Effect of Transmission Constraints on a Single Price Electricity Market

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Abstract

In this paper, I analyze the effect of transmission constraints on a single price electricity market. When the transmission line is congested, the equilibrium price and firms' payoff increase. An increase in transmission capacity reduces equilibrium prices and the payoff of the firm located in the high demand region, by contrast the effect on the payoff function of the firm located in the low demand region is non-monotone. I have characterized the equilibrium for uniform and discriminatory price auctions and I have introduced the concept of efficiency in transmission to analyze the performance of both types of auctions when the transmission line is congested.

KEYWORDS: electricity auctions, transmission constraint, market design.

1 Introduction

Electricity transmission facilities have long been recognized as central elements in the efficient planning and operation of electricity systems. Traditionally, the role of large, interutility transmission paths has been to permit transactions between utilities that exploit regional differences in consumption seasonality and generation costs. As the electricity generation industry is deregulated, however, transmission facilities will also provide important competitive links between potentially isolated markets, thus mitigating the potential for exercise of market power. In particular, transmission line congestion increases equilibrium prices around 5% (Offer 1995; Bohn et al., 1999).

There exists an open debate to analyze the perform of uniform and discriminatory auctions in electricity markets. When the firms are asymmetric in generation cost and the transmission line is not congested,[1] the uniform price auction generates higher equilibrium prices but the most efficient firm is dispatched first with certainty. By contrast, the discriminatory price auction generates lower equilibrium prices but the most efficient firm

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[1] The term "congested" is used throughout this article in the electrical engineering sense: a line is congested when the flow of power is equal to the line's capacity, as determined by engineering standards.

is dispatched last in the auction with some positive probability Fabra et al. (2006). However, when the transmission line is congested and there are losses during the transmission process, a new criteria to evaluate the performance of auctions need to be introduced. In particular, the auction that prioritizes in the dispatch the firm located in the high demand region performs better because less electricity flow through the grid (and so less losses).[2] I name this criteria, efficiency in transmission criteria

In this paper, based on efficiency in transmission and equilibrium prices' criteria, I analyze and classify the performance of uniform and discriminatory auctions in the presence of transmission constraint on single price electricity markets.

My analysis proceeds by first extending a simple duopoly model similar to the one in Fabra et al. (2006), which is then varied in several directions. In the basic set up, two suppliers with symmetric capacities and (marginal) costs, are allocated in two different regions[3] (North and South) connected by a transmission line. The two firms face a demand in each region that is assumed to be perfectly inelastic and known with certainty when suppliers submit their offer prices. Each supplier must submit a single price offer for its entire capacity. The assumption of price-inelastic demand can be justified by the fact that the vast majority of consumers purchase electricity under regulated tariffs that are independent of the prices negotiated in the wholesale market, at least in the short run.

The assumption that suppliers have perfect information concerning market demand is reasonable when applied to markets in which offers are "short lived", such as in Spain, where there are 24 hourly day-ahead markets each day. In such markets suppliers can be assumed to know the total demand they face in any period with a high degree of certainty. In markets in which offer prices remain fixed for longer periods, e.g., a whole day, like in Australia and in the former markets in England and Wales, on the other hand, it is more accurate to assume that suppliers face some degree of demand uncertainty, or volatility, at the time they submit their offers.

Expectedly, when the transmission line is congested, the equilibrium price and firms' payoff increases. An increase in transmission capacity induces a reduction on equilibrium prices and on the payoff of the firm located in the high demand region. By contrast, an increase in transmission capacity modifies the payoff of the firm located in the low demand region in a non-monotonic pattern. When the transmission capacity is low, an increase in transmission capacity increases the size of the market that the firm located in the low demand region faces. The increase in market size dominates the decrease in expected bid and the payoff increases. However, when the transmission capacity is big enough, the decrease in bids dominates and the payoff decreases. This non-monotonic pattern on the payoff function could have important implications on generation capacity

[2]In this paper, I assume that there is no loss of electricity during the transmission process. However, it can be easily modified to introduce losses, in such a case, the auction that prioritize in the dispatch the firm located in the high demand region performs better. Therefore, when the transmission line is congested, in addition to the criteria of low price equilibrium and efficiency in generation established in Fabra et al. (2006) to evaluate the performance of an auction, a new criteria, efficiency in transmission, should be introduced to evaluate the performance of an auction.

[3]In this paper, I have assumed a single price electricity market. Therefore, there exists a single market and so, it has no sense to speak of different markets. Hence, throughout the paper, I refer always to regions instead of markets.

investment decisions and so on equilibrium outcomes.

When the transmission line is not congested, as in Fabra et al. (2006), I have found that the discriminatory price auction outperforms uniform price auction because it generates lower equilibrium prices. When the transmission line is congested, the uniform auction generates higher equilibrium prices and the firm located in the high demand region is dispatched last with certainty. By contrast, the discriminatory auction generates lower equilibrium prices and the firm located in the high demand region is dispatched first with positive probability. In case of loss during the transmission, the discriminatory price auction performs better both in terms of prices equilibrium and in terms of efficiency in transmission (lower transmission through the grid and so lower electricity loss).

Pioneering research on electricity markets in which transmission lines are congested was done by Schweppe et al. (1988). They concluded that the short-term price of transmission services between any two locations is the difference of spot prices between those two points. Hogan (1992) introduces the concept of contract network which provides a mechanism for allocating long-term transmission capacity rights subject to maintaining short-run price efficiency. Chao and Peck (1996) use the physical rights approach to incorporate network externality impacts into the competitive trading mechanism. When competition in the spot electricity market is perfect, the mechanisms proposed by (Hogan, 1992) and (Chao and Peck, 1996) generates the efficient equilibrium predicted by Schweppe et al. (1988). However, as Joskow and Tirole (2000) have shown, when competition in the spot electricity market is imperfect, the way in which transmission rights are assigned modifies the equilibrium outcome on the spot electricity market.

Joskow and Tirole (2000) assume in their analysis that the equilibrium price in one of the markets is a parameter. Under this assumption, they work out the equilibrium in the other market using two types of transmission rights, financial and physical. Borenstein et al. (2000) work out the equilibrium when the firms compete in quantities and financial transmission rights are assigned to the grid operator.

The previous models work out the equilibrium assuming congestion in the transmission line and nodal pricing[4]. By contrast, Fabra et al. (2006) characterize the equilibrium in a single price electricity market when uniform and discriminatory auctions are run by the auctioneer and the firms are asymmetry in capital and generation costs and the transmission line is not congested. In this paper, I characterize the equilibrium in a single price electricity market when firms are symmetric and the transmission line is congested.

The article proceeds as follows, in section two, I describe the model. In section three, I characterize the equilibrium when the transmission line is and it is not congested. Section four concludes. Proofs are in the Appendix.

2 The model

Set up of the model. There exist two electricity regions, region North and region South, that are connected by a transmission line with capacity T.

[4]Nodal electricity markets are the ones in which the equilibrium price differ across markets when the transmission line is congested

There exist two duopolists with capacities k_n and k_s, where subscript n means that the firm is located in region North and subscript s means that the firm is located in region South. Suppliers' marginal costs of production are c_n and c_s. In this paper I analyze the effect that asymmetries in the access to demand has on equilibrium. In order to focus in this effect, I will assume that firms are symmetric in capital $k_n = k_s = k > 0$ and symmetric in costs $c_n = c_s = c = 0$. The level of demand in any period, θ_n in region North and θ_s in region South, is a random variable uniformly distributed that is independent across markets[5] and independent of market price, i.e., perfectly inelastic. In particular, $\theta_i \in [\underline{\theta}_i, \overline{\theta}_i] \subseteq [0, k+T]$ is distributed according to some known distribution function $G(\theta_i)$, $i = n, s, i \neq j$

The capacity of the transmission line can be lower than the installed capacity in each market $T \leq k$, i.e. the transmission line could be congested for some realization of demands (θ_s, θ_n).

Timing of the game. Having observed the realization of demands $\theta \equiv (\theta_s, \theta_n)$, each supplier simultaneously and independently submits a bid specifying the minimum price at which it is willing to supply up to its capacity, $b_i \leq P$, $i = n, s$, where P denotes the "market reserve price", possibly determined by regulation.[6] Let $b \equiv (b_s, b_n)$ denote a bid profile. On the basis of this profile, the auctioneer calls suppliers into operation. If suppliers submit different bids, the lower-bidding supplier's capacity is dispatched first. Without loss of generality, assume that $b_n < b_s$. If the capacity of supplier n is not sufficient to satisfy the total demand $(\theta_s + \theta_n)$ in the case of the transmission line not congested, or $(\theta_n + T)$ in the case of the transmission line congested,[7] the higher-bidding supplier's capacity, firm s, is then dispatched to serve residual demand, $(\theta_s + \theta_n - k)$ if $(\theta_s > k - \theta_n$ and $\theta_n \in [k - T, k])$, or $(\theta_s - T)$ if $(\theta_s > T$ and $\theta_n \in [0, k - T])$. If the two suppliers submit equal bids, then supplier i is ranked first with probability ρ_i, where $\rho_n + \rho_s = 1$, $\rho_i = 1$ if $\theta_i > \theta_j$, and $\rho_i = \frac{1}{2}$ if $\theta_i = \theta_j$, $i = n, s$, $i \neq j$. The tie breaking rule implemented is such that if the bids of both firms are equal and the demand in region i is greater than the demand in region j, the auctioneer dispatches first the supplier located in region i.

The output allocated to supplier $i, i = n, s$, denoted by $q_i(\theta, b)$, is given by

[5] In the majority of electricity markets, demand in one market is higher than in the other market. Moreover, there exists the possibility of some type of correlation between demands across markets. In this paper, I assume uniform distribution and independence of demand. However, the model can be easily modified to introduce different distributions of demand and correlation between demands across markets.

[6] P can be interpreted as the price at which all consumers are indifferent between consuming and not consuming, or a price cap imposed by the regulatory authorities. See von der Fehr and Harbord (1993, 1998).

[7] When the demand in market South is larger than the transmission line capacity $\theta_s > T$, firm n can only satisfy the demand in its own region and T units of demand in market South $(\theta_n + T)$. Below in this section, I will explain with detail the total demand that can be satisfied by each firm and the residual demand that can be satisfied by each firm.

$$q_i(b;\theta,T) = \begin{cases} min\{\theta_i+\theta_j, \theta_i+T, k_i\} & \text{if } b_i < b_j \\ \rho_i min\{\theta_i+\theta_j, \theta_i+T, k_i\} + & \\ \qquad [1-\rho_i] max\{0, \theta_i - T, \theta_i+\theta_j-k_j\} & \text{if } b_i = b_j \\ max\{0, \theta_i - T, \theta_i+\theta_j-k_j\} & \text{if } b_i > b_j \end{cases} \tag{1}$$

The output function has an important role determining the equilibrium, therefore I will explain it in greater detail. Below, I describe the construction of firm n's output function, the one for firm s is symmetric.

The total demand that can be satisfied by firm n when it submits the lower bid ($b_n < b_s$) is defined by $min\{\theta_n+\theta_s, \theta_n+T, k\}$. The realization of (θ_s, θ_n) determines three different areas (left panel in figure 1).

$$min\{\theta_n+\theta_s, \theta_n+T, k\} = \begin{cases} \theta_s+\theta_n & \text{if } \theta_n \leq k-\theta_s \text{ and } \theta_s < T \\ \theta_n+T & \text{if } \theta_n < k-T \text{ and } \theta_s > T \\ k & \text{if } \theta_n > k-\theta_s;\ \theta_s \in [0,T] \\ & \text{or if } \theta_n > k-T;\ \theta_s \in [T, k+T] \end{cases}$$

When demand in both regions is low, firm n can satisfy the total demand $(\theta_s+\theta_n)$. If the demand in region South is greater than the transmission capacity $\theta_s > T$, firm n cannot satisfy the demand in region South even when it has enough generation capacity to do so, therefore the total demand that firm n can satisfy is (θ_n+T). Finally, if the demand is big enough the total demand that firm n can satisfy is its own generation capacity.

