows for an endogenous degree of cooperation between the partner firms. In a two-stage game, firms decide how much information to share about the new product. The extent of the cooperation determines the degree of product differentiation. In the second stage, firms compete in prices. The paper addresses the questions of the optimal degree of cooperation, the effect of cooperation on prices and quality, and the welfare aspects of cooperation. The results of the basic models are compared to two alternative scenarios: a situation in which firms are allowed to jointly sell the new product and a two-country world with import tariffs/ export taxes.

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[4] Joanna Poyago-Theotoky (1994) introduces a model of product innovation in which quality can be improved if firms cooperate. Cooperation implies that the firms jointly sell the new product, thus reducing the number of available products.

Why or why not do firms cooperate in the development of new products? Looking at the business literature or the announcements of cooperative agreements in the press it becomes apparent that there are numerous motives for firms to engage in joint product development. One of the most prominent arguments is that firms join forces to develop a higher quality product. Cooperative R&D might result in a higher quality product through pooling of R&D resources, technological know-how, consumer research, specialization etc. Cost savings or a minimum required investment that exceeds the financial possibilities of a single firm are other arguments for cooperation. Against all these compelling reasons for cooperation stand arguments against it. The inherent danger of R&D cooperation lies in allowing a competitor access to core technologies thereby potentially weakening the own strategic position in the future. If the RJV is not allowed to market and sell the product, i.e. firms will compete after the development phase is completed, cooperation might increase competition by making the products better substitutes. The fact that not all firms engage in cooperative R&D and that RJV partners also tend to retain some independent R&D capacities indicates that there are at least two effects pulling in opposite directions[5].

The model in this paper is based on the idea of improved product quality through cooperative R&D, representing the pro-cooperation effect. Equivalently, firms could develop the same quality product at lower cost. However, the quality

[5] An observation that can support this argument comes from the IUI database “Activities of Swedish Multinational Enterprises Abroad”. The data show that firms who are engaged in a RJV, tend to allocate less than 100% of their R&D expenditure to the joint venture. This indicates that firms choose to do some of their research independently.

approach is chosen because developing a better product seems to be the more intuitive story. In a simple world, there are two firms playing a two-stage game. In the first stage, firms can do research and develop a certain new product. The product is indeed new, not just a better version of an existing product. Examples of this type of product innovation could be the introduction of TVs, VCRs, microwaves, video cameras, copying machines, and PCs. The firms can choose to develop the product on their own or to cooperate in R&D. Whether or not they cooperate, the firms will compete in prices in the second stage. R&D cooperation affects the attributes/quality of the product. Therefore, the firms can affect the demand side by cooperating. The degree of cooperation is a choice variable for the firms. The decision on R&D cooperation in the first stage determines the intensity of competition in the production stage. As firms cooperate, their products become more similar intensifying the competition in the production stage. The increased substitutability of the product constitutes the downside of cooperative R&D.

Demand conditions play an important role in the model. Consumers favor either one or the other firm and they incur switching costs if they do not buy their preferred product. However, if the firms cooperate and the products become closer substitutes, switching costs for the consumers are reduced as they perceive the products as being more similar.

The main findings are that in equilibrium, firms sell a higher quality product at a price that is higher than without cooperation but lower than the maximum possible price. If switching costs can go to zero, firms will never fully cooperate. If consumers' valuation for the new product is high relative to the switching costs, firms will not cooperate at all. If firms are allowed to jointly sell the new product, welfare can be larger than under product market competition. This somewhat counterintuitive result arises because firms cooperate fully if they do not have to compete selling the product. This cooperation does not only increase the price and firm profits but it also results in the highest quality product.

The following section describes the model in detail. Section three and four analyze the price game in the second stage and the choice of the degree of cooperation in the first stage, respectively. In section five, the basic model is extended to allow for tariffs in a world where the firms are located in different countries. Section six concludes.

2. The Model

The following section translates the scenario described above into a simple model based on consumer switching costs. An alternative approach using a "Hotelling" type location model is described in the Appendix.

2.1. The Firm Side

There are two firms that do not currently compete in the same market. The firms intend to develop a new product, which is unrelated to their current products. This ensures that they do not pursue any strategies that relate to their current market or products. The new product consists of many parts, say n components. The R&D cost for the new product is fixed and equal to F for both firms. The components can be ordered from one to n according to some technical characteristics. Each firm can develop its own n components and complete the new product. In that case, the products perform the same basic function but remain incomplete substitutes.

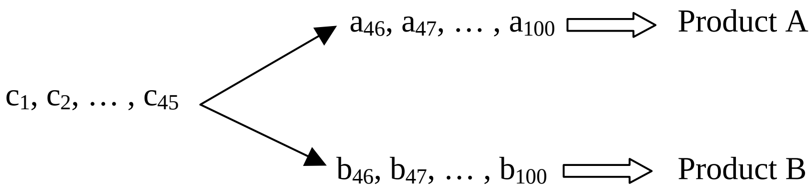
Assume that firms A and B are equally capable of developing the product as a whole, but the quality and production technology of the components varies between the firms. For example, if the new product is an airplane, firm A might be an expert on wing design and fuel systems but firm B has the better expertise on cockpit electronics and landing gear hydraulics. The firms can cooperate such that they compare the components and use the ones that are of better quality (or less costly produced). The firms decide up to which component number they share information.

An Example

- 100 components   n=100
- Firm A could develop all 100 parts on its own: $a_1, a_2, \ldots, a_{100}$
- Firm B could do the same: $b_1, b_2, \ldots, b_{100}$

If both firms decide to complete their product on their own, the result would be "Product A" and "Product B" which consist of completely different parts but are incomplete substitutes. This situation represents the maximum degree of differentiation.

On the other hand, firms could choose to share information on a certain number of components, say the first 45 components such that both products contain the same first 45 parts and after that the firms add their own components.



The percentage of common components will be called α and α is defined as the degree of cooperation.[6]

2.2. The Demand Side

As mentioned earlier, it seems that many products that come to mind when thinking of cooperative R&D are durable goods, which are bought in small quantities. Therefore, in this model consumers buy at most one unit of the new good. A consumer buys either from firm A, from firm B, or not at all. Consumers' valuation for the product is uniformly distributed in an interval from 0 to $\bar{u}$. The quality of the product increases with the degree of cooperation. Therefore, $\bar{u}$ is an increasing function of α, $h(\alpha)$ with $\frac{\partial h(\alpha)}{\partial \alpha} > 0$ and $\frac{\partial^2 h(\alpha)}{\partial \alpha \partial \alpha} \leq 0$, e.g. $\bar{u} = 1+\alpha$. This could be seen as follows: Each consumer i has a valuation u_i which equals u_i^0 if firms do not cooperate. The valuation increases with the quality of the product. As the

[6] Of course there are many alternative ways of modeling cooperative product development. One could imagine that firms are free to choose the optimal degree of product differentiation along with the degree of cooperation. In this paper, however, the focus will be on a situation in which the degree of product differentiation is determined by technological constraints. Examples that might fit this situation are described in Degryse and Irmen (1996). One example is car manufacturing. To achieve better gas mileage, cars are made more aerodynamic but only certain shapes guarantee better aerodynamics. Hence, by improving this quality aspect of cars, the cars become more similar in their appearance.

quality is affected by cooperation, an individual's base utility increases with the degree of cooperation such that $u_i(\alpha)=u_i^0(1+\alpha)$. The u_i^0 are uniformly distributed among consumers, which results in a uniform distribution of the u_i between 0 and $\bar{u}$.

