Voter turnout in small referendums

by

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Abstract: We analyse voter turnout using data from 309 local Norwegian language referendums carried out during 1965-2005, emphasising the effect of referendum type and changes in suffrage. The largest determinant of voter participation is suffrage: The contraction of voting rights to parents with children in school leads to an 18 percentage point average increase in turnout, even after controlling for electoral size effects. The data fail to corroborate our prediction that turnout is significantly higher in semi-binding than advisory referendums. The evidence confirms our hypotheses that turnout is negatively correlated with electoral size and positively correlated with electoral competition.

Keywords: Rational choice; referendum design; suffrage; voter turnout.

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1. Introduction

Referendums typically are employed to decide on moral, territorial and constitutional issues (Gallagher and Uleri, 1996) and are becoming increasingly popular as a supplement to representative democracy. Nearly half of the referendums[1] recorded worldwide have been conducted since 1970 (Bjørklund, 1997). In her survey of 20 OECD countries, Scarrow (2001) finds that many of them reformed their legislation at the end of the 20th century to allow for an expanded use of referendums.

Referendums display a significant amount of variation across countries both in terms of voting rights and the legal restrictions surrounding the referendums, see, *e.g.*, Suksi (1993) for a classification. Some countries allow for direct democracy only at the local level, others even at the regional and national levels (Scarrow, 2001). Moreover, in Switzerland and in the US, referendums are binding. Conversely, in Finland, Norway and Sweden referendums are advisory in the sense that the outcome of the referendum places no legal restrictions on the subsequent decision of the policy maker (see, *e.g.*, Stordrange, 1991). Between these extremes are what we term semi-binding referendums; voting mechanisms that produce binding results provided specified threshold and/or majority quorums are satisfied.

The objective of this paper is to analyse the determinants of voter turnout in small referendums, in particular the effects of referendum type and changes in voting rights on voter participation. Our empirical examination is based on a sample of 309 local Norwegian language referendums conducted during the period 1965-2005.[2] Norway has two official languages (*bokmål* and *nynorsk)*, and in each referendum voters had to choose between these linguistic alternatives in order to decide which language should constitute the primary language in their particular school district.[3] Prior to 1985, referendums were of a *semi-binding* type; the majority vote was binding conditional on at least one of the alternatives being supported by 40% or more of the electorate. In 1985, Stortinget (the Norwegian parliament) changed the referendum rules. From this point on, referendums have been de jure *advisory*

[1] We use the word referendums, not referenda, as a plural form meaning ballots on one issue. In constrast, the Latin plural gerundive referenda ("things to be referred") connotes a plurality of issues (cf. Nairne, 1996, p. 100).

[2] The Norwegian parliament introduced the referendum institution in the form of local language referendums in 1892.

[3] Norway was part of a union with Denmark between 1536 and 1814. Gradually, Danish was established as the only written language in Norway. Danish prevailed during the subsequent union with Sweden, which ended in 1905. A Norwegianised version of Danish – bokmål ("book language") - nowadays constitutes the majority language in Norway. However, in 1885, the Norwegian parliament established nynorsk ("new Norwegian") as the second official language in addition to bokmål.

(Bjørklund, 2004). Finally, and more important from an empirical point of view, between 1971 and 1985 and in the period 1999 to 2000, suffrage in language referendums was not universal but restricted to parents with children attending school(s) in the relevant school district.

We expand the pivotal voter model of Palfrey and Rosenthal (1983 and 1985) to incorporate the possibility of semi-binding and advisory referendums.[4] Voters simultaneously and independently decide whether to vote or to abstain. They participate if and only if their perceived influence over the outcome is sufficiently large relative to the cost of voting. As there is a positive probability of the losing alternative being implemented after an advisory referendum, *ceteris paribus* individual voter influence decreases as we go from binding via semi-binding to advisory referendums. Moreover, voter influence in the form of the probability of being pivotal is lower the higher is the expected voter turnout, the larger is the electorate and the more even the distribution of political opinions across the electorate. Utilising an approximate probability of being pivotal, we obtain the following predictions: Voter turnout in advisory referendums is lower than under binding and semi-binding referendums, decreasing in the size of the electorate and increasing in the degree of electoral competition, *i.e.*, the expected closeness of the referendum outcome.