The residual demand that firm n satisfies when it submits the higher bid ($b_n > b_s$) is defined by $max\{0, \theta_n-T, \theta_s+\theta_n-k\}$. The realization of (θ_s, θ_n) determines three different cases (right panel in figure 1).

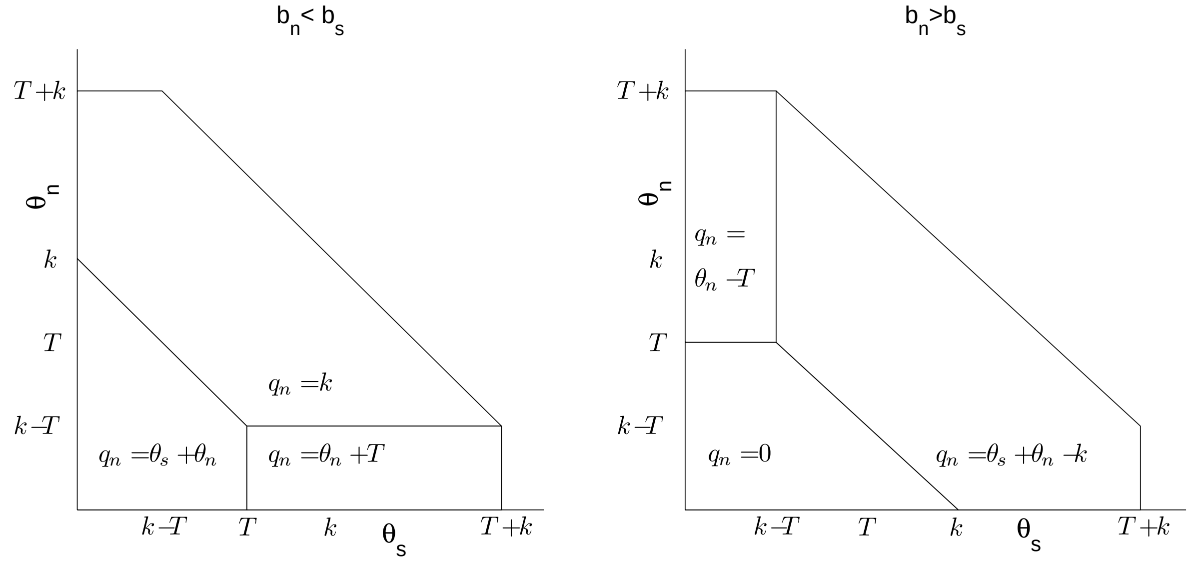
$$max\{0, \theta_n-T, \theta_s+\theta_n-k\} = \begin{cases} 0 & \text{if } \theta_n < T;\ \theta_s \in [0, k-T] \\ & \text{or } \theta_n < k-\theta_s;\ \theta_s \in [k-T, k] \\ \theta_n - T & \text{if } \theta_n > T \text{ and } \theta_s \in [0, k-T] \\ \theta_s+\theta_n-k & \text{if } \theta_n > k-\theta_s;\ \theta_s \in [k-T, T+k] \end{cases}$$

When demand in both regions is low, firm s satisfies the total demand, therefore the residual demand that remains to firm n is zero. When the total demand is large enough, firm s cannot satisfy the total demand and some residual demand $(\theta_s+\theta_n-k)$ remains to firm n. Due to the transmission constraint, the total demand that firm s can satisfy diminishes. As soon as demand in region North is larger than the transmission capacity $(\theta_n > T)$, firm s cannot satisfy it, therefore some residual demand $(\theta_n - T)$ remains to firm n.

Finally, the payments are worked out by the auctioneer. I will work out the equilibrium when the auctioneer runs a uniform price auction and a discriminatory price auction.

When the auctioneer runs a uniform price auction, the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to the higher accepted bid in the auction. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

Figure 1: Output function for firm n. ($k_n = k_s = 60, T = 40$)



$$\pi_i^u(b;\theta,T) = \begin{cases} (b_j - c_i)min\,\{\theta_i + T, k\} & \text{if } b_i < b_j \text{ and } \theta_i + \theta_j \geq k_i \text{ or } \theta_j \geq T \\ (b_i - c_i)q_i(b;\theta,T) & \text{otherwise} \end{cases} \tag{2}$$

As in the case of the production function, the payoff function has an important role determining the equilibrium, therefore I will explain it in greater detail. Below, I describe the construction of firm n's payoff function when the auction is uniform, the one for firm s is symmetric.

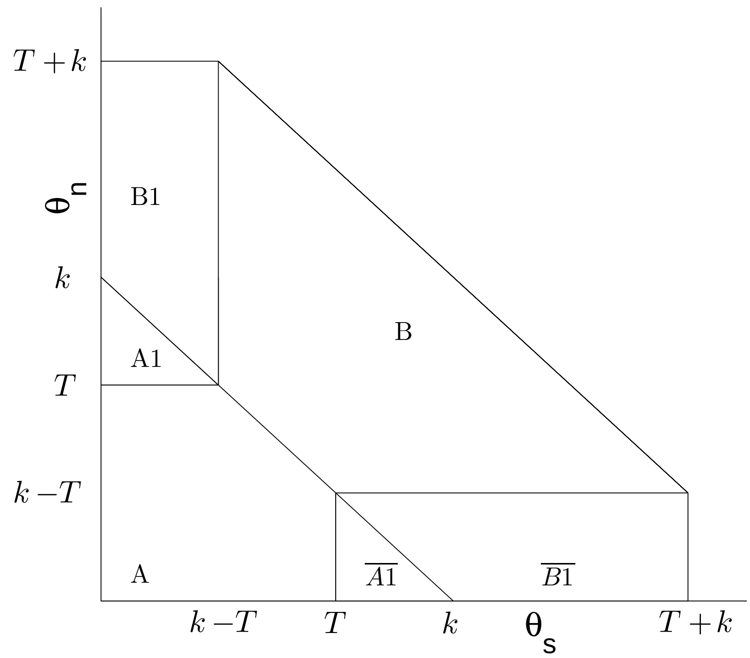
If $b_n < b_s$ and $\theta_n + \theta_s \geq k$ and $\theta_s \geq T$. Firm n submits the lower bid, has not enough capacity to satisfy the demand in both regions or the demand in region South is higher than the transmission capacity. In such a case, firm s sets the price in the auction. Hence, the payoff function for firm n is equal to $\pi_n^u(b;\theta,T) = (b_s - c_n)k$. In the rest of the cases, firm n's payoff is its own bid multiply by its dispatch, i.e., the payoff in case of discriminatory auction.

When the auctioneer runs a discriminatory price auction, the price received by a supplier for any positive quantity dispatched by the auctioneer is equal to its own bid. Hence, for a given realization of $\theta \equiv (\theta_s, \theta_n)$ and a bid profile $b \equiv (b_s, b_n)$, supplier n's profits, $i = n, s$, can be expressed as

$$\pi_i^d(b;\theta,T) = (b_i - c_i)q_i(b;\theta,T) \tag{3}$$

To conclude this section, I explain firms' payoff function for both types of auctions. Equations 2 and 3 define firms' payoff function for both types of auctions. Depending on the realization of demand and agents' bids, these equations define different areas (figure 2). In particular, area A is the area in which the realization of demand in each market is lower than the transmission line capacity and both firms have enough generation capacity to satisfy the total demand in both markets, I named this area "low demand"; areas $A1$

Figure 2: Single price market. Uniform and discriminatory auction. Payoff areas



and $B1$ (or its symmetric $\overline{A1}$ and $\overline{B1}$) are the areas in which the transmission line is congested when the firm located in the high demand market submits the higher bid, I named this area "intermediate"; finally area B is the area in which the realization of demand in higher than the installed generation capacity of the firms and the transmission line is not congested, I named this area "high demand". Based on the realization of demand, table 1 defines firm n's payoff function. As can be observed in table 1 firm n's payoff function coincide for uniform and discriminatory only in area A.

Table 1: Single price market. Uniform and discriminatory auction. Payoff function

Area	$b_n < b_s$	$b_n > b_s$
Area A (low)	$\pi_n^u = b_n(\theta_n + \theta_s)$ $\pi_n^d = b_n(\theta_n + \theta_s)$	$\pi_n^u = 0$ $\pi_n^d = 0$
Area $\overline{A1}$ (intermediate)	$\pi_n^u = b_s(\theta_n + T)$ $\pi_n^d = b_n(\theta_n + T)$	$\pi_n^u = 0$ $\pi_n^d = 0$
Area $\overline{B1}$ (intermediate)	$\pi_n^u = b_s(\theta_n + T)$ $\pi_n^d = b_n(\theta_n + T)$	$\pi_n^u = b_n(\theta_s + \theta_n - k)$ $\pi_n^d = b_n(\theta_s + \theta_n - k)$
Area B (high)	$\pi_n^u = b_s k$ $\pi_n^d = b_n k$	$\pi_n^u = b_n(\theta_s + \theta_n - k)$ $\pi_n^d = b_n(\theta_s + \theta_n - k)$

3 Equilibrium analysis

In this section, I characterize the equilibrium when the transmission line is not congested and when it is congested. I work out the equilibrium when the auctioneer runs an uniform price auction and a discriminatory price auction.

3.1 No transmission constraint

In this section, I characterize the equilibrium for both types of auctions when the transmission line is not congested $T > k$. The main results coincide with the ones presented in Fabra et al. (2006). However, Fabra et al. (2006) characterize the equilibrium when firms are asymmetric both in generation capacity and cost. By contrast, I have assumed that firms are symmetric in generation capacity and cost. Even when the procedure that I have used to characterize the equilibrium is similar to the one presented in Fabra et al. (2006), I have decided to characterize the equilibrium in the case in which the transmission line is not congested to facilitate the comprehension of the characterization of the equilibrium in the next section, in which I will introduce congestion in the transmission line.

Lemma 1. When the realization of demands (θ_s, θ_n) is low (area A), the equilibrium is in pure strategies for both types of auctions. When the realization of demands (θ_s, θ_n) is high (area B), a pure strategy equilibrium exist for the uniform price auction, but not for the discriminatory price auction (figure 3).

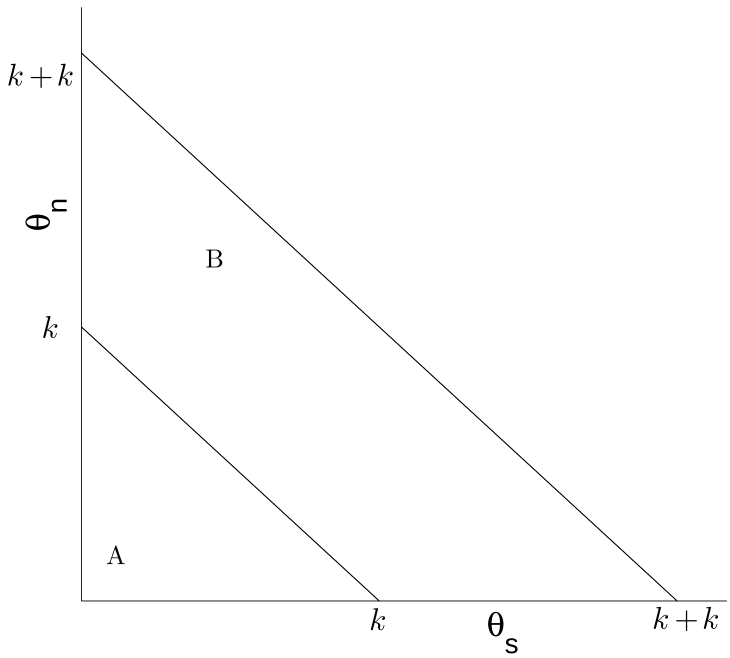
Proof. When the realization of demands (θ_s, θ_n) is low (area A), the two producers have enough capacity to satisfy the demand in both regions. Therefore, they compete fiercely to be dispatched first in the auction. Hence, the equilibrium is the typical Bertrand equilibrium in which both firms submit bids equal to their marginal cost.