Consumers have brand loyalties, which could be interpreted as consumers having switching cost s. Those switching cost could arise if the firms are located in different regions, A and B. A citizens have usually bought from firm A and would like to continue buying there. Similarly, B citizens feel that firm B products best satisfy their needs. Therefore, some consumers prefer to buy from firm A and some from firm B but if the competitor's price is low enough, they will buy from the other firm. The loyalty depends on the perceived difference between the products, i.e. on α. As firms cooperate, their products become more similar, decreasing switching costs for consumers. Hence, switching costs are a decreasing function of α, $\bar{s} = k(\alpha)$ with $\frac{\partial k(\alpha)}{\partial \alpha} < 0$ and $\frac{\partial^2 k(\alpha)}{\partial \alpha \partial \alpha} \underset{>}{\leq} 0$. More formally:

Consumers valuation: $u \sim \text{unif}\ [0\ ;\ \bar{u}\,] \rightarrow f(u) = \frac{1}{\bar{u}}$ where $\bar{u} = h(\alpha)$ e.g. $\bar{u} = 1+\alpha$

Switching cost: $s \sim \text{unif}\ [0\ ;\ \bar{s}\,] \rightarrow g(s) = \frac{1}{\bar{s}}$ where $\bar{s} = k(\alpha)$ e.g. $\bar{s} = 1-\alpha$

3. The second-stage price game

Solving the game backwards, we begin with the price game in the production stage. Given preferences and switching cost, the demand for the product of each firm can be derived using the following table. It describes the buying behavior of the people preferring firm A (called A citizens) and firm B (called B citizens). The two groups are of measure one each and are identical except for their preference for one firm.

	A citizens	**B citizens**
Case $p_A \leq p_B$		
buy good **A**	$u \geq p_A$	$u \geq p_A + s$ and $u - p_A - s > u - p_B$ $\rightarrow p_B - p_A > s$
buy good **B**	_	$u \geq p_B$ and $p_B - p_A \leq s$
Case $p_A \geq p_B$		
buy good **A**	$u \geq p_A$ and $p_A - p_B \leq s$	_
buy good **B**	$u \geq p_B + s$ and $u - p_B - s > u - p_A$ $\rightarrow p_A - p_B > s$	$u \geq p_B$

Given the described buying behavior, the demand function for firm A is[7]:

$$
(1)\ Q_A = \begin{cases} \int_{p_A}^{\bar{u}} f(u)du + \int_{0}^{p_B - p_A} \left[\int_{p_A + s}^{\bar{u}} f(u)du \right] g(s)ds & \text{if } p_A \leq p_B \\ \int_{p_A - p_B}^{\bar{s}} \left[\int_{p_A}^{\bar{u}} f(u)du \right] g(s)ds & \text{if } p_A \geq p_B \end{cases}
$$

[7] A similar demand structure can be found in Chen (1998).

$$\text{(2) } Q_A = \begin{cases} 1-\dfrac{p_A}{\bar{u}}+\left(1-\dfrac{p_A+p_B}{2\bar{u}}\right)\left(\dfrac{p_B-p_A}{\bar{s}}\right) & \text{if } p_A \leq p_B \\ \left(1-\dfrac{p_A}{\bar{u}}\right)\left(1-\dfrac{p_A-p_B}{\bar{s}}\right) & \text{if } p_A \geq p_B \end{cases}$$

Similarly for firm B.[8]

Note that the demand for the firm with the lower price can be decomposed in three terms.

$$\text{(3) } Q_i = 1-\frac{p_i}{\bar{u}}+\left(1-\frac{p_j}{\bar{u}}\right)\left(\frac{p_j-p_i}{\bar{s}}\right)+\frac{(p_i-p_j)^2}{\bar{s}} \qquad p_i < p_j$$

The first term is the demand in firm i's "own" market. The second term represents the demand taken away from firm j, i.e. people who would have bought from j but now buy from i because of the lower price. The third term is the additional demand from firm j's market, that is, people who would not have bought the product from j at p_j but they buy from firm i. In this case, p_i+s<u but p_j>u. If the prices of the two firms are equal, both serve their "own" customers only.

It should be pointed out that in this model firms are not allowed to price discriminate between consumers of the two markets. This assumption is not problematic if we think of a situation in which switching costs arise from consumers' brand loyalties or if consumers live in different regions of one country such that firms cannot distinguish between customers and arbitrage is easily possible. The assumption will be carried over into section 5 that introduces tariffs into the game.

In a two-country world it is more disputable to assume that firms cannot charge different prices in the two markets. However, the extension of the model to allow for price discrimination will be left for future research[9].

For computational convenience and without loss of generality the production costs of the new product are assumed to be equal to zero. Then the profit functions of the firms are $\Pi_A = p_A * Q_A - F$ and $\Pi_B = p_B * Q_B - F$.

The first order conditions in the price game for firm A are:

(4) $$\frac{\partial \Pi_A}{p_A} = 1 - \frac{2p_A}{\bar{u}} + \frac{p_B - 2p_A}{\bar{s}} + \frac{3p_A^2 - p_B^2}{2\bar{u}\bar{s}} \leq 0 \qquad \text{for } p_A \leq p_B$$

(5) $$\frac{\partial \Pi_A}{p_A} = 1 - \frac{2p_A}{\bar{u}} + \frac{p_B - 2p_A}{\bar{s}} + \frac{3p_A^2 - 2p_A p_B}{\bar{u}\bar{s}} \leq 0 \qquad \text{for } p_B \leq p_A$$

$p_A \geq 0$, $p_B \geq 0$, $\bar{s} \geq 0$, $\bar{u} \geq 0$

Similar equations hold for firm B. Solving for the prices yields the equations for the reaction functions:

(6) $$p_A = \begin{cases} \frac{2}{3}\bar{s} + \frac{2}{3}\bar{u} - \frac{1}{3}\sqrt{4\bar{s}^2 + 2\bar{u}\bar{s} + 4\bar{u}^2 - 6\bar{u}p_B + 3p_B^2} & \text{if} \quad p_A \leq p_B \\ \frac{1}{3}\bar{s} + \frac{1}{3}\bar{u} + \frac{1}{3}p_B - \frac{1}{3}\sqrt{\bar{s}^2 - \bar{u}\bar{s} + 2\bar{s}p_B + \bar{u}^2 - \bar{u}p_B + p_B^2} & \text{if} \quad p_A > p_B \end{cases}$$

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[8] The demand functions for firm B are found in the Appendix.

[9] In a different context, a comparison of the cases with and without price discrimination in a switching cost model can be found in Chen (1998).

$$(7)\ p_B = \begin{cases} \frac{2}{3}\bar{s}+\frac{2}{3}\bar{u}-\frac{1}{3}\sqrt{4\bar{s}^2+2\bar{u}\bar{s}+4\bar{u}^2-6\bar{u}p_A+3p_A^2} & \text{if} \quad p_B \le p_A \\ \frac{1}{3}\bar{s}+\frac{1}{3}\bar{u}+\frac{1}{3}p_A-\frac{1}{3}\sqrt{\bar{s}^2-\bar{u}\bar{s}+2\bar{s}p_A+\bar{u}^2-\bar{u}p_A+p_A^2} & \text{if} \quad p_B > p_A \end{cases}$$

The system of equations has a unique solution in the relevant parameter space. Only pure strategies are considered in this paper. The second-stage equilibrium prices are[10]:

$$(8)\ p_A^* = p_B* = p* = \bar{s}+\frac{1}{2}\bar{u}-\frac{1}{2}\sqrt{4\bar{s}^2+\bar{u}^2}$$

Proposition 1: There exists a unique Nash equilibrium in the second stage price game with both firms charging the same price. In equilibrium, no consumers switch. The equilibrium price is increasing in the switching costs and the maximum utility value[11].

Substituting the specific functional forms $\bar{s}=1-\alpha$ and $\bar{u}=1+\alpha$ into the equilibrium price, we can see that the price is a concave function of the degree of cooperation. For $\alpha\in[0, 0.2)$ p* is increasing in α, reaching its maximum at α=0.2

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[10] This is the price obtained from solving the first order conditions for firm i if $p_i \le p_j$. The price of the other firm, p_j for $p_i \le p_j$ is equal to $p_j^* = \bar{s}+\frac{1}{2}\bar{u}-\frac{1}{6}\sqrt{4\bar{s}^2+\bar{u}^2}-\frac{1}{3}\sqrt{5\bar{s}^2-2\bar{s}\sqrt{4\bar{s}^2+\bar{u}^2}+\bar{u}^2}$ which can be shown to equal p_i for s≥0, u≥0.