The pivotal voter model is reasonable only to the extent that voters perceive their individual influence over the outcome to be significant.[5] This approach seems justifiable here in as much as the electorates in our sample were generally small, ranging in size from 6 to 4121 whereas the average electorate consisted of 428 eligible voters. Furthermore, several of the referendums were decided with a small majority; eleven of these with the smallest possible margin (one vote). Two of the referendums were actually ties. Under such circumstances, voters may well perceive their influence to be non-negligible.

Our key findings are, first, that semi-binding referendums display a higher voter turnout than advisory referendums. However, the statistical significance of the observed difference vanishes once we control for changes in voting rights. Hence, the empirical evidence does not corroborate the prediction that turnout is increasing in the decisiveness of the referendum, but instead suggests that voters act as if

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Nynorsk is a written code based on a wide range of Norwegian dialects. Currently, approximately 15% of the Norwegians are taught and use nynorsk as their primary language (Grepstad, 2006).

[4] We thus adapt the standard "calculus of voting" approach taken by Downs (1957), Tullock (1967), Riker and Ordeshook (1968) and subsequently refined; see Aldrich (1997), Blais (2000), Grossman and Helpman (2001) for surveys. The theoretical literature on direct democracy has focused exclusively on binding referendums.

[5] The rationality of voting has been questioned ever since Downs (1957) first applied rational choice theory to the analysis of voter turnout (see, *e.g.*, Green and Shapiro, 1994 and Aldrich, 1997). In large elections, it is argued, the impact of any single voter is so small that the benefit of voting, *i.e.*, the possibility of affecting the outcome of the election, is typically outweighed

advisory referendums resemble binding ones. Note, however, that local authorities have overruled the majority of voters in nearly 12% of the advisory referendums. Apparently, the degree of uncertainty concerning the implementation of the majority alternative is not sufficient to affect voter turnout in advisory referendums. Second, and consistent with our predictions, voter turnout depends negatively on the size of the electorate and positively on electoral competition. These effects hold for both advisory and semi-binding referendums, and are highly statistically significant.[6] Third, the contraction of voting rights to parents with children in school is the single largest determinant of voter turnout in our sample. It leads to an 18 percentage point average increase in turnout, even after controlling for electoral size effects. Fourth, referendums held at the same date as a local or national election display significantly higher turnout than those held at unique dates.

The remainder of our paper is organised as follows. Section 2 reviews recent studies on referendum design and determinants of voter turnout in elections and places our contribution in its relevant context. Section 3 presents the theoretical framework used to generate hypotheses against which the data are evaluated. The fourth section contains the empirical analysis. Concluding remarks are outlined in the last section. Some tedious calculations are relegated to appendix A and a proof to appendix B.

2. Related literature

Our paper adds to the empirical literature studying the effects on voter behaviour of the institutional constraints surrounding referendums. A recent contribution is Hug (2002), who utilises a panel of referendums held on European integration to analyse how the distribution of votes and implemented policies are affected by whether the referendum is required or non-required and whether it is initiated by the government or the opposition. In Norway, local language referendums are initiated either by the municipal board or by the eligible voters if at least 25% of them have signed a petition. Unfortunately, we do not possess information as to how the referendums in our data were triggered, hence we are prevented from investigating the effects of referendum initiation on voter behaviour.

During the last 30 years, 10 European countries have introduced or expanded opportunities for holding local referendums (Scarrow, 2001). Austria and Germany have authorised the use of binding local referendums, whereas Belgium, Finland, France, Italy, Luxembourg, the Netherlands, Spain and

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by the cost of voting. Hence, the rational choice pivotal model predicts low turnout in large elections. The fact that voters nevertheless turn up in significant numbers has come to be known as the "turnout paradox".

[6] Employing a subset of our data, Kaniovski and Mueller (2005) have subsequently corroborated these results.

Sweden have passed legislation providing for conducting advisory (consultative) local referendums. However, to the best of our knowledge, ours is the first study of the effects of referendum type on voter turnout, *i.e.*, the extent to which voter participation is affected by whether a referendum is binding, semi-binding or merely advisory.

Regarding the other explanatory variables in our model, electoral size and closeness, some previous studies have produced similar correlations as ours. Of the 36 studies surveyed by Blais (2000), the hypothesis that turnout decreases with the size of the electorate has been confirmed (disconfirmed) in eight (five) studies. The verdict is clearer with regard to closeness: In 27 out of 32 studies, increased closeness was found to positively affect turnout. In a recent meta analysis of 83 studies, Geys (2006) concludes that election size and closeness should be included in any "core" model of voter turnout.