When the realization of demands (θ_s, θ_n) is high (area B) and the auction is uniform, one of the firms (without loss of generality, I assume that is firm i) submits a bid equal to the maximum price allowed by the auctioneer and the other firm (firm j) submits a bid that made firm i be indifferent between submits the maximum bid allowed by the auctioneer and satisfy the residual demand or undercut firm j and satisfy the total demand. By the way in which the equilibrium has been constructed, firm i has no incentive to deviate. Firm j has no incentive to deviate because in the equilibrium, it is dispatched first and sells its generation capacity at the maximum price allowed by the auctioneer. Therefore, when the auction is uniform, a pure strategies equilibrium exists in which one of the firms submit the maximum price[8] allowed by the auctioneer.

When the realization of demands (θ_s, θ_n) is high (area B) and the auction is discriminatory, a pure equilibrium does not exist. First, an equilibrium such that $b_i = b_j = c$ does not exist because at least one firm has incentive to deviate and satisfy the residual demand. Second, an equilibrium such that $b_i = b_j > c$ does not exist because at least one firm has incentive to undercut the other to be dispatched first. Finally, an equilibrium such that $b_j > b_i > c$ does not exist because firm i has incentive to shade the bid submit-

[8] In pure strategies equilibrium, the firm that satisfies the residual demand always submits the maximum price allowed by the auctioneer. Otherwise, it can increases its expected payoff by increasing its bid.

Figure 3: Single price market. No transmission constraint. Equilibrium areas.



ted by firm j. $\square$

When the auction is discriminatory, a pure strategies equilibrium does not exist. However, the model satisfies the properties[9] established by Dasgupta and Maskin (1986) that guarantees that a mixed strategy equilibrium exists.

Lemma 2. In a mixed strategy equilibrium none firm submits a bid lower than bid $(\underline{b}_i)$ such that $\underline{b}_i min\{\theta_i+\theta_j,\theta_i+T,k\} = Pmax\{0,\theta_i-T,\theta_i+\theta_j-k\}$. Moreover, the support for the mixed strategies equilibrium for both firms is $S=\left[max\left\{\underline{b}_i,\underline{b}_j\right\},P\right]$.

Proof. Each firm can guarantees to itself the payoff $Pmax\{0,\theta_i-T,\theta_i+\theta_j-k\}$, because each firm always can submit the highest bid and satisfies the residual demand. Therefore, in a mixed strategies equilibrium, none firm submits a bid that generate a payoff equilibrium lower than $Pmax\{0,\theta_i-T,\theta_i+\theta_j-k\}$. Hence, none firm submits a bid lower than $\underline{b}_i$, where $\underline{b}_i$ solves $\underline{b}_i min\{\theta_i+\theta_j,\theta_i+T,k\} = Pmax\{0,\theta_i-T,\theta_i+\theta_j-k\}$.

None firm can rationalize submit a bid lower than $\underline{b}_i, i=n,s$. In the case that $\underline{b}_i=\underline{b}_j$, the mixed strategy equilibrium and the support is symmetric. In the case that $\underline{b}_i<\underline{b}_j$, firm i knows that firm j never submits a bid lower than $\underline{b}_j$. Therefore, in a mixed strategy equilibrium, firm i never submits a bid b_i such that $b_i\in\left(\underline{b}_i,\underline{b}_j\right)$, because firm i can increases its expected payoff choosing a bid b_i such that $b_i\in\left[\underline{b}_j,P\right]$. Hence, the equilibrium strategies support for both firms is $S=\left[max\left\{\underline{b}_i,\underline{b}_j\right\},P\right]$ $\square$

Using Lemmas one and two, I establish the main result of this section.

[9] In the Annex, Proposition 1. I proof that the model satisfies the properties established by Dasgupta and Maskin that guarantee that a mixed strategy equilibrium exists.

Proposition 1. When the auction is uniform, the characterization of the equilibrium strategies fall into one of the next two categories (figure 3).

i Low demand (area A) and high demand (area B). The equilibrium is in pure strategies.

When the auction is discriminatory, the characterization of the equilibrium fall into one of the next two categories (figure 3).

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii High demand (area B). The equilibrium strategies pair is in mixed strategies.

When the realization of demands (θ_s, θ_n) is low, both firms have enough capacity to satisfy the total demand. Therefore, the competition to be dispatched first is fierce and both firms submit a bid equal to its marginal cost. The pure strategies equilibrium is the same for both types of auctions.

When the realization of demands (θ_s, θ_n) is high and the auction is uniform, multiplicity of pure strategies equilibrium exists where one firm submits the higher bid allowed by the auctioneer.

When the realization of demands (θ_s, θ_n) is high and the auction is discriminatory, the equilibrium is in mixed strategies. The equilibrium price is lower than when the auction is uniform. Given that the mixed strategy equilibrium is symmetric, the probability that one of the firm submits the lower bid is 0.5.

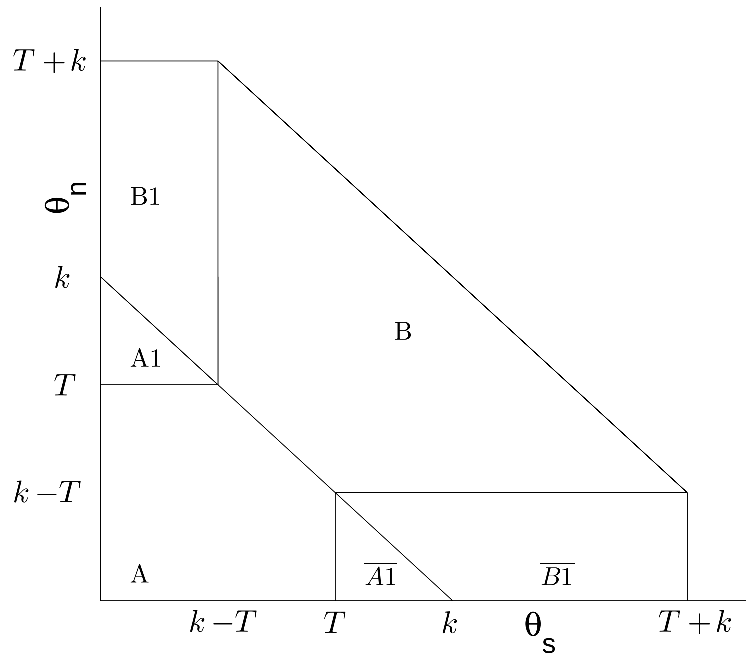
The discriminatory price auction generates lower equilibrium prices. Therefore, when the firms are symmetric in cost and capital and the transmission line is not congested, discriminatory price auction performs better than uniform price auction.[10]

Corollary 1. When the auction is discriminatory and the realization of demands (θ_s, θ_n) is high (area B), an increase in θ_n and/or θ_s increases the lower bound of the support $\underline{b}$, the payoff and the expected bids for both firms.

The lower bound of the support determines the bid that made firms be indifferent between satisfy the residual demand at the maximum price allowed by the auctioneer and satisfy the total demand at a bid equal to the lower bound of the support. An increase in demand increases the residual demand an so the lower bound of the support. An increase in the support made that firms randomize in higher prices, therefore the equilibrium prices and the payoff are higher.

[10] Fabra et al. (2006) established that when firms are asymmetric in cost. Uniform price auction generates higher equilibrium prices. However, the most efficient firm, the one with lower marginal cost, is dispatched first in the equilibrium. Therefore, a clear rank between uniform and discriminatory price auction can not be established.

Figure 4: Single price market. Transmission constraint. Equilibrium areas.



3.2 Transmission constraint

In this section, I characterize the equilibrium for both types of auctions when the transmission line is congested[11] $T \leq k$.

Using lemmas one and two, I establish the main result of this section.

Proposition 2. When the auction is uniform, the characterization of the equilibrium strategies fall into one of the next three categories (figure 4).

i Low demand (area A), intermediate demand (areas $A1$, $B1$) and high demand (area B). The equilibrium strategies pair is in pure strategies.

When the auction is discriminatory, the characterization of the equilibrium fall into one of the next two categories (figure 4).

i Low demand (area A). The equilibrium strategies pair is in pure strategies.

ii Intermediate demand (areas $A1$, $B1$) and high demand (area B). The equilibrium strategies pair is in mixed strategies.

[11]In the introduction section, I have defined the term "congested" in the electrical engineering sense: a line is congested when the flow of power is equal to the line's capacity, as determined by engineering standards. In this section, I introduce a more accurate definition, in particular: the transmission line is congested when the firm that is dispatched first in the auction, after satisfy the demand in its own market, can not sell its remain residual generation capacity in the other region because of the transmission constraint.

When the realization of demands (θ_s, θ_n) is low, as in the case in which the transmission line is not congested, both firms have enough capacity to satisfy the total demand. Therefore the competition to be dispatched first is fierce and both firms submit a bid equal to its marginal cost. The pure strategies equilibrium is the same for both types of auctions.

When the realization of demands (θ_s, θ_n) is high and the auction is uniform, two new types of pure strategies equilibrium emerge, areas $A1$ and $B1$. Where areas $A1$ and $B1$ are the ones for which, the transmission line is congested as defined in footnote 11. In particular, area $A1, (\overline{A1})$ is the one in which the firm located in high demand market can satisfy the total demand in case of be dispatched first in the auction and area $B1, (\overline{B1})$ is the one in which the firm located in the high demand market has not enough installed generation capacity to satisfy total demand in case of be dispatched first in the auction. As in the case in which the transmission line is not congested, one firm submits the higher bid allowed by the auctioneer. When the realization of demands (θ_s, θ_n) belong to area $A1$, the firm located in the high demand region submits the higher bid with probability 1. When the realization of demands (θ_s, θ_n) belong to area $B1$, no prior probability can be established about the type of equilibrium that emerge on equilibrium. In area B, the equilibrium characterization coincide with the one established in proposition one.

When the realization of demands (θ_s, θ_n) is high and the auction is discriminatory, two new types of mixed strategies equilibrium emerge, areas $A1$ and $B1$. As in the case in which the transmission line is not congested, the equilibrium is in mixed strategies. However, due to the transmission constraint, the equilibrium is asymmetric. The expected equilibrium price is lower than the maximum price allowed by the auctioneer.

When firms are symmetric and the demand is intermediate or high, the uniform price auction generates higher prices in the equilibrium. Moreover, when the realization of demands (θ_s, θ_n) is intermediate (area $A1$) and the auction is uniform, the firm located in the high demand region is dispatched last with probability 1, by contrast, when the auction is discriminatory, the firm located in the high demand region is dispatched first with a positive probability. Hence, discriminatory auction generates lower equilibrium prices and the firm located in the high demand region is dispatched first with some positive probability.