[11] The derivatives of p* with respect to $\bar{s}$ and $\bar{u}$ are:

$$\frac{\partial p*}{\partial \bar{s}} = 1-\frac{2\bar{s}}{\sqrt{4\bar{s}^2+\bar{u}^2}} > 0 \qquad \frac{\partial p*}{\partial \bar{u}} = \frac{1}{2}-\frac{\bar{u}}{2\sqrt{4\bar{s}^2+\bar{u}^2}} > 0$$

and decreasing for $\alpha \in (0.2, 1]$. In the next section, the first stage of the game is studied. It will be shown that the optimal degree of cooperation does not maximize the price in the production stage.

4. The R&D Stage

The following section is divided into three subsections. In the first subsection, the first stage of the game is solved for the optimal degree of cooperation using the specific functional forms $\bar{s} = 1 - \alpha$ and $\bar{u} = 1 + \alpha$. It follows a discussion about how the characteristics of the equilibrium depend on the functional forms. The subsection on welfare effects compares the solution of the basic game to a social planner's solution as well as a situation in which the joint venture can sell the new product as a monopolist.

4.1. Choosing the Optimal Degree of Cooperation

When choosing the optimal degree of cooperation, α*, the firms take the second-stage equilibrium profits as given. Due to the symmetric nature of the problem, α* that maximizes joint profit also maximizes profit of the individual firm. Furthermore, as equilibrium prices are the same for both firms, the profit function simplifies to $\Pi = p^*\left(1 - \frac{p^*}{\bar{u}}\right) - F$. Using the second-stage equilibrium prices, we get the following maximization problem:

$$(9)\quad \max_{\alpha} \Pi = \bar{s} + \frac{1}{2}\bar{u} - \frac{1}{2}\sqrt{4\bar{s}^2 + \bar{u}^2} - \frac{1}{\bar{u}}\left(\bar{s} + \frac{1}{2}\bar{u} - \frac{1}{2}\sqrt{4\bar{s}^2 + \bar{u}^2}\right)^2$$

which simplifies to

(10) $$\max_{\alpha} \Pi = \frac{\bar{s}}{\bar{u}}\sqrt{4\bar{s}^2 + \bar{u}^2} - \frac{2\bar{s}^2}{\bar{u}}$$

Both $\bar{s}$ and $\bar{u}$ are functions of the cooperation variable α. Before solving for the optimal α, it might be useful to isolate the effects of α on profit through the switching costs and the utility. If we look at the partial derivatives of the profit function with respect to α, taking either $\bar{s}$ or $\bar{u}$ as determined exogenously, we get for $\bar{s} = k(\alpha), \bar{u} = \text{constant}:$

(11) $$\frac{\partial \Pi}{\partial \alpha} = \frac{\partial k(\alpha)}{\partial \alpha} \frac{8k^2(\alpha) + \bar{u}^2 - 4k(\alpha)\sqrt{4k^2(\alpha) + \bar{u}^2}}{\bar{u}\sqrt{4k^2(\alpha) + \bar{u}^2}}$$

which can be shown to be negative for all $k(\alpha)$ with $\frac{\partial k(\alpha)}{\partial \alpha} < 0$. As expected, if utility were to remain constant, firms would never cooperate if cooperation reduces switching cost. Similarly, if utility is an increasing function of α and switching costs remained unchanged, firms would cooperate 100%[12].

Returning to question of the optimal degree of cooperation α*, initially let $\bar{s} = k(\alpha)$ and $\bar{u} = h(\alpha)$. Then α satisfies the first order condition:

(12) $$\frac{\partial \Pi}{\partial \alpha} =$$

[12] The derivative for $\bar{u} = h(\alpha), \bar{s} = \text{constant}$ is:

$$\frac{\partial \Pi}{\partial \alpha} = -2\bar{s}\,\frac{\partial h(\alpha)}{\partial \alpha}\,\frac{2\bar{s} - \sqrt{4\bar{s}^2 + h^2(\alpha)}}{h^2(\alpha)\sqrt{4\bar{s}^2 + h^2(\alpha)}} > 0 \quad \text{for } \frac{\partial h(\alpha)}{\partial \alpha} > 0$$

$$\frac{8k^2(\alpha)h(\alpha)\frac{\partial k(\alpha)}{\partial\alpha}+\frac{\partial k(\alpha)}{\partial\alpha}h^3(\alpha)\frac{\partial h(\alpha)}{\partial\alpha}-4k^3(\alpha)\frac{\partial h(\alpha)}{\partial\alpha}+\left(2k^2(\alpha)\frac{\partial h(\alpha)}{\partial\alpha}-4k(\alpha)\frac{\partial k(\alpha)}{\partial\alpha}h(\alpha)\right)\sqrt{4k^2(\alpha)+h^2(\alpha)}}{h^2(\alpha)\sqrt{4k^2(\alpha)+h^2(\alpha)}}\leq 0$$

$$\frac{\partial k(\alpha)}{\partial\alpha}<0,\ \frac{\partial h(\alpha)}{\partial\alpha}>0,\ 0\leq\alpha\leq 1$$

To solve for α*, assume a specific functional form.

Assumption A1: $\bar{u}=1+\alpha$ and $\bar{s}=1-\alpha$

Under A1, α* is the solution to

$$(13)\ \ 0=-\frac{13-17\alpha+7\alpha^2+5\alpha^3-\left(6-4\alpha-2\alpha^2\right)\sqrt{5-6\alpha+5\alpha^2}}{(1+\alpha)^2\sqrt{5-6\alpha+5\alpha^2}}\qquad \text{with}\ 0\leq\alpha^*\leq 1.$$

$$\alpha^*=0.364$$

The optimal degree of cooperation α*=0.364 exceeds the price maximizing α of 0.2. The resulting second stage price of 0.385 is higher than the price without cooperation. Thus we have:

Proposition 2: Under A1, if firms are allowed to cooperate in R&D, they will offer a higher quality product at a price that is higher than without cooperation but lower than the maximum possible price.

Equilibrium profit is a concave function of α with its maximum at α*. One implication of this is that there exists are range for the fixed R&D cost F in which firms need to cooperate in order to innovate at all. For

$\sqrt{5}-2 < F < \frac{1-\alpha^*}{1+\alpha^*}\sqrt{4(1-\alpha^*)^2+(1+\alpha^*)^2}$ firms will innovate and produce the new product only if they are allowed to cooperate.

The result in proposition 2 is limited to specific functional forms for the maximum value of utility and switching costs. The next section investigates to what extent the qualitative results depend on the functional forms.

4.2. The Role of Functional Forms

So far it was assumed that $\bar{u}=1+\alpha$ and $\bar{s}=1-\alpha$, implying that the switching costs approach zero as α approaches one. Furthermore, these functional forms also fix the size of the impact cooperation has on consumers' valuation and switching costs. This assumption is now relaxed.

Assumption A2: $\bar{u}=x+\alpha$ and $\bar{s}=t-\alpha$ with $x \geq 0,\ t \geq 1$

Recall that under A2, the individual's utility is given by $u_i(\alpha)=u_i^0(1+\alpha/x)$. The parameter x in the utility function thus provides a measure of the relative importance of cooperation for the consumer. Similarly, t represents the relative importance of α for reducing an individual's switching cost. Of course there is an infinite number of alternative formulations so any specification is restrictive. However, in order to get any results, a functional form has to be assumed. The goal of this analysis is merely to develop a possible scenario for product innovation and to draw some conclusions in the light of this limited framework.

There is no closed form solution for α* for the generalized functional forms $\bar{u} = x + \alpha$ and $\bar{s} = t - \alpha$. Therefore, the derivative of profit with respect to α is evaluated at the extreme points of α, zero and one. The questions to be answered are when do firms cooperate and if so, when do they cooperate with α*=1, i.e. sharing all of the R&D output. Furthermore, the effect of α on equilibrium price is determined.

