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Efficiency Wages, Urban Unemployment and Housing Consumption

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Abstract

The aim of this paper is to introduce endogenous housing consumption in an efficiency wage model in which two cases are considered: very high and zero relocation costs. First, in both cases, we are able to totally characterize the efficiency wage for any preferences that are quasi-linear with respect to the composite good consumption. Second, in each case, we show how endogenous housing consumption affects the value of the efficiency wage (compared to the case of fixed housing consumption) and demonstrates the existence and the uniqueness of a steady-state equilibrium. Finally, we compare the two models and show how mobility costs affect the efficiency wage setting.

Key words: relocation costs, location-dependent wages, urban labor markets.

JEL Classification: J41, R14.

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1 Introduction

Over the last ten years, a growing literature on the theoretical study of urban labor markets has emerged. One of the popular approach has been the efficiency wage model in which firms set high wages to deter shirking. To the best of our knowledge, Zenou and Smith (1995) were the first to study this issue by putting together the standard urban model (Brueckner, 1987, Fujita, 1989) and the standard efficiency wage approach (Shapiro and Stiglitz, 1984). As a result, in their model, workers' location and land prices as well as wages and unemployment are endogenously determined in equilibrium. The interaction between land and labor markets is then crucial to understand the way equilibrium emerges. Further developments on urban efficiency wage models have been undertaken (for a recent survey see Zenou, 2004).¹ However, in all these approaches, two important aspects have been neglected: The housing consumption is always exogenous and normalized to 1 (endogenous housing consumption has never been modelled in an urban labor market) and relocation costs are assumed to be zero (exceptions with high-relocation costs include Brueckner and Zenou, 2003, and Zenou, 2003).

The aim of this paper is to relax these two assumptions and to study their consequences on the land and labor markets.

To be more precise, we define a steady-state equilibrium of an urban labor market when both the urban land use and the labor market equilibrium are solved simultaneously. Because housing consumption is endogenous, in the urban land use equilibrium, both the city fringe and the border between the employed and unemployed become endogenous and makes the analysis quite cumbersome; in particular the exact calculation of the instantaneous utilities becomes extremely difficult. As a result, because the determination of the efficiency wage requires a closed-form solution of the variables determined at the urban land use equilibrium (in particular the instantaneous utilities), we assume throughout preferences for all workers that are quasi-linear with respect to the composite good consumption.²

We consider two cases. In the first one, relocation costs are assumed to

¹There is also a literature on urban labor markets that considers the search-matching model instead of the efficiency wage model. See in particular, Coulson, Laing and Wang, 2001, Sato, 2001 and Wasmer and Zenou, 2002.

²In fact, in the case of zero relocation costs, it is not possible to obtain an explicit solution for the efficiency wage with other types of preferences. In particular, Cobb-Douglas preferences cannot lead to an explicit solution of the efficiency wage.

be zero (workers change location as soon as they change employment status) whereas in the second one they are extremely high (workers never change location, even when they change employment status). First, we obtain a general result. Indeed, for any preferences that are quasi-linear with respect to the composite good consumption, we are able to totally characterize the efficiency wage, both in the case of high and zero relocation costs. Second, in the no-relocation cost model, we show how endogenous housing consumption affects the value of the efficiency wage (compared to the case of fixed housing consumption) and demonstrate the existence and the uniqueness of the steady-state equilibrium. Third, in the case of high-relocation costs, we determine the exact value of the efficiency wage and show that workers reduce their composite consumption when they are unemployed in order to stay in the same location. We also demonstrate the existence and uniqueness of the steady-state equilibrium. Finally, we compare the two models and show how mobility costs affect the efficiency wage setting.

The rest of the paper is presented as follows. In the next section, we describe the general model. Section 3 deals with the no-relocation case whereas the high-relocation model is analyzed in section 4. Section 5 concludes.

2 The model

There is a continuum of ex ante identical workers whose mass is N and a continuum of M identical firms. Among the N workers, there are L employed and U unemployed so that $N = L + U$. The workers are uniformly distributed along a linear, closed and monocentric city. Their density at each location is taken to be 1. All land is owned by absentee landlords and all firms are exogenously located in the Central Business District (CBD hereafter) and consume no space. Workers are assumed to be infinitely lived and decide their optimal place of residence between the CBD and the city fringe.

Each individual is identified with one unit of labor. As in the standard efficiency wage model without space (see Shapiro and Stiglitz, 1984), there are only two possible levels of effort that a worker can exert: either the worker shirks, has zero effort, $e = 0$ and contributes to zero production or he/she does not shirk, provides full effort, $e > 0$ and contributes to e production. Each employed worker goes to the CBD to work and incurs a fixed commuting cost τ per unit of distance. When living at a distance x from the CBD, he/she also pays a land rent $R(x)$, consumes h unity of land and earns a wage w_L (that

will be determined at the labor market equilibrium).

The steady-state equilibrium of this economy requires solving *simultaneously* two problems:

(i) a location and rental price outcome (referred to as an urban land use equilibrium).

(ii) a (steady state) labor market equilibrium with determines wages and unemployment (referred to as a labor market equilibrium).

We will consider two extreme opposite cases. In the first one, workers have no relocation costs whereas in the second one, these costs are very high.

3 The case of no-relocation costs

Let us start with the case of no relocation costs, either in terms of time or money. This is a simplifying assumption, which is quite standard in urban economics. It implies that workers change location as soon as they change employment status.

3.1 The urban land use equilibrium

In order for the model to be tractable, we assume quasi-linear preferences for all workers. For worker with employment status $k = L, U$, we have the following preferences:

$$\Omega(h_k, z_k) = z_k + h_k^{1/2} \tag{1}$$

where z_k and h_k are respectively the *composite good* and *housing* consumptions for a worker with employment status $k = L, U$. It is also assumed that housing h_k is a normal good so richer workers consume more land. The budget constraint of a non-shirker employed workers is given by:

$$h_L R(x) + \tau x + z_L = w_L - e \tag{2}$$

where the composite good is taken as the numeraire good with unit price

Concerning the unemployed, they commute less often to the CBD since they mainly go there to search for jobs. So, we assume that they incur a commuting cost $s \tau$ per unit of distance, with $0 < s < 1$. For example, $s = 1/2$ implies that the unemployed make only half as many CBD-trips as the employed workers. Each unemployed worker earns a fixed unemployment benefit $w_U > 0$, pays a land rent $R(x)$, consumes h_U unit of land. The budget constraint of an

unemployed worker is thus equal to:

$$h_U R(x) + s\tau x + z_U = w_U \quad (3)$$

Maximizing utility (1) subject to (2) yields the following *housing(land) demand for non-shirker employed workers* at x :³

$$h_L^{NS}(x) = \frac{1}{4[R(x)]^2} \quad (4)$$

Similarly, maximizing (1) subject to (3) yields the following *housing (land) demand for unemployed workers* at x :

$$h_U(x) = \frac{1}{4[R(x)]^2} \quad (5)$$

Using (1) and (4), we can now derive the following instantaneous indirect utility

$$W_L^{NS}(x) = w_L - e - \tau x + \frac{1}{4R(x)} \quad (6)$$

for each *employed* worker at x , and, using (1) and (5), we have the following instantaneous indirect utility

$$W_U(x) = w_U - s\tau x + \frac{1}{4R(x)} \quad (7)$$

for each *unemployed* worker at x .