Previous studies of voter turnout typically build on observations sampled from general elections across time, or on referendums on different topics. Sometimes cross-country data have been used. *A priori* we would expect members of the electorate to be better able to calculate the benefit of voting in a referendum on two well-defined uni-dimensional policy alternatives than in a general election where two or more parties or candidates promote multi-dimensional platforms.[7] Our sample consists entirely of *binary referendums*. In binary referendums, the incentive to vote might vary considerably depending on the issue to be decided upon. Our data set controls for such unobserved heterogeneity insofar as all referendums are on the same topic; nynorsk vs. bokmål as the primary school language. Coate and Conlin (2004) and Coate, Conlin and Moro (2004) use a high quality data set to analyse voter turnout, in their case Texas liquor referendums. They find voter turnout to be decreasing in the number of eligible voters, but do not analyse the effect of closeness on voter turnout. Furthermore, their data set does not offer the opportunity to investigate the effect of referendum design on voter turnout, as all the Texas liquor referendums were binding. Moreover, the policy issues differed (in the degree to which restrictions were relaxed) and the level of jurisdictions varied across elections, thereby creating a degree of heterogeneity not present in our data set.

3. Theory

The following model is a variant of the one introduced by Palfrey and Rosenthal (1983 and 1985). Consider a referendum where voters either support or oppose a given project. Each member V of the electorate values her most preferred policy at $b > 0$, and the alternative at 0. There is a net cost c

[7] In such elections, voter participation may be plagued by the possibility that eligible voters simply cannot make up their mind for which candidate to vote, whence opting to abstain from voting. Abstentions are thus due to incomplete preferences.

associated with voting, which represents the cost of obtaining information about the referendum alternatives, plus "foot-sole" costs minus the intrinsic benefit of voting. Observe that the net cost of voting could be negative. This happens if the direct benefit of voting is larger than the participation cost. Denote the relative cost of voting $\gamma = c/b$. Each voter knows her own relative cost of voting, but not that of the others. However, it is common knowledge that $\gamma \in [\gamma_l, \gamma_h]$ is identically and independently distributed across electors according to the continuous, differentiable and strictly increasing cumulative distribution $G(\gamma)$.

V's expected net utility of voting is $u = pb - c$ where p measures expected influence on the electoral outcome. A rational voter trades off her influence against the cost of voting when making a decision. She participates if and only if her perceived influence over the outcome of the election is sufficiently high, *i.e.*, $p \geq \gamma$. In this model p is the expected probability of being pivotal. The pivotal probability depends on the number of electors sharing *V*'s political views and the probability of these electors actually voting, the number of electors who do not share *V*'s political views and the likelihood of voting in that group, as well as referendum type. In order to make the model meaningful, we assume that $\gamma_l < 1$ and $\gamma_h > 0$. With $\gamma_h \leq 0$, the net cost of voting would be non-positive for all electors and voter turnout 100% in every referendum no matter how uninfluential each voter perceived herself to be. Conversely, $\gamma_l \geq 1$ would imply a net cost of voting at least as high as the value of the most preferred policy for all electors; hence nobody would participate in any referendum, even if any voter could be certain to cast a decisive vote.

Referendum design

The likelihood of becoming pivotal crucially depends on the referendum design. A *binding* referendum is decided by simple majority rule. We assume the winning alternative to be implemented with exogenous probability $k \in (0.5,1)$ under an *advisory* referendum. A *semi-binding* referendum is binding on the proviso that at least one of the alternatives attracts more than a certain threshold θ of the *eligible* votes and otherwise remains advisory. The semi-binding referendum rule used in local language referendums in Norway between 1965 and 1985 had a 40% threshold. The semi-binding referendum contains the other two referendum types as special cases; hence, we focus our subsequent analysis on this referendum type.