Looking at the derivative of profit[13] at α = 0, we find that firms are unwilling to cooperate if x is more than twice as large as t. In that case, the negative effect of decreasing switching costs outweighs the positive effect of increasing quality. This indicates that in markets where the valuation for the new product is high relative to the switching cost, firms will pursue the innovation on their own. The equilibrium price is a concave function of α with its maximum at $\alpha = 3/5t - 2/5x$. If $t/x < 2/3$ the maximum equilibrium price is a corner solution at α = 0. Because the condition $t/x < 2/3$ is always satisfied in the case when α* = 0, the firms will charge the maximum possible price without cooperation. For $t/x > 0.466$ firms will always choose an α > 0. Up to $t/x = 2/3$ the equilibrium price with cooperation will be lower than without. For $t/x > 2/3$, the price is higher than without cooperation but it is never optimal for the firms to choose α to maximize equilibrium price. The results are summarized in propositions 3 and 4:

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[13] See the Appendix for all mathematical details of this section.

Definition:

Let p^* = equilibrium price in the second stage price game

p^*_{max} = the maximum equilibrium price in the second stage price game

p_0 = price without cooperation

Proposition 3: Under A2, firm behavior in equilibrium can be divided into three categories depending on the relative size of t and x:

(i)	$t < 0.466x$	No cooperation, $p^* = p^*_{max} = p_0$
(ii)	$0.466x \leq t \leq 2/3x$	Cooperation, $p^* < p_0$
(iii)	$t > 2/3x$	Cooperation, $p_0 < p^* \leq p^*_{max}$

Proof: See Appendix

For region (iii) it can be shown that the derivative of profit with respect to α, evaluated at the α that maximizes price, is larger than zero for all t and x. Then it must be the case that α* always exceeds the price maximizing α until α* is a corner solution with α*=1. Only if α*=1 the maximum equilibrium price is reached along the line t =5/3 + 2/3x. Proposition four states under what conditions firms will choose to cooperate to the fullest extend and share all information.

Proposition 4: Under A2, firms will never fully cooperate if switching costs go to zero as α goes to one. Furthermore, for all t > 1.47 there exists an x^{max} such that for all x with $x^{max} > x > 0$ firms will choose $\alpha^* = 1$, i.e. firms optimal degree of cooperation is 100%.

Proof: See Appendix.

Firms' cooperation decisions vary depending on the relative importance of consumers' valuations and switching costs, covering a wide range of equilibrium prices and qualities. The next section describes and compares welfare implications in two different organizational settings. One scenario is the game presented in the previous sections in which firms cooperate in R&D but later compete in prices. The other is a situation in which firms are allowed to jointly sell the new product. A joint venture in the latter sense would be a separate entity developing and producing the product acting as a single firm.

4.3. Welfare Analysis

Welfare is defined as the sum of consumer surplus and firm profits. Recall that the firms aim to develop a completely new product. Therefore, there does not exist an alternative lower quality product and welfare is equal to zero if firms decide not to develop the product.[14] Given demand in this market, welfare is equal to

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[14] A welfare analysis of a situation in which consumers could have chosen a lower quality product if firms had not innovated is found in J. Poyago-Theotoky (1994).

$$(14)\quad W(\alpha) = \frac{1}{2}\left(\bar{u}(\alpha) - p(\alpha)\right)\left(1 - \frac{p(\alpha)}{\bar{u}(\alpha)}\right) + p(\alpha)\left(1 - \frac{p(\alpha)}{\bar{u}(\alpha)}\right)$$

which simplifies to

$$(15)\quad W(\alpha) = \frac{\bar{u}^2(\alpha) - p^2(\alpha)}{2\bar{u}(\alpha)}$$

The model is set up such that cooperation increases competition as well as the value of the product to consumers.[15] Consequently, a social planner aiming to maximize welfare in both regions would choose α^{SP} as high as possible. If the switching costs go to zero as α goes to 1, $\alpha^{SP}=1$ implies that the new product has the highest possible quality and the price is equal to marginal cost. This is a first-best outcome with the only problem that firms, in order to innovate, have to be able to recover their R&D expenditure. To solve this problem, the social planner could use some mechanism to transfer money from consumers to firms in a non-distortionary way to cover the R&D expenses. If that is not possible, α^{SP} could be chosen such that the profit from the sales of the new product just covers the R&D cost.

It is more interesting to compare the outcome of the basic game in which firms compete in stage two to an alternative scenario in which the joint venture is allowed to sell the product. In this case, the RJV would act as a separate entity or profit center. The problem of consumers switching from one product to the other does not arise. As the firms' profit is strictly increasing in α if there are no switching

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[15] As a direct result of the model setup, firms optimal level of cooperation always stays below the socially optimal level unless $\alpha^* = \alpha^{SP} = 1$. This model design is in accordance with results from a large body of literature showing that private incentives for cooperation fall short of the social incentives. See for example Choi (1993), Kamien et al. (1992).

costs, firms will choose α = 1 and sell the product at the monopoly price $\bar{u}(1)/2$. Then welfare under the joint selling arrangement, W_m, is given by $W_m = 3/8\,\bar{u}(1)$. To compare welfare under monopoly to welfare under product market competition, W(α*), it is necessary to find the numerical solution for α* in order to calculate W(α*). In the case of $\bar{s} = 1 - \alpha$ and $\bar{u} = 1 + \alpha$ we get the following result:

Proposition 5: Under A1, overall welfare is higher in the joint selling setup than under product market competition. A reduction in consumer surplus is more than offset by the increase in firm profits.

Proof: See Appendix.

This result might seem somewhat counterintuitive – product market collusion that hurts consumers should not outperform competition in terms of overall welfare. In the present case, however, collusion gives the firms the incentive to produce the highest quality product. In the case of A1, where switching costs approach zero as α goes to 1, the quality increasing effect of collusion on welfare outweighs the negative price effect. Does this result carry over to other functional forms?

It is not possible to come to a general conclusion for A2 given that there is no closed form solution for α*. But using the results from propositions 3 and 4, we can compare welfare at the extremes of α, α*=0 and α*=1. If firms choose α*=1, demand conditions are such that firms have enough incentive to develop the highest quality product despite product market competition. In that case, consumers' maximum valuation under competition $\bar{u}(\alpha^*)$ equals $\bar{u}(1)$, the outcome under monopoly. As the competitive equilibrium price p*(α) is always less than the monopoly price, welfare must be higher under product market competition.

Now let us turn to the case in which firms do not want to cooperate at all if they compete in the product market. If α*=0, the jump in quality from competition to collusion is the largest. It can be shown that there exists a region in the x-t-space such that $W(\alpha^*) < W_m$ if α*=0. Recall from proposition 3 that firms are unwilling to cooperate if $t \leq 0.466x$. Furthermore, at α*=0 we have $W(\alpha) - W_m < 0$ if the following condition is satisfied:

$$(16)\quad -\frac{1}{4}x - t - \frac{2}{x}t^2 + \left(\frac{1}{2} + \frac{t}{x}\right)\sqrt{(4t^2 + x^2)} - \frac{3}{4} < 0$$

These two conditions are represented graphically in Figure 1, which is mapping out the x-t space for the functional forms in A2, $\bar{u} = x + \alpha$ and $\bar{s} = t - \alpha$. The purpose of the figure is to show the relative sizes of the parameter regions in which $W(\alpha^*) < W_m$ and α*=0, marked M, relative to the area in which product market competition yields higher welfare (the striped area open to the right in Figure 1). Equation (16) is represented by the function labeled "$W(\alpha^*) = W_m$ at α*=0". Given that α*=0, $W(\alpha^*) > W_m$ to the left of this line. This division is only meaningful in the area where α*=0, i.e. below the line t=0.466x and above t=1[16]. The size of area M compared to the stripe area $W(\alpha^*) > W_m$ shows that for α*=0, given any x-t combination, it is much more likely that product market competition leads to higher welfare. This illustrates that the situation in Proposition 5, when a joint-

[16] In order for switching costs not to become negative as α goes to one, it is necessary that t is greater or equal to one.

selling agreement increases welfare, is therefore just a special case that might occur under specific demand conditions.