Since there are no relocation costs, the urban equilibrium is such that all the employed (in equilibrium the efficiency wage is such that nobody shirks) enjoy the same level of utility $W_L^{NS}(x) \equiv W_L$ while all the unemployed obtain W_U . The bid rents of the (non-shirking) employed workers and the unemployed are thus equal to:

$$\Psi_L(x, W_L) = \frac{1}{4(W_L - w_L + e + \tau x)} \quad (8)$$

$$\Psi_U(x, W_U) = \frac{1}{4(W_U - w_U + s\tau x)} \quad (9)$$

They are both decreasing and convex in x . We can now calculate the housing consumption of each worker:

³There is obviously a unique solution to the maximization problem of both the employed and the unemployed since the second order condition is given by:

$$\frac{\partial^2 \Omega(h_k, z_k)}{\partial h_k^2} = -\frac{1}{4} h_k^{-3/2} < 0$$

$$h_L(x, W_L) = 4(W_L - w_L + e + \tau x)^2 \quad (10)$$

$$h_U(x, W_U) = 4(W_U - w_U + s\tau x)^2 \quad (11)$$

We have the following straightforward result:

Proposition 1 *With quasi-linear preferences and endogenous housing consumption, the employed reside close to jobs whereas the unemployed live at the periphery of the city.*

Proof. Let us take a pair of bid-rent curves $\Psi_L(x_b, W_L)$ and $\Psi_U(x_b, W_U)$, and suppose that they intersect at some distance x_b (which is the border between the employed and the unemployed), i.e. $\Psi_L(x_b, W_L) = \Psi_U(x_b, W_U) = R_b$. By using (4) and (5), we have:

$$h_L(x, W_L) = h_L^{NS}(x_b) = \frac{1}{4R_b^2} = h_U(x, W_U) = h_U(x_b) = \frac{1}{4R_b^2}$$

i.e. the housing consumption of the employed and the unemployed is the same at x_b . But by differentiating both (8) and (9), and since $s < 1$, we have:

$$-\frac{\partial \Psi_L(x, W_L)}{\partial x} \Big|_{x=x_b} = \frac{-\tau}{h_L(x_b, W_L)} > \frac{-s\tau}{h_U(x_b, W_U)} = -\frac{\partial \Psi_U(x, W_U)}{\partial x} \Big|_{x=x_b}$$

which implies that the employed's bid rent is steeper than the unemployed's. As a result, they occupy the core of the city and bid away the unemployed to the periphery of the city. ■

When housing consumption is endogenous, then one would expect that no clear urban pattern would emerge. Indeed, on the one hand, the employed workers want to be closer to jobs than the unemployed because of higher commuting costs. On the other, because housing is a normal good, they consume more land and therefore prefer to be further away from the CBD since housing is cheaper there. However, because we assume quasi-linear preferences, the second effect is nil since at x_b both the employed and the unemployed consume the same amount of land because housing consumption is independent of net income (see (4) and (5)). As a result, there is only the commuting cost effect and thus the employed locate close to jobs. Interestingly, this result is robust if one uses any quasi-linear utility function in which the non-linearity is on h .

Because housing consumption is endogenous, we cannot as in the case of fixed housing consumption (see e.g. Zenou and Smith, 1997, or Brueckner and Zenou, 1999) equate x_b to L and x_f to N , but we have to determine

them. We focus on a closed-city model with absentee landlords. Without loss of generality and to simplify the calculus, we normalize the outside land rent (the agricultural land rent) R_A to $1/4$. Since density is 1 at each location, we have the following definition

Definition 1 *An urban-land use equilibrium with no relocation costs is a 5-tuple $(x_b, x_f, W_L, W_U, R(x))$ such that:*

$$\Psi_L(x_b, W_L) = \Psi_U(x_b, W_U) \quad (12)$$

$$\Psi_U(x_f, W_U) = R_A = \frac{1}{4} \quad (13)$$

$$\int_0^{x_b} \frac{1}{h_L(x, W_L)} dx = L \quad (14)$$

$$\int_{x_b}^{x_f} \frac{1}{h_U(x, W_U)} dx = N - L \quad (15)$$

$$R(x) = \begin{cases} \Psi_L(x_b, W_L) & \text{for } x \leq x_b \\ \Psi_U(x_f, W_U) & \text{for } x_b < x \leq x_f \\ R_A = \frac{1}{4} & \text{for } x > x_f \end{cases} \quad (16)$$

Equations (12) and (13) reflect equilibrium conditions in the land market. Equation (12) says that, in the land market, at the frontier x_b , the bid rent offered by the employed is equal to the bid rent offered by the unemployed. Equation (13) in turn says that the bid rent of the unemployed must be equal to the agricultural land at the city fringe. Equations (14) and (15) give the two population constraints. Finally, equation (16) defines the equilibrium land rent as the upper envelope of the equilibrium bid rent curves of all workers' types and the agricultural rent line.

Solving (12)-(15) is quite cumbersome. Fortunately, after tedious calculus and using (8)-(11), we are able to obtain closed-form solutions. Denoting

$$A \equiv \frac{1}{1 + 4s\tau(N - L)} \quad (17)$$

we obtain the following equilibrium values:⁴

$$x_b = \frac{4LA}{1/A + 4\tau L} \quad (18)$$

$$x_f = 4(N - L)A + \frac{4LA}{1/A + 4\tau L} \quad (19)$$

⁴See Appendix 1 for the proof of these results.

$$W_L^{NS} = w_L - e + \frac{1}{1/A + 4\tau L} \quad (20)$$

$$W_U = w_U + 1 - 4s\tau A \left[\frac{L}{1/A + 4\tau L} + N - L \right] \quad (21)$$

$$R(x) = \begin{cases} \frac{1}{4} \left(\frac{1}{1/A + 4\tau L} + \tau x \right)^{-1} & \text{for } x \leq x_b \\ \frac{1}{4} \left(1 - 4s\tau A \left[\frac{L}{1/A + 4\tau L} + N - L \right] + s\tau x \right)^{-1} & \text{for } x_b < x \leq x_f \\ \frac{1}{4} & \text{for } x > x_f \end{cases} \quad (22)$$

Because, this will be important below, we study the properties of x_b . We have:

Proposition 2 *The border between the employed and the unemployed x_b is equal to 0 when $L = 0$ and increases with L , i.e.*

$$x_b(L = 0) = 0 \text{ and } \frac{\partial x_b}{\partial L} > 0$$

Moreover, for a given L , we have:

$$\frac{\partial x_b}{\partial s} < 0$$

$$\frac{\partial x_b}{\partial \tau} \geq 0 \Leftrightarrow \tau \leq \frac{1}{4\sqrt{s(N-L)}[L + s(N-L)]}$$

Proof. See Appendix 1.

