Urban Labor Economics

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June 27, 2006

Part 2: Urban Efficiency Wages

Chapter 5: Extensions of Urban Efficiency Wage Models

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1. Introduction

We extend the benchmark urban efficiency wage model developed in section 2 of chapter 4 in several directions. First, we endogeneize the effort function, which becomes a negative function of distance to jobs. Second, we introduce leisure in the benchmark model by considering that effort and leisure are not independent activities. Third, we relax the assumption of no-relocation costs and consider instead the other extreme assumption that costs of relocation are infinite. Finally, we develop a model where both leisure and relocation costs are present.

Because in each model workers can provide different effort or utility levels depending on their location, the information available to firms about workers' residence will matter in the process of wage formation. As a result, in all models, we consider two different information structures. In the first one, firms can perfectly observe the residential location of all workers. In the second one, firms cannot observe workers' residence because either such information would be too costly to obtain or workers would have an incentive not to truthfully reveal where they live. Because the city size as well as population density are fixed in our model, differences in information structure could be justified either in terms of country differences (in some countries it may be more costly to obtain information about the residential location of all workers than others) or culture/race differences (for example, in the US, African Americans may not want to reveal where they live if it implies spatial discrimination or redlining). In other words, workers who live in "stigmatized" areas (like for example the area of Queens or Harlem in New York city) may not want to give their true address to their employers while those who live in good neighborhoods may have the incentive to do so.

2. Effort as a function of distance to jobs

Consider the benchmark model (of chapter 4) in which each individual can provide two possible levels of effort: either the worker shirks, exerting zero effort, $e = 0$, and contributing zero to production, or he/she does not shirk, providing full effort. Now, contrary to the benchmark model, we endogeneize effort for non-shirker workers in the following way. Effort equals $e(x) > 0$, $\forall x \in [0, x_f]$ (with $e(0) = e_0 > 0$), and a worker located at x contributes to $e(x)$ units of production. We assume that $e'(x) < 0$ and $e''(x) \geq 0$ so that the greater the distance to work, the lower the effort level. For remote locations, the marginal difference in effort is quite small. The former assumption is to capture the fact that workers who have longer commuting trips are more tired and are thus less able to provide higher levels of effort than those who reside closer to jobs. We discuss in depth the empirical validity of this assumption in chapter 7 of this book.

Apart from this endogeneization, all the other aspects of the benchmark model are the same. The worker's behavior can now be seen as a two-stage decision problem. First, depending on their residential location, each worker must decide to shirk or not. Since effort is costly, it is clear that workers who live the closest to jobs will be more inclined to shirk that those residing further away. Thus, contrary to the benchmark model, *the moral hazard problem is here locationally dependent.* Second, once the worker has decided not to shirk (this is the behavior that will emerge in equilibrium), he/she must decide how much effort he/she provides. This decision is also locationally dependent since we assume that workers who have longer commutes are more tired and provide less effort than those who live closer to jobs.

As stated in the introduction, we consider two different information structures. For the sake of the presentation, let us first consider the imperfect information case and then the other case.

2.1. Firms do not observe the residential location of all workers[1]

As stated above, this model is identical to the benchmark model with the exception that workers' effort e is now a negative function of distance to jobs, i.e. $e(x)$, with $e'(x) < 0$. How does this affect the urban land use equilibrium?

As in the benchmark model, the bid rent functions are given by

$$\Psi_L(x, W_L) = w_L - e(x) - \tau\, x - W_L \tag{2.1}$$

$$\Psi_U(x, W_U) = w_U - s\,\tau\, x - W_U \tag{2.2}$$

Now, the main difference is that, for all workers to reach the same utility level, the land rent needs to compensate not only for commuting costs but also for effort costs. First, to guarantee that the bid rent curve of the employed workers is downward sloping, we assume that

$$\tau + e'(x) > 0 \tag{2.3}$$

which means that $\tau x + e(x)$ is increasing in x despite $e'(x) < 0$. To understand this, observe that commuting cost τx includes more than just money costs.

[1] This section is based on Zenou (2002).

It also includes these negative effects of a longer commute such as non-work-related fatigue. So even though people benefit from working less hard on the job as x goes up, the other effects of tiredness (along with the money and time outlay on commuting) make the person worse off overall.

Now, by denoting by x_b the recruitment area of firms, which is equal here to the border between the employed and the unemployed, we easily obtain the following result:

Proposition 1. *Assume that firms do not observe the residential location of all workers. Assume also that*

$$\tau > -e'(x_b)/(1-s) \tag{2.4}$$

Then, in equilibrium, the employed reside close to the CBD whereas the unemployed live at the outskirts of the city.

The intuition of this result is as follow. An increase in distance x has offsetting effects on employed workers: they pay higher commuting costs but lower effort is exerted on the job. The net effect is thus less than the pure commuting cost effect, and the question is whether this net effect is stronger than the shrunken commuting cost effect for unemployed workers, which is smaller than that of the employed worker because $s < 1$. In this context, when the commuting cost τ is high enough, the employed workers reside close to jobs by outbidding the unemployed.

Using the same definition for the urban land use as in the benchmark model, we easily obtain the following equilibrium values:

$$x_b^* = L^* \tag{2.5}$$

$$x_f^* = N \tag{2.6}$$

$$W_U^* = w_U - s\,\tau\,N \tag{2.7}$$

$$W_L^* = w_L - e(L^*) - \tau L^* - s\tau\ (N - L^*) \tag{2.8}$$

$$R^*(x) = \begin{cases} \tau\,[(1-s)L^* + s\,N] + e(L^*) - [e(x) + \tau x] & \text{for} \quad 0 \leq x \leq L^* \\ s\,\tau\,(N - x) & \text{for} \quad L^* < x \leq N \\ 0 & \text{for} \quad x > N \end{cases} \tag{2.9}$$

The Bellman equations are the same as in the benchmark model except for the fact that

$$\begin{aligned} W_L^S(x) &= W_L^{NS} + e(x) \\ &= w_L - e(L^*) - \tau\, L^* - s\tau\, (N - L^*) \end{aligned} \tag{2.10}$$

i.e. the instantaneous utility of the employed shirker workers is now distant dependent.

The efficiency wage must be set to make workers indifferent between shirking and not shirking. However, contrary to the benchmark model, *the utility of shirkers is not constant over locations whereas it is constant for non-shirkers.* It is in fact easy to see that the utility of shirkers increases as x, the distance to the CBD, decreases. The intuition is straightforward. Since the land rent compensates for both commuting costs and effort levels, then shirkers, who do not provide effort, have a higher utility when residing closer to the CBD (since their commuting costs are lower). Formally, we have:

$$\frac{\partial W_L^S}{\partial x} = -\tau - R'(x) = -\tau + e'(x) + \tau = e'(x) < 0$$

This implies, in particular, that the highest utility that a shirker can reach is at $x = 0$ (the CBD) and the lowest is at $x = x_b$. As a result, the efficiency wage must be set to make workers indifferent between shirking at location $x = 0$ and not shirking since if the worker at $x = 0$ does not shirk, then all workers located further away will not shirk. In other words, the condition that determines the efficiency wage is given by

$$I_L^{NS} = I_L^S(0) = I_L$$

We first obtain the following incentive condition:

$$I_L - I_U = \frac{e(0)}{m} > 0$$

that guarantees that the surplus of being employed is positive. From this equation, we can easily calculate the efficiency wage, which is equal to:

$$w_L^* = w_U + e(L^*) + \frac{e_0}{m}\left(\frac{\delta\, N}{N - L^*} + r\right) + (1-s)\tau\, L^* \tag{2.11}$$

where $e(0) \equiv e_0$.

As in the benchmark model of Chapter 4, the urban efficiency wage has two roles: to prevent shirking (incentive component) and to ensure that workers are locationally indifferent (spatial compensation component). The key relation here is the interaction between the effort function $e(\cdot)$ and the location of workers.

Our setting thus implies that there is a fundamental *asymmetry* between workers and firms. Because firms cannot observe workers' residential location, all workers obtain the same efficiency wage whatever their location. However, they do *not* contribute the same level of production because effort decreases with distance to jobs. In other words, even though the wage cost is location independent the production is not. This implies that the per-worker profit decreases with distance to jobs. The next natural step of our analysis is thus to calculate the per-worker profit for each firm and to determine the employment level in the city.

There are M identical firms ($j = 1, ..., M$) in the economy. We assume that all jobs are obtained through an employment agency that coordinates workers in such a way that each firm employs only one worker at each location.[2] Since all firms and workers are (ex ante) identical and since the density of workers at each location is M, we focus on a symmetric equilibrium in which each firm sets the same $x_b = L$. This is quite reasonable since, ex ante (before location), all workers are equally productive (location is not a characteristic of a worker), and, ex post (once located), they are all indifferent to work in any of the M firms since all firms are located at 0 and offer the same wage w_L. In this context, since all firms are identical, the employment level in each firm j is equal to: $l_j = l = L/M = x_b$.