So far, I have assumed that there is no loss of electricity during the transmission process. However, in case of loss of electricity during the transmission process, the auction that prioritize in the dispatch the firm located in the high demand region performs better (less electricity flows through the grid and so less losses). Therefore, when the transmission line is congested, to evaluate the performance of an auction, in addition to the criteria of low price equilibrium and efficiency in generation established in Fabra et al. (2006), a new criteria, efficiency in transmission, should be introduced to evaluate the performance of an auction. Hence, under these criteria, the auction design that generates lower equilibrium prices and prioritizes in the dispatch the firm located in the high demand region performs better.

To conclude this section, I introduce corollary two and corollary there that analyze the effect that an increase in demand has on equilibrium and proposition three that analyzes the effect that an increase in transmission has on equilibrium.

Table 2: Numerical example: increase in θ_n ($\theta_s = 5, k = 60, T = 40, c = 0, P = 7$)

Area	θ_n	$\underline{b}$	$F_n(P)$	$\overline{\pi}_s$	$\overline{\pi}_n$	$E(b_s)$	$E(b_n)$
Area A1	41	0.15	0.746	6.84	7	0.59	2.22
	45	0.7	0.736	31.5	35	1.79	3.16
	50	1.27	0.736	57.2	70	2.65	3.79
	55	1.75	0.75	78.7	105	3.23	4.17
Border A1-B1	60	2.33	0.75	105	140	3.84	4.63
Area B1	61	2.45	0.75	110.25	147	3.95	4.71
	65	2.91	0.75	131.25	175	4.37	5.03
	70	3.5	0.75	157.5	210	4.85	5.38

The cumulative distribution function for firm n is discontinuous at P, $1 - F_n(P)$ represents the probability that firm n assigns to the maximum bid allowed by the auctioneer.

Corollary 2. An increase in θ_n increases the lower bound of the support $\underline{b}$, the payoff and the expected bids for both firms. When the realization of demands (θ_s, θ_n) belongs to area $B1$, an increase in θ_n does not change the probability that the firm located in the high demand region assigns to the maximum bid allowed by the auctioneer. However when the realization of demands (θ_s, θ_n) belongs to area $A1$, an increase in θ_n modifies in a non-monotonic pattern the probability that the firm located in the high demand region assigns to the maximum bid allowed by the auctioneer (table 2 and figure 5).

When the realization of demands (θ_s, θ_n) belongs to areas $A1$ or $B1$, an increase in the demand in the high demand region increases the lower bound of the support. An increase of θ_n increases the residual demand and so the bid that made the firms be indifferent between satisfy the residual demand at the maximum price allowed by the auctioneer and satisfy the total demand at a bid equal to the lower bound of the support. Consequently the equilibrium expected price and the payoff increase.

When the realization of demands (θ_s, θ_n) belongs to area $B1$, the probability that the firm located in the high demand region assigns to the maximum bid allowed by the auctioneer does not change. In area $B1$, the probability that the firm located in the high demand region assigns to the maximum bid allowed by the auctioneer represents the opportunity cost of submit high bids for the firm located in the low demand region. When θ_n increases and θ_s remain fix, the total demand that firm s can satisfy does not change and so the opportunity cost of submit high bids for the firm located in the low demand region does not change. However, in area $A1$, the probability that the firm located in the high demand market assigns to the maximum bid allowed by the auctioneer changes in a non-monotonic pattern. In area $A1$, an increase in θ_n modifies the total and the residual demand that firm n can satisfy. Therefore, an increase in θ_n modifies the opportunity cost of submit high bids for the firm located in the high demand region.

Corollary 3. When the realization of the demands (θ_s, θ_n) belongs to area $A1$, an increase in θ_s reduces the lower bound of the support $\underline{b}$; increases the probability that the firm located in the high demand region assigns to high bids; does not modify the payoff of the

Figure 5: Effect of an increase in θ_n on the main variables. Area $A1, (\theta_n < 55)$ and area $B1, (\theta_n > 55)$

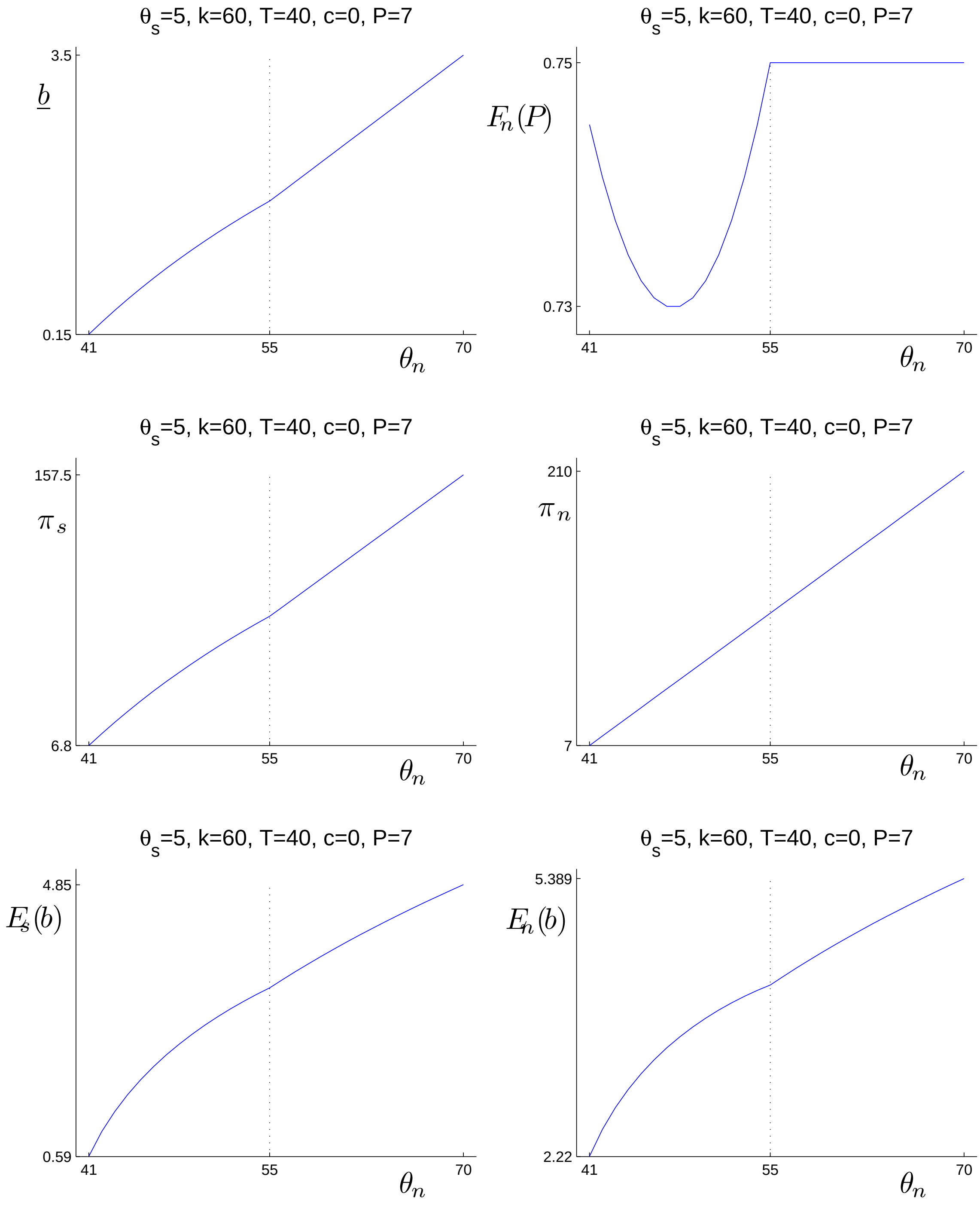


Table 3: Numerical example: increase in θ_s ($\theta_n = 45, k = 60, T = 40, c = 0, P = 7$)

Area	θ_s	$\underline{b}$	$F_n(P)$	$\overline{\pi}_s$	$\overline{\pi}_n$	$E(b_s)$	$E(b_n)$
Area A1	1	0.76	0.66	31.99	35	1.89	3.6
	5	0.7	0.73	31.5	35	1.79	3.16
	10	0.63	0.82	31.81	35	1.67	2.6
	14	0.59	0.89	32.03	35	1.59	2.14
Border A1-B1	15	0.58	0.91	32.08	35	1.58	2.03
Area B1	16	0.58	0.93	32.66	35	1.58	1.94
	20	0.58	1	35	35	1.58	1.58

The cumulative distribution function for firm n is discontinuous at P, $1 - F_n(P)$ represents the probability that firm n assigns to the maximum bid allowed by the auctioneer.

firm located in the high demand region, increases the payoff of the firm located in the low demand region and decreases the expected bid of both firms. When the realization of demands (θ_s, θ_n) belongs to area $B1$, an increase in θ_s does not modify the lower bound of the support $\underline{b}$, the probability that the firm located in the high demand region assign to high bids and the payoff of the firm located in the high demand region; increases the payoff of the firm located in the low demand region; decreases the expected bid of the firm located in the high demand region and does not modify the expected bid of the firm located in the low demand region (table 3 and figure 6).

When the realization of the demands (θ_s, θ_n) belongs to Area $A1$. An increase in θ_s increases the total demand that firm n can satisfy in case of be dispatched first in the auction and so decreases the lower bound of the support. The probability that the firm located in the high demand region assigns to the maximum bid allowed by the auctioneer represents the opportunity cost of submit high bids for firm s. An increase in θ_s increases the residual demand for firm s and so reduces the probability that firm n assigns to the maximum bid allowed by the auctioneer. An increase in θ_s increases the total demand that both firms can satisfy. In the case of firm n, this increase is compensated with the decrease in expected bids due to the decrease in the lower bound of the support, therefore, its payoff does not change. By contrast, the increase in the demand for firm s dominates the decrease in expected bids and so its payoff increases. Finally, an increase in θ_s induces a decrease in expected bids for both firms because the lower bound of the support decreases and the probability that the firm located in the high demand market assigns to high bids decreases.

When the realization of the demands (θ_s, θ_n) belongs to Area $B1$. An increase in θ_s does not modify the total demand that firm n can satisfy in case of be dispatched first in the auction and so the lower bound of the support does not change. As in Area $A1$, an increase in θ_s increases the residual demand for firm s and so reduces the probability that firm n assign to the maximum bid allowed by the auctioneer. An increase in θ_s does not change the total demand that firm n can satisfy and so its payoff does not change, by contrast, an increase in θ_s increases the total demand that firm s can satisfy and so its payoff increases. Finally, an increase in θ_s reduces the expected bids for firm n because the lower bound of the support decreases and the probability that the firm located in the

Table 4: Numerical example: increase in T ($\theta_n = 55, \theta_s = 5, k = 60, c = 0, P = 7$)

T	$\underline{b}$	$F_n(P)$	$\overline{\pi}_s$	$\overline{\pi}_n$	$E(b_s)$	$E(b_n)$
0	7	0	35	385	7	7
10	5.25	0.25	78.75	315	6.04	6.76
20	4.08	0.41	102.08	245	5.28	6.28
30	2.91	0.58	102.08	175	4.37	5.47
40	1.75	0.75	78.75	105	3.23	4.17
50	0.58	0.91	32.083	35	1.58	2.03
55	0	1	0	0	0	0

The cumulative distribution function for firm n is discontinuous at P, $1 - F_n(P)$ represents the probability that firm n assigns to the maximum bid allowed by the auctioneer.

high demand region assigns to high bids decreases. By contrast the expected bid for firm s does not change because its cumulative distribution function and the lower bound of the support does not change.