This proposition says that if L the level of employment in this economy increases, then more workers are employed and thus richer, and, therefore, the space they occupy increases. It also says that, at a given L (at the labor equilibrium L will itself depend on s and τ), the higher is the percentage of unemployed CBD-trips, the lower is x_b . This is because if the unemployed goes more often to the CBD, their commuting costs increase and, thus, their willingness to pay for land decreases (see (9)). Since s does not affect the bid rent of the employed, the border x_b decreases. Finally, when τ increases, the impact on x_b is ambiguous. Indeed, the commuting cost τ negatively affects both bid rents (see (8) and (9)) so that an increase of τ reduces the willingness to pay for land for both the employed and the unemployed workers. However, if τ is small enough, then a rise in τ increases x_b , while we have the reverse if τ is large enough. This is because the employed have higher total commuting costs than the unemployed (τx versus $s\tau x$) so, if τ is already large, then when τ increases, they will increase less their bid rent than the unemployed so that x_b decreases. When τ is small, we have the reverse result.

3.2 The steady-state equilibrium

We are now able to solve the labor market equilibrium and thus the steady-state equilibrium. Observing that $W_L^S = W_L + e$ and using (59) and (60), we can write the steady-state Bellman equations for respectively the non-shirkers, the shirkers and the unemployed as follows:

$$r I_L^{NS} = w_L - e + 1 - s\tau (x_f - x_b) - \tau x_b - \delta (I_L^{NS} - I_U) \quad (23)$$

$$r I_L^S = w_L + 1 - s\tau (x_f - x_b) - \tau x_b - (\delta + m) (I_L^S - I_U) \quad (24)$$

$$r I_U = w_U + 1 - s\tau x_f + a(I_L - I_U) \quad (25)$$

where r is the discount rate, m , the monitoring rate, δ , the job destruction rate, a , the job acquisition rate, I_L^{NS} , I_L^S and I_U respectively represent the expected lifetime utility of a non-shirker, a shirker and an unemployed worker. The first equation that determines I_L^{NS} states that a non-shirker obtains today W_L^{NS} but can lose his/her job with a probability δ and then obtains a negative surplus of $I_U - I_L^{NS}$. For I_L^S , we have the same interpretation, except for the fact that a shirker can lose his/her job for two reasons: either the job is destroyed or if he/she is caught shirking. The last equation has a similar interpretation.

Firms must pay enough to prevent shirking, i.e. $I_L^{NS} \geq I_L^S$; otherwise workers will provide $e = 0$ and produce nothing. However, there is no need to pay more than the minimum needed to induce effort. Therefore, firms will choose a wage w_L so that $I_L^{NS} = I_L^S = I_L$, i.e. the efficiency wage must be set to make workers indifferent between shirking and not shirking. By equating (24) and (23), we easily obtain:

$$I_L - I_U = \frac{e}{m} \quad (26)$$

The surplus of being employed only depends on the monitoring technology and on the effort level provided by workers.

Let us now calculate the efficiency wage. We can rewrite (23) as follows:

$$\begin{aligned} w_L &= e + r I_L + \delta (I_L - I_U) - 1 + s\tau (x_f - x_b) + \tau x_b \\ &= e + r I_U + (\delta + r)(I_L - I_U) - 1 + s\tau (x_f - x_b) + \tau x_b \end{aligned}$$

Furthermore, using (25) and (26), this can be rewritten as:

$$w_L = w_U + e + \frac{e}{m}(a + \delta + r) + 1 - s\tau x_f - 1 + s\tau (x_f - x_b) + \tau x_b$$

which is equivalent to

$$w_L = w_U + e + \frac{e}{m}(a + \delta + r) + (1 - s)\tau x_b \quad (27)$$

Finally, at the steady state, flows out of unemployment equal flows into unemployment, i.e.

$$a(N - L) = \delta L \quad (28)$$

so that, using (18), the efficiency wage is finally given by:⁵

$$w_L^{nr} = w_U + e + \frac{e}{m} \left(\frac{\delta N}{N - L} + r \right) + \Delta SC \quad (29)$$

where

$$\begin{aligned} \Delta SC &= (1 - s) \tau x_b \\ &= (1 - s) \left[\frac{1}{1 + 4s\tau(N - L)} - \frac{1}{1 + 4s\tau(N - L) + 4\tau L} \right] \end{aligned}$$

is the spatial-cost differential between the employed and the unemployed.

Equation (29) is referred to as the Urban Non Shirking Constraint (UNSC) since it is the lowest wage at each level of employment that is necessary to induce workers not to shirk and to stay in the city. The properties of this efficiency wage are given by the following proposition.

Proposition 3 *The properties of the efficiency wage w_L defined by (29) are as follows:*

- (i) *The efficiency wage w is increasing in L ;*
- (ii) *When L tends to N , we obtain:*

$$\lim_{L \rightarrow N} w_L^{nr} = +\infty$$

- (iii) *When $L = 0$, we have:*

$$w_L^{nr}(L = 0) = w_U + e + \frac{e}{m} (\delta + r) > 0$$

- (iv) *The efficiency wage is increasing in w_U , e , δ and r and decreasing in m .*
- (v) *We also have:*

$$\begin{aligned} \frac{\partial w_L^{nr}}{\partial \tau} \geq 0 &\Leftrightarrow \tau \leq \frac{1}{4\sqrt{s(N - L)} [L + s(N - L)]} \\ \frac{\partial w_L^{nr}}{\partial s} &< 0 \end{aligned}$$

⁵The superscript *nr* refers to the no-relocation case.

Proof. See Appendix 1.