We can calculate the total production (or effort) level provided in each firm. It is given by:

$$e^{to} = \int_0^L e(x)dx \tag{2.12}$$

It is interesting to observe that the average production (or effort) in each firm

[2]In the first period, the timing is as follows. All N workers apply for a job in the employment agency and only L of them obtain a job and locate somewhere in the city (since they are indifferent between all locations between 0 and x_b). Then, the employment agency allocates workers to firms in such a way that each firm recruits one worker at each location. This is true at any moment of time (and in particular in the steady state) since, at each period, some workers with different locations lose their job and new workers obtain a job and reside somewhere in the city between 0 and x_b. Then again, the employment agency allocates these new workers to firms in such a way that each firm (those who has lost workers) employs only one worker at each location.

is given by $e^{av} = \frac{1}{L}\int_0^L e(x)dx$, with

$$\frac{\partial e^{av}}{\partial L} = -\frac{1}{L}\left[e^{av} - e(L)\right] < 0 \tag{2.13}$$

In words, a larger recruitment area decreases the average effort level in each firm since new hired workers produce less effort because they live further away. The aggregate profit function can be written as: $F(e^{to}) - w_L L$, and therefore the equilibrium labor demand is given by:

$$\frac{w_L}{e(L^*)} = F'(e^{to}) \tag{2.14}$$

Indeed, when deciding the optimal L, each firm takes into account the fact that a higher L increases its workforce but reduces its average effort level and thus average production (see 2.13). It is easy to verify that the labor demand L is downward sloping in the plane (L, w_L). It can then be shown that there exists a unique labor market equilibrium, which is given by:

$$w_U + e(L^*) + \frac{e_0}{m}\left(\frac{\delta\, N}{N - L^*} + r\right) + (1-s)\tau\, L^* = e(L^*)F'(e^{to}) \tag{2.15}$$

Observe that L^* affects both the quality of the workers and the efficiency wage. Indeed, when L^* increases, the average effort level in each firm decreases but the equilibrium efficiency wage w_L^* increases. The latter result is true only if the effort of the workers is not too sensitive to the length of the recruitment area. Let us now investigate the properties of the equilibrium. We have:

Proposition 2. *Assume that firms do not observe the residential location of all workers. Assume also that* $\tau > -e'(L^*)/(1-s)$*. Then, the equilibrium recruitment area* $x_b^* = L^*$ *is increasing in the monitoring rate* m*, in the percentage of CBD-trips of the unemployed* s *and in the size of the active population* N*, but it is decreasing in the unemployment benefit* w_U*, the unit commuting cost* τ*, the effort level* e_0 *provided at location* $x = 0$*, the discount rate* r *and the job destruction rate* δ*. For the equilibrium wage* w_L^**, we have exactly the reverse effects.*

The following comments are in order. First, a rise in the unemployment benefit shifts upward the UNSC since, at each recruitment area level x_b (or equivalently employment level), the efficiency wage must increase to deter shirking. This is the standard outside option effect generated by the unemployment benefit. Because wages are higher, it is more costly for firms to hire new workers since they are less productive (they live further away from jobs) and cost more. As a result, firms reduce their recruitment area. In other words, when w_U increases, firms employ workers who live closer to jobs that are therefore more productive but pay them more. Second, we have the opposite result concerning the monitoring technology m. Indeed, if firms monitor their workers more, the efficiency wage is lower so that firms extend their recruitment area. Third, increasing the unit commuting cost τ borne by workers or decreasing the number of CBD-trips s reduces the recruitment area x_b. The intuition is exactly the same as for w_U but here the efficiency wage must increase not to deter shirking but to spatially compensate employed workers (this is the compensation effect mentioned above). Fourth, when the maximum effort provided in the city, e_0, increases, the efficiency wage increases because globally all workers provide more effort (a rise in the intercept e_0 shifts upward the non-shirking effort curve $e(x)$). As a result, firms hires less workers and thus reduce x_b. Finally, when there are more technological shocks in the economy so that jobs are destroyed more often (δ increases), then firms have to increase their wages to deter shirking and thus to reduce their recruitment area.

2.2. Firms perfectly observe the residential location of all workers

Assume now that firms perfectly observe workers' residential location. In that case, they can pay workers according to their location (assuming that there are no anti-discrimination laws that prevent this type of policy). This means, in particular, that the wage is now a function of distance to job, i.e. $w_L(x)$. Thus, bid rents are now given by:

$$\Psi_L(x, W_L) = w_L(x) - e(x) - \tau\, x - W_L \tag{2.16}$$

$$\Psi_U(x, W_U) = w_U - s\,\tau\, x - W_U \tag{2.17}$$

Observe that

$$\frac{\partial \Psi_L(x, W_L)}{\partial x} = w_L'(x) - e'(x) - \tau$$

Of course, at this stage, we do not know the sign of $w_L'(x)$ so we cannot sign this derivative. This is because the land rent here compensates not only for commuting costs but for wage differences between location. If $w_L'(x) < 0$, then the land rent will always be lower farther away from the CBD because remote locations imply both longer commutes and lower wages. If, on the contrary, $w_L'(x) > 0$, then land rents can increase with distance to jobs because if wages are sufficiently high far away from the CBD. We show below, however, that the latter is never possible.

Let us calculate the efficiency wage. The Bellman equations are given by:

$$r\, I_L^{NS}(x) = w_L - e(x) - \tau\, x - R(x) - \delta\,(I_L^{NS} - I_U) \tag{2.18}$$

$$r\, I_L^{S}(x) = w_L - \tau\, x - R(x) - (\delta + m)\,(I_L^{S} - I_U) \tag{2.19}$$

$$r\, I_U(x) = w_U - s\,\tau\, x - R(x) + a(I_L - I_U) \tag{2.20}$$

The efficiency wage is such that, *at each location*, we have

$$I_L^{NS}(x) = I_L^{S}(x) = I_L(x)$$

which is equivalent to

$$I_L(x) - I_U = \frac{e(x)}{m} > 0,\ \forall x \in [0, N] \tag{2.21}$$

with $\partial\,(I_L - I_U)\,/\partial x < 0$. This is an interesting and new result since it shows that the surplus to be employed is a function of distance to jobs. Indeed, since firms observe where workers reside and since they know that effort is inversely related to distance to jobs, they will pay less workers who reside in remote locations compared to those locating closer to jobs. So, the gain to be employed is lower but still positive for remote workers.

Using this relationship and solving for the efficiency wage in the usual way, we easily obtain the following efficiency wage at each x:

$$w_L(x) = w_U + e(x) + \frac{e(x)}{m}\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau\, x \tag{2.22}$$

with

$$w_L'(x) = e'(x)\left[1 + \frac{1}{m}\left(\frac{\delta N}{N-L} + r\right)\right] + (1-s)\,\tau \gtrless 0$$

The intuition runs as follows. Firms have to decide which wage to pay to each worker, given that they reside in different locations. There are two opposite forces. On the one hand, remote workers providing less effort should be paid less (see (2.21)). This is a pure incentive effect. On the other, remote workers should be compensate more to accept to stay in the city. This is a pure spatial compensation effect and it is given by $(1-s)\,\tau\, x$. So the net effect is ambiguous. If one plugs (2.22) in (2.16), one obtains

$$\Psi_L(x, W_L) = w_U + \frac{e(x)}{m}\left(\frac{\delta N}{N-L} + r\right) - s\tau\, x - W_L$$

with

$$\frac{\partial \Psi_L}{\partial x} = \frac{e'(x)}{m}\left(\frac{\delta N}{N-L} + r\right) - s\tau < 0$$

So, whatever the sign of $w_L'(x)$, land rents decrease with distance to jobs. When $w_L'(x) < 0$, employed workers living further away from jobs pay lower land rents to compensate for their low wages and their high commuting costs compared to workers residing closer to jobs. When $w_L'(x) > 0$, the wage effect is not strong enough to outweight the commuting cost effect so that the land rent is still decreasing. We thus have:

Proposition 3. *Assume that firms perfectly observe the residential location of all workers. Then, in equilibrium, the employed reside close to the CBD whereas the unemployed live at the outskirts of the city.*

By solving the urban-land use equilibrium, it is easy to see that x_b^*, x_f^*, W_U^* and $R^*(x)$ are still respectively given by (2.5), (2.6), (2.7) and (2.9), while W_L^* is now equal to:

$$W_L^* = w_U + \frac{e(L^*)}{m}\left(\frac{\delta N}{N-L^*} + r\right) - s\,\tau\,N \tag{2.23}$$

We can finally determine the equilibrium L^*. Since all firms are totally identical, let us focus on a symmetric (steady-state) equilibrium in which each firm employs the same number of workers and pays the same total wage cost than any other firm. As above, we assume that all jobs are obtained through an employment agency that coordinates workers in such a way that each firm employs only one worker at each location. The aggregate profit is thus given by: $F(e^{to}) - w_L^{to}$, where

$$\begin{aligned} w_L^{to} &= \int_0^L w_L(x)dx \\ &= w_U L + \left[1 + \frac{1}{m}\left(\frac{\delta N}{N-L} + r\right)\right] e^{to} + (1-s)\,\tau\,\frac{L^2}{2} \end{aligned}$$

and e^{to} is defined by (2.12). The equilibrium employment level L^* is thus determined by:

$$w_U + e(L^*) + \frac{e(L^*)}{m}\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau L^* = e(L^*)F'(e^{to}) \tag{2.24}$$

Compared to (2.15), the only difference is in the value of the effort function $e(.)$ in front of $\left(\frac{\delta N}{N-L} + r\right)$. So, basically, the results of Proposition 2 are basically the same.

3. Effort and leisure

We would like now to introduce leisure in the model by assuming that leisure and effort are not independent activities. For the employed, the utility function is assumed to be separable and given by:

$$z_L + \Gamma(\zeta, e)$$

where z_L is the quantity of a (non-spatial) composite good (taken as the numeraire) consumed by the employed and ζ denotes leisure. $\Gamma(.)$ is assumed to be increasing in ζ and decreasing in the effort e, and concave in both arguments. This choice of the utility function aims at capturing the fact that effort and leisure are not independent activities. Indeed, if one interprets $-e$ as the leisure activity on the job (shirking), then the benefits arising from additional *leisure activity on the job* is obviously related to the extent of *leisure activity at home* and visa-versa.

We are now able to write the budget constraint of an employed worker. Each worker purchases the good z produced and incurs τx in monetary commuting costs when he/she lives at distance x from the BD. Letting $R(x)$ denote rent per unit of land, the budget constraint of an employed worker at distance x can be written as follows:

$$w_L \digamma = z_L + R(x) + \tau x \tag{3.1}$$

where w_L is the per-hour wage and $\digamma$ denotes the amount of working hours. $\digamma$ is assumed to be the same and constant across workers, an assumption that agrees with most jobs in the vast majority of developed countries.

Each worker provides a fixed amount of labor time $\digamma$ so that the time available for leisure ζ depends solely on commuting time. Thus, denoting by ιx the commuting time from distance x (where $\iota > 0$ is the time commuting cost per unit of distance), the time constraint of an employed worker at distance x can be written as:

$$1 - \digamma = \zeta + \iota x \tag{3.2}$$

in which the total amount of time is normalized to 1 without loss of generality.

By plugging (3.1) and (3.2) into the utility function, we obtain the following indirect utility for the employed:

$$W_L(x, e) = z_L + \Gamma(\zeta, e) = w_L \digamma - R(x) - \tau x + \Gamma\left(1 - \digamma - \iota x, e\right) \tag{3.3}$$

Let us now focus on the unemployed. Their budget constraint is given by:

$$w_U = z_U + R(x) + s\tau x \tag{3.4}$$

To keep the analysis manageable and to be consistent with the utility of the employed, we assume that the unemployed's utility function is given by: $z_U + \Gamma_U$.[3] In this formulation, Γ_U is a constant utility benefit arising to all who are unemployed. Basically, Γ_U is introduced to recognize the inherent disutility to being at work and commuting to work since it assures that when people have exactly the same z, the one working can receive less utility.

