Microeconomic Theory EC104
Problem Set 4

(* is easy, ** is difficult, *** is more difficult)

1. ** (The Bayes' rule)

One evening, individual A was knocked over by a taxi driver who run away. The informations gathered by the police show that, among the taxis that were out that night, 75% were green and 25% were blue (in this city, taxis can only be blue or green). However, an eyewitness has been found and he asserts that it was a blue taxi. Several experiments have been undertaken and they show that under identical circumstances to that of the accident, an eyewitness sees the correct color in 80% of the cases.

1a. Without the information about the eyewitness, which company should the lawyers of individual A prosecute?

1b. With the information about the eyewitness but without the experiments, which company should the lawyers of individual A prosecute?

1c. With the informations about the eyewitness and the experiments, lawyers will update their beliefs by using the Bayes' rule.. In this case, which company should the lawyers of individual A prosecute?

1d. Assume now that, among the taxis that were out that night, 85% were green and 15% were blue. With the exactly the same informations about the eyewitness and the experiments, lawyers will again update their beliefs by using the Bayes' rule. In this case, which company should the lawyers of individual A prosecute?

2. ** (Exercise 282.1 Osborne) (Fighting an opponent of unknown strength)

Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; she assigns probability α to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Each person's preferences are represented by the expected utility of a Bernoulli payoff function that assigns the payoff of 0 if she yields (regardless of the other person's action) and a payoff of 1 if she fights and her opponent yields. If both persons fight then their payoffs are $(-1, 1)$ if person 2 is strong and $(1, -1)$ is person 2 is weak.

2a. Formulate this situation as a Bayesian game.

2b. Find its Nash equilibria if $\alpha < 1/2$ and if $\alpha > 1/2$.

3. *** (Certification by a monopolist)

Consider a seller of a good of quality v, with $v \in [a, b]$. The seller knows v, which is drawn from a uniform distribution on $[a, b]$. The seller has no production cost. The consumers are heterogenous and characterized by their willingness to pay for the good; this willingness is denoted by θ. The parameter θ is uniformly distributed on $[0, 1]$. The utility function of a consumer of type θ who buys a good of quality v is given by

$$U(\theta, v) = \theta\, v - p$$

where p is the price by for the acquisition of the good. If the consumer does not buy the good, his utility is equal to 0. Let us denote k the cost of certification. Certification consists in revealing in a credible way the value of v.

3a. Calculate the total demand under *perfect information* for a good of quality v sold at a price p.

3b. Calculate the profit function under *perfect information* of a seller of a good of quality v.

3c. Find the Nash equilibrium of the Bayesian game (*imperfect information* on the quality v of the good) with certification by assuming that if type v is certified, then any type $v' > v$ is also certified and if type v is not certified, then any type $v' < v$ is also not certified.

4. *** Consider a first-price sealed bid auction model with private value information. Assume that there are three bidders and that the valuations of the bidders are independent random variables uniformly distributed over an interval $[\underline{v}\ ,\overline{v}]$. We assume that each bidder bids independently, a reasonable assumption in the case of sealed-bid auction.

Show that the linear bidding rules

$$b_i(v_i) = \frac{1}{3}\underline{v} + \frac{2}{3}v_i$$

for $i = 1, 2, 3$ form a symmetric Bayesian-Nash equilibrium.

5. *** Signs of oil have been discovered in Country A. This country decided to perform a drilling in order to check if there is a substantial amount of oil. The results of the oil drilling are Country A's private information. Only two states can be realized: either state p, which means "positive drilling results" or state n, which means "negative drilling results". Country B believes that each state has equal probability to occur. After the drilling, Country A and Country B make simultaneous armament decisions, in preparation for a war over the oil field. Given that Country A invests $\$x > 0$ in armament, and Country B invests $\$y > 0$ in armament, Country A gains control over the oil field with probability $x/(x + y)$, or with probability 1 if $x = y = 0$, and Country B gains control with the remaining probabilities, i.e. probability $y/(x + y)$ if each invests $\$x > 0$ and $\$y > 0$ in armament, and 0 if $x = y = 0$. The value of gaining control over the oil field is \$1 if the state is p and 0 is the state is n.

5a. Formulate the interaction as a Bayesian game (i.e., define the players, the states, the actions and the action spaces, the signals, the beliefs, and the payoffs).

5b. Determine the (pure-strategy) Bayesian-Nash equilibrium of this game. Show that it is unique.

6. *** (Exercise 307.1 Osborne) (Swing voter's curse)

Whether candidate 1 or candidate 2 is elected depends on the votes of two citizens. The economy may be in one of two states, A and B. The citizens agree that candidate 1 is best if the state is A and candidate 2 is best if the state is B. Each citizen's preferences are represented by the expected value of a Bernoulli payoff function that assigns a payoff of 1 if the best candidate for the state wins (obtains more votes than the other candidate), a payoff of 0 if the other candidate wins, and payoff of 1/2 if the candidates tie. Citizen 1 is informed of the state, whereas citizen 2 believes it is A with probability 0.9 and B with probability 0.1. Each citizen may either vote for candidate 1, vote for candidate 2, or not vote.

6a. Formulate this situation as a Bayesian game. (Construct the table of payoffs for each state.)

6b. Show that the game has exactly two pure Nash equilibria, in one of which citizen 2 does not vote and in the other of which she votes for 1.

6c. Show that one of the player's actions in the second of these equilibria is weakly dominated.

6d. Why is the "swing voter's curse" an appropriate name for the determinant of citizen 2's decision in the second equilibrium?

7. ** (Joint densities)

A study claims that the daily number of hours, X, a teenager watches television and daily number of hours, Y, she works on her homework are approximated by the joint probability density function (pdf),

$$f_{XY}(x,y) = x\,y\,e^{-(x+y)}\ , \qquad x > 0, \qquad y > 0$$

What is the probability a teenager chosen at random spends at least twice as much time watching television as she does working on her homework?

8. ** Consider the following asymmetric-information model of Bertrand duopoly with differentiated products. The demand for firm $i = 1, 2$, $j \neq i$, is given by:

$$q_i(p_i, p_j) = a - p_i - b_i \, p_j$$

Costs of producing the good is zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm i, $b_i = b_L$ with probability θ and $b_i = b_H$ with probability $1-\theta$, independent of the realization of b_i All of this is common knowledge.

8a. Formulate the interaction as a Bayesian game (i.e., define the players, the states, the actions and the action spaces, the signals, the beliefs, and the payoffs).

8b. What conditions define a symmetric pure-strategy Bayesian Nash equilibrium of this game?

8c. Determine this symmetric pure-strategy Bayesian Nash equilibrium.