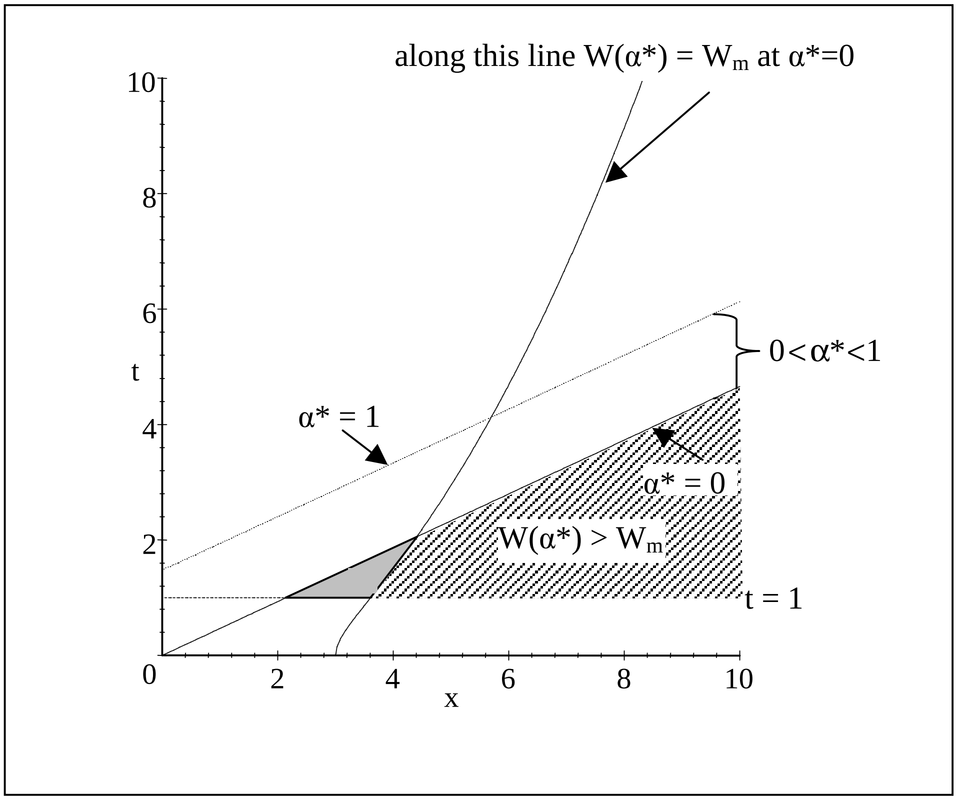

Figure 1: Welfare Comparison at α=0*

In general, the relationship between $W(\alpha)$ and W_m under A2 depends on the exact values for t and x, but the closer α^* is to 1, the more likely it is that $W(\alpha^*) > W_m$.

5. R&D Cooperation under Tariff Protection

In this section, assume that firms A and B are located in two countries, A and B, and each country has the opportunity to impose an import tariff. Note that the analysis also applies to other situations of decreased competition between the firms. The same effects would result from an export tax or a per unit transport cost that each consumer has to pay. Tariffs are just chosen to represent an added per unit cost to the consumer. The basic game from section 3 changes such that A citizens who wish to buy product B have to pay A's tariff, t_A, in addition to their switching costs. Similarly, B citizens have to pay t_B if they decide to switch. The introduction of a tariff, which is equivalent to increasing switching cost per consumer by a fixed amount, results in discontinuous reaction functions for the firms. The firms' altered maximization problems and the solutions are found in the Appendix. Figure 2 illustrates how the situation for the firms has changed with the introduction of a tariff.

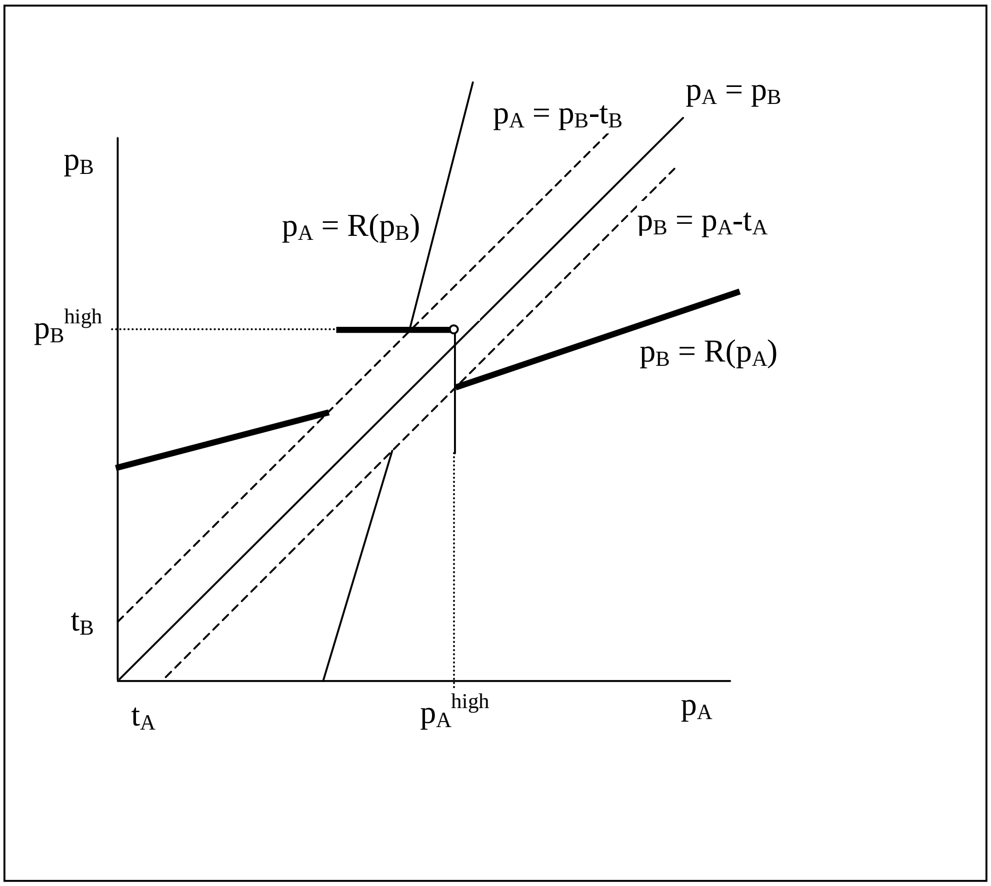

Figure 2: Reaction Functions with Tariffs

The discontinuous reaction functions shown in Figure 2 result because the tariff provides a price range in which the firms can change their price without triggering a response from their rival. In this area, a small price change will only affect consumers in the own market without inducing any consumers to switch. In particular, if $p_B - t_B < p_A < p_B + t_A$ a reduction in p_A will not attract B customers and an increase in p_A will not result in A customers to switch to B as long as the new price remains in the interval. Similarly, if $p_A - t_A < p_B < p_A + t_B$ a change in p_B will not induce any consumers to switch. Let p_A^{high} and p_B^{high} be the prices that firms A and B can charge, respectively, without triggering a reaction from the other firm. Let p^m denote the monopoly price with $p^m = \bar{u}/2$.

Proposition 6: For $t_A \geq 0$, $t_B \geq 0$, there exists a unique Nash equilibrium in the second stage game with prices $p_A = \min\left(p_A^{high}, p^m\right)$ and $p_B = \min\left(p_B^{high}, p^m\right)$. The firms A and B will charge the monopoly price if $t_A \geq \frac{1}{2}\frac{\bar{u}^2}{\bar{u}+4\bar{s}}, t_B \geq \frac{1}{2}\frac{\bar{u}^2}{\bar{u}+4\bar{s}}$, respectively.

Proof: See Appendix.