By using (3.4), we obtain the following indirect utility function for the unemployed:

$$W_U(x) = z_U + \Gamma_U = w_U - R(x) - s\tau x + \Gamma_U$$

As above, let is consider two cases: Firms do or do not observe the residential location of all workers.

3.1. Firms do not observe the residential location of all workers

The bid rent functions of all workers are given by:

$$\Psi_L(x, W_L) = w_L F - \tau x + \Gamma\left(1 - F - \iota x, e\right) - W_L \tag{3.5}$$

$$\Psi_U(x, W_U) = w_U - s\tau x + \Gamma_U - W_U \tag{3.6}$$

with

$$\frac{\partial \Psi_L}{\partial x} = -\tau - \iota\,\Gamma_\zeta < 0$$

$$\frac{\partial \Psi_U}{\partial x} = -s\tau < 0$$

For the employed, the land rent compensates both commuting costs and leisure costs. Indeed, workers living farther away have longer commutes, which, in turn, implies that they have less time for leisure. For the unemployed, as usual, it compensates for commuting costs only.

Proposition 4. *The employed reside close to jobs while the unemployed live at the periphery of the city.*

This result is straightforward since the employed have higher commuting cost than the unemployed and they suffer from distant location because of the

[3] This formulation assumes that there is no search behavior from the unemployed. They just obtain a job randomly. This is consistent with the standard assumptions of exogenous reemployment probability in the efficiency wage model.

resulting loss of leisure. The equilibrium values of the land-use equilibrium are thus given by

$$W_U^* = w_U - s\tau N + \Gamma_U \tag{3.7}$$

$$W_L^* = w_L \digamma + \Gamma^e(L) - \tau L - s\tau\,(N - L) \tag{3.8}$$

$$R^*(x) = \begin{cases} \tau\,(L - x) + \Gamma^e(x) - \Gamma^e(L) + s\tau\,(N - x) & \text{for} \quad 0 \le x \le L \\ s\tau\,(N - x) & \text{for} \quad L < x \le N \\ 0 & \text{for} \quad x > N \end{cases} \tag{3.9}$$

where

$$\Gamma^e(x) \equiv \Gamma\,(1 - \digamma - \iota x, e)$$

Observe that, since $\Gamma(.)$ is decreasing in x, the highest utility value is at $x = 0$ while the lowest one is at $x = L$. Thus, $\Gamma^e(x) > \Gamma^e(L)$.

Let us calculate the efficiency wage. The Bellman equations of all workers are equal to:

$$r\,I_L^{NS} = w_L \digamma + \Gamma^e(L) - \tau L - s\tau\,(N - L) - \delta\,(I_L^{NS} - I_U) \tag{3.10}$$

$$r\,I_L^S = w_L \digamma + \Gamma^0(L) - \tau L - s\tau\,(N - L) - (\delta + m)\,(I_L^S - I_U) \tag{3.11}$$

$$r\,I_U = w_U - s\tau N + \Gamma_U + a(I_L - I_U) \tag{3.12}$$

where

$$\Gamma^0(x) \equiv \Gamma\,(1 - \digamma - \iota x, 0)$$

Indeed, someone who decides to shirk provides no effort and thus his/her instantaneous utility at x is given by $\Gamma^0(x)$ while a non-shirker enjoys $\Gamma^0(x)$. Solving $I_L^{NS} = I_L^S = I_L$ leads to

$$I_L - I_U = \frac{\Gamma^0(L) - \Gamma^e(L)}{m} > 0$$

This is the incentive condition to prevent shirking since it gives the surplus obtained from employment. Indeed, workers will be even more induced to work the larger the utility difference between shirking and non-shirking. This condition is calculated at location L since it is exactly where workers and non-workers pay the same land rent.

Proposition 5. *At location L, we have:*

(i) *If effort and leisure are substitutes, i.e.*

$$\frac{\partial^2 \Gamma(L)}{\partial \zeta \partial e} < 0 \tag{3.13}$$

then

$$\frac{\partial\,(I_L - I_U)}{\partial L} < 0$$

(*ii*) *If effort and leisure are complements, i.e.*

$$\frac{\partial^2 \Gamma(L)}{\partial \zeta \partial e} > 0 \tag{3.14}$$

then

$$\frac{\partial \left(I_L - I_U\right)}{\partial L} > 0$$

This is an interesting and new result because, for the first time, the surplus of being employed (based on an incentive condition) depends on L, the number of employed workers (which is equal here to the location of the last employed in the city). Indeed, when L increases, workers located at L are worse off since they are further away from the CBD and thus have to spend more time in commuting, which in turn implies that they have less leisure and therefore less utility. Now, when L increases, in order to increase the surplus and thus the incentive not to shirk, it has to be that

$$\frac{\partial \Gamma^0(L)}{\partial \zeta} < \frac{\partial \Gamma^e(L)}{\partial \zeta}$$

that is, at a given location L, the marginal increase in utility following a rise in leisure is lower for shirkers than nonshirkers. In other words, effort and leisure have to be complement. Otherwise, if they are substitute, we have the reverse effect, i.e. $\partial \left(I_L - I_U\right)/\partial L < 0$.

Let us be more precise about these assumptions. Low leisure at home may imply that the worker has less time for rest and relaxation and more pressed for time at home (less time for relaxation or errands), and as a result the benefit of taking leisure or conducting home production (relaxation or errands) while at work rises. This story is consistent with increasing disutility from e as leisure falls or $\partial^2\Gamma(L)/\partial\zeta\partial e > 0$, and the level of e will fall as commutes increase. On the other hand, if someone's leisure time at home ζ is reduced, social life may suffer substantially, which in turn reduces the benefits derived from leisure and leads to less planned activities at home. This decline in quality of social life is likely to reduce the overall demand for personal time and activities. As a result, the benefit from doing home production at work falls because in the case of errands the worker has less overall demand for those activities and in the case of relaxation a substantial amount of time at home is already available for relaxation. Thus, the worker provides higher effort e at work. In the extreme case, the worker has less leisure time at home so his/her wife divorces him/her. Once the divorce goes through, the worker has less household errands to run and most of his/her time at home is spent watching TV and relaxing, which is consistent with $\partial^2\Gamma(L)/\partial\zeta\partial e < 0$.

We can now calculate the efficiency wage. We obtain:

$$w_L F = w_U + \Gamma_U - \Gamma^e(L) + \left[\frac{\Gamma^0(L) - \Gamma^e(L)}{m}\right]\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau L \quad (3.15)$$

This efficiency wage is a generalization of the one obtained in the previous section and in the benchmark case. We find the same effects of w_U, δ, m, e, s and τ on the efficiency wage w_L. However, the impact of L on w_L is not as straightforward as before. In particular, a sufficient condition for $\partial w_L/\partial L > 0$, which is the essence of the efficiency wage to act as a worker's discipline device, is clearly (3.14), i.e. effort and leisure are complement. Indeed, if (3.14) holds, then when there is less unemployment, the surplus of being employed increases (pure incentive effect) and the spatial compensation increases. If (3.13) prevails, i.e. effort and leisure are substitute, then it has to be that the spatial costs (commuting costs) are sufficiently large for $\partial w_L/\partial L > 0$.

There is also a new effect, the impact of time cost ι on the efficiency wage. We have:

$$\frac{\partial w_L}{\partial \iota} = L\,\frac{\partial \Gamma^e(L)}{\partial \zeta} + \frac{L}{m}\left(\frac{\delta N}{N-L} + r\right)\left[\frac{\partial \Gamma^e(L)}{\partial \zeta} - \frac{\partial \Gamma^0(L)}{\partial \zeta}\right]$$

Thus, if (3.14) holds, an increase in transportation time increases the efficiency wage. Indeed, when transport time increases (it takes more time to travel the same distance), leisure is reduced. But if it is relatively more reduced for shirkers (effort and leisure are complement), then firms raise the efficiency wage to meet the Urban Non Shirking Curve. If it is not the case, then the net effect is ambiguous.

The labor equilibrium is then calculated in a standard way (the labor demand is decreasing) and is given by:

$$w_U + e + \frac{e}{m}\frac{\delta N}{N-L} + (1-s)\,\tau N = F'(L) \quad (3.16)$$

3.2. Firms perfectly observe the residential location of all workers

In that case, firms can pay workers according to their location. The bid rent functions of all workers are given by:

$$\Psi_L(x, W_L) = w_L(x) F - \tau x + \Gamma^e(x) - W_L$$

$$\Psi_U(x, W_U) = w_U - s\tau x + \Gamma_U - W_U$$

with

$$\frac{\partial \Psi_L}{\partial x} = w'_L(x)\digamma - \tau - \iota \frac{\partial \Gamma^e(x)}{\partial \zeta}$$

Here, the land rent compensates for (pecuniary) commuting cost, wage and leisure differences between location. If $w'_L(x) < 0$, then the land rent will always be lower farther away from the CBD because remote locations imply both longer commutes, lower wages and less leisure (more time spent in commuting). If, on the contrary, $w'_L(x) > 0$, then land rents can increase or decrease with distance to jobs.