The following comments are in order. First, (i), (ii) and (iii) are standard results in the non-spatial efficiency wage literature (Shapiro and Stiglitz, 1984). Indeed, w_L^{nr} increases with L , goes to infinity when L tends to N and has a positive value when $L = 0$. This is because unemployment acts as a worker discipline device so that lower employment level, or equivalently higher unemployment level, reduces the efficiency wage because the outside option of workers is lower since it is more difficult to find a job. When there is full employment, $L = N$, then no efficiency wage can be implemented. Indeed, efficiency wages are not compatible with full employment since, in this case, workers will always shirk because, if caught and fired, they will immediately find another job. When there is full unemployment, $U = N$ or $L = 0$, the efficiency wage is strictly positive because firms have still to induce workers to take a job and leave unemployment. Second, (iv), the effects of the non-spatial variables on the efficiency wage are also standard. The efficiency wage is indeed increasing in w_U , e and δ , and decreasing in r and m . This is because an increase in w_U , e , δ or r shifts upward the UNSC curve because workers have higher outside option and thus firms must increase wages in order for workers not to shirk. On the contrary, an increase in the monitoring rate m shifts downward the UNSC curve because firms invest more money in controlling their workers. As a result, they can reduce wages because the chance to be caught increases and thus the outside option decreases.

Let us now interpret (v), which are the effects of the *spatial* variables, s and τ , on the efficiency wage w_L^{nr} . In fact, the effects of s and τ on w_L^{nr} are exactly the ones of s and τ on x_b (see Proposition 2). Since $\Delta SC = (1 - s) \tau x_b$ is the spatial compensation that firms must give to their employed workers, an increase in s , the percentage of unemployed CBD-trips relative to employed CBD-trips, will always reduce the efficiency wage. Indeed, firms need to compensate less the employed for spatial costs since ΔSC decreases with s . However, when the pecuniary commuting cost per unit of distance τ increases, the effect on wages is ambiguous. This is the effect mentioned in Proposition 2 where an increase in τ raises x_b only if τ is small enough.

Let us now compare our efficiency wage (29) with the efficiency wage obtained where housing consumption is fixed and equal to 1 (Zenou and Smith, 1997). In the latter, the efficiency wage is given by:

$$\begin{aligned}
w_L^{sz}(L) &= w_U + e + \frac{e}{m} \left(\frac{\delta N}{N-L} + r \right) + (1-s) \tau x_b \\
&= w_U + e + \frac{e}{m} \left(\frac{\delta N}{N-L} + r \right) + (1-s) \tau L
\end{aligned}$$

It is easy to see that the only difference between these two wages is on the spatial-cost differential between the employed and the unemployed ΔSC , i.e. the spatial component of the wage that firms must set in order to compensate the employed workers for their spatial costs (commuting and land rent costs). In fact, since all employed workers obtain the same utility level and since all unemployed workers also obtain the same utility level, we can compare the employed and the unemployed who live exactly at a distance x_b (i.e. the border between the employed and the unemployed) from the CBD since at x_b they both pay the same land rent. As a result, in the case of fixed housing (normalized to 1), $x_b = L$ and the only difference between these two workers is the commuting cost differential, which is equal to $\Delta SC = (1-s) \tau L$. However, in the case of endogenous housing consumption, the spatial difference is also $(1-s) \tau x_b$ but the value of x_b is now more complicated because it takes into account the competition between the employed and the unemployed in the land market.

Observe that this result is obtained because the housing consumption only depends on the land rent $R(x)$ and not on income (see (4) and (5)). As a result, at x_b , where the land rent between the employed and the unemployed is the same, the only spatial difference between them is the commuting cost difference $(1-s) \tau x_b$. This is true for any quasi-linear function. Indeed,

Proposition 4 *Assume no relocation costs. Then, if the preferences of the employed and unemployed workers are quasi-linear with respect to z_k , i.e.*

$$\Omega(h_k, z_k) = z_k + g(h_k) \quad , \quad k = L, U$$

where $g(\cdot)$ is any increasing function in h_k , then the employed reside close to the CBD and the unemployed at the periphery of the city and the efficiency wage is given by

$$w_L = w_U + e + \frac{e}{m} \left(\frac{\delta}{u} + r \right) + (1-s) \tau x_b \quad (30)$$

The value of x_b depends on the specific form taken by $g(\cdot)$.

Proof. See Appendix 1.

This result is due to the fact that, when preferences are quasi-linear, the demand function for the good h does not depend on the individual's wealth (see e.g. Mas-Colell, Whinston and Green, 1995) but only on the price of h . As a result, at the same location and thus at the same price $R(x)$, the consumption of h will be the same for the employed and the unemployed. So, at the same location, which in our case can only be x_b since the employed and the unemployed are totally separated in the city, the only spatial cost difference between the employed and the unemployed is the commuting cost difference, that is $(1 - s) \tau x_b$. Since the efficiency wage has two roles: to prevent shirking (which is not spatially related) and to compensate for spatial cost differences, the efficiency wage will always be given by (30). Observe that the case of fixed housing consumption normalized to 1 (Zenou and Smith, 1997) is a special case of this proposition since it assumes that $g(h_k) = g(1) = 1$. In that case, $x_b = L$.

We can now determine the labor market equilibrium. As stated above, there are M identical firms ($j = 1, \dots, M$) in the economy. All firms produce the same composite good and sell it at a fixed market price p (this good is taken as the numeraire so that its price p is set to 1). All workers whatever their location contribute to one unit of production. The production function of each firm is denoted by $f(l_j)$ and it is assumed that $f(\cdot)$ is twice differentiable with $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, and it satisfies the Inada conditions, i.e. $f'(0) = +\infty$ and $f'(+\infty) = 0$.

Because all firms are identical, the employment level in each firm j is equal to: $l_j = l = L/M$. As a result, each firm adjusts employment until the marginal product of an additional worker equals the average efficiency wage so that we have

$$w_L^{nr} = f'(l) \tag{31}$$

This determines the labor demand in each firm. Since there are M firms in the economy, the aggregate production function $F(L) = M f(L/M)$ and the total labor demand in the economy is equal to $L = Ml$. The aggregate equivalent of (31) is thus given by:⁶

$$w_L^{nr} = F'(L) \tag{32}$$

Using (29), this is equivalent to:

⁶Observe that

$$F'(L) = M f'(L/M)(1/M) = f'(l)$$

$$w_U + e + \frac{e}{m} \left(\frac{\delta N}{N - L} + r \right) + \Delta SC = F'(L) \quad (33)$$

Proposition 5 *Assume no relocation costs. Then, there exists a unique labor market equilibrium in which the labor demand is (implicitly) defined by (33) and the equilibrium efficiency wage w_L^{rr} by (29).*

Proof. See Appendix 1.

4 The case of high-relocation costs

In this section, we assume that mobility costs are so high that once someone is located somewhere the worker never moves. As a result, *a worker's residential location and housing consumption remain fixed as he/she enters and leaves unemployment.*

In the present model, because housing consumption is endogenous, when workers decide their optimal location they also decide their optimal housing consumption (this is referred to as the delayed risk model; see in particular Drèze and Modigliani, 1972, and Zenou and Eeckhoudt, 1997). We assume perfect capital markets with a zero interest rate ($r \rightarrow 0$). When there is a zero interest rate, workers have no intrinsic preference for the present so that they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent. We will thus consider the average expected utility of a worker rather than, as in the previous section, the lifetime expected utilities of employed and unemployed workers. At any moment of time, the disposable utility of a worker is thus equal to that worker's average utility over the job cycle.