As shown in the Appendix, p_A^{high} is an increasing function of t_A and p_B^{high} of t_B. The import tariff imposed by the country in which the firm resides allows that firm to charge a higher price in equilibrium. For a tariff higher or equal to $\bar{u}^2 \big/ 2(\bar{u} + 4\bar{s})$, the firm can act as a monopolist. If $t_A \neq t_B$, the equilibrium prices for the firms will differ such that the firm in the country with the higher tariff will charge the higher price. Even if prices are different in the two countries, there will be no trade in equilibrium. If the prices and profits for the firms are not equal, the α that maximizes profit for each of the two firms will be different as well. This introduces a new type of coordination problem for the firms. Although their cost structures are identical, they now prefer different degrees of cooperation. The tariffs in the two countries determine the equilibrium prices but the firms have to negotiate the extent of their cooperation. One possibility for the firms is an agreement at the lowest common denominator, i.e. firms choose the lower α that maximizes the profit of the firm in the low-tariff country. Alternatively, the firms could choose α to maximize joint profit and bargain over the gains. The bargaining game will not be modeled in this paper. However, independent of the specific outcome of the negotiations we know that prices will increase and α will be equal to or higher than its non-tariff level[17]. Ultimately, the firms' agreement on the degree of cooperation will determine to what extent the tariff affects welfare in each country.

The protection of the domestic firms by a tariff allows the firms to cooperate more extensively, potentially offsetting part of the negative welfare effects of the tariff by increasing product quality. It should be pointed out, however, that a unilateral increase in t would inevitably raise the price in that country but might not affect cooperation at all. If the other firm whose profit maximizing α remains unchanged is not willing to increase cooperation, the quality of the product will remain at the non-tariff level. Furthermore, if demand conditions were such that firms were willing to cooperate at 100 percent without a tariff, introducing a tariff would result in a pure price effect.

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[17] From section 4.1 we know that an increase in switching costs holding utility constant will result in an increase in α. The introduction of a tariff represents an increase in switching costs for consumers. As utility remains unaffected, an increase in t will increase the willingness to cooperate.

6. Summary

In previous research, R&D cooperation has already been looked at from many different angles. This paper contributes to this line of literature by presenting a model of cooperative product development that allows for an endogenous degree of cooperation. It was shown that demand side conditions could drive firms' decisions to cooperate. In particular, firms could raise the quality of a new product by cooperating in product development. The firms were thus able to increase the value of the product to the consumer. On the other hand, R&D cooperation led to a higher substitutability of the final product, which increased competition between firms. It was shown that firms facing this trade-off might not choose to fully cooperate even though all consumers prefer the higher quality good. If switching costs can go to zero, firms will never cooperate at 100 percent.

In addition to the basic model, two modified versions were discussed. If instead of product market competition firms are allowed to jointly sell the new product, they will always cooperate to the fullest extent. Depending on demand conditions, the joint selling arrangement might yield a higher welfare. This is the case if a large quality gain is possible by cooperation, the gain is important to the consumer, and firms would not be willing to fully cooperate due to low switching costs.

The second alternative model introduced tariffs or other additional per unit costs to the consumer. The reduced competition between the firms under a tariff regime has similar effects as the joint selling arrangement. It allows the firms to raise the price of the new product and to cooperate more intensively in R&D.

Functional forms played a major role in the analysis. Depending on the specific demand conditions a wide variety of outcomes is possible. In general it can be said that firms are more willing to cooperate in R&D if competition between them is reduced. Several conclusions can be drawn from the link between competition and R&D cooperation. We could expect to see more R&D cooperation in less competitive industries, controlling for R&D intensity of that industry. If a joint product development is observed between competitors, this might indicate that the firms found a way to avoid fierce competition after the development is completed. On the other hand, it might also be the case that possible quality improvements are not realized because a high level of competition deters firms from cooperating in R&D.

References

Caballero-Sanz, F.; Moner-Colenques, R.; Sempere-Monerris, J.
"Market Structure and R&D Joint Ventures: The Case of Product Innovation", *European Journal of Law and Economics*, Vol. 5, 1998, 51-66

Chen, Yongmin
"Paying Customers to Switch", *Journal of Economics and Management Science*, Vol. 6, No. 4, Winter 1997, 877-897

Degryse, Hans; Irmen, Andreas
"R&D Decisions when Quality and Variety Interact", University of Lausanne Working Paper No. 9614, June 1996

Poyago-Theotoky, Joanna
"Research Joint Ventures and Product Innovation: Part II Some Welfare Aspects", University of Bristol Discussion Paper No. 94/378, June 1994

Veugelers, Reinhilde
"Collaboration in R&D: An Assessment of Theoretical and Empirical Findings"*De Economist* v146, n3 October 1998

A3. Appendix

A3.1. Demand functions for firm B

$$Q_B = \begin{cases} \int_{p_B}^{\bar{u}} f(u)du + \int_{0}^{p_A - p_B} \left[\int_{p_B+s}^{\bar{u}} f(u)du \right] g(s)ds & \text{if } p_B \leq p_A \\ \int_{p_B - p_A}^{\bar{s}} \left[\int_{p_B}^{\bar{u}} f(u)du \right] g(s)ds & \text{if } p_B \geq p_A \end{cases}$$

$$Q_B = \begin{cases} 1 - \frac{p_B}{\bar{u}} + \left(1 - \frac{p_A + p_B}{2\bar{u}}\right)\left(\frac{p_A - p_B}{\bar{s}}\right) & \text{if } p_B \leq p_A \\ \left(1 - \frac{p_B}{\bar{u}}\right)\left(1 - \frac{p_B - p_A}{\bar{s}}\right) & \text{if } p_B \geq p_A \end{cases}$$

A3.2. Proofs of Propositions

Proof of Proposition 3: Firm behavior under A2

The derivative of the profit function under the assumption that $\bar{u} = x + \alpha$ and

$\bar{s} = t - \alpha$ is: $\frac{\partial \Pi}{\partial \alpha} =$

$$\frac{8(t-\alpha)^2(x-\alpha) - (x+\alpha)^3 - 4(t-\alpha)^3 + \left(2(t-\alpha)^2 + 4(t-\alpha)(x+\alpha)\right)\sqrt{4t^2 - \alpha(8t-2x) + 5\alpha^2 + x^2}}{(x+\alpha)^2\sqrt{4t^2 - 8t\alpha + 5\alpha^2 + x^2 + 2x\alpha}}$$

At α = 0, setting the derivative equal to zero yields the only real solution t=0.4662656125x. For t<0.4662656125x, the derivative is negative at α=0. Hence, firms will not cooperate if t<0.4662656125x.

The derivative of the equilibrium price p* with respect to α is:

$$\frac{\partial p^*}{\partial \alpha}=-\frac{1}{2}\frac{\sqrt{(4t^2-8t\alpha+5\alpha^2+x^2+2x\alpha)}-4t+5\alpha+x}{\sqrt{(4t^2-8t\alpha+5\alpha^2+x^2+2x\alpha)}}$$

$$\frac{\partial p^*}{\partial \alpha}=0 \text{ at } \alpha=\frac{3}{5}t-\frac{2}{5}x$$

Therefore, if α=0 and x>3/2t the derivative of the equilibrium price is negative. Increasing α above 0 will decrease the equilibrium price. Combining the conditions

t<0.4662656125x → $\alpha^*=0$ and $t\leq\frac{2}{3}$ → $p^*=p_{max}$ at $\alpha^*=0$ yield Proposition 3.

Proof of Proposition 4: Maximum degree of cooperation

Derivative of the profit function at α = 1

$$\left.\frac{\partial \Pi}{\partial \alpha}\right|_{\alpha=1}=$$

$$\frac{8(t-1)^2(x-1)-(x+1)^3-4(t-1)^3+\left(2(t-1)^2+4(t-1)(x+1)\right)\sqrt{4t^2-(8t-2x)+5+x^2}}{(x+1)^2\sqrt{4t^2-8t+5+x^2+2x}}$$

This derivative is negative for all x if t=1, i.e. firms will never choose $\alpha^*=1$ if switching cost can go to zero. From t=1.466265 there exists an $x_{max}\geq 0$ such that $\left.\frac{\partial \Pi}{\partial \alpha}\right|_{\alpha=1}=0$. For all $x<x_{max}$ the derivative is greater than zero and firms choose to cooperate 100 per cent.