Let us calculate the efficiency wage. The Bellman equations are:

$$r\, I_L^{NS}(x) = w_L \digamma - R(x) - \tau x + \Gamma^e(x) - \delta\,(I_L^{NS} - I_U)$$

$$r\, I_L^{S}(x) = w_L \digamma - R(x) - \tau x + \Gamma^0(x) - (\delta + m)\,(I_L^{S} - I_U)$$

$$r\, I_U(x) = w_U - R(x) - s\tau x + \Gamma_U + a(I_L - I_U)$$

$$\Gamma^e(x) - \delta\,(I_L^{NS} - I_U) = \Gamma^0(x) - (\delta + m)\,(I_L^{S} - I_U)$$

Since firms observe workers' residential location, the efficiency wage is such that, at each location, we have

$$I_L^{NS}(x) = I_L^{S}(x) = I_L(x)$$

which is equivalent to

$$I_L(x) - I_U = \frac{\Gamma^0(x) - \Gamma^e(x)}{m} > 0,\ \forall x \in [0, N] \qquad (3.17)$$

Proposition 6. *At each $x \in [0, N]$, we have:*

(i) *If effort and leisure are substitute, i.e.*

$$\frac{\partial^2 \Gamma(x)}{\partial \zeta \partial e} < 0 \qquad (3.18)$$

then

$$\frac{\partial\,(I_L(x) - I_U)}{\partial L} < 0$$

(ii) *If effort and leisure are complement, i.e.*

$$\frac{\partial^2 \Gamma(L)}{\partial \zeta \partial e} > 0 \qquad (3.19)$$

then

$$\frac{\partial\,(I_L(x) - I_U)}{\partial L} > 0$$

The intuition is close to that of the previous section. When x increases, workers have less leisure but, for the surplus to increase, it has to be that it affects more the shirkers than the non shirkers. Using this relationship, we easily obtain the efficiency wage at each x:

$$w_L(x)F = w_U + \Gamma_U - \Gamma^e(x) + \left[\frac{\Gamma^0(x) - \Gamma^e(x)}{m}\right]\left(\frac{\delta N}{N-L} + r\right) + (1-s)\,\tau x \tag{3.20}$$

We have:

$$w_L'(x)F = \iota\frac{\partial\Gamma^e(x)}{\partial\zeta} - \frac{\iota}{m}\left(\frac{\delta N}{N-L} + r\right)\left[\frac{\partial\Gamma^0(x)}{\partial\zeta} - \frac{\partial\Gamma^e(x)}{\partial\zeta}\right] + (1-s)\,\tau \tag{3.21}$$

Now the relationship between wages and distance to jobs is endogenous and determined by (3.21). Thus, as above, if (3.19) holds, then $w_L'(x) > 0$. If on the contrary, (3.18) prevails, the sign of $w_L(x)$ is ambiguous.

Let us now determine the urban land use equilibrium. Using (3.21), we obtain

$$\frac{\partial\Psi_L}{\partial x} = -\frac{\iota}{m}\left(\frac{\delta N}{N-L} + r\right)\left[\frac{\partial\Gamma^0(x)}{\partial\zeta} - \frac{\partial\Gamma^e(x)}{\partial\zeta}\right] - s\tau$$

The sign is ambiguous because of the three effects (pecuniary commuting, wage and leisure) mentioned above. If (3.19) holds, i.e. effort and leisure are complement, then $w_L'(x) > 0$ and the bid rent can be downward or upward sloping. Indeed, workers living further away have less leisure and more expenses in commuting but have higher wages. On the other hand, if $w_L'(x) < 0$, then bid rents are decreasing. Let us focus on this case. Compared to the unemployed, the bid rent of the employed is steeper and thus they reside closer to jobs. We can calculate the equilibrium values, which are given by:

$$W_L = w_U + \Gamma_U + \left(\frac{\Gamma^0(L) - \Gamma^e(L)}{m}\right)\left(\frac{\delta N}{N-L} + r\right) - s\tau N$$

$$W_U = w_U - s\tau N + \Gamma_U$$

The equilibrium land rent is thus equal to:

$$R^*(x) = \begin{cases} \left(\frac{\delta N}{N-L} + r\right)\frac{1}{m}\left[\Gamma^0(x) - \Gamma^0(L) - (\Gamma^e(x) - \Gamma^e(L))\right] & \\ +s\tau\,(N-x) & \text{for } \; 0 \le x \le L \\ s\tau\,(N-x) & \text{for } \; L < x \le N \\ 0 & \text{for } \; x > N \end{cases}$$

4. Relocation costs[4]

The model developed so far had no relocation costs. Although this assumption is quite frequent in urban economics, its relevance may depend on the nature of the labor market. Indeed, when unemployment and employment spells are short (like e.g. in the U.S), it is not necessarily appealing: low-income households do not necessarily change their residential location as soon as they change their employment status. However, in a European context, long spells of employment and unemployment make it more likely that relocation and labor transitions coincide, in which case our benchmark assumption of absence of mobility costs is relevant.

We assume that mobility costs are so high (which is the case for most low-income households, especially if they live in council houses or housing projects) that once someone is located somewhere he/she never moves. As a result, *a worker's residential location remains fixed as he/she enters and leaves unemployment.* We also assume perfect capital markets with a zero interest rate. When there is a zero interest rate, workers have no intrinsic preference for the present, and thus they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent.

As a result, we consider the average expected utility of a worker rather than the lifetime expected utilities of employed and unemployed workers. At any moment of time, the disposable income (since workers are risk neutral utilities are equal to income) of a worker is thus equal to that worker's average income over the job cycle. The intuition is straightforward. Since workers do not discount, only their average net revenue (or equivalently their utility) matters. Indeed, over their lifetime, workers are totally indifferent between being employed today for T periods and then become unemployed for T' periods, and being unemployed today for T periods and then become employed for T' periods (which is not true if $r > 0$ since the present and the future are discounted differently). In other words, the only thing that matters for them is the fraction of their lifetime they are employed and the fraction of their lifetime they are unemployed and not the order of their employment career. This is why the average net revenue is the same whatever the present employment status of the worker (employed or unemployed).

As before , we assume that changes in the employment status (employment versus unemployment) are governed by a continuous-time Poisson (Markov)

[4] This section is based on Zenou (2005).

process. Therefore, for the unemployment rate of non-shirkers is given by:

$$u \equiv u^{NS} = \frac{\delta}{a+\delta} \tag{4.1}$$

while for the one of shirkers, we have

$$u^{S} = \frac{\delta+m}{a+\delta+m} \tag{4.2}$$

with $u^S > u^{NS}$, $\forall a, \delta, m > 0$.

Since workers have a zero discount rate, they only care about the average income over time. For a non shirker located at a distance x from the CBD, it is equal to:

$$EW_L^{NS}(x) = \left(1-u^{NS}\right)\left[w_L - e - \tau x - R(x)\right] + u^{NS}\left[w_U - s\tau x - R(x)\right] \tag{4.3}$$

whereas for a shirker residing at a distance x from the CBD, it is given by:

$$EW_L^{S}(x) = \left(1-u^{S}\right)\left[w_L - \tau x - R(x)\right] + u^{S}\left[w_U - s\tau x - R(x)\right] \tag{4.4}$$

As above, because information about residential location matters in the wage setting, we will now consider two different cases and thus two different equilibria.

4.1. Firms do not observe the residential location of all workers

Let us calculate the efficiency wage. Since firms know that wages are increasing with x the distance from the CBD, they will set the highest possible wage to prevent shirking, i.e.

$$w_L = w_U + e + \frac{e}{m}\frac{\delta}{u} + (1-s)\,\tau N \tag{4.5}$$

Now wages are not location-dependent and land rents will only compensate for commuting costs. Let us solve the urban land use equilibrium. By plugging (4.5) in (4.3), we obtain the following expected utility of a (non-shirker) worker located at a distance x from the CBD:

$$EW(x) = w_U + \frac{e}{m}\frac{\delta\,(1-u)}{u} + (1-u)\,(1-s)\,\tau N - \left[1-(1-s)\,u\right]\tau x - R(x)$$

If we denote that I the (expected) utility reached by all workers in the city in the unconstrained equilibrium, then the bid rent is equal to

$$\Psi(x, I) = w_U + \frac{\delta\,(1-u)}{u} + (1-u)\,(1-s)\,\tau N - \left[1-(1-s)\,u\right]\tau x - I \tag{4.6}$$

The role of the land rent in the constrained equilibrium is to compensate workers only for commuting costs. We have indeed:

$$\frac{\partial \Psi(x,I)}{\partial x} = -\left[1-(1-s)\,u\right]\tau < 0$$

As above, the utility I is determined by the fact that the bid rent at the city-fringe is equal to zero. We obtain:

$$I = w_U + \frac{e}{m}\frac{\delta\,(1-u)}{u} - s\tau N \tag{4.7}$$

Furthermore, plugging (4.7) in (4.6), we obtain the equilibrium land rent, which is given by:

$$R(x) = \left[1-(1-s)\,u\right]\tau\,(N-x) \tag{4.8}$$

We can now determine the labor market equilibrium. Each firm adjusts employment until the marginal product of an additional worker equals the efficiency wage (4.5) so that

$$w_U + e + \frac{e}{m}\frac{\delta}{u} + (1-s)\,\tau N = F'((1-u)N) \tag{4.9}$$

4.2. Firms perfectly observe the residential location of all workers

Firms know that workers have a zero discount rate so they solve $EV^{NS}(x) = EV^{S}(x)$ at each x, i.e. the average income over time of a non shirker is equal to the one of a shirker. By using (4.3) and (4.4), we easily obtain:[5]

$$w_L(x) = w_U + e + \frac{e}{m}\,(\delta + a) + (1-s)\,\tau x \tag{4.10}$$

Since, at the efficiency wage, no worker shirks, we can use the value of a in (4.1) and plug in (4.10) to obtain:

$$w_L(x) = w_U + e + \frac{e}{m}\frac{\delta}{u^{NS}} + (1-s)\,\tau x \tag{4.11}$$

This efficiency wage has the standard effects of both non-spatial and spatial models (see chapter 4). This implies, as in Chapter 4, that the efficiency wage has two roles: to prevent shirking and to compensate workers for commuting costs.

Now, the main difference here is that mobility costs are very high so that workers always stay in the same location. So when a worker makes his/her

[5] See the Appendix A.2.1 at the end of this chapter for a more rigorous demonstration of the derivation of the efficiency wage (4.10).

decision to shirk or not, he/she trades off longer spells of unemployment but lower effort and lower commuting (if he/she decides to shirk) with longer spells of employment but higher effort and higher commuting costs (if he/she decides not to shirk). As a result, at each location x, the setting of the efficiency wage needs to include these two elements (incentive and spatial compensation). For the spatial compensation element (which varies with x), it has to be that, at each x, the compensation is equal to $\tau x\left(1-u^{NS}+su^{NS}\right)-\tau x\left(1-u^{S}+su^{S}\right)$, i.e. the average commuting cost when non shirking minus the average commuting cost when shirking, which, using (4.1) and (4.2), is exactly equal to $(1-s)\,\tau x$.