Since housing is the same whether a worker is employed or unemployed but composite consumption is different, the budget constraints a non-shirker employed worker and an unemployed worker are respectively given by:

$$h R(x) + \tau x + z_L = w_L - e \quad (34)$$

$$h R(x) + s\tau x + z_U = w_U \quad (35)$$

It is easy to see that

$$z_L - z_U = w_L - w_U - e - (1 - s)\tau x \quad (36)$$

which is in general strictly positive (this will be verified in equilibrium). This means that workers, who have fixed location, always propose the same bid rent (to stay at this location) and consume the same amount of land but adjust their composite consumption good over time. Indeed, since $z_L - z_U > 0$, in order to stay in the same location and to consume the same amount of land, workers reduce their composite consumption when unemployed.

Workers can either be employed or unemployed. As above, we assume that changes in the employment status (employment versus unemployment) are governed by a continuous-time Markov process. Firms cannot perfectly monitor workers so that there is a probability of detected shirking, denoted by θ . If a worker is caught shirking, he/she is automatically fired. Job contacts (that is the transition rate from unemployment to employment) randomly occur at an endogenous rate a while the exogenous job separation rate is δ . In this context, the expected duration of employment is given by $1/\delta$ for non-shirkers and $1/(\delta + \theta)$ for shirkers whereas the expected duration of unemployment amounts to $1/a$. It then follows that a worker spends a fraction $a/(a + \delta)$ if non-shirker and $a/(a + \delta + \theta)$ if shirker of his/her lifetime employed and a fraction $\delta/(a + \delta)$ if non-shirker and $(\delta + \theta)/(a + \delta + \theta)$ if shirker of his/her lifetime unemployed. In steady state, flows into and out of unemployment are equal. Therefore, for the unemployment rate of non-shirkers is given by:

$$u \equiv u^{NS} = \frac{\delta}{a + \delta} \quad (37)$$

while for the one of shirkers, we have

$$u^S = \frac{\delta + m}{a + \delta + m} \quad (38)$$

with $u^S > u^{NS}$, $\forall a, \delta, \theta > 0$.

We assume the same quasi-linear preferences. Combining (1) with (34) gives the utility of the non-shirker employed, which is equal to

$$\Omega^{NS}(h^{NS}) = w_L - e - \tau x - h^{NS}R(x) + (h^{NS})^{1/2}$$

The utility of the employed shirker is thus:

$$\Omega^S(h^S) = w_L - \tau x - h^S R(x) + (h^S)^{1/2}$$

Combining (1) with (35) determines the utility of the unemployed. It is given by:

$$\Omega^U(h^U) = w_U - s\tau x - h^U R(x) + (h^U)^{1/2}$$

As a result, the expected utility of a non-shirker and a shirker are respectively given by:

$$\begin{aligned} E\Omega^{NS}(x) &= (1 - u^{NS}) \Omega^{NS}(h^{NS}) + u^{NS} \Omega^U(h^{NS}) \\ &= (1 - u^{NS}) \left[w_L - e - \tau x - h^{NS} R(x) + (h^{NS})^{1/2} \right] \\ &\quad + u^{NS} \left[w_U - s\tau x - h^{NS} R(x) + (h^{NS})^{1/2} \right] \end{aligned} \quad (39)$$

$$\begin{aligned} E\Omega^S(x) &= (1 - u^S) \Omega^S(h^S) + u^S \Omega^U(h^S) \\ &= (1 - u^S) \left[w_L - \tau x - h^S R(x) + (h^S)^{1/2} \right] \\ &\quad + u^S \left[w_U - s\tau x - h^S R(x) + (h^S)^{1/2} \right] \end{aligned} \quad (40)$$

Each worker chooses h that maximizes his/her utility. First order condition for non-shirkers and shirkers give:⁷

$$h^{NS}(x) = \frac{1}{4 [R(x)]^2} \quad (41)$$

$$h^S(x) = \frac{1}{4 [R(x)]^2} \quad (42)$$

Thus, $h^{NS} = h^S$ and that housing consumption only depends on the price of housing $R(x)$ and is exactly the same as in the case of no relocation costs. This is because quasi-linear preferences imply that the demand for housing is independent of income so, here, the employment status does not matter in this decision.

The (expected) indirect utility for a non-shirker worker located at x is given by:

$$\begin{aligned} EV^{NS}(x) &= (1 - u^{NS}) (w_L - e) + u^{NS} w_U - \tau x [1 - (1 - s) u^{NS}] + \frac{1}{4R(x)} \\ &= \frac{a}{a + \delta} (w_L - e) + \frac{\delta}{a + \delta} w_U - \left(\frac{a + s\delta}{\delta + a} \right) \tau x + \frac{1}{4R(x)} \end{aligned} \quad (43)$$

whereas for a shirker residing at a distance x from the CBD, it is equal to:

$$\begin{aligned} EV^S(x) &= (1 - u^S) w_L + u^S w_U - \tau x [1 - (1 - s) u^S] + \frac{1}{4R(x)} \\ &= \frac{a}{\delta + m + a} w_L + \frac{\delta + m}{\delta + m + a} w_U - \left[\frac{a + s(\delta + m)}{\delta + m + a} \right] \tau x + \frac{1}{4R(x)} \end{aligned} \quad (44)$$

⁷It is easy to verify that there is a unique solution to each maximization problem since the second order condition is always satisfied.

Let us calculate the efficiency wage. To prevent shirking, firms solve $EV^{NS}(x) = EV^S(x)$ at each x , i.e. the average income over time of a non shirker is equal to the one of a shirker. By using (43) and (44), we easily obtain:

$$w_L(x) = w_U + e + \frac{e}{m}(\delta + a) + (1 - s)\tau x \quad (45)$$

Since, at the efficiency wage, no worker shirks, we can use the value of a in (37) and plug in (45) to obtain:⁸

$$w_L^r(x) = w_U + e + \frac{e}{m} \frac{\delta}{u} + (1 - s)\tau x \quad (46)$$

This efficiency wage has the standard effects of both non-spatial (Shapiro and Stiglitz, 1984) and spatial models (Zenou and Smith, 1995). Indeed, when b , e , or δ increases or θ or u decreases (these are the non-spatial effects), the efficiency wage has to increase in order to prevent shirking. When τ increases or s decreases (these are the spatial effects), the efficiency wage has to increase in order to compensate for spatial costs. This implies (as in Zenou and Smith, 1995) that the efficiency wage has two roles: to prevent shirking and to compensate workers for commuting costs.