Equilibrium

We consider (Bayesian) Nash equilibria in which all voters simultaneously and independently decide whether to vote. Each voter participates if and only if her relative cost of voting sinks below the

pivotal probability p. Thus, p defines the threshold cost below which she participates in the referendum. The rational voter anticipates all other electors to pursue such trigger strategies, too. Following Palfrey and Rosenthal (1985), we restrain attention to *symmetric* strategies; every group member pursues the same strategy as everybody else in her own group. Consider a pair of thresholds (γ_M, γ_m), and let $q_M = G(\gamma_M)$ and $q_m = G(\gamma_m)$ be the unconditional probabilities that a member of the majority and minority, respectively, participate in the referendum. Denote by $p_M(q_M, q_m)$ and $p_m(q_M, q_m)$ the respective pivotal probabilities. Any pair (q_M, q_m) of participation probabilities satisfying

$$p_M(q_M, q_m) = G^{-1}(q_M) \text{ and } p_m(q_M, q_m) = G^{-1}(q_m)$$

constitutes a Nash equilibrium. These expressions are complicated because the pivotal probabilities contain multiples of binomial probabilities. To find solutions, one has to rely on numerical methods, except in special cases.[8] We circumvent this problem by approximating the pivotal probability. As shown in Appendix A, p_M and p_m converge to

$$\phi = \frac{e^{-\frac{N}{2}\frac{(\varpi q_M - (1-\varpi)q_m)^2}{\varpi q_M(1-q_M)+(1-\varpi)q_m(1-q_m)}}}{\sqrt{2\pi N(\varpi q_M(1-q_M)+(1-\varpi)q_m(1-q_m))}}(1-2(1-k)F(N\theta \mid \mu, \sigma^2)$$

as the size N of the electorate grows to infinity.[9] ϖ denotes the fraction of the electorate belonging to the majority. F is the (Normally distributed) probability of the referendum being advisory, conditional on the legal threshold θ, the expectation μ and variance σ^2 (see appendix A for the details). If $\theta > 1-\omega$, $F = 1$ and the semi-binding referendum is equivalent to an advisory referendum.

The first thing to note about ϕ is that the pivotal probability is the same for both groups in the limit. Generally, the pivotal probabilities differ between the two groups, all else equal. Let $p_M(M+1,m)$ be the pivotal probability of a majority voter in a referendum with majority size $M+1$ and minority size m. Similarly, $p_m(M,m+1)$ is the pivotal probability of a minority voter in a referendum with majority size M and minority size $m+1$. One can easily verify that $p_M(M+1,m) = p_m(M,m+1)$ under binding or advisory referendums. Hence, the difference in individual influence reduces to the advantage of having one more elector in one's group.[10] Unless the electorate is very small, shifting one voter back and forth between the two groups has little effect on pivotal probabilities. In the limit the difference vanishes completely. Thus, assuming that individual influence does not vary across groups seems a

[8] One such exception appears when the electorate is exactly divided in two, see Palfrey and Rosenthal (1983).

[9] Owen and Grofman (1984) and Mueller (1989) derive a similar approximation in a binding referendum with $q_M = q_m = 0.5$.

[10] This is not the case in a semi-binding referendum. In a semi-binding referendum each individual gains additional influence through the ability to sometimes swing the referendum from advisory to binding. This happens when exactly θN-1 of the fellow group members and θN-1 or less of the opposers participate. The likelihood of this event may differ between the two groups. However, in the limit they are the same since the probability of any single event is near zero.

reasonable approximation. Second, as both p_M and p_m converge to zero, any ϕ that converges to zero approximates the pivotal probabilities in the limit. A test of the appropriateness of the chosen approximation is to study the limits of the likelihood ratio p_m/ϕ. We show in Appendix A that p_m/ϕ converges to unity in the limit. Hence, the chosen approximation improves as N increases. Finally, it may appear contradictory to use a large sample approximation to analyse equilibrium in small referendums. However, "large" is not particularly large in this setting. To obtain ϕ, we use the Normal as an approximation to the Binomial distribution. The approximation is quite good already for N as small as 20 (see, *e.g.*, DeGroot, 1989). Numerical simulations indicate that p_m/ϕ converges to unity at least at a rate inversely proportional to the square of size of the electorate.[11] Hence, convergence is in fact faster when the electorates are small.

In the remainder of the analysis, we assume voters to behave as if the expected pivotal probability is exactly equal to ϕ. Equivalence of voter influence across electors has the implication that all eligible voters use the same equilibrium trigger strategy. Hence, $q_M = q_m = q$ and the implicit solution to

$$\phi(q) = \frac{e^{-2N\frac{q}{1-q}(\varpi-0.5)^2}}{\sqrt{2\pi Nq(1-q)}}\left(1-2(1-k)F(N\theta \mid \mu,\sigma^2)\right) = G^{-1}(q).$$

By differentiating ϕ, it is easy to establish that voter influence, *ceteris paribus*, is decreasing in the size of the electorate, increasing in the closeness of electoral competition, increasing in the probability of the majority decision being implemented and decreasing in the threshold required for a referendum to be binding.