It is interesting to compare this efficiency wage with the one obtained in the case of no-relocation costs (chapter 4) and the case of high relocation costs (like here) but same commuting costs for employed and unemployed workers, i.e. $s=1$. In both these models, the equilibrium is such that firms do not observe the residential location of all workers. Recall that in the former, the efficiency wage is equal to:

$$w_L = w_U + e + \frac{e}{m}\frac{\delta}{u^{NS}} + (1-s)\,\tau\left(1-u^{NS}\right)N \tag{4.12}$$

whereas, in the latter, it is given by:

$$w_L = w_U + e + \frac{e}{m}\frac{\delta}{u^{NS}} \tag{4.13}$$

The main difference between (4.11) and (4.12) is that, in the benchmark model of chapter 4, workers are perfectly mobile, thus change location as soon as they change their employment status, and thus employed and unemployed have different lifetime utilities. As a result, because of perfect mobility, all the employed workers are indifferent to reside between 0 and $(1-u)\,N$ (this is the area of the employed) and firms set an efficiency wage at $(1-u)\,N$ since, at this location, the employed and the unemployed pay exactly the same land rent but do not have the same commuting costs, the difference being precisely $(1-s)\,\tau\,(1-u)\,N$. This is the spatial compensation role of the efficiency wage highlighted above. The main difference between (4.11) and (4.13) is that, because employed and unemployed workers bear exactly the same commuting costs, the efficiency wage do not need anymore to compensate for spatial costs and its only role is to prevent shirking. This is why they obtain exactly the same wage as in Shapiro and Stiglitz (1984).

Let us now solve the urban land use equilibrium. We are able to calculate the bid rent of all workers in the city (at the efficiency wage, nobody will shirk in equilibrium). By plugging (4.11) in (4.3), we obtain the following expected utility of a (non-shirker) worker located at x:

$$EW(x) = w_U + \frac{e}{m}\frac{\delta\left(1-u^{NS}\right)}{u^{NS}} - s\tau x - R(x)$$

If we denote that I the (expected) utility reached by all workers in the city, then the bid rent is equal to

$$\Psi(x,I) = w_U + \frac{e}{m}\frac{\delta\left(1-u^{NS}\right)}{u^{NS}} - s\tau x - I \tag{4.14}$$

with

$$\frac{\partial \Psi(x,I)}{\partial x} = -s\tau < 0$$

This equation highlights the role of the land rent in the unconstrained equilibrium, which is to compensate workers for commuting costs and wages. Indeed, workers living further away obtain higher (efficiency) wages but pay higher commuting costs whereas those living closer to the CBD have the reverse trade-off.

In equilibrium, we obtain:

$$I = w_U + \frac{e}{m}\frac{\delta\left(1-u^{NS}\right)}{u^{NS}} - s\tau N \tag{4.15}$$

$$R(x) = s\tau\left(N-x\right) \tag{4.16}$$

We can now determine the labor market equilibrium. For that, we further assume that, when a job is vacant, a firm is always willing to hire a worker whatever his/her location. This means that, once a firm has a vacant job, it is always more profitable to hire the first worker that 'knocks at its door' rather than to wait for the next worker. This is true for all workers, even for the one located at N, i.e. the worker who obtains the highest wage and lives the furthest away from firms.[6] A consequence of this assumption is that each position within a firm will be filled totally randomly by a worker residing between 0 and N.

Since all firms are totally identical, let us focus on a symmetric (steady-state) equilibrium in which each firm employs the same number of workers and pays the same total wage cost than any other firm. This means that each firm

[6] Formally, it suffices to assume that the cost of waiting is sufficiently high compared to the profit that a firm makes on a worker located at N.

j hires $l_j = l = L/M$ workers and pays a total wage cost of $\int_0^N \mathbf{1}_{jx} w_L(x)dx = \frac{L}{M} w_L(\frac{N}{2})$, where $\mathbf{1}_{jx} = 1$ if a worker is hired by firm j at x and zero otherwise.[7]

As a result, each firm adjusts employment until the marginal product of an additional worker equals the average efficiency wage so that we have

$$w_L(N/2) = f'(l)$$

This determines the labor demand in each firm. Using (4.11), the aggregate equivalent is thus given by:

$$w_U + e + \frac{e}{m}\frac{\delta N}{N-L} + (1-s)\frac{\tau N}{2} = F'(L) \tag{4.17}$$

4.3. Comparison between the two equilibria

Let us determine the total welfare in each equilibrium. In the perfect information equilibrium, by taking the sum of all (expected) utilities (workers, absentee landlords and firms), it is easy to verify that the total welfare is given by:[8]

$$\mathcal{W}^{pi} = \left(w_U - s\tau\frac{N}{2}\right)\left(N - L^{pi}\right) - L^{pi}e + F\left(L^{pi}\right) - \tau\frac{N}{2}L^{pi} \tag{4.18}$$

where L^{pi} is given by (4.17). Similarly, for the imperfect information equilibrium, we have:

$$\mathcal{W}^{ii} = \left(w_U - st\frac{N}{2}\right)\left(N - L^{ii}\right) - L^{ii}e + F\left(L^{ii}\right) - \tau\frac{N}{2}L^{ii} \tag{4.19}$$

where $L^{ii} < L^{pi}$ is given by (3.16). Let us compared these two equilibria (perfect and imperfect information). We have the following result.

Proposition 7.

(*i*) *The unemployment rate in the perfect information equilibrium is lower than in the imperfect information equilibrium, i.e.* $0 < u^{pi*} < u^{ii*} < 1$.

(*ii*) *Wages are such that* $w_L^{pi*}(N/2) < w_L^{ii*}$ *and* $w^{pi*}(x) \gtreqqless w^{ii*} \Leftrightarrow x \gtreqqless x_d$, *where*

$$x_d \equiv N - \frac{e\delta}{m(1-s)\tau}\left(\frac{1}{u^{pi*}} - \frac{1}{u^{ii*}}\right) > \frac{N}{2}$$

[7] An easy interpretation is that firms only hire workers from an employment agency, which coordinates firms in a way that they hire workers in a symmetric way.

[8] The superscripts *pi* and *ii* refer to the perfect and imperfect information cases respectively.

Furthermore, when N is small enough, $Ew_L^{pi*}(x) > Ew_L^{ii*}(x)$, $\forall x \in [0, N]$, whereas, when N is large enough, $Ew_L^{pi*}(x) \gtreqless Ew_L^{pi*}(x) \Leftrightarrow x \gtreqless x_{Ed}$, where

$$x_{Ed} = \left(\frac{1-u^{ii*}}{1-u^{pi*}}\right) N - \frac{e\delta}{m\,(1-s)\,\tau}\left[\frac{1}{u^{pi*}} - \frac{(1-u^{ii*})}{(1-u^{pi*})\,u^{ii*}}\right]$$

(iii) *The equilibrium land rent in the imperfect information equilibrium is always higher than in the perfect information equilibrium, i.e. $R^{ii*}(x) > R^{pi*}(x)$, $\forall x \in [0, N[$, while the expected commuting cost is always higher in the perfect information equilibrium, i.e. $E\tau^{pi*}(x) > E\tau^{ii*}(x)$, $\forall x \in [0, N]$.*

(iv) *Workers' expected utility in the perfect information equilibrium is higher than in the imperfect information equilibrium, i.e. $I^{pi*} > I^{ii*} > 0$.*

(v) *In terms of total welfare, we have:*

$$\mathcal{W}^{pi*} \gtreqless \mathcal{W}^{ii*} \Leftrightarrow \frac{F\left(L^{pi*}\right) - F\left(L^{ii*}\right)}{L^{pi*} - L^{ii*}} \gtreqless w_U + e + (1-s)\,\tau\frac{N}{2}$$

The first result (i) is straightforward. Indeed, since firms pay *on average* higher wage costs in the imperfect information equilibrium, they hire less workers and thus the unemployment rate is lower in the perfect information equilibrium. Even though $w_L^{pi*}(N/2) < w^{ii*}$, this does not mean that wages or expected wages are always higher in the imperfect information equilibrium. Figures 5.1a and 5.1b illustrate (ii). Indeed, since $u^{pi*} < u^{ii*}$, then because of the nature of efficiency wages (for which the equilibrium unemployment acts as a worker discipline device), wages should be higher in the perfect information case. However, because firms do not observe residential location, they set the same high wage to all their workers, which implies that wages should be higher in the imperfect information case. So the result is ambiguous. For wages, for residential location close to jobs, w_L^{ii} is higher than w_L^{pi} because the first effect dominates the second one; the reverse is true for remote locations. For expected wages, more weight is put on the first effect since the unemployment rate measures the time spent in unemployment over workers' lifetime. When N is sufficiently small, the second effect is sufficiently reduced so that $Ew_L^{pi}(x) > Ew_L^{ii}(x)$, $\forall x$. When N is larger, there is threshold distance x_{Ed} above which $Ew_L^{pi}(x) > Ew_L^{ii}(x)$.

Figure 5.2 helps us to understand result (iii). In the two panels, we have drawn the expected values of land rents and commuting costs for each equilibrium. First, whatever the residential location, workers bear on average higher commuting costs (basically because they are less often unemployed and commuting is more costly when employed) in the perfect information equilibrium. Second, even if expected wages tend be higher in the perfect information equilibrium, because of higher commuting costs, their capacity to bid for land is reduced and thus the housing price is everywhere higher in the city with imperfect information.

The next result is interesting since it shows that the expected utility of workers is always higher in the perfect information equilibrium, even though their wage is in general lower and their expected commuting costs higher. It is really the sharp reduction in land rents that makes them better off.

Finally, not surprising, this proposition shows that, in terms of welfare, none of the equilibria Pareto dominates the other one. Indeed, workers are better off but absentee landlords are worse off in the perfect information equilibrium and, for firms, it is ambiguous since they hire more people and thus have higher production but pay on average higher wages.

$$[Insert\ Figures\ 5.1a,\ 5.1b\ and\ 5.2\ here]$$

Let us now examine the properties of these two equilibria.

Proposition 8.

(i) *The comparative-statics effects of the unemployment rates and the expected utilities are the same in the two equilibria and are as follows. An increase in the unemployment benefit w_U, the effort e, the job-destruction rate, δ or a decrease in the monitoring rate m, the relative number of CBD-trips for the unemployed s, increases the unemployment rate but has an ambiguous effect on the expected utility in both equilibria while an increase in the commuting cost τ rises unemployment but reduces the expected utility in both equilibria.*

(ii) *The comparative statics effects of the land rent are different. Indeed, in the perfect information equilibrium, an increase in s or τ increases $R(x)$. In the imperfect information equilibrium, because $R(x)$ is affected by the unemployment rate, an increase in w_U, e, δ or a decrease in m, s, reduces the land rent while τ has an ambiguous effect.*

The following comments are in order. First, consider the common effects in both equilibria of the *non-spatial exogenous variables*, w_U, e, δ and m on the *non-spatial endogenous variables,* the unemployment rate and the expected utility. When w_U, e or δ increases or m decreases, the efficiency wage is augmented in order to prevent shirking and thus the unemployment rate increases. However, the effect on the expected utility is ambiguous because there are two opposite forces at work. On the one hand, there is a direct positive effect since wages increase. But, on the other, there is an indirect negative effect since it raises unemployment, which implies that workers will experience more unemployment spells. It is clearly ambiguous because, over their lifetime, workers are richer when employed but spend more time unemployed.