Proof of Proposition 5: Welfare comparison under A1

Product market competition

$\alpha^* = 0.364$

Consumer surplus: $$CS(\alpha^*) = \frac{1}{2}\left(\bar{u}(\alpha^*) - p(\alpha^*)\right)\left(1 - \frac{p(\alpha^*)}{\bar{u}(\alpha^*)}\right) = 0.351$$

Firm profit: $$\Pi(\alpha^*) = p(\alpha^*)\left(1 - \frac{p(\alpha^*)}{\bar{u}(\alpha^*)}\right) = 0.276$$

Welfare: $\underline{W(\alpha^*) = 0.627}$

Joint selling agreement

$\alpha^* = 1$

Consumer surplus: $$CS(1) = \frac{1}{4}\left(\bar{u}(1) - \frac{\bar{u}(1)}{2}\right) = 0.25$$

Firm profit: $$\Pi(1) = \frac{\bar{u}(1)}{4} = 0.5$$

Welfare: $\underline{W(1) = 0.75}$

Proof of Proposition 6: Nash equilibrium with tariffs

Reaction functions:

$$p_A = \begin{cases} \frac{2}{3}\bar{s} + \frac{2}{3}\bar{u} - \frac{2}{3}t_B - \frac{1}{3}\sqrt{4\bar{s}^2 + 2\bar{u}\bar{s} + 4\bar{u}^2 - 6\bar{u}p_B + 3p_B^2 - 8\bar{s}t_B - 2\bar{u}t_B + t_B^2} \\ \quad \text{if } \; p_A \le p_B - t_A \\ \frac{1}{3}\bar{s} + \frac{1}{3}\bar{u} + \frac{1}{3}p_B + \frac{1}{3}t_A - \frac{1}{3}\sqrt{\bar{s}^2 - \bar{u}\bar{s} + 2\bar{s}p_B + \bar{u}^2 - \bar{u}p_B + p_B^2 + 2\bar{s}t_A - \bar{u}t_A + 2p_B t_A - t_A^2} \\ \quad \text{if } \; p_A > p_B + t_A \end{cases}$$

$$p_B = \begin{cases} \frac{2}{3}\bar{s} + \frac{2}{3}\bar{u} - \frac{2}{3}t_A - \frac{1}{3}\sqrt{4\bar{s}^2 + 2\bar{u}\bar{s} + 4\bar{u}^2 - 6\bar{u}p_A + 3p_A^2 - 8\bar{s}t_A - 2\bar{u}t_A + t_A^2} \\ \quad \text{if } p_B \le p_A - t_A \\ \frac{1}{3}\bar{s} + \frac{1}{3}\bar{u} + \frac{1}{3}p_A + \frac{1}{3}t_B - \frac{1}{3}\sqrt{\bar{s}^2 - \bar{u}\bar{s} + 2\bar{s}p_A + \bar{u}^2 - \bar{u}p_A + p_A^2 + 2\bar{s}t_B - \bar{u}t_B + 2p_B t_B - t_B^2} \\ \quad \text{if } p_B > p_A + t_B \end{cases}$$

$$p_A^{high} = \bar{s} + \frac{1}{2}\bar{u} + \frac{1}{2}t_A - \frac{1}{2}\sqrt{4\bar{s}^2 + \bar{u}^2 - 2\bar{u}t_A - 4\bar{s}t_A + t_A^2}$$

$$p_B^{high} = \bar{s} + \frac{1}{2}\bar{u} + \frac{1}{2}t_B - \frac{1}{2}\sqrt{4\bar{s}^2 + \bar{u}^2 - 2\bar{u}t_B - 4\bar{s}t_B + t_B^2}$$

Recall that the equilibrium price without a tariff is equal to

$p_A* = p_B* = p* = \bar{s} + \frac{1}{2}\bar{u} - \frac{1}{2}\sqrt{4\bar{s}^2 + \bar{u}^2}$. If we compare to initial equilibrium price without the tariff, we see that p_A^{high}, p_B^{high} go to $p_A*, p_B* = p*$ as t_A and t_B go to zero. The maximum prices p_A^{high}, p_B^{high} are increasing in t_A and t_B, respectively.

$$\frac{\partial p_A^{high}}{\partial t_A} = \frac{1}{2} - \frac{t_A - \bar{u} - 2\bar{s}}{\sqrt{4\bar{s}^2 + \bar{u}^2 - 2\bar{u}t_A - 4\bar{s}t_A + t_A^2}} > 0$$

A3.3. An Alternative Model of R&D Cooperation

The following model is an alternative approach to the model on cooperative R&D presented in the main part of the paper. The firms' interactions are represented by a "Hotelling" type location model. In this world, consumers are uniformly distributed along a line of infinite length. Positions on the line correspond to different product characteristics. A consumer's position on the line corresponds to the consumer's most preferred variety. A consumer demands one or zero units of the product.

Let t be the distance from the consumer's most preferred variety to the nearest variety. The consumer at a distance t from firm i has to pay γt in addition to the product price p_i, i.e. the "delivered price"[18] is equal to $p_i + \gamma t$. Define the "choke price" $\bar{p}(\alpha)$ as the maximum delivered price any consumer would be willing to pay for the product. The parameter α is the endogenous degree of cooperation as defined in section 2.1. of this paper. The choke price $\bar{p}(\alpha)$ is an increasing concave function of α. The consumer's demand is one if the price is less than $\bar{p}(\alpha) - \gamma t$. The firms be located at points A and B on the characteristics line with a distance of two units at α = 0. The distance shrinks to zero at α = 1. The situation is shown in Figure A1.

[18] Using quadratic transport cost (or cost of not having the most preferred variety) would be an alternative formulation that eliminates discontinuities in the model. However, the linear transport cost approach was chosen to make the model more tractable.

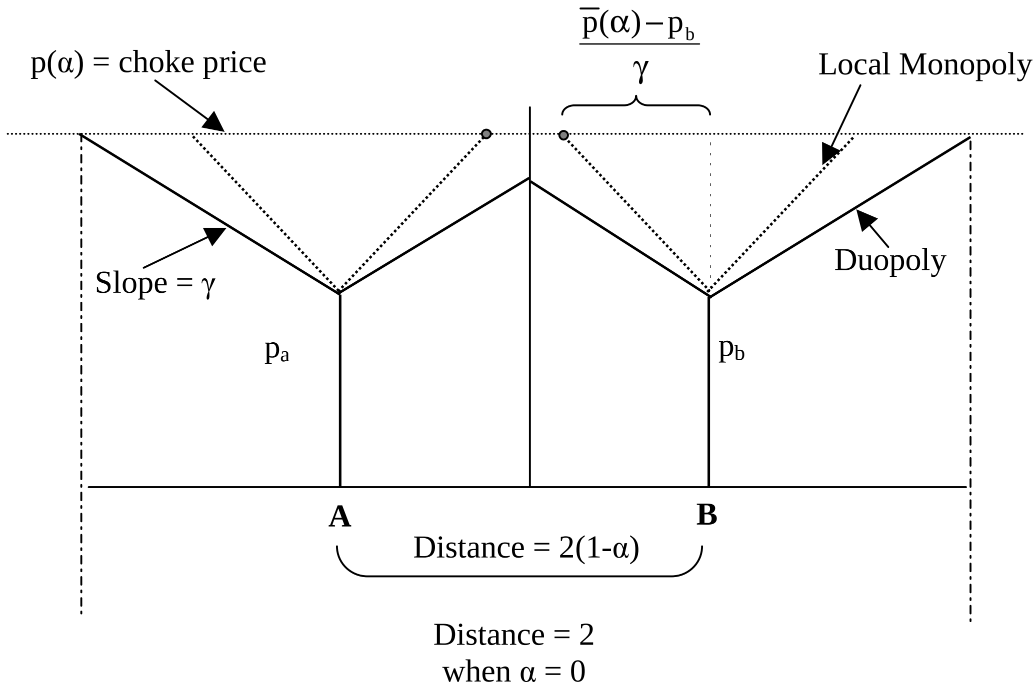

Figure A1: A Location Model of Cooperation

The lines shown in Figure A1 with the slope γ are the delivered price. A consumer will buy a unit as long as the delivered price is less than $\bar{p}(\alpha)$. Thus beginning at A, the limit of customers served is $t = (\bar{p}(\alpha) - p_a)/\gamma$. In the initial situation, α = 0 and the firms are located at the maximum distance from each other. If the firms choose to cooperate, α increases and the goods move closer together. With the increase in α the choke price increases as well, thus extending the range of customers served.