Assumption I $4\varpi(1-\varpi) \leq (2N-1)/2N$

The above assumption is sufficient (but not necessary) to guarantee that voter influence is a decreasing function of voter turnout under binding and advisory referendums. It is satisfied if either preferences are sufficiently homogeneous, *i.e.*, ω is very small or very large, or the electorate is sufficiently large.

Proposition 1: *Under assumption I, binding and advisory referendums have a unique symmetric equilibrium.*[12] *Expected voter turnout is higher in the semi-binding than in the advisory referendum, but lower in the semi-binding than in the binding referendum.*

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[11] Formally, $N^2(p_M/\phi - 1) \to 0$ as $N \to \infty$.

[12] Asymmetric, mixed-pure strategy equilibria (MPSE) may exist. In an MPSE, electors are partitioned into three groups, those who randomise, those who always abstain and those who always vote (Palfrey and Rosenthal, 1983). A stronger set of assumptions on γ, namely, $\gamma_l < 0$ and $\gamma_h > 1$, would eliminate all MPSE in this model since there is always a positive

Ceteris paribus, the expected voter influence is higher under a binding than under an advisory referendum. The reason is that there is a certain probability (1 - k) of the losing alternative being implemented following an advisory referendum. Hence, the net benefit of voting is higher under a binding than under an advisory referendum, which drives up voter turnout in the former case. In a semi-binding referendum, the outcome is sometimes binding and sometimes not, depending on voter participation. Voter influence is thus higher in the semi-binding than in the advisory referendum, but lower than in the binding referendum. Hence, voter turnout lies somewhere between the two extremes. Upon implicit differentiation of the equilibrium condition, the following is easily established:

Proposition 2: *Under assumption I in binding and advisory referendums, symmetric equilibrium expected voter turnout is (i) increasing in the "expected closeness"* $-|\omega - 0.5|$ *of the election and (ii) decreasing in the size N of the electorate.*

These results are fairly intuitive. Consider, for example, the effect of increasing the majority's share of the total voting population, *i.e.*, reducing the expected closeness of the election. When the majority is large relative to the minority, the majority alternative is likely to win the referendum, everything else held equal. The minority voter perceives her influence over the outcome to be small as the referendum is likely to be *lost* independently of her voting decision. The majority voter estimates her chances of being decisive as low because the referendum is likely to be *won* no matter what she decides to do. Hence, both minority and majority electors vote with a lower probability when the voting population is skewed very much in favour of the majority alternative.

4. Evidence

Empirical overview

We have data from 309 referendums carried out between 1965 and 2005. The data have been collected by Noregs Mållag, a pro-nynorsk organization that gathers such data on a private basis.[13] Observations are available from 1965, but during the period 1965-1969 the data are incomplete in as much as

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probability $G(0) > 0$ that a voter will participate and a positive probability that she will abstain $1\text{-}G(1) > 0$ no matter how insignificant (influential) she is, see Palfrey and Rosenthal (1985) for the details. Palfrey and Rosenthal (1983 and 1985) contain no general results on equilibria in which everybody votes with a probability between zero and one.

[13] There exist no official statistics pertaining to local referendums in Norway. Statistics Norway provides information on nationwide referendums only. However, Noregs Mållag publishes local language referendum statistics in their annual reports ("årsmeldingar"), *cf.* http://www.nm.no. Also, Adamiak (2001) shows that more than 500 local referendums were conducted in Norway between 1970 and 2000. Nearly half of these were language referendums.

information regarding the number of eligible voters is lacking for 31 referendums. Also, 7 referendums between 1970 and 2005 are excluded from the data set for the same reason. Four referendums conducted in 1981, 1991, 1994 and 2005 were deemed invalid, and are not listed in our data set.[14]

In general, language referendums may be initiated in two ways. Either the municipal board decides to conduct a referendum, or at least 25% of the eligible voters sign a petition demanding a referendum to be held. Unfortunately, we do not possess information as to how the referendums in our data were triggered, as one might expect participation to depend, in part, on whether the referendum was demand driven or "forced upon" the electorate. Throughout the considered time period 5 years had to pass before another referendum could be held in a given school district, regardless of the manner in which the previous referendum was initiated. Referendums may be conducted at any time during a given year, and have not been systematically linked to, *e.g.*, ordinary municipal elections, although in nine instances the referendum coincided with either a local (six times) or a general election (three times).