Second, consider the effects of the *spatial exogenous variables*, τ and s on all the endogenous variables, i.e. the unemployment rate, the land rent and the expected utility. In both equilibria, when the commuting cost τ increases (the relative number of CBD-trips for the unemployed s decreases), firms raise (reduce) the efficiency wage to deter shirking. As a result, the unemployment rate increases (decreases). Concerning the land rent, the effects are different between equilibria. In the perfect information equilibrium, the land rent only depends on s and τ and the relationship is positive because an increase in either s or τ rises the competition for land since the access to the CBD becomes more costly. Thus, in the perfect information equilibrium, when τ increases, wages are higher but both the land rent and the unemployment increase. Since these two negative effects dominate the positive effect on wages, the expected utility decreases. For s the effect on expected utility is ambiguous because a rise in s decreases wages and increases land rents but reduces unemployment. Concerning the imperfect information equilibrium, the effects of τ and s are more subtle. Indeed, inspection of (4.8) shows that the land rent is both a direct and indirect (via the unemployment rate) function of τ and s. So, when τ increases, there is a direct effect, which leads to more competition and thus a higher land rent at each location but there is also an indirect effect since it also implies higher wages and thus higher unemployment, which reduces the capacity to bid for land. The net effect is thus ambiguous. However, the effect of τ on the expected utility is negative because the negative unemployment effect dominates the ambiguous land rent effect. For s the two effects on $R(x)$ go in the same direction. Indeed, an increase in s leads to more competition and thus higher land rent, but also lower wage and lower unemployment and thus a higher capacity to bid for land rents. However, the effect of s on the expected utility is ambiguous. Indeed, a rise in s has a negative effect via the increase in land rent but has a positive effect via the decrease in unemployment.

Finally, consider the effects of the *non-spatial exogenous variables*, w_U, e, δ and m on the *spatial endogenous variable*, the land rent $R(x)$. They have no impact in the perfect information equilibrium whereas they strongly affect $R(x)$ in the imperfect information equilibrium via the unemployment rate. Indeed, in the latter, when w_U, e or δ increases or m decreases, the unemployment rate increases. Because workers anticipate that they will experience longer unemployment spells, their capacity to bid for land is lower. There is thus less competition in the land market and, as a result, the land rent at each location decreases.

4.4. Free entry and long-run equilibrium

Let us study the long-run properties of the two equilibria so that the total number of firms M is not fixed by determined by the free-entry condition, i.e. firms freely enter the labor market up to point where profits equal to zero. If c denotes the fixed-entry cost, then, for each equilibrium $k = pi, ii$, the zero-profit condition can be written as:

$$f(l^k) - w_L^k l^k - c = 0 \tag{4.20}$$

Observe that, for the perfect information equilibrium, the free-entry condition is set for $w_L^{pi} = w_L(N/2)$. As a result, as in Chapter 4, for each equilibrium $k = pi, ii$, the long-run equilibrium is characterized by four unknowns, l^k, w_L^k, u^k, M^k and the four following equations:

$$w_L^k = f'(l^k) \tag{4.21}$$

$$f(l^k) - w_L^k l^k - c = 0 \tag{4.22}$$

$$w_L^k = w_U + e + \frac{e}{m}\frac{\delta}{u^k} + \mathbf{B}^k (1-s)\tau N \tag{4.23}$$

$$\left(1-u^k\right) N = M^k l^k \tag{4.24}$$

where $\mathbf{B}^{pi} = 1/2$ and $\mathbf{B}^{ii} = 1$ and $w_L^{pi} = w_L(N/2)$. Here is the way each equilibrium is solved. First, (4.21) gives the labor demand of each firm as a function of wage, $l^k(w_L^k) = f'^{-1}(w_L^k)$. Then, by plugging this value in (4.22), we obtain the wage $w_L^k(c)$. Observe that, in the perfect information case, it is only the wage at $x = N/2$, that is $w_L^{pi}(N/2)$, which is a function of c; all wages for other locations x are still given by (4.11). Next, by plugging $w_L^k(c)$

in (4.23), we obtain $u^k(c, w_U, e, m, \delta, s, \tau, N)$ and finally, by using (4.24), we get $M^k(c, w_U, e, m, \delta, s, \tau, N)$.

We have the following straightforward result.

Proposition 9. *In the long run, we have:*

$$w_L^{pi}(N/2) = w_L^{ii} \equiv w_L(c)$$

$$u^{ii} > u^{pi}$$

$$M^{ii} < M^{pi}$$

First, the reason why the long-run wage is constant and only depends on c in the imperfect information equilibrium is that, in the long-run, M adjusts via entry and exit to give zero profits. So, for example, if τ, the commuting cost increases, because the free-entry condition has to hold, aggregate labor demand shifts in tandem with the urban non-shirking curve in such a way as to keep the equilibrium efficiency wage constant. For the perfect information equilibrium, we still have a wage distribution because of workers' heterogeneity in terms of location but, the wage at $x = N/2$, i.e. $w_L^{pi}(N/2)$, which is used for the profit function and thus the free entry condition, is constant. So, if τ increases, the whole wage distribution shifts in such a way as to keep the wage cost per firm equal to $l^{pi} w_L^{pi}(N/2) = l^{pi} w_L(c)$, and the aggregate labor demand shifts in tandem with the urban non-shirking curve in such a way as to keep the equilibrium efficiency wage $w_L^{pi}(N/2)$ constant. Second, because on average wage costs are higher in the imperfect information equilibrium, unemployment is higher, which leads to less entry so that the total number of firms is less than in the perfect information equilibrium. Finally, all the other results obtained in the short-run case are still valid but the mechanisms are different because of the free-entry condition, which implies that, in the long run, firms adjust via entry and exit.

5. Effort, leisure and relocation costs[9]

We would like to reconsider the model of section 3 by introducing high-relocation costs as in section 4. This will, in particular, allows us to derive rather than to assume the fact that $e'(x) < 0$, i.e. the greater the distance to work, the lower the effort level.

[9] This section is based on Ross and Zenou (2005).

Here again we consider two cases, depending on the structure of information. Observe that, since a worker spends a fraction $1-u=a/(a+\delta)$ of his/her lifetime employed and a fraction $u=\delta/(a+\delta)$ unemployed, at any moment of time, the disposable utility of a worker located at x is thus equal to that worker's average utility over the job cycle and is given by

$$\begin{aligned} I &= (1-u)W_L(x,e)+uW_U(x) \\ &= (1-u)\left[w_L F - R(x) - \tau x + \Gamma^e(x)\right] - R(x) - \tau x + u\left[w_U + \Gamma_U\right] \end{aligned} \tag{5.1}$$

Also, because we would consider cases where workers may shirk in equilibrium, we now assume that shirkers provide an effort of $\underline{e}>0$, instead of zero as in the previous cases, while for non-shirkers, the effort is $\overline{e}>\underline{e}>0$. The contribution to production is equal to $\overline{e}$ and $\underline{e}$ respectively.

5.1. Firms do not observe the residential location of all workers

All employed workers in the city have to obtain the same utility level. Let us focus on an equilibrium in which some workers may shirk in equilibrium. Using (5.1), and given that all workers obtain the same wage, this implies that the expected indirect utilities of non-shirker and shirker workers are respectively equal to:

$$I^{NS}(x,\overline{e}) = (1-u^{NS})\left[wF + \Gamma^{\overline{e}}(x)\right] - R(x) - \tau x + u^{NS}\,\Gamma_U$$

$$I^{S}(x,\underline{e}) = (1-u^{S})\left[wF + \Gamma^{\underline{e}}(x)\right] - R(x) - \tau x + u^{S}\,\Gamma_U$$

where

$$\Gamma^{\overline{e}}(x) \equiv \Gamma\left(1 - F - \iota x, \overline{e}\right) \text{ and } \Gamma^{\underline{e}}(x) \equiv \Gamma\left(1 - F - \iota x, \underline{e}\right)$$

These are simply weighted averages of the utility levels when employed and unemployed where the share of time spent unemployed is used to form the weights.

All workers (shirking or not shirking) must in equilibrium obtain the same utility level I, which is location independent. Since workers stay in the same location all their life, bid rents are given by:

$$\Psi^{NS}(x,I) = (1-u^{NS})\left[wF + \Gamma^{\overline{e}}(x)\right] - \tau x + u^{NS}\,\Gamma_U - I \tag{5.2}$$

$$\Psi^{S}(x,I) = (1-u^{S})\left[wF + \Gamma^{\underline{e}}(x)\right] - \tau x + u^{S}\,\Gamma_U - I \tag{5.3}$$

Inspection of these two equations shows that, as usual, the bid-rent functions are decreasing in x, with $\partial\Psi^{NS}(x,I)/\partial x < 0$ and $\partial\Psi^{S}(x,I)/\partial x < 0$. In the present model, this reflects the combined influence of the time cost of commuting and the monetary cost. Let us denote by x_s the border between non-shirkers and shirkers. We have the following result.

Proposition 10.

(i) *If effort and leisure are complements, i.e.*

$$\frac{\partial^2 \Gamma(\zeta, e)}{\partial \zeta \partial e} > 0 \tag{5.4}$$

then, workers who reside close to jobs will choose not to shirk whereas workers located farther away will shirk.

(ii) *If effort and leisure are substitutes, i.e.*

$$\frac{\partial^2 \Gamma(\zeta, e)}{\partial \zeta \partial e} < 0 \tag{5.5}$$

then the location pattern of shirkers and non-shirkers is indeterminate. However, if we assume something stronger, that is:

$$(a + m + \delta) \frac{\partial \Gamma^{\overline{e}}}{\partial \zeta} |_{x=x_s} < (a + \delta) \frac{\partial \Gamma^{\underline{e}}}{\partial \zeta} |_{x=x_s} \tag{5.6}$$

then workers who reside close to jobs will choose to shirk whereas workers located farther away will not shirk.