Proposition 3. An increase in T reduces the lower bound of the support $\underline{b}$ and reduces the probability that the firm located in the high demand region assigns to high bids. Moreover, an increase in T reduces the payoff of the firm located in the high demand region. However, an increase in T modifies in a non-monotonic pattern the payoff of the firm located in the low demand region. Finally, an increase in T reduces the expected bids for both firms.

An increase in T reduces the residual demand that each firm faces in case of be dispatched last in the auction and so reduces the lower bound of the support. Moreover, an increase in T increases the total demand that each firm can satisfy in case of be dispatched first in the auction and so reduces the probability that the firm located in the high demand region assigns to high bids. Consequently, the expected bid for both firms and the payoff of the firm located in the high demand region decreases. By contrast, the payoff of the firm located in the low demand region changes in a non-monotonic pattern. When the transmission capacity is low, an increase in transmission capacity increases the total demand that the firm located in the low demand region faces. The increase in demand effect dominates the decrease in expected bid effect and the payoff increases. However, when the transmission capacity is big enough the decrease in bids effect dominates and the payoff decreases.

The literature that analyzes the effect that an increase in transmission capacity has on equilibrium outcomes usually focus in the effect on competition between regions. However, as I have shown in proposition three, an increase in transmission capacity modifies the payoff of the firm located in the low demand region. This could have important implications in generation capacity investment decisions. To motivate the argument, I introduce the next example: imagine that small hydro-power plant that faces a fix entry cost would like to install some generation capacity in the low demand region. When there is no transmission capacity between regions, due to the reduced size of the market, the firm can not cover its fix entry cost. However, if the transmission line increases, the size of the market increases and the firm could enter in the low demand region. This entry could increase the competition within the low demand region.

Figure 6: Effect of an increase in θ_s on the main variables. Area $A1, (\theta_n < 15)$ and area $B1, (\theta_n > 15)$

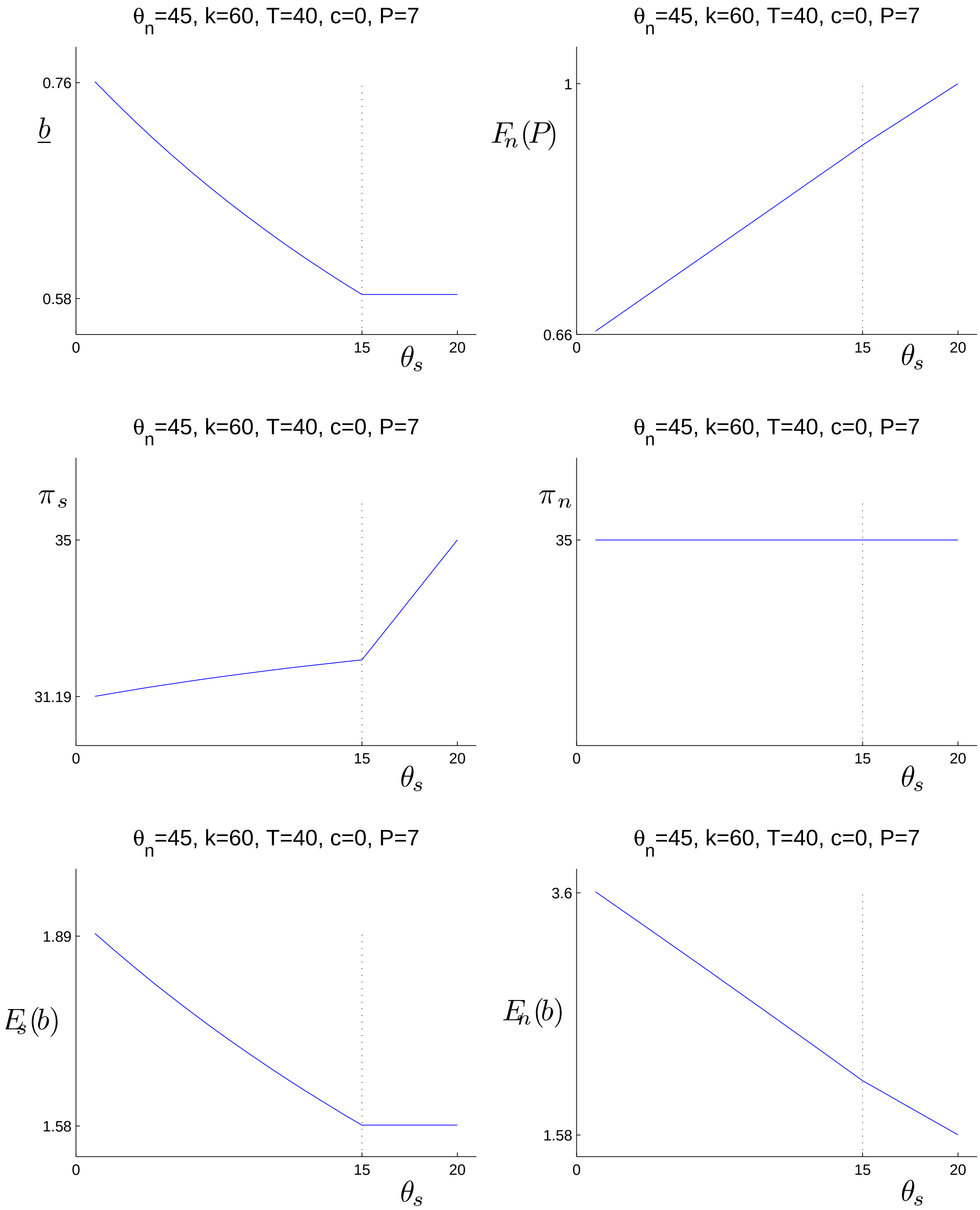
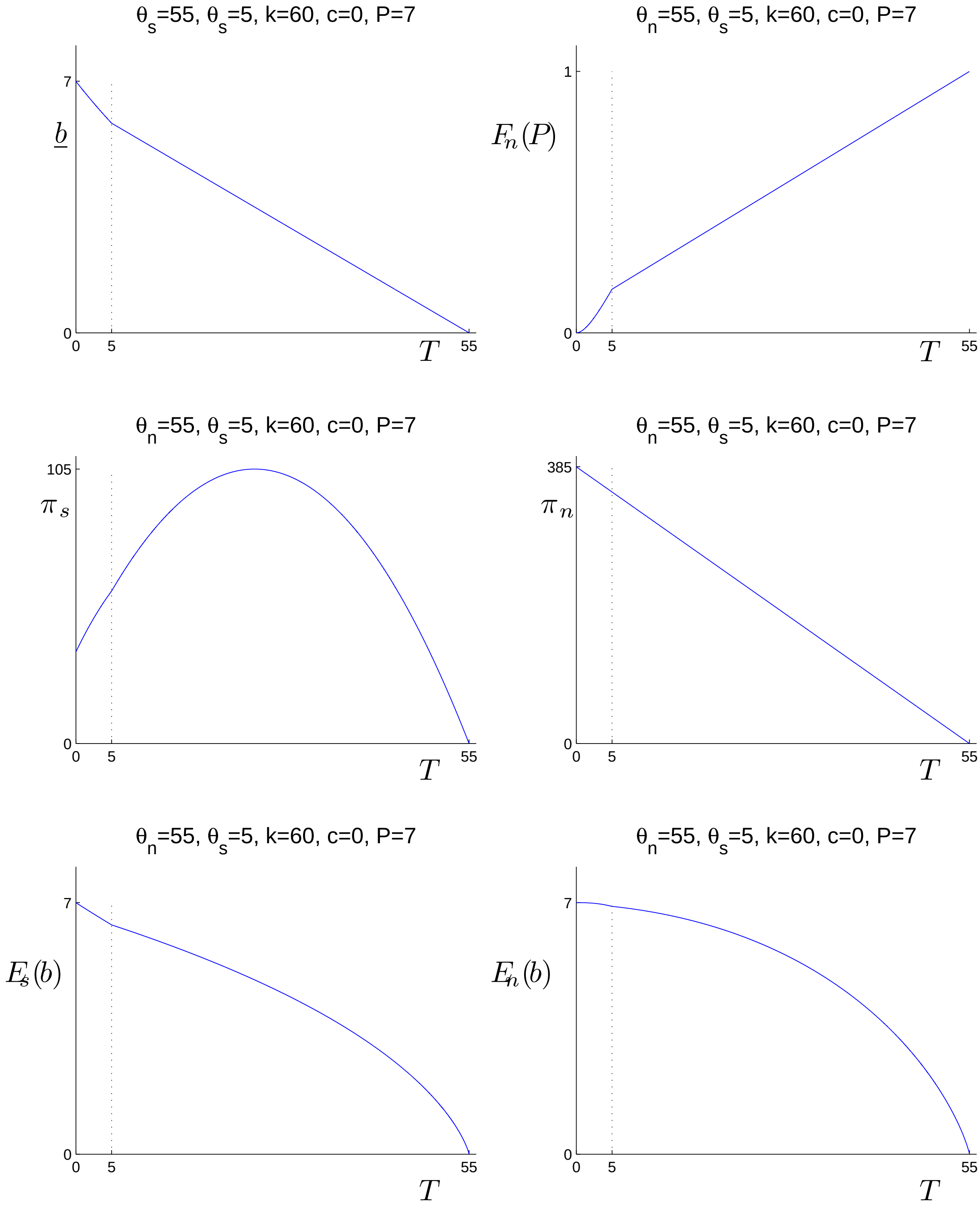


Figure 7: Effect of an increase in T on the main variables. Border $A1 - B1$



4 Conclusions

In this paper, I have characterized the equilibrium in a single price electricity market when the transmission line is congested and it is not congested. I have characterized the equilibrium for uniform and discriminatory auctions.

When the transmission line is not congested, as in Fabra et al. (2006), discriminatory price auction generates lower equilibrium prices. Given that both firms are equally efficient in generation, discriminatory price auction outperforms uniform price auction.

When the transmission line is congested, uniform auction generates higher equilibrium prices and the firm located in the high demand region is dispatched last with probability one. By contrast, discriminatory auction generates lower equilibrium prices and the firm located in the high demand region is dispatched first with positive probability. In case of loss during transmission, discriminatory price auction performs better both in terms of prices equilibrium and in terms of efficiency in transmission (lower transmission through the grid and so lower electricity loss).

It is important to remark that in case that the transmission line is congested and firms are asymmetric in generation cost, a more detailed analysis is necessary to analyze the performance of both types of auctions in terms of equilibrium prices, efficiency in generation and efficiency in transmission. I also would like to analyze the effect that different tie breaking policy rules[12] has on equilibrium outcomes when the transmission line is congested.

Finally, an increase in transmission capacity increases competition between regions. Moreover, as I have shown in proposition three, an increase in transmission capacity could have important implications in generation capacity decisions and so in competition within a single market. In the next future, I would like to analyze the effect that the transmission line capacity have on generation capacity investment decisions.

[12]In case of a tie, it is necessarily to determine which firm is dispatched first. In order to determine the priority in dispatch, the efficiency in generation and in transmission should be take into consideration. The weigh assigned to efficiency in transmission and efficiency in generation could have important implications on equilibrium outcomes.

Annex

Proposition 1. Uniform Price Auction. Using lemma one, the proof is straight forward.

When the demand is low: $b_n = b_s = c = 0$. The equilibrium payoff is zero for both firms. No electricity flows through the grid.