Table 1. Summary statistics

	Voter turnout	Voter turnout (advisory)	Voter turnout (semi-binding)	Size of electorate (# individuals)	Nynorsk% of votes
Mean	59.91	50.18	65.14	428	44.16
Median	63.00	51.48	72.37	246	46.07
St.dev.	23.27	20.49	23.03	537	19.58
Max	100	100	100	4121	89.29
Min	8.09	9.40	8.09	6	0
No. obs.	309	108	201	309	309

Table 1 presents summary statistics relating to the key variables identified by our theoretical discussion: Voter turnout (%), the size of the electorate and the closeness of electoral competition (at this point simply measured by nynorsk's proportion of the registered votes).

Voter turnout is defined as the percentage of eligible voters within the relevant school district who actually vote in the referendum. In general, the mean voter turnout was about 60%, but markedly lower in the advisory referendums in which average turnout was just above 50%. One third of the observations in our sample consist of *de jure* advisory referendums. On average approximately 65% of

[14] The data set is available from the authors and at http://www.riie.se/thomast.

the eligible voters participated in the 201 semi-binding referendums, 73 of which turned out to be *de facto* advisory since turnout did not exceed the required threshold for them to be binding.[15]

The data reveal that the electorates were small. On average only 428 individuals were eligible to vote. Moreover, the referendums tended to yield close calls. On average, 44% (56%) voted for nynorsk (bokmål). Eleven of the referendums were decided with the smallest possible majority (one vote) and two referendums were actually ties. At the other extreme, a unanimous vote was recorded four times.

Figure 1. Voter turnout (%) in semi-binding and advisory referendums

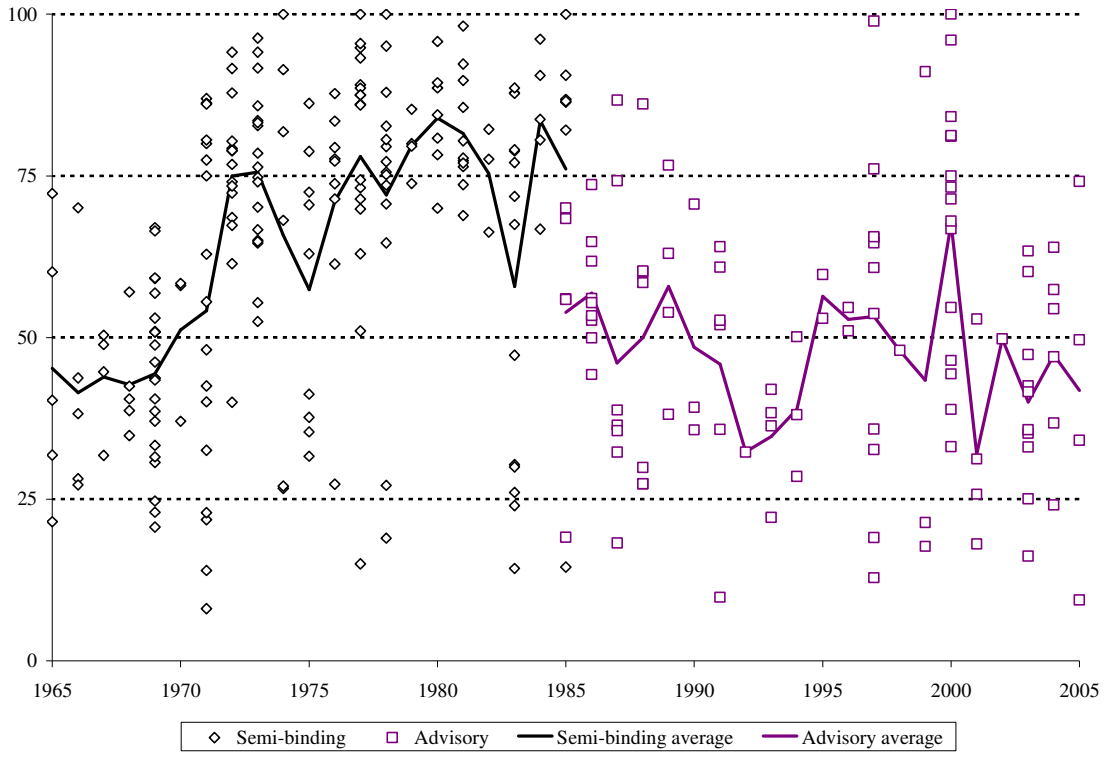

Figure 1 depicts voter turnout in semi-binding and advisory referendums over time. Since August 1st 1985 all referendums are *de jure* advisory. Voter participation in each referendum is marked by dots, and the solid lines show average turnout. The frequency of conducted referendums declines during the