As it can be seen from this proposition, the crucial assumption is whether $\partial^2 \Gamma(\zeta, e) / \partial \zeta \partial e$ is positive or negative. Neither sign can be ruled out using reasonable restrictions on preferences.[10] If $\partial^2 \Gamma(\zeta, e) / \partial \zeta \partial e > 0$, then disutility from e increases as leisure falls. If this assumption holds, workers residing close to jobs will provide more effort than those residing further away from jobs because they have lower commuting time and thus more leisure time at home. This provides a rationale of the assumption $e'(x) < 0$ made in section 2.

On the other hand, if $\partial^2 \Gamma(\zeta, e) / \partial \zeta \partial e < 0$ is assumed, the location pattern is less obvious. Indeed, workers residing close to jobs have lower commute time and thus more leisure time at home and, because $\partial^2 \Gamma(\zeta, e) / \partial \zeta \partial e < 0$, provide less effort. So they are more likely to be shirkers and spend a greater fraction of their time unemployed. On the other hand, unemployment offers the consumers a savings in terms of no commutes during the spell of

[10] See Section 3 for more intuition on these assumptions.

unemployment, and the benefit of these savings are larger away from the CBD. Accordingly, the overall unemployment cost of shirking is lower near the edge of the urban area, which implies less effort in those locations. The net sign is thus ambiguous. If however (5.6) holds, which means that the unemployment spells are not too long (because for example the monitoring rate m is quite low), then the shirkers will live close to jobs.

Let us now determine x_s, the boundary between shirking and non-shirking workers where a consumer is indifferent between high and low levels of effort at work. To obtain the value of x_s, we have to solve: $\Psi^{NS}(x_s, I) = \Psi^S(x_s, I)$, which is equivalent to:

$$(1 - u^S)\Gamma^{\underline{e}}(x_s) - (1 - u^{NS})\Gamma^{\overline{e}}(x_s) = \left(u^S - u^{NS}\right)\left(w\digamma - \Gamma_U\right) \tag{5.7}$$

showing a clear trade off between shirking (higher utility when employed since less effort but more unemployment spells during the lifetime) and nonshirking. We have the following result.

Proposition 11.

(*i*) *If (5.4) holds, then, higher wages implies less shirking in the city, i.e.* $\partial x_S / \partial w_L > 0$.

(*ii*) *If (5.6) holds, then higher wages reduces shirking in the city, i.e.* $\partial x_s / \partial w_L < 0$.

This proposition states that wages affect x_s the border between shirkers and non-shirkers. Indeed, if (5.4) holds, i.e. effort and leisure are substitutes, then when wages are higher, less workers shirk (the fraction of shirkers $N - x_s$ decreases) since there are more incentive not to shirk (the average wage difference $w\digamma(u^S - u^{NS})$ between shirkers and non-shirkers increases). If effort and leisure are complements and the difference in employment rates between the shirkers and the nonshirkers is not too large (5.6), then shirkers outbid non-shirkers for central locations and higher wages reduce the faction of shirkers.

There are thus two types of urban land use equilibria. If (5.4) holds, then the first emerges in which nonshirkers reside close to jobs while shirkers reside further away from jobs. If (5.6) holds, then the other equilibrium prevails in which shirkers live close to the CBD while nonshirkers reside at the outskirts of the city.

Let us now determine the labor market equilibrium for each type of equilibrium. There are M firms in the economy. The profit function of a typical firm can be written as:

$$\Pi = F\left(\phi\left[L^{NS}\,\overline{e} + L^{S}\,\underline{e}\right]\right) - w_L \phi L$$

where ϕ is the fraction of workers hired by each firm and where the total number of non-shirkers in the economy is given by

$$L^{NS} = x_s(1 - u^{NS}) \tag{5.8}$$

the total number of non-shirkers is

$$L^{S} = (N - x_s)(1 - u^{S}) \tag{5.9}$$

and the total number of employed workers is $L = L^S + L^{NS}$. Since firms do not observe the residential location, all workers, whatever their location, obtain the same wage. We focus on a symmetric equilibrium in which each firm employs the same fraction ϕ of workers (shirkers and nonshirkers). Thus, even if firms know that all workers residing beyond x_s will shirk, they have to pay them the same wage as the ones who live between 0 and x_s (nonshirkers). Let us now solve the firm's program. By taking $\overline{e}$, $\underline{e}$, u^S and u^{NS} as given, each firm chooses w_L and ϕ that maximize its profit. When choosing w_L firms will face the following trade off. Because it affects x_s, higher wages implies that the fraction of shirkers hired will be lower and thus total output increases but labor costs are also higher since the wage given to workers is the same. When choosing ϕ firms face the following trade off. Higher ϕ means that more workers are higher; thus higher output but also higher labor costs.

It can be shown that the optimal wage is given by:

$$w\frac{\partial x_s}{\partial w} = \frac{L\left[(N - x_s)(1 - u^S)\,\overline{e} + x_s(1 - u^{NS})\,\underline{e}\right]}{(\overline{e} - \underline{e})\left[(1 - x_s)(1 - u^S)^2 + x_s(1 - u^{NS})^2\right]} \tag{5.10}$$

In equilibrium, it has to be that labor supply equals labor demand for nonshirkers and shirkers respectively. These conditions can be written as:

$$\phi M L^{NS} = (1 - u^{NS})x_s$$

$$\phi M L^{S} = (N - x_s)(1 - u^{S})$$

Using (5.8) and (5.9), this implies that

$$M = \frac{1}{\phi}$$

5.2. Firms perfectly observe the residential location of all workers

The utility of each worker is still given by (5.1). Now, as we will show in the labor market analysis, there will be no shirking in equilibrium. This implies that the unemployment rate of the economy is given by

$$u^{NS} = \frac{\delta}{\theta + \delta} \tag{5.11}$$

Furthermore, the bid rent of a (non-shirker) worker is equal to

$$\Psi(x, I) = (1 - u^{NS}) \left[w_L(x) F + \Gamma^{\overline{e}}(x) \right] - \tau x + u^{NS} \, \Gamma_U - I \tag{5.12}$$

Inspection of this equation shows that

$$\frac{\partial \Psi(x, I)}{\partial x} = (1 - u^{NS}) \left[w'(x) F - \frac{\partial \Gamma^{\overline{e}}}{\partial \zeta} \iota \right] - \tau \tag{5.13}$$

which can be positive or negative depending on the sign of $w'(x)$ (it will determined below in the labor market analysis).

Since all workers provide the same effort level and are identical in all respects, they just locate anywhere in the city and enjoy the same utility level I, the land rent adjusting for commuting cost differences between different locations.

To close the urban equilibrium, we have to check that $\Psi(N, I) = 0$, which is equivalent to:

$$I = (1 - u^{NS}) \left[w(N) F + \Gamma^{\overline{e}}(N) \right] - \tau + u^{NS} \, \Gamma_U \tag{5.14}$$

Let us now solve the labor market equilibrium. At each location in the city ($0 \leq x < N$), each firm has to set a NSC (that equates shirking and non-shirking utilities) to prevent shirking. At each x, we obtain:

$$w_L(x) = \frac{(1 - u^S)\Gamma^{\underline{e}}(x) - (1 - u^{NS})\Gamma^{\overline{e}}(x)}{F (u^S - u^{NS})} + \frac{\Gamma_U}{F} \tag{5.15}$$

This is the standard Shapiro-Stiglitz style non-shirking condition evaluated in equilibrium for every residential location x. It should be clear here that, when firms observe the residential location of workers, it is optimal for them not to allow shirking in equilibrium. In the previous model, this was not possible since each firm had to give to all its workers the same wage and thus it was somehow optimal to let some workers shirk.

Proposition 12.

(*i*) *Assume (5.4). Then,* $w_L'(x) > 0$.

(*ii*) *Assume (5.6). Then,* $w_L'(x) < 0$.

This result is quite intuitive. If leisure and effort are substitutes (i.e. (5.4) holds), then wages have to compensate workers who live further away since they commute more and thus have less time for leisure at home. If this is not the case and (5.6) holds (which is more that leisure and effort are complements), then firms have to compensate workers who live closer to jobs for the time they spend employed because they value less leisure.

Using (5.13), Proposition 12 implies that when (5.4) holds, $w_L'(x) > 0$ and thus the sign of $\partial\Psi(x, I)/\partial x$ is ambiguous. This is because there are two opposite effects. On the one hand, workers residing far away have higher wages. On the other, they have higher monetary commuting costs and also higher time costs and thus lower leisure. The compensation of the land rent is therefore not straightforward. The following condition guarantees that land rents to decrease from the center to the periphery:

$$0 < \frac{\partial\Gamma^{\overline{e}}}{\partial\zeta} - \frac{\partial\Gamma^{\underline{e}}}{\partial\zeta} < \frac{\tau\, m}{\iota\, a} \tag{5.16}$$

This condition encompasses (4.22).

Here again there are two equilibria. If (5.16) hold, we have Equilibrium 1 while if (5.6) holds, we have Equilibrium 2. Let us determine the labor market equilibrium in each urban land use pattern. Let us define the labor demand ϕ of each firm. Firms solve the following program:

$$\max_{\phi} \left[\Pi = F(\phi L\overline{e}) - \phi L \int_0^N w_L(x)dx \right]$$

First order condition yields

$$\overline{e}F'(\phi L\overline{e}) = \int_0^N w_L(x)dx \tag{5.17}$$

Equilibrium condition (labor demand equals labor supply):

$$L = 1 - u^{NS} = \frac{a}{a + \delta} \tag{5.18}$$

We focus on a symmetric labor market equilibrium in which each firm employs the same number of workers $\phi L = L/M$ so that, again,

$$M = \frac{1}{\alpha} \tag{5.19}$$

6. Conclusion

We have developed different extensions of the benchmark urban efficiency wage model (Chapter 4) to see how results change with different assumptions. One of the main implication is that the structure of information on workers' residential location crucially matters in the efficiency wage setting. Also, introducing high-relocation costs has important implication both in terms of urban and land-market equilibria.

References

[1] Ross, S. and Y. Zenou (2005), "Shirking, commuting and labor market outcomes," mimeo.

[2] Zenou, Y. (2002), "How do firms redline workers?" *Journal of Urban Economics* 52, 391-408.

[3] Zenou, Y. (2005), "Efficiency wages and unemployment in cities: The case of high-relocation costs," *Regional Science and Urban Economics*, forthcoming.

