STOCKHOLM UNIVERSITY
Department of Economics

Course name:	**Game Theory**
Course code:	**EC7101**
Examiner:	**Yves Zenou**
Number of credits:	**7,5 credits**
Date of exam:	**Saturday 8 June 2013**
Examination time:	**3 hours**

Write your identification number on each paper and cover sheet (the number stated in the upper right hand corner on your exam cover).

If you think that a question is vaguely formulated, specify the conditions used for solving it. Only legible exams will be marked. No aids are allowed.

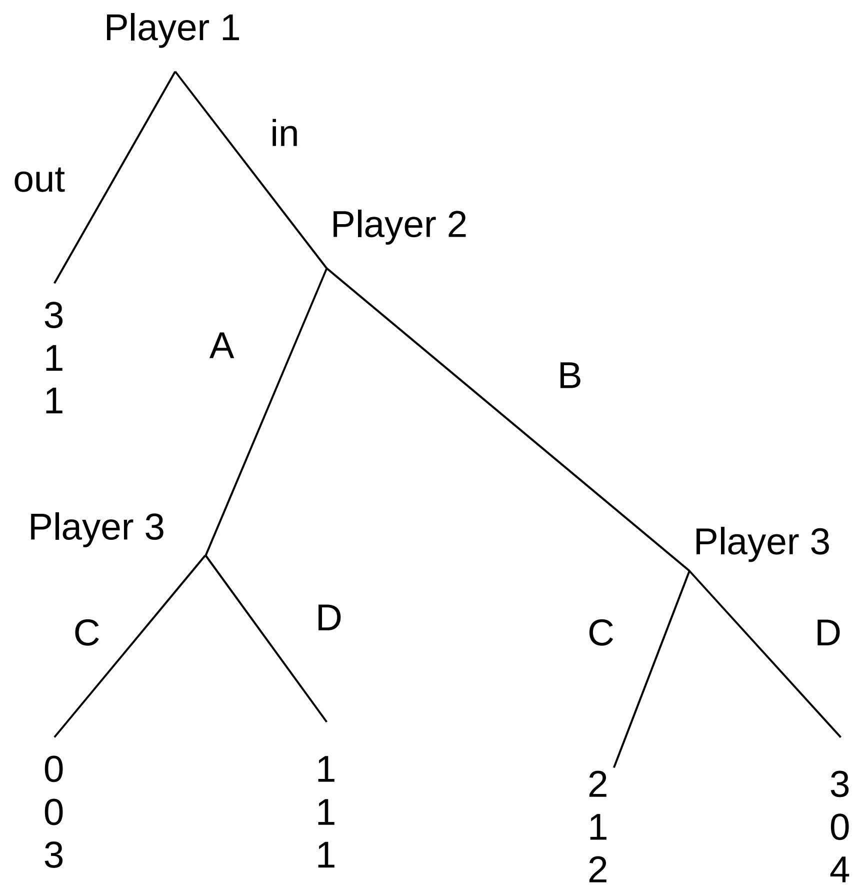
The exam consists of 3 questions. Question 1 is worth 35 points. Question 2 is worth 25 points. Question 3 is worth 40 points. 100 points in total. For the grade E 45 points are required, for D 50 points, C 60 points, B 75 points and A 90 points.

Your results will be made available on your "My Studies" account (www.mitt.su.se), on June 27, 2013, at the latest.

Good luck!

Question 1 (35 points)

Consider the following figure:



By convention, the payoff of player 1 is the first payment, the payoff of player 2 is the second payment and the payoff of player 3 is the third payment.

1a) How many subgames can you identify? Draw a figure showing all the subgames.

1b) Show that there does not exist a Subgame-Perfect Nash equilibrium in pure strategies.

1c) Calculate the unique Subgame-Perfect Nash equilibrium in mixed strategies.

Question 2 (25 points)

Consider the following game of the policeman and the thief. Player 1 (the policeman) can monitor (*M*) or not (*NM*) the thief. If the policeman monitors the thief, it costs him $c > 0$, whereas if he does not monitor the thief, the cost is zero. Player 2 (the potential thief) has two possible strategies: steal (*S*) or not to steal (*NS*).

We assume that if the policeman monitors the thief while the latter is stealing, then the probability that the policeman catches the thief is 1. In that case, if player 2 is caught while

stealing because he was monitored by the policeman, he has to pay a fine of $A > c$ to the policeman. If the policeman monitors the thief and the latter is not stealing, then the policeman obtains a negative utility equals to $-c < 0$.
If the policeman does not monitor the thief and the latter steals, then of course the thief is not caught and the policeman has to give $B > 0$ to the thief.
Finally, whether he is monitored or not, player 2 gets a utility of zero if he does not steal.

2a) Represent this game in normal form, i.e. draw a matrix with the strategies and the payoffs of the players.

2b) Determine the Nash equilibria in pure strategies and in mixed strategies of this game. Explain the results.

Question 3 (40 points)

The decision of an action $a \geq 0$ of a child affects both his own revenue and that of his parent. The child is selfish: he only cares of the amount of money he has. On the contrary, his parent cares of how much money he and the child have. The parent has to decide how much money to transfer to his child. This transfer is denoted by $t \geq 0$. The preferences of the parent are described by the following utility function:

$$U_p(a,t) = \log a + \log t - a\,t$$

The action of the child can be interpreted as the number of hours he spends doing his homeworks. He has 24 hours available per day and sleeps 8 hours per day. His budget constraint can be written as :

$$16 = a + l$$

where l indicates the number of hours he spends in leisure and 24 – 8 = 16 hours in the time available he has when he is not sleeping. The utility function of the child is given by:

$$U_c(a,t,l) = \log a + \log t + a\,l$$

Remember that for any function $\log[f(x)]$, we have the following derivative:

$$\frac{d(\log[f(x)])}{dx} = \frac{f'(x)}{f(x)}$$

where $f'(x)$ is the derivative of $f(x)$ with respect to *x*. In particular,

$$\frac{d(\log x)}{dx} = \frac{1}{x}$$

The timing of the game is as follows. First, the child decides the optimal *a*, that is the optimal number of hours spent in doing homeworks. Then, the parent decides the optimal amount of transfer $t \geq 0$, that is how much money he wants to transfer to his child.

3a) Model this situation as a game in extensive form.

3b) Solve the second stage of this game, that is the optimal choice of parents in terms of t. Denote the optimal choice by t^*. Show that it is unique.

3c) Calculate the Subgame-Perfect Nash Equilibrium of this game. Determine the equilibrium values of t (parents' transfer), a (the child's number of hours spent doing homeworks), l (leisure), $U_c(a,t,l)$ (utility of the child), $U_p(a,t)$ (utility of the parent).