Now, the main difference here is that mobility costs are very high so that workers always stay in the same location. So when a worker makes his/her decision to shirk or not, he/she trades off longer spells of unemployment but lower effort and lower commuting (if he/she decides to shirk) with longer spells of employment but higher effort and higher commuting costs (if he/she decides not to shirk). As a result, at each location x , the setting of the efficiency wage needs to include these two elements (incentive and spatial compensation). For the spatial compensation element (which varies with x), it has to be that, at each x , the compensation is equal to $\tau x (1 - u^{NS} + su^{NS}) - \tau x (1 - u^S + su^S)$, i.e. the average commuting cost when non shirking minus the average commuting cost when shirking, which, using (37) and (38), is exactly equal to $(1 - s)\tau x$.

The fact that wages increase with distance to jobs (or equivalently with commuting time) is a well-established empirical fact. For example, Manning (2003) using British data (the Labour force Survey for 1993-2001 and the British Household Panel Survey for 1991-2000) shows that an extra hour of commuting each day is associated, on average, with an increase in wages of 27 log points. These results are consistent with the ones found in the US (see the survey by White, 1999). For instance, Madden (1985) uses the PSID to

⁸The superscript r refers to the relocation cost case.

investigate how wages vary with distance to the CBD. She finds that, for all workers who changed job, there is a positive relationship between wage change and change in commute. Zax (1991), who uses data from a single company and regresses wages on commutes, also finds a positive relationship. Compared to (29), this wage (46) means that, when relocation costs are very high, wages increase with distance to jobs whereas it is not the case if relocation costs are sufficiently low.

Finally, observe that, plugging (46) in (36), yields:

$$z_L - z_U = \frac{e \delta}{m u} > 0 \quad (47)$$

Not surprisingly, compared to employment, workers reduce their composite consumption when unemployed in order to stay at the same location and to consume the same amount of land. Interestingly, the difference in composite good consumption between employment and unemployment is positively correlated with effort and the job destruction rate and negatively correlated with the monitoring rate and unemployment. Indeed, if for example the unemployment rate increases, then workers are less paid (see (46)) and thus consume less composite good when employed. As a result, the difference between z_L and z_U is smaller. This result is reversed for δ since when it increases, wages increase and therefore $z_L - z_U$ rises.

As in the previous section, we can have the following general result:

Proposition 6 *Assume high relocation costs and $r \rightarrow 0$. Then, if the preferences of the employed and unemployed workers are quasi-linear with respect to z_k , i.e.*

$$\Omega(h_k, z_k) = z_k + g(h_k) \quad , \quad k = L, U$$

where $g(\cdot)$ is any increasing function in h_k , then the efficiency wage is given by

$$w_L^r = w_U + e + \frac{e \delta}{m u} + (1 - s) \tau x \quad (48)$$

Proof. See Appendix 2.

This is a quite general result since in case high relocation costs, as long as the demand for housing is independent of income, we obtain the efficiency wage (48). It encompasses, in particular, the case of fixed housing consumption normalized to 1, i.e. $g(h_k = 1) = 1$ (Zenou, 2003). For example, if, as in Brueckner and Zenou (2003), one normalized h_k to 1 and assume that the employed and the unemployed workers incur the same commuting costs, i.e.

$s = 1$, then the efficiency wage is exactly equal to that of Shapiro and Stiglitz (1984) and given by:

$$w_L^{ss} = w_U + e + \frac{e \delta}{m u}$$

4.1 The urban-land use equilibrium

Let us now solve the urban land use equilibrium. For that we have to take a specific form. We use the one of the previous section, i.e. $g(h_k) = (h_k)^{1/2}$. We are able to calculate the bid rent of all workers in the city (at the efficiency wage, nobody will shirk in equilibrium). By plugging (46) in (43), we obtain the following expected utility of a (non-shirker) worker located at x :⁹

$$\begin{aligned} EV(x) &= \frac{a}{a + \delta} [w_L(x) - e] + \frac{\delta}{a + \delta} w_U - \left(\frac{a + s\delta}{\delta + a} \right) \tau x + \frac{1}{4R(x)} \\ &= w_U + \frac{e \delta (1 - u)}{m u} - s\tau x + \frac{1}{4R(x)} \end{aligned}$$

If we denote that EV the (expected) utility reached by all workers in the city, then the bid rent is equal to

$$\Psi(x, EV) = \frac{1}{4 \left[EV - w_U - \frac{e \delta (1 - u)}{m u} + s\tau x \right]} \quad (49)$$

with

$$\begin{aligned} \frac{\partial \Psi(x, EV)}{\partial x} &= \frac{-s\tau}{4 \left[EV - w_U - \frac{e \delta (1 - u)}{m u} + s\tau x \right]^2} < 0 \\ \frac{\partial^2 \Psi(x, EV)}{\partial x^2} &= \frac{s^2 \tau^2}{2 \left[EV - w_U - \frac{e \delta (1 - u)}{m u} + s\tau x \right]^3} > 0 \end{aligned}$$

The first line of this equation highlights the role of the land rent in the unconstrained equilibrium, which is to compensate workers for commuting costs and wages. Indeed, workers living further away obtain higher (efficiency) wages but pay higher commuting costs whereas those living closer to the CBD have the reverse trade-off.

We can now calculate the equilibrium housing consumption for all workers. Using (41) and (49), it is given by:

$$h(x, EV) = 4 \left[EV - w_U - \frac{e \delta (1 - u)}{m u} + s\tau x \right]^2 \quad (50)$$

⁹Recall that $u = \delta/(\delta + a)$ or equivalently $a = \delta(1 - u)/u$.

Contrarily to the case with no-relocation costs, it is interesting to observe that now housing demand depends on unemployment. Not surprisingly, this relationship is positive. If unemployment is low, then a worker anticipates that the fraction of his/her lifetime spent unemployed is lower and thus can increase housing price as well as housing space.

Definition 2 *An urban-land use equilibrium with high-relocation costs is a triple $(x_f, EV, R(x))$ such that:*

$$\Psi(x_f, EV) = R_A = \frac{1}{4} \quad (51)$$

$$\int_0^{x_f} \frac{1}{h(x, EV)} dx = N \quad (52)$$

$$R(x) = \begin{cases} \Psi(x, EV) & \text{for } 0 < x \leq x_f \\ R_A = \frac{1}{4} & \text{for } x > x_f \end{cases} \quad (53)$$

Solving these equations yields

$$x_f = \frac{4N}{1 + 4s\tau N} \quad (54)$$

$$EV = w_U + \frac{e \delta (1 - u)}{m u} + \frac{1}{1 + 4s\tau N} \quad (55)$$

Interestingly, an increase in s or τ reduces the city-size but decreases expected utility. Indeed, an increase in s or τ rises total commuting costs and thus reduces expected utility but also reduces bid rent (see 49) so that the city is less spread.