[15] Albeit not directly comparable, nationwide average voter turnout in ordinary municipal elections during 1967-2003 was 68.2%. Such elections are conducted every 4 years. However, the trend is one of falling voter turnout. In 1967 the nationwide average turnout was nearly 75%, whereas turnout reached an all-time low in 2003; 59%. We have not found evidence of any systematic time trend in voter turnout in local referendums.

time relevant period. The average number of referendums was nearly 10 per year during 1965-1985, and just above 5 annually during 1985-2005. In 1992, 1998 and 2002 there was just one referendum.

Both the summary statistics depicted in Table 1 and an eyeballing of the figure appear to corroborate our theoretical prediction (*cf*. Proposition 1 above) that voter turnout is lower in advisory than in semi-binding referendums. However, the Norwegian Parliament implemented noticeable changes of voting rights during the considered time period. Until July 1st 1971, all those living within the borders of the school district in question who were eligible to vote in a general election, could participate in the language referendum, *cf*. "Lov 10. april 1959 om folkeskolen" (School Law of April 10th 1959). Thereafter, voting rights were restricted to parents with children attending school(s) in the relevant school district in accordance with "Lov 13. juni 1969 om grunnskolen" (School Law of June 13th 1969). This regime lasted until August 1st 1985, when the Norwegian parliament reintroduced universal suffrage (as well as made local language referendums *de jure* advisory), *cf*. "Lov av 14. juni 1985 nr. 72" (Law no. 14 of June 14th 1985). Moreover, voting rights were once again restriced to parents only between August 1st 1999 and July 31st 2000, *cf*. "Forskrift til opplæringslova 28. juni 1999" (School Law Regulation of June 28th 1999) and "Lov 30. juni 2000 nr. 63 (Law no. 63 of June 30th 2000) respectively. [16]

Thus, suffrage was restricted to a subset of the adult population of the school districts during July 1st 1971 - August 1st 1985 and August 1st 1999 - July 31st 2000. Confining voting rights to parents only appears to coincide with an increase in voter turnout, confer the marked increase in average voter turnout depicted in Figure 1 between, first, 1970 and 1971 and, second, 2000 compared with 1999 and 2001.

This visual impression is confirmed by the summary statistics shown in Table 2. Average voter turnout is positively correlated with both changes in referendum type from advisory to semi-binding and restrictions of voting rights to parents only. Moreover, confining voting rights to parents affects the average number of eligible voters per referendum. When universal voting rights apply, the average electorate has 671 eligible voters. In the restricted voting rights case, the average size is 213.

[16] Bjørklund (2004) argues that, historically, opponents of nynorsk have wanted to restrict voting rights to parents, whereas the nynorsk movement has favoured universal suffrage. Politically, Labour and the centrist parties have supported universal suffrage.

Table 2. Average voter turnout (VT) and average number of eligible voters (N) in local referendums

	Restricted voting rights	Universal voting rights	Aggregate average
Advisory	VT: 65.4%	VT: 47.3%	VT: 50.2%
referendums	N: 278	N: 829	N: 742
	# obs.: 17	# obs.: 91	# obs.: 108
Semi-binding	VT: 73.3%	VT: 42.9%	VT: 65.1%
referendums	N: 205	N: 404	N: 259
	# obs.: 147	# obs.: 54	# obs.: 201
Aggregate average	VT: 72.5	VT: 45.7%	VT: 59.9%
	N: 213	N: 671	N: 428
	# obs.: 164	# obs.: 145	# obs.: 309

Econometric analysis

We now turn to a reduced form linear econometric analysis of our theoretical predictions. Proposition 1 asserts that, in equilibrium, voter turnout is higher in semi-binding than in advisory referendums, whereas Proposition 2 predicts turnout to be increasing in the expected closeness of the referendum outcome and decreasing in the size of the electorate when the referendum is either advisory or binding.[17,18] No such predictions apply to the semi-binding referendums.[19]

Below we employ a random effects panel data model to analyse whether the data corroborate these predictions. The model can be formulated as

$$VT_{ij} = \alpha + \beta' x_{ij} + u_i + \varepsilon_{ij}$$

where $VT \in [0,1]$ measures voter turnout, subscript i refers to year during the period 1965 to 2005, whereas subscript j denotes the jth observation within any given year. The term u_i is the random

[17] Equivalently, the theoretical hypotheses do not relate to voter turnout per se, but are evaluated to changes in voter turnout. Also see Grofman (1993).