When the demand is high:

The pure strategies equilibrium is defined by

$$b_i = P; \;\; b_j = \frac{P(\theta_i + \theta_j - k)}{k} \;\; \forall i, j = s, n$$

The equilibrium price is P.

The payoff function is defined by

$$\overline{\pi}_i = P(\theta_i + \theta_j - k); \;\; \overline{\pi}_j = Pk \;\; \forall i, j = s, n$$

The probability that firm i submits the lower bid depend of which type of equilibrium emerge and can not be determined a priori.

Discriminatory Price Auction.

Proof:

The model presented in section two satisfies the properties established by Dasgupta and Maskin (1986), that guarantee that a mixed strategy equilibrium exists. In particular, the discontinuities of $\pi_i, \forall i, j$ are restricted to the strategies such that $b_i = b_j$. Furthermore, it is simple to confirm that by lowering its price from a position where $b_i = b_j$, a firm discontinuously increases its profit. Therefore, $\pi_i(b_i, b_j)$ is everywhere left lower semi-continuous in b_i, and hence weakly lower semi-continuous. Obviously $\pi_i(b_i, b_j)$ is bounded. Finally, $\pi_i(b_i, b_j) + \pi_j(b_i, b_j)$ is continuous, because discontinuous shifts in clientele from one firm to another occur only where both firms derive the same profit per customer. Therefore, theorem five in Dasgupta and Maskin (1986) applies, hence a mixed strategy equilibrium exists. However, Dasgupta and Maskin (1986) did not provide an algorithm to work out the equilibrium. Nevertheless, using the approach proposed by (Karlin, 1959; Beckmann, 1965; Shapley, 1957; Shilony, 1977; Varian, 1980; Deneckere and Kovenock, 1986; Osborne and Pitchik, 1986; Fabra et al., 2006), the equilibrium characterization is guaranteed by construction. I will use the approach proposed by this branch of the literature to work out the mixed strategy equilibrium.

First, I work out the cumulative distribution function.

First step, the payoff function for any firm is:

$$\begin{aligned} \pi_i(b) &= b\left[F_j(b) max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\} + (1 - F_j(b)) min\left\{\theta_i + \theta_j, \theta_i + T, k\right\}\right] = \\ &= -bF_j(b)\left[min\left\{\theta_i + \theta_j, \theta_i + T, k\right\} - max\left\{0, \theta_i - T, \theta_i + \theta_j - k\right\}\right] + \\ &\quad bmin\left\{\theta_i + \theta_j, \theta_i + T, k\right\} \end{aligned} \tag{4}$$

Second step, $\pi_i(b) = \overline{\pi}_i \forall b \in S_i, i = n, s$, where S_i is the support of the mixed strategies. Then,

$$\begin{aligned}\overline{\pi}_i &= -bF_j(b)\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right] + \\ &\quad bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \Rightarrow \\ F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - \overline{\pi}_i}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} \end{aligned} \quad (5)$$

Third step, at $\underline{b}$, $F_i(\underline{b}) = 0 \forall i = n, s$. Then,

$$\overline{\pi}_i = \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} \quad (6)$$

Fourth step, Plug in 6 into 5, I obtain the mixed strategies for both firms.

$$\begin{aligned}F_j(b) &= \frac{bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - \underline{b}min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{b\left[min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}\right]} = \\ &= \frac{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\}}{min\left\{\theta_i+\theta_j,\theta_i+T,k\right\} - max\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}}\frac{b-\underline{b}}{b} \quad \forall i = n, s \end{aligned} \quad (7)$$

For further reference:
$L_i(b) = bmin\left\{\theta_i+\theta_j,\theta_i+T,k\right\}$ and
$H_i(b) = bmax\left\{0,\theta_i-T,\theta_i+\theta_j-k\right\}$.

It is easy to verify that equation $F_j(b)\forall i, j$ is indeed a cumulative distribution function. First, in the third step, I have established that $F_j(\underline{b}) = 0$. Second, $F_j(b)$ is an increasing function in b. At $\underline{b}$, $L_i(\underline{b}) = H_i(b)$, for any $b > \underline{b}$, $L_i(\underline{b}) < H_i(b)$; moreover, $\frac{\partial L_i(b)}{\partial b} > 0$, $\frac{\partial L_i(\underline{b})}{\partial b} = 0$ and $\frac{\partial H_i(b)}{\partial b} > 0$, therefore, $\frac{\partial\left(L_i(b) - L_i(\underline{b})\right)}{\partial b} > \frac{\partial\left(L_i(b) - H_i(b)\right)}{\partial b}$. Third, $F_j(b) \leq 1 \forall b \in S_i$. Finally, $F_j(b)$ is continuous in the support because $L_i(b) - L_i(\underline{b})$ and $L_i(b) - H_i(b)$ are continuous functions in the support.

Using equation 7, the cumulative distribution function in area B is defined by:

$$F_i(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \frac{k}{2k-\theta_i-\theta_j}\frac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \quad \forall i = s, n \\ 1 & \text{if } b = P \end{cases}$$

Second, I work out the support of the mixed strategy equilibrium. It is straight forward to check that in area B, $\underline{b}_s = \underline{b}_n = \frac{P(\theta_s+\theta_n-k)}{k}$. Therefore, in area B, $S = \left[max\left\{\underline{b}_n, \underline{b}_s\right\}, P\right] = \left[\frac{P(\theta_s+\theta_n-k)}{k}, P\right]$.

Third, I work out the expected bid in Area B.

$$f_i(b) = \frac{\partial F_i(b)}{\partial b} = \frac{k}{2k - \theta_i - \theta_j} \frac{\underline{b}}{b^2} \ \ \forall i = s, n$$

$$E(b_i) = \int_{\underline{b}}^{P} b f_i(b_i) \partial b = \int_{\underline{b}}^{P} \frac{k}{2k - \theta_n - \theta_s} \frac{\underline{b}}{b} \partial b = \frac{k}{2k - \theta_n - \theta_s} \underline{b} \left[ln(b)\right]_{\underline{b}}^{P}$$

Fourth, I work out the payoff function. Using 6, the payoff function in area B is:

$$\overline{\pi}_i = \underline{b}k \ \ \forall i = s, n$$

Finally, due to the symmetry in the strategies, the probability that firm i submit the lower bid is $prob(b_i < b_j) = \frac{1}{2}$.

Corollary 1.

$$\frac{\partial \underline{b}}{\partial \theta_i} = \frac{P}{k} > 0 \ \ \forall i = s, n$$

$$\frac{\partial F_i(P)}{\partial \theta_i} = 0 \ \ \forall i = s, n$$

$$\begin{aligned} \frac{\partial E(b_i)}{\partial \theta_i} = \ = \ & \frac{k}{(2k - \theta_i - \theta_j)^2} \frac{P(\theta_i + \theta_j - k)}{k} ln\left(\frac{P}{\underline{b}}\right) + \\ & \frac{k}{(2k - \theta_i - \theta_j)} \frac{P}{k} ln\left(\frac{P}{\underline{b}}\right) + \\ & \frac{k}{(2k - \theta_i - \theta_j)} \frac{P(\theta_i + \theta_j - k)}{k} \frac{\underline{b}}{P} \frac{P}{k} > 0 \ \ \forall i = s, n \end{aligned}$$

$$\frac{\partial \overline{\pi}_i}{\partial \theta_i} = P > 0 \ \ \forall i = s, n$$

Proposition 2. When the auction is uniform. Using lemma one, the proof is straight forward.

When the demand is low: $b_n = b_s = c = 0$. The equilibrium payoff is zero for both firms. No electricity flows through the grid.

When the demand is intermediate:

The pure strategies equilibrium is defined by

$$b_s = \frac{P(\theta_n - T)}{\theta_s + \theta_n}; \ \ b_n = P$$

The equilibrium price is P.

The payoff function is defined by

$$\overline{\pi}_s = P(\theta_s + T); \;\; \overline{\pi}_n = P(\theta_n - T)$$

The probability that firm s submits the lower bid is

$$prob(b_s < b_n) = 1$$

When the demand is high:

The pure strategies equilibrium is defined by

$$b_i = P; \;\; b_j = \frac{Pmax\left\{\theta_i - T, \theta_j + \theta_i - k\right\}}{min\left\{\theta_i + T, k\right\}} \;\; \forall i, j = s, n$$

The equilibrium price is P.

The payoff function is defined by either

$$\overline{\pi}_i = Pmax\left\{\theta_i - T, \theta_j + \theta_i - k\right\}; \;\; \overline{\pi}_j = Pk \;\; \forall i, j = s, n$$

The probability that firm i submits the lower bid depend of which type of equilibrium emerge and can not be determined a priori.

When the auction is discriminatory:

First, I work out the cumulative distribution function.

In area $A1$, using equation 7, the cumulative distribution function for both firms is defined by:

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_n + \theta_s}{\theta_s + T} \dfrac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s + T}{k + T - \theta_n} \dfrac{b - \underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{\theta_n + \theta_s}{\theta_s + T} \frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = 1 \\ F_n(P) &= \frac{\theta_s + T}{k + T - \theta_n} \frac{P - \dfrac{P(\theta_n - T)}{\theta_n + \theta_s}}{P} = \frac{(\theta_s + T)^2}{(\theta_n + \theta_s)(k + T - \theta_n)} < 1 \end{aligned}$$

In area $B1$, using equation 7, the cumulative distribution function for both firms is defined by:

$$F_s(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{k}{T+k-\theta_n}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

$$F_n(b) = \begin{cases} 0 & \text{if } b < \underline{b} \\ \dfrac{\theta_s+T}{T+k-\theta_n}\dfrac{b-\underline{b}}{b} & \text{if } b \in (\underline{b}, P) \\ 1 & \text{if } b = P \end{cases}$$

Moreover,

$$\begin{aligned} F_s(P) &= \frac{k}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P} = 1 \\ F_n(P) &= \frac{\theta_s+T}{T+k-\theta_n}\frac{P-\dfrac{P(\theta_n-T)}{k}}{P} = \frac{\theta_s+T}{k} < 1 \end{aligned}$$

Second, I work out the support of the mixed strategy equilibrium.

In the border between areas $B1$ and B, $\theta_s = k - T$. In the border, $\underline{b}_n$ solves $\underline{b}_n min\left\{\theta_n+\theta_s, \theta_n+T, k\right\} = Pmax\left\{0, \theta_n-T, \theta_s+\theta_n-k\right\}$, therefore $\underline{b}_n = \dfrac{P(\theta_n-T)}{k}$ and $\underline{b}_s$ solves $\underline{b}_s min\left\{\theta_n+\theta_s, \theta_s+T, k\right\} = Pmax\left\{0, \theta_s-T, \theta_s+\theta_n-k\right\}$, therefore $\underline{b}_s = \dfrac{P(\theta_n+\theta_s-k)}{\theta_s+T}$. Plug in the value of θ_s in the border between these areas into $\underline{b}_s$ formula, I obtain $\underline{b}_s = \dfrac{P(\theta_n+k-T-k)}{k-T+T} = \dfrac{P(\theta_n-T)}{k} = \underline{b}_n$. Therefore, in the border between these areas, $\underline{b}_s = \underline{b}_n = \dfrac{P(\theta_n-T)}{k}$.

In areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. In area $A1$, taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial \theta_s} = \dfrac{-P(\theta_n-T)}{(\theta_n+\theta_s)^2} < 0$ and $\dfrac{\partial \underline{b}_s}{\partial \theta_s} = \dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2} > 0$. In area $B1$, taking partial derivatives $\dfrac{\partial \underline{b}_n}{\partial \theta_s} = 0$ and $\dfrac{\partial \underline{b}_s}{\partial \theta_s} = \dfrac{P(k+T-\theta_n)}{(\theta_s+T)^2} > 0$. Therefore, in areas $A1$ and $B1$, $\underline{b}_n > \underline{b}_s$. Hence, $S = \left[max\left\{\underline{b}_n, \underline{b}_s\right\}, P\right] = [\underline{b}_n, P]$. In particular, in area $A1$, $S = \left[\dfrac{P(\theta_n-T)}{(\theta_n+\theta_s)}, P\right]$ and in area $B1$, $S = \left[\dfrac{P(\theta_n-T)}{k}, P\right]$.

Third, I work out the expected bid.

In Area $A1$,

$$
\begin{aligned}
f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{\theta_n+\theta_s}{\theta_s+T}\frac{\underline{b}}{b^2} \\
f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{k+T-\theta_n}\frac{\underline{b}}{b^2}
\end{aligned}
$$

$$
\begin{aligned}
E(b_s) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{\theta_n+\theta_s}{\theta_s+T}\frac{\underline{b}}{b}\partial b = \frac{\theta_n+\theta_s}{\theta_s+T}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\
E(b_n) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\underline{b}}{b^2}\partial b = \frac{\theta_s+T}{k+T-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1-F_n(P))\,P
\end{aligned}
$$

In area $B1$,

$$
\begin{aligned}
f_s(b) &= \frac{\partial F_s(b)}{\partial b} = \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b^2} \\
f_n(b) &= \frac{\partial F_n(b)}{\partial b} = \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b^2}
\end{aligned}
$$

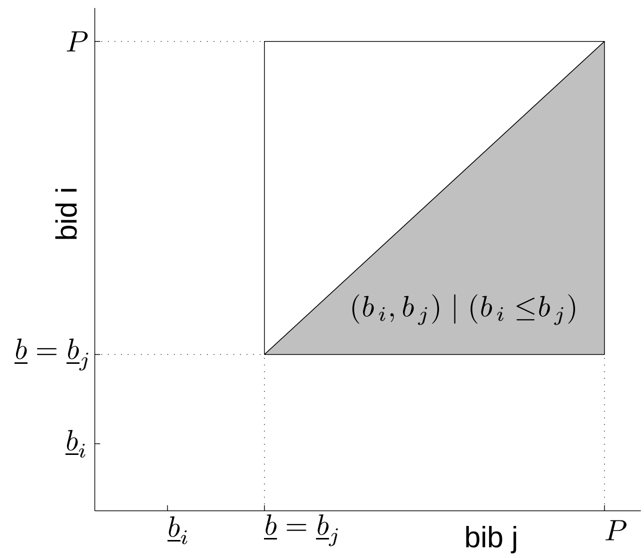
$$
\begin{aligned}
E(b_s) &= \int_{\underline{b}}^{P} b f_s(b_s)\partial b = \int_{\underline{b}}^{P} \frac{k}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b = \frac{k}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} \\
E(b_n) &= \int_{\underline{b}}^{P} b f_n(b_n)\partial b = \int_{\underline{b}}^{P} \frac{\theta_s+T}{T+k-\theta_n}\frac{\underline{b}}{b}\partial b = \\
&\quad \frac{\theta_s+T}{T+k-\theta_n}\underline{b}\left[ln(b)\right]_{\underline{b}}^{P} + (1-F_n(P))\,P
\end{aligned}
$$

Finally, I work out the $Prob(b_i < b_j)$. This probability is determined by the integral of the joint distribution in the grey area in figure 8.

$$
\begin{aligned}
prob(b_i < b_j) &= \left(\int_{\underline{b}}^{P} f_j(b_j)\left(\int_0^{b_j} f_i(b_i)\partial b_i\right)\partial b_j\right) + F_i(P) - F_j(P) = \\
&= \int_{\underline{b}}^{P} f_j(b_j)F_i(b_j)\partial b_j + F_i(P) - F_j(P) = \\
&= \int_{\underline{b}}^{P} \frac{min\{\theta_i+\theta_j, \theta_i+T, k\}}{min\{\theta_i+\theta_j, \theta_i+T, k\} - max\{0, \theta_i-T, \theta_i+\theta_j-k\}}\frac{\underline{b}}{b^2} \\
&\quad \frac{min\{\theta_i+\theta_j, \theta_j+T, k\}}{min\{\theta_i+\theta_j, \theta_j+T, k\} - max\{0, \theta_j-T, \theta_i+\theta_j-k\}}\frac{b-\underline{b}}{b}\partial b + \\
&\quad 1 - F_j(P) \\
&= \frac{min\{\theta_i+\theta_j, \theta_i+T, k\}}{min\{\theta_i+\theta_j, \theta_i+T, k\} - max\{0, \theta_i-T, \theta_i+\theta_j-k\}} \\
&\quad \frac{min\{\theta_i+\theta_j, \theta_j+T, k\}}{min\{\theta_i+\theta_j, \theta_j+T, k\} - max\{0, \theta_j-T, \theta_i+\theta_j-k\}}\underline{b} \\
&\quad \left[\int_{\underline{b}}^{P} \frac{\partial b}{b^2} - \int_{\underline{b}}^{P} \frac{\underline{b}}{b^3}\partial b\right] + 1 - F_j(P) = \\
&= \frac{min\{\theta_i+\theta_j, \theta_i+T, k\}}{min\{\theta_i+\theta_j, \theta_i+T, k\} - max\{0, \theta_i-T, \theta_i+\theta_j-k\}} \\
&\quad \frac{min\{\theta_i+\theta_j, \theta_j+T, k\}}{min\{\theta_i+\theta_j, \theta_j+T, k\} - max\{0, \theta_j-T, \theta_i+\theta_j-k\}}\underline{b} \\
&\quad \left[\frac{\underline{b}}{4b^4} - \frac{1}{3b^3}\right]_{\underline{b}}^{P} + 1 - F_j(P) = \\
&= \frac{min\{\theta_i+\theta_j, \theta_i+T, k\}}{min\{\theta_i+\theta_j, \theta_i+T, k\} - max\{0, \theta_i-T, \theta_i+\theta_j-k\}} \\
&\quad \frac{min\{\theta_i+\theta_j, \theta_j+T, k\}}{min\{\theta_i+\theta_j, \theta_j+T, k\} - max\{0, \theta_j-T, \theta_i+\theta_j-k\}}\underline{b} \\
&\quad \left[\frac{3\underline{b}-4b}{12b^4}\right]_{\underline{b}}^{P} + 1 - F_j(P) = \\
&= \frac{min\{\theta_i+\theta_j, \theta_i+T, k\}}{min\{\theta_i+\theta_j, \theta_i+T, k\} - max\{0, \theta_i-T, \theta_i+\theta_j-k\}} \\
&\quad \frac{min\{\theta_i+\theta_j, \theta_j+T, k\}}{min\{\theta_i+\theta_j, \theta_j+T, k\} - max\{0, \theta_j-T, \theta_i+\theta_j-k\}}\underline{b} \\
&\quad \left[\frac{3\underline{b}-4P}{12P^4} + \frac{1}{12\underline{b}^3}\right]_{\underline{b}}^{P} + 1 - F_j(P) = \\
&= \frac{min\{\theta_i+\theta_j, \theta_i+T, k\}}{min\{\theta_i+\theta_j, \theta_i+T, k\} - max\{0, \theta_i-T, \theta_i+\theta_j-k\}} \\
&\quad \frac{min\{\theta_i+\theta_j, \theta_j+T, k\}}{min\{\theta_i+\theta_j, \theta_j+T, k\} - max\{0, \theta_j-T, \theta_i+\theta_j-k\}}\underline{b} \\
&\quad \frac{(3\underline{b}-4P)\underline{b}^3 + P^4}{12P^4\underline{b}^3} + 1 - F_j(P)
\end{aligned} \tag{8}
$$

Using 8, the $prob(b_s < b_n)$ in Area $A1$ is equal to,

Figure 8: $(b_i, b_j) \mid b_i < b_j$



$$prob(b_s < b_n) \quad = \quad \frac{(\theta_s + \theta_n)}{(k + T - \theta_n)} \underline{b} \frac{(3\underline{b} - 4P)\underline{b}^3 + P^4}{12P^4\underline{b}^3} + 1 - F_n(P)$$

Using 8, the $prob(b_s < b_n)$ in Area $B1$ is equal to,

$$prob(b_s < b_n) \quad = \quad \frac{(\theta_s + T)k}{(k + T - \theta_n)^2} \underline{b} \frac{(3\underline{b} - 4P)\underline{b}^3 + P^4}{12P^4\underline{b}^3} + 1 - F_n(P)$$

Corollary 2. Area $A1$

$$\frac{\partial \underline{b}}{\partial \theta_n} \quad = \quad \frac{P(\theta_s + T)}{(\theta_n + \theta_s)^2} > 0$$

$$\begin{aligned} \frac{\partial F_n(P)}{\partial \theta_n} &= \frac{-\left[(k + T - \theta_n) - (\theta_n + \theta_s)\right]}{\left[(\theta_n + \theta_s)(k + T - \theta_n)\right]^2} \\ &= \frac{2\theta_n + \theta_s - k - T}{\left[(\theta_n + \theta_s)(k + T - \theta_n)\right]^2} > 0 \Longleftrightarrow \theta_n > \frac{k + T - \theta_s}{2} \end{aligned}$$

$$\begin{aligned} \frac{\partial E(b_n)}{\partial \theta_n} = \; & \frac{\theta_s + T}{(k + T - \theta_n)^2} \underline{b} ln \left(\frac{P}{\underline{b}} \right) + \\ & \frac{\theta_s + T}{k + T - \theta_n} \frac{\partial \underline{b}}{\partial \theta_n} ln \left(\frac{P}{\underline{b}} \right) + \\ & \frac{\theta_s + T}{k + T - \theta_n} \underline{b} \frac{\underline{b}}{P} \frac{\partial \underline{b}}{\partial \theta_n} - \frac{\partial F_n(P)}{\partial \theta_n} > 0 \end{aligned}$$

Where all the elements are positive and $-\frac{\partial F_n(P)}{\partial \theta_n} > 0$ when $\theta_n < \frac{k+T-\theta_s}{2}$.

$$\begin{aligned}\frac{\partial E(b_s)}{\partial \theta_n} &= \frac{1}{(T+\theta_s)}\underline{b}ln\left(\frac{P}{\underline{b}}\right)+ \\ &\frac{1}{(T+\theta_s)}\frac{\partial \underline{b}}{\partial \theta_n}ln\left(\frac{P}{\underline{b}}\right)+ \\ &\frac{1}{(T+\theta_s)}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial \theta_n} > 0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial \theta_n} = P > 0$$

$$\frac{\partial \overline{\pi}_s}{\partial \theta_n} = \frac{P}{k}(\theta_s+T) > 0$$

Area $B1$

$$\frac{\partial \underline{b}}{\partial \theta_n} = \frac{P}{k} > 0$$

$$\frac{\partial F_n(P)}{\partial \theta_n} = 0$$

$$\begin{aligned}\frac{\partial E(b_n)}{\partial \theta_n} &= \frac{\theta_s+T}{(k+T-\theta_n)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right)+ \\ &\frac{\theta_s+T}{k+T-\theta_n}\frac{\partial \underline{b}}{\partial \theta_n}ln\left(\frac{P}{\underline{b}}\right)+ \\ &\frac{\theta_s+T}{k+T-\theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial \theta_n} > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial E(b_s)}{\partial \theta_n} &= \frac{k}{(k+T-\theta_n)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right)+ \\ &\frac{k}{k+T-\theta_n}\frac{\partial \underline{b}}{\partial \theta_n}ln\left(\frac{P}{\underline{b}}\right)+ \\ &\frac{k}{k+T-\theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial \theta_n} > 0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial \theta_n} = P > 0$$

$$\frac{\partial \overline{\pi}_s}{\partial \theta_n} = \frac{P}{k}(\theta_s+T) > 0$$