Finally, by plugging (55) in (49) and using (53), we obtain the following equilibrium land rent for $0 < x \leq x_f$:

$$R(x) = \frac{1}{4[s\tau x + 1/(1 + 4s\tau N)]} \quad (56)$$

4.2 The steady-state equilibrium

We can now determine the labor market equilibrium. As stated above, there are M identical firms ($j = 1, \dots, M$) in the economy. All firms produce the same composite good and sell it at a fixed market price p (this good is taken as the numeraire so that its price p is set to 1). All workers whatever their location contribute to one unit of production. The production function of each firm is denoted by $f(l_j)$ and it is assumed that $f(\cdot)$ is twice differentiable with

$f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, and it satisfies the Inada conditions, i.e. $f'(0) = +\infty$ and $f'(+\infty) = 0$.

We further assume that, when a job is vacant, a firm is always willing to hire a worker whatever his/her location. This means that, once a firm has a vacant job, it is always more profitable to hire the first worker that ‘knocks at its door’ rather than to wait for the next worker. This is true for all workers, even for the one located at N , i.e. the worker who obtains the highest wage and lives the furthest away from firms.¹⁰

A consequence of our assumption is that each position within a firm will be filled totally randomly by a worker residing between 0 and N . Because of the law of large numbers, after a sufficient long period of time (i.e. at the steady state), each position within a firm will be paid at the average-distance wage, i.e. $w_L(N/2)$. Moreover, because all firms are identical, the employment level in each firm j is equal to: $l_j = l = L/M$. As a result, each firm adjusts employment until the marginal product of an additional worker equals the average efficiency wage so that we have

$$w_L(N/2) = f'(l) \quad (57)$$

This determines the labor demand in each firm. Since there are M firms in the economy, the aggregate production function $F(L) = M f(L/M)$ and the total labor demand in the economy is equal to $L = Ml$. The aggregate equivalent of (57) is thus given by:

$$w_L^r(N/2) = F'(L)$$

Since $L = (1 - u)N$, using (46), this is equivalent to:

$$w_L^r(N/2) = w_U + e + \frac{e \delta}{m u} + (1 - s) \frac{tN}{2} = F'((1 - u)N) \quad (58)$$

Proposition 7 *Assume high relocation costs. Then, there exists a unique labor market equilibrium in which unemployment u is (implicitly) defined by (58) and the equilibrium efficiency wage w_L^{nr} by (46).*

Proof. See Appendix 2.

5 Conclusion

In this paper, we have developed two urban efficiency wage models with endogenous housing consumption. In the first one in which relocation costs were

¹⁰Formally, it suffices to assume that the cost of waiting is sufficiently high compared to the profit that a firm makes on a worker located at N .

assumed to be zero, we derive the efficiency wage and show its properties. Housing consumption does affect wages and thus unemployment because the part of the efficiency wage that is needed to compensate workers for spatial costs depends on the competition in the land market between the employed and the unemployed. In the second model in which relocation costs were very high, the efficiency wage becomes location-dependent and the housing consumption is strongly affected by the unemployment rate. When we compare the two models, we show that wages increase with distance to jobs only if relocation costs are large enough. We also show that, when workers are not mobile, they reduce their composite good consumption when unemployed and their housing size depends on the state of the economy whereas, when relocation costs are zero, they essentially reduce their bid rents when unemployed.

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Appendix 1: Proofs for the no-relocation cost model

Proof of (18)-(21)

Using (9) in (13) yields:

$$W_U - w_U = 1 - s\tau x_f \quad (59)$$

Using (8) and (9) in (12) yields:

$$W_U - w_U + s\tau x_b = W_L - w_L + e + \tau x_b$$

which, by using (59), gives:

$$W_L - w_L + e = 1 - s\tau (x_f - x_b) - \tau x_b \quad (60)$$

Now, using (11) in (15) gives:

$$\int_{x_b}^{x_f} \frac{1}{4(W_U - w_U + s\tau x)^2} dx = N - L$$

which is equivalent to:

$$-\frac{1}{4s\tau} \left[\frac{1}{W_U - w_U + s\tau x} \right]_{x_b}^{x_f} = N - L$$

or

$$\frac{1}{W_U - w_U + s\tau x_b} - \frac{1}{W_U - w_U + s\tau x_f} = 4s\tau (N - L)$$

Now, using (59), this rewrites:

$$\frac{1}{1 - s\tau (x_f - x_b)} - 1 = 4s\tau (N - L)$$

which is equivalent to:

$$x_f - x_b = \frac{4(N - L)}{1 + 4s\tau (N - L)} \quad (61)$$

Now, using (10) in (14) gives:

$$\int_0^{x_b} \frac{1}{4(W_L - w_L + e + \tau x)^2} dx = L$$

which is equivalent to:

$$-\frac{1}{4\tau} \left[\frac{1}{W_L - w_L + e + \tau x} \right]_0^{x_b} = L$$

or

$$\frac{1}{W_L - w_L + e} - \frac{1}{W_L - w_L + e + \tau x_b} = 4\tau L$$

Now, using (60), this rewrites:

$$\frac{1}{1 - s\tau (x_f - x_b) - \tau x_b} - \frac{1}{1 - s\tau (x_f - x_b)} = 4\tau L$$

which is equivalent to:

$$x_b [1 + 4\tau L (1 - s\tau (x_f - x_b))] = 4L [1 - s\tau (x_f - x_b)]^2$$

Now using (61), we have:

$$x_b \left[1 + \frac{4\tau L}{1 + 4s\tau (N - L)} \right] = 4L \left[\frac{1}{1 + 4s\tau (N - L)} \right]^2$$

or equivalently

$$x_b = \frac{4LA^2}{1 + 4\tau LA} \quad (62)$$

$$\text{where } A = 1 - s\tau (x_f - x_b) = \frac{1}{1 + 4s\tau (N - L)}$$

Now, plugging (62) in (61) gives:

$$x_f = 4(N - L)A + \frac{4LA^2}{1 + 4L\tau A} \quad (63)$$

Using (59), we easily obtain:

$$\begin{aligned} W_U &= w_U + 1 - 4s\tau A \left[\frac{L}{1/A + 4\tau L} + N - L \right] \\ &= w_U + 1 - \frac{4s\tau}{1 + 4s\tau (N - L)} \left[\frac{L}{1 + 4\tau [sN + (1 - s)L]} + N - L \right] \end{aligned} \quad (64)$$

Finally, using (61), we have $1 - s\tau (x_f - x_b) = A$. Thus, using (60) and (62), we obtain:

$$\begin{aligned} W_L &= w_L - e + \frac{1}{1/A + 4\tau L} \\ &= w_L - e + \frac{1}{1 + 4\tau [sN + (1 - s)L]} \end{aligned} \quad (65)$$