A. Appendix A.2.1. Bellman equations and efficiency wages when $r \to 0$

Let us calculate the efficiency wage (4.10) using the Bellman equations. When $r > 0$, the standard (steady-state) Bellman equations for the non-shirkers, the shirkers and the unemployed are respectively given by:

$$r\,I_L^{NS} = W_L^{NS} - \delta\,(I_L^{NS} - I_U) \tag{A.1}$$

$$r\,I_L^{S} = W_L^{S} - (\delta + \theta)\,(I_L^{S} - I_U) \tag{A.2}$$

$$r\,I_U = W_U + a(I_L^{NS} - I_U) \tag{A.3}$$

where r is the discount rate, I_1^{NS}, I_1^{S} and I_0 represent respectively the expected lifetime utility of a non-shirker, a shirker and an unemployed worker, and where $W_L^{NS} = w_L - e - \tau x - R(x)$, $W_L^{S} = w_L - \tau x - R(x)$ and $W_U = w_U - s\tau x - R(x)$.

First, observe that when $r \to 0$, workers are infinitely patient and the value functions of I_1^{NS}, I_1^{S} and I_0 become infinite since there is no more discounting. However, when $r \to 0$, rI_1^{NS} and rI_0 have finite values and in fact $rI_1^{NS} = rI_0$. Indeed, by combining (A.1) and (A.3), we obtain:

$$rI_L^{NS} = \frac{(r+a)(w_L - e) + \delta\, w_U - (r + \delta\, s + a)\tau x}{r + \delta + a} - R(x) \tag{A.4}$$

$$rI_U = \frac{a(w_L - e) + (r+\delta)\, w_U - (r\, s + \delta\, s + a)\tau x}{r + \delta + a} - R(x) \tag{A.5}$$

so that

$$\lim_{r \to 0} rI_L^{NS} = \lim_{r \to 0} rI_U = \frac{a(w_L - e) + \delta\, w_U - (\delta\, s + a)\tau x}{\delta + a} - R(x) \tag{A.6}$$

which has obviously a finite value. This is very logical since, when $r \to 0$, what matters is not the current employment status but the fraction of time one spends in each state.

The efficiency wage is set as follows. Firms set a wage such that the lifetime discounted expected utility of non-shirking is equal to that of shirking, i.e. $rI_L^{NS} = rI_L^{S} = rI_U$. Let us first derive rI_L^{S}. By combining (A.2) and (A.3), we have:

$$rI_L^{S} = \frac{(r+a)w_L + (\delta+\theta)\, w_U - (r + \delta\, s + a + \theta\, s)\tau x}{r + \delta + a + \theta} - R(x) \tag{A.7}$$

with

$$\lim_{r \to 0} rI_L^{S} = \frac{aw_L + (\delta+\theta)\, w_U - (\delta\, s + a + \theta\, s)\tau x}{\delta + a + \theta} - R(x) \tag{A.8}$$

In our context, rI_L^{NS} and rI_L^S are well-defined functions since when $r \to 0$ they have finite values. By combining (A.6) and (A.8), we obtain the following efficiency wage:

$$w_L(x) = w_U + e + \frac{e}{\theta}(\delta + a) + (1 - s)\,\tau x \tag{A.9}$$

Figure 5.1a: Comparison of wages between equilibria

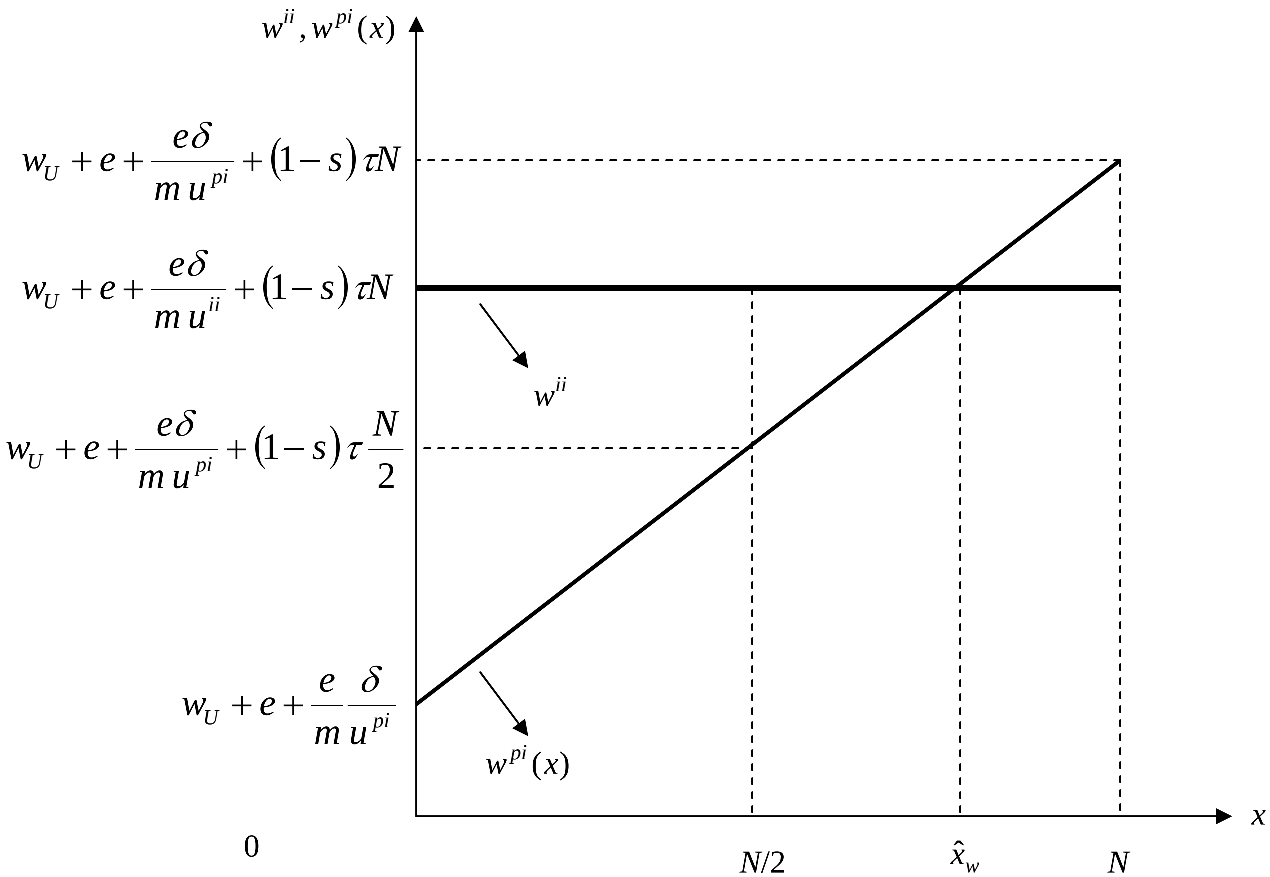
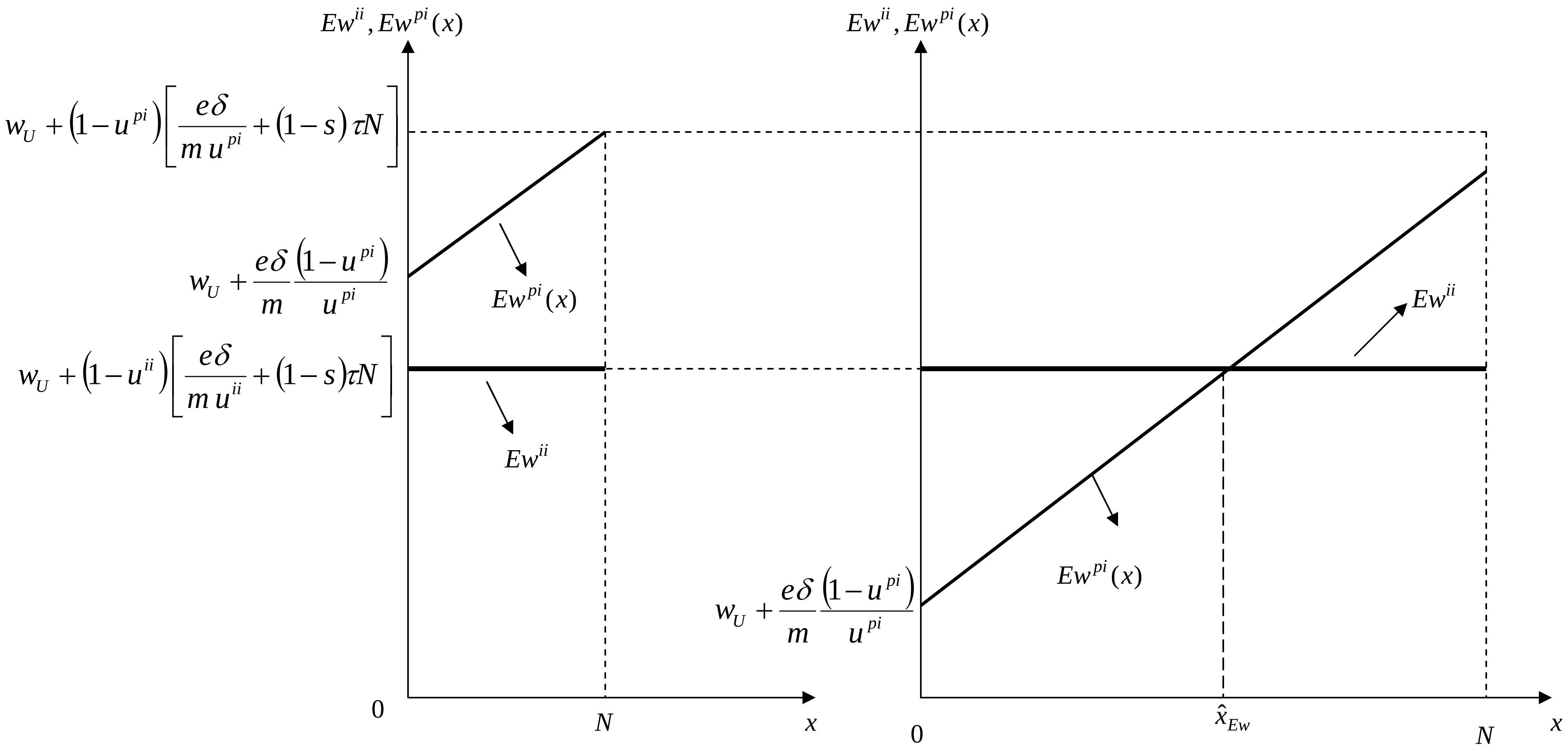


Figure 5.1b: Comparison of expected wages between equilibria



Case of *N* small

Case of *N* large

Figure 5.2: Comparison of (expected) land rents and expected commuting costs between equilibria