Corollary 3. Area $A1$.

$$\frac{\partial \underline{b}}{\partial \theta_s} = \frac{-P(\theta_n - T)}{(\theta_n + \theta_s)^2} < 0$$

$$\begin{aligned}\frac{\partial F_n(P)}{\partial \theta_s} &= \frac{2(\theta_s + T)\left[(\theta_n + \theta_s)(k + T - \theta_n)\right] - (\theta_s + T)^2\left[k + T - \theta_n\right]}{\left[(\theta_n + \theta_s)(k + T - \theta_n)\right]^2} \\ &= \frac{(\theta_s + T)(k + T - \theta_n)(2\theta_n + \theta_s - T)}{\left[(\theta_n + \theta_s)(k + T - \theta_n)\right]^2} > 0\end{aligned}$$

$$\begin{aligned}\frac{\partial E(b_n)}{\partial \theta_s} = & \frac{1}{k + T - \theta_n}\underline{b}ln\left(\frac{P}{\underline{b}}\right) + \\ & \frac{\theta_s + T}{k + T - \theta_n}\frac{\partial \underline{b}}{\partial \theta_s}ln\left(\frac{P}{\underline{b}}\right) \\ & \frac{\theta_s + T}{k + T - \theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial \theta_s} - \frac{\partial F_n(P)}{\partial \theta_s} < 0\end{aligned}$$

Where, $\frac{\theta_s + T}{k + T - \theta_n}\frac{\partial \underline{b}}{\partial \theta_s}ln\left(\frac{P}{\underline{b}}\right)$, $\frac{\theta_s + T}{k + T - \theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial \theta_s}$ and $\frac{\partial F_n(P)}{\partial \theta_s}$ are negative and $\frac{1}{k + T - \theta_n}\underline{b}ln\left(\frac{P}{\underline{b}}\right)$ is positive.

$$\begin{aligned}\frac{\partial E(b_s)}{\partial \theta_s} = & \frac{T - \theta_n}{(T + \theta_s)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right) + \\ & \frac{\theta_s + \theta_n}{T + \theta_s}\frac{\partial \underline{b}}{\partial \theta_s}ln\left(\frac{P}{\underline{b}}\right) \\ & \frac{\theta_s + \theta_n}{T + \theta_s}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial \theta_s} < 0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial \theta_s} = 0$$

$$\frac{\partial \overline{\pi}_s}{\partial \theta_s} = \frac{-P(\theta_n - T)}{(\theta_n + \theta_s)^2}(\theta_s + T) + \frac{-P(\theta_n - T)}{(\theta_n + \theta_s)} > 0$$

Area $B1$.

$$\frac{\partial \underline{b}}{\partial \theta_s} = 0$$

$$\frac{\partial F_n(P)}{\partial \theta_s} = \frac{1}{k} > 0$$

$$\begin{aligned}\frac{\partial E(b_n)}{\partial \theta_s} &= \frac{1}{k+T-\theta_n}\frac{P(\theta_n-T)}{k}ln\left(\frac{P}{\underline{b}}\right)-\frac{P}{k}>0 \Leftrightarrow \\ & \frac{(\theta_n-T)}{k+T-\theta_n}ln\left(\frac{P}{\underline{b}}\right)>1\end{aligned}$$

$$\frac{\partial E(b_s)}{\partial \theta_s} = 0$$

$$\frac{\partial \overline{\pi}_n}{\partial \theta_s} = 0$$

$$\frac{\partial \overline{\pi}_s}{\partial \theta_s} = \underline{b}>0$$

Proposition 3. Area $A1$

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{(\theta_n+\theta_s)}<0$$

$$\begin{aligned}\frac{\partial F_n(P)}{\partial T} &= \frac{2(\theta_s+T)(\theta_n+\theta_s)(k+T-\theta_n)-(\theta_s+T)^2(\theta_n+\theta_s)}{(\theta_n+\theta_s)^2(k+T-\theta_n)^2}= \\ &= \frac{2(\theta_s+T)(\theta_n+\theta_s)\left[2(k+T-\theta_n)-(\theta_s+T)\right]}{(\theta_n+\theta_s)^2(k+T-\theta_n)^2}>0\end{aligned}$$

$$\begin{aligned}\frac{\partial E(b_n)}{\partial T} &= \frac{(k+T-\theta_n)-(\theta_s+T)}{(k+T-\theta_n)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right) \\ & \frac{(\theta_s+T)}{k+T-\theta_n}\frac{\partial \underline{b}}{\partial T}ln\left(\frac{P}{\underline{b}}\right)+ \\ & \frac{(\theta_s+T)}{k+T-\theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial T}-\frac{\partial F_n(P)}{\partial T}?0\end{aligned}$$

Where, $\frac{(\theta_s+T)}{k+T-\theta_n}\frac{\partial \underline{b}}{\partial T}ln\left(\frac{P}{\underline{b}}\right)$ and $\frac{(\theta_s+T)}{k+T-\theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial T}-\frac{\partial F_n(P)}{\partial T}$ are negative and $\frac{(k+T-\theta_n)-(\theta_s+T)}{(k+T-\theta_n)^2}$ is positive.

$$\begin{aligned}\frac{\partial E(b_s)}{\partial T} &= \frac{-(\theta_s+\theta_n)}{(k+T-\theta_n)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right) \\ & \frac{-(\theta_s+\theta_n)}{(k+T-\theta_n)}\frac{\partial \underline{b}}{\partial T}ln\left(\frac{P}{\underline{b}}\right)+ \\ & \frac{-(\theta_s+\theta_n)}{(k+T-\theta_n)}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial T}<0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{k}(\theta_s + T) + \frac{P(\theta_n - T)}{k} = \frac{P(\theta_n - 2T - \theta_s)}{k} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

Area $B1$

$$\frac{\partial \underline{b}}{\partial T} = \frac{-P}{k} < 0$$

$$\frac{\partial F_n(P)}{\partial T} = \frac{1}{k} > 0$$

$$\begin{aligned}\frac{\partial E(b_n)}{\partial T} = & \frac{(k + T - \theta_n) - (\theta_s + T)}{(k + T - \theta_n)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right) \\ & \frac{(\theta_s + T)}{k + T - \theta_n}\frac{\partial \underline{b}}{\partial T}ln\left(\frac{P}{\underline{b}}\right) + \\ & \frac{(\theta_s + T)}{k + T - \theta_n}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial T} - \frac{\partial F_n(P)}{\partial T} < 0\end{aligned}$$

$$\begin{aligned}\frac{\partial E(b_s)}{\partial T} = & \frac{-k}{(k + T - \theta_n)^2}\underline{b}ln\left(\frac{P}{\underline{b}}\right) \\ & \frac{-k}{(k + T - \theta_n)}\frac{\partial \underline{b}}{\partial T}ln\left(\frac{P}{\underline{b}}\right) + \\ & \frac{-k}{(k + T - \theta_n)}\underline{b}\frac{\underline{b}}{P}\frac{\partial \underline{b}}{\partial T} < 0\end{aligned}$$

$$\frac{\partial \overline{\pi}_n}{\partial T} = -P < 0$$

$$\frac{\partial \overline{\pi}_s}{\partial T} = \frac{-P}{k}(\theta_s + T) + \frac{P(\theta_n - T)}{k} = \frac{P(\theta_n - 2T - \theta_s)}{k} > 0 \Leftrightarrow \theta_n > 2T + \theta_s$$

References

Bhagwati, J. N., 1968, "More on the Equivalence of Tariffs and Quotas," *American Economic Review*, 142-46.

Boiteux, M., 1949, "La tarification des demandes en pointe: Application de la théorie de la vente au cout marginal" *Revue Générale d'Electricité*, 321-40.

Borenstein, S., Bushnell J. and Knittel C. R., 1999, "Market Power in Electricity Markets: Beyond Concentration Measures," *Energy Journal*, 20(4).

Borenstein, S., Bushnell J. and Stoft S., 2000, "The Competitive effects of transmission capacity in a deregulated electricity industry," *Rand Journal of Economics*, 31 (2), 294-325.

Deneckere, R. and Kovenock, D., 1996, "Bertrand-Edgeworth Duopoly with Unit Cost Asymmetry," *Economic Theory*, 8, 1-25.

Fabra, N., von der Fehr N. H. and Harbord D., 2006, "Designing Electricity Auctions," *Rand Journal of Economics*, 37 (1), 23-46.

Gilbert, R., Neuhoff K. and Newbery D., 2004, "Allocating Transmission to Mitigate Market Power in Electricity Networks," *The RAND Journal of Economics*, 35(4), 691-709.

Green, R. and Newbery D., 1992, "Competition in the British Electricity Spot Market," *Journal of Political Economy*, 100(5), 929-53.

Hotelling, H., 1929, "The Stability of Competition," *Economic Journal*, 39, 41-57.

Itoh, M. and Ono M., 1982, "Tariffs, Quotas, and Market Structure," *Quarterly Journal of Economics*, 97, 295-305.

Joskow, P. L., Tirole J., 2000, "Transmission rights and market power on electric power networks," *RAND Journal of Economics*, 31(3), 450-487.

Klemperer, P., 2004 "Auctions: Theory and Practice," 1^{st} ed., Priceton University Press.

Klemperer, P. and Meyer M., 1989, "Supply Function Equilibria in Oligopoly under Uncertainty," *Econometrica*, 57(6), 1243-77.

Dasgupta, P., Maskin E., 1986, "The Existence of Equilibrium in Discontinuous Economic Games, II: Applications", *The Review of Economic Studies*, 53 (1), 27-41.

Neuhoff, K., 2003, "Integrating Transmission and Energy Markets Mitigates Market Power," *DAE Woriking Paper WP 0310.*

Newbery, D.M., 1992, "Capacity-Constrained Supply Function Equilibria: Competition and Entry in the Electricity Spot Market," Department of Applied Economics *Working*

Paper no. 9208, Cambridge University.

Newbery, D., 1998, "Competition, contracts, and entry in the electricity spot market," *RAND Journal of Economics*, 29 (4), 726-749.

Osborne, M. and Pitchik, C., 1986, "Price Competition in a capacity-constrained duopoly," *Journal of Economic Theory*, 38, 283-260.

Shibata, H., 1968, "A Note on the Equivalence of Tariffs and Quotas," *American Economic Review*, 137-42.

Shitovitz, B., 1973, "Oligopoly in Markets with Continuum of Traders," *Econometrica*, 41, 467-501.

Varian, H., 1980, "A Model of Sales," *American Economic Review*, 70, 651-659.

von der Fehr, N.H. and Harbord D., 1993, "Spot Market Competition in the UK Eletricity Industry," *Economic Journal*, 103(418), 531-46.

Wilson, R., 2002, "Architecture of Power Markets," *Econometrica*, 70(4), 1299-1340.

Wolfram, C.D., 1998, "Stategic Bidding in a Multi-Unit Auction: An Empirical Analysis of Bids to Supply Electricity in England and Wales," *RAND Journal of Economics*. Vol. 29 (1998), pp. 703-725.

Yadav, G., 1968, "A Note on the Equivalence of Tariffs and Quotas," *Canadian Journal of Economics*, 105-110.