■

Proof of Proposition 2

By differentiating (18), we obtain:

$$\frac{\partial x_b}{\partial L} = \frac{4s(1-s)\tau}{[1+4s\tau(N-L)]^2} + \frac{4(1-s)^2\tau}{[1+4\tau[sN+(1-s)L]]^2} > 0$$

$$\frac{\partial x_b}{\partial s} = -\frac{1+4\tau(N-L)}{[1+4s\tau(N-L)]^2} + \frac{1+4\tau N}{[1+4s\tau(N-L)+4\tau L]^2}$$

Thus

$$\frac{\partial x_b}{\partial s} < 0 \Leftrightarrow \frac{1+4\tau N}{[1+4s\tau(N-L)+4\tau L]^2} < \frac{1+4\tau(N-L)}{[1+4s\tau(N-L)]^2}$$

$$\Leftrightarrow [1+4s\tau(N-L)]^2 < 2[1+4\tau(N-L)][1+4s\tau(N-L)+2\tau L]$$

$$\Leftrightarrow [1+4s\tau(N-L)][1+4\tau(N-L)(2-s)] + 4\tau L[1+4\tau(N-L)] > 0$$

which is always true.

Finally, by differentiating (18), we have:

$$\frac{\partial x_b}{\partial \tau} \geq 0 \Leftrightarrow \frac{s(N-L)+L}{[1+4s\tau(N-L)+4\tau L]^2} \geq \frac{s(N-L)}{[1+4s\tau(N-L)]^2}$$

$$\Leftrightarrow [s(N-L)+L][1+4s\tau(N-L)]^2 \geq s(N-L)[1+4s\tau(N-L)+4\tau L]^2$$

$$\Leftrightarrow \tau \leq \frac{1}{4\sqrt{s(N-L)[L+s(N-L)]}}$$

■

Proof of Proposition 3

(i) By differentiating (29) and by using Proposition 2, we have:

$$\frac{\partial w_L^{nr}}{\partial L} = \frac{e}{m} \frac{\delta N}{(N-L)^2} + \frac{\partial x_b}{\partial L} > 0$$

(ii), (iii) and (iv): Using (27), these results are straightforward.

(v) By differentiating (29), we have

$$\frac{\partial w_L^{nr}}{\partial \tau} = \frac{\partial x_b}{\partial \tau}$$

which by using Proposition 2 leads to

$$\frac{\partial w_L^{nr}}{\partial \tau} \geq 0 \Leftrightarrow \tau \leq \frac{1}{4\sqrt{s(N-L)[L+s(N-L)]}}$$

Finally, by differentiating (29) and using Proposition 2, we obtain:

$$\frac{\partial w_L^{nr}}{\partial s} = \frac{\partial x_b}{\partial \tau} < 0$$

■

Proof of Proposition 4

Using Proposition 1, it is obvious that the employed reside close to the CBD and the unemployed at the periphery of the city.

The composite good for the employed and the unemployed, z_L and z_U , are respectively given by (2) and (3). Thus, when calculating the housing consumption h_L and h_U , it should be clear that, for worker $k = L, U$, the first order conditions will be

$$-h_k R(x) + g'(h_k) = 0$$

As a result, h_L and h_U will only depend on $R(x)$. Thus, when firms calculate the efficiency wage, and since there are no relocation costs, at x_b , the border between the employed and the unemployed, the two types of workers pay the same land rent $R(x)$ and consume the same amount of land $h_L = h_U$. Thus, at x_b , the only spatial difference between the employed and the unemployed is the commuting cost and it is equal to $\Delta SC = (1 - s)\tau x_b$. Finally, since the spatial part of the efficiency wage, ΔSC , is to compensate workers's cost differential, the result follows. ■

Proof of Proposition 5

We have seen in Proposition 3 that w_L^{nr} is an increasing function and $\lim_{L \rightarrow N} w_L^{nr} = +\infty$ and $w_L^{nr}(L = 0) = w_U + e + \frac{e}{m}(\delta + r) > 0$. Furthermore, it is easy to verify that, because $F''() < 0$, in (32), w_L^{nr} is a decreasing function of L . As a result, because of the Inada condition $\lim_{L \rightarrow 0} F'(L) = +\infty$, there exists a unique solution w_L^{nr} and L . ■

Appendix 2: Proofs for the high-relocation cost model

Proof of Proposition 6

The expected utility of non-shirkers and shirkers are given by:

$$E\Omega^{NS}(x) = (1 - u^{NS}) [w_L - e - \tau x - h^{NS}R(x) + g(h^{NS})] \\ + u^{NS} [w_U - s\tau x - h^{NS}R(x) + g(h^{NS})]$$

$$E\Omega^S(x) = (1 - u^S) [w_L - e - \tau x - h^S R(x) + g(h^S)] \\ + u^S [w_U - s\tau x - h^S R(x) + g(h^S)]$$

The optimal choice of h in the delayed uncertainty model leads to:

$$-R(x) + g'(h^{NS}) = 0$$

$$-R(x) + g'(h^S) = 0$$

which implies that $h^{NS}(R(x)) = h^S(R(x)) = h(R(x))$.

The (expected) indirect utility for a non-shirker worker located at x is thus given by:

$$EV^{NS}(x) = \frac{a}{a + \delta} (w_L - e) + \frac{\delta}{a + \delta} w_U - \left(\frac{a + s\delta}{\delta + a} \right) \tau x \\ - h(R(x)) R(x) + g(h(R(x)))$$

whereas the one for a shirker is equal to:

$$EV^S(x) = \frac{a}{\delta + \theta + a} (w_L - e) + \frac{\delta + \theta}{\delta + \theta + a} w_U - \left[\frac{a + s(\delta + \theta)}{\delta + \theta + a} \right] \tau x \\ - h(R(x)) R(x) + g(h(R(x)))$$

The efficiency wage at each x is determined by $EV^{NS}(x) = EV^S(x)$. By solving this equation, we easily obtain (48). ■

Proof of Proposition 7

It is easy to see that w_L^r , defined by:

$$w_L^r(N/2) = w_U + e + \frac{e}{m} \frac{\delta}{u} + (1-s) \frac{tN}{2}$$

is decreasing in u and

$$\lim_{u \rightarrow 0} w_L^r = +\infty$$

whereas $F'((1-u)N)$ is increasing in u , since $F''(\bullet) < 0$ and

$$\lim_{u \rightarrow 0} F'(\bullet) = F'(N) < +\infty$$

As a result, there exists a unique solution in u to the equation

$$w_U + e + \frac{e}{m} \frac{\delta}{u} + (1-s) \frac{tN}{2} = F'((1-u)N)$$

■