In the model, there are two possible competition scenarios. In the beginning of the game when α is equal to zero and the distance between A and B is equal to two, firms are either duopolists or monopolists depending on p_a, p_b, γ, and p(0).

Note that the condition for a duopoly is that $\frac{2\bar{p}(a) - p_a - p_b}{\gamma} > 2(1-\alpha)$.

To solve the second-stage price game and determine the form of competition, two constrained optimization problems have to be set up. For firm A, the Lagrangian for the duopoly case is:

$$L = p_a\left[(\bar{p}(\alpha) - p_a)/\gamma + (1-\alpha) + (p_b - p_a)/2\gamma\right] + \lambda\left[2\bar{p}(a) - p_a - p_b - 2\gamma(1-\alpha)\right]$$

$$\frac{\partial L}{\partial p_a} = (\bar{p}(\alpha) - 2p_a)/\gamma + (1-\alpha) + (p_b - 2p_a)/2\gamma - \lambda \le 0 \qquad \lambda \le 0 \text{ c.s.}$$

$$\frac{\partial L}{\partial \lambda} = 2\bar{p}(a) - p_a - p_b - 2\gamma(1-\alpha) \le 0$$

$$\left[2\bar{p}(a) - p_a - p_b - 2\gamma(1-\alpha)\right]\lambda = 0$$

Solving for p_a we get:

$$p_a = \begin{cases} \frac{p_b}{6} + \frac{\bar{p}(\alpha)}{3} + \frac{\gamma(1-\alpha)}{3} & \text{if } p_b \le \frac{10}{7}\bar{p}(\alpha) - 2\gamma(1-\alpha) \\ p_b \text{ with } p_a = p_b = \bar{p}(\alpha) - \gamma(1-\alpha) & \text{if } p_b > \frac{10}{7}\bar{p}(\alpha) - 2\gamma(1-\alpha) \end{cases}$$

If the constraint is binding, the firms are local monopolists and solve a different maximization problem. For the monopoly case, we have:

$$L = p_a\left[2(\bar{p}(\alpha) - p_a)/\gamma\right] + \lambda\left[2\gamma(1-\alpha) - 2\bar{p}(a) + p_a + p_b\right]$$

Solving for p_a yields:

$$p_a = \begin{cases} \frac{\bar{p}(\alpha)}{2} & \text{if } p_b \ge \frac{3}{2}\bar{p}(\alpha) - 2\gamma(1-\alpha) \\ p_b \text{ with } p_a = p_b = \bar{p}(\alpha) - \gamma(1-\alpha) & \text{if } p_b < \frac{3}{2}\bar{p}(\alpha) - 2\gamma(1-\alpha) \end{cases}$$

Combining the two problems and solving for the symmetric Nash equilibria yields the following prices and conditions:

Monopoly	$p_a = p_b = \frac{\bar{p}(\alpha)}{2}$	if	$2\gamma(1-\alpha) \geq \bar{p}(\alpha)$
Binding Constraint	$p_a = p_b = \bar{p}(\alpha) - \gamma(1-\alpha)$	if	$\frac{7}{3}\gamma(1-\alpha) > \bar{p}(\alpha) > 2\gamma(1-\alpha)$
Duopoly	$p_a = p_b = \frac{2}{5}\bar{p}(\alpha) + \frac{2}{5}\gamma(1-\alpha)$	if	$\bar{p}(\alpha) \geq \frac{7}{3}\gamma(1-\alpha)$

If firms are local monopolists in the beginning of the game, they could cooperate and increase $\bar{p}(\alpha)$ without becoming competitors. Therefore, only the duopoly case is interesting for the analysis of cooperation. To guarantee that the firms are competing in the beginning of the game when $\alpha = 0$, we need $\gamma \leq \frac{3}{7}\bar{p}(0)$.

Given the linear nature of the problem, it is possible that the duopoly price equilibrium is not stable for certain α-γ-combinations. When $\alpha = 0$ and $\gamma \leq \left(\frac{5}{67}\sqrt{10} - \frac{7}{67}\right)\bar{p}(0)$, the best response to the duopoly price is to undercut that price by $2\gamma(1-\alpha)$ and to capture the entire market. In order to have a stable duopoly equilibrium in prices in the case of no cooperation with prices $p_a = p_b = \frac{2}{5}\bar{p}(0) + \frac{2}{5}\gamma$ it is necessary for γ to be in the interval $0.1315\,\bar{p}(0) < \gamma < 0.4286\,\bar{p}(0)$. If firms cooperate, i.e. α>0, the lower bound on γ increases.

In the first stage, assuming the conditions for a duopoly are satisfied, the firms maximize profit as a function of α taking the price game in the second stage as given. Profit as a function of α is given by:

$$\Pi_a(\alpha) = \left(\frac{2}{5}\bar{p}(\alpha) + \frac{2}{5}\gamma(1-\alpha)\right)\left(\frac{\frac{3}{5}\bar{p}(\alpha) - \frac{2}{5}\gamma(1-\alpha)}{\gamma} + 1 - \alpha\right)$$

The FOC is:

$$\frac{\partial \Pi_a}{\partial \alpha} = \frac{12}{25}\left(\gamma \frac{\partial \bar{p}(\alpha)}{\partial \alpha}\right)\left(\frac{\gamma\alpha - \gamma - \bar{p}(\alpha)}{\gamma}\right) \le 0 \qquad 0 \le \alpha \le 1$$

To solve for α, assume a specific strictly concave function $\bar{p}(\alpha)$ of the form $\bar{p}(\alpha) = x - (s - s\alpha)^2$ with x > 0, s > 0 and $x - s^2 > 0$. The intercept of this function with the price axis is equal to the choke price without cooperation $\bar{p}(0)$. The parameter s captures the curvature and reflects how important cooperation is in raising the choke price relative to $\bar{p}(0)$. The maximum $\bar{p}(\alpha)$ is reached at α=1. Substituting the specific $\bar{p}(\alpha)$ function into the FOC and solving for α yields the optimal degree of cooperation α* with:

$$\alpha^* = \frac{1}{2}\frac{2s^2 - \gamma}{s^2} \text{ for } \gamma \le 2s^2$$

$$\alpha^* = 0 \text{ for } \gamma > 2s^2$$

Second order conditions are satisfied.

For this solution to be stable in the sense that firms will not undercut each other in the second stage price game, it is necessary that $\gamma > 0.4968 s\sqrt{x}$. In contrast to the switching cost model this alternative approach results in a concise closed form solution. A disadvantage of this model is that there are numerous conditions that have to hold in order for the solution to be stable. The overall interpretation of the result is similar to the original model.

The optimal degree of cooperation depends on the curvature of the $\bar{p}(\alpha)$ function and γ. As expected α is increasing in s because the steeper the $\bar{p}(\alpha)$ function, the more is to gain by cooperation. The fact that α is decreasing in γ seems somewhat counterintuitive at first. Firms cooperate more if the market is more competitive, thus increasing competition even further. In the original model, firms cooperated more if competition was less intense, i.e. switching costs were high. However, in the alternative model γ not only captures the intensity of competition but also the size of the market. With a low γ, firms get more consumers on the side that they are not competing on. This indicates that the effect of increasing the limit of consumers served by increasing $\bar{p}(\alpha)$ outweighs the competition effect.