[18] The validity of Propositions 1 and 2 requires that Assumption I is satisfied. 274 of our 309 observations are consistent with Assumption I, but the econometric analysis reported below is based on the complete sample. However, the empirical estimates are neither markedly nor significantly affected by our using the entire data set. Nevertheless, regression analyses based on data consistent with Assumption I are available from the authors upon request. Also see footnote 20 below.

[19] This is due to the fact that semi-binding referendums may have multiple equilibria under the proposed set-up. If we restrict the attention to the set of *stable* equilibria, the predictions that apply to advisory referendums apply to semi-binding referendums as well.

disturbance characterising any observation in year *i*. Thus, the choice of a random effects modelling framework is consistent with controlling for (randomly distributed) annual time effects affecting voter turnout. α is a constant term, and ε_{ij} a disturbance term to which conventional assumptions apply. x_{ij} is a vector which contains the following regressors:

- A dummy variable equal to 1 if the referendum is semi-binding and 0 if not.
- Closeness, measured by the negative of the absolute value of the actual split of vote shares; -|ϖ-0.5|, where ϖ measures the nynorsk share of registered votes.[20]
- Electoral size, *i.e.*, the number of eligible voters.
- A dummy variable equal to 1 if voting rights are restricted to parents only and 0 if not.
- A dummy variable equal to 1 if the referendum is held concurrently with either a local or a general election and 0 if not.

We present the results from three different regressions in Table 3. The first regression estimates the effects on voter turnout from the variables identified as relevant by our theoretical discussion: Referendum design, (expected) closeness and electoral size. The model as fitted explains approximately 48% of the variation in voter turnout levels. The principal result is that substituting semi-binding referendums rules for advisory referendums causes a statistically significant increase in voter turnout by an average of 16.55 percentage points. This estimate provides prima facie evidence in support of our Proposition 1. Moreover, in line with Proposition 2, the estimates reveal voter turnout in advisory referendums to be negatively correlated with both decreasing closeness and increases in the number of eligible voters. Similar effects hold true in the case of semi-binding referendums.

Regardless of referendum design the closeness effect dominates that of electoral size. To wit, when referendums are advisory, an increase in the size of the electorate by a hundred eligible voters leads to an average increase in voter turnout by merely 1.6 percentage points. In comparison, a reduction in the difference in popularity between the two alternatives by two percentage points, say from 55-45 to 54-46, yields an approximately equal increase in voter turnout; 1.8 percentage points.

[20] In the theoretical discussion in the previous section, the term ϖ measures the fraction of the entire electorate who prefers nynorsk (or equivalently, bokmål). In symmetric equilibrium, both nynorsk and bokmål voters vote with the same probability. Hence, the expected split in the referendum is identical to the split in the electorate. In the regression analysis we use the actual (ex post) split as a measure of "expected closeness" of ϖ. Thus, we assume that voters have rational expecations in terms of the referendum outcome. Schachar and Nalebuff (1999) state that ex post closeness is typically a function of the dependent variable, namely voter turnout, the implication of which is biased estimates. Therefore, ideally we

Table3. Regression results

Independent variable	**Regression 1**	**Regression 2**	**Regression 3**
	Coefficient (p-value)	Coefficient (p-value)	Coefficient (p-value)
Constant	0.6892	0.6560	0.5770
	(0.0000)	(0.0000)	(0.0000)
Referendum design binary variable:			
Semi-binding (1) vs. advisory referendums (0)	0.1655	0.0322	0.0254
	(0.0000)	(0.4139)	(0.5030)
Advisory referendum variables:			
Closeness	0.5415	0.6047	
	(0.0003)	(0.0000)	
Electoral size	-0.00016	-0.00015	-0.00015
	(0.0000)	(0.0000)	(0.0000)
Semi-binding referendum variables:			
Closeness	0.5355	0.4468	
	(0.0000)	(0.0000)	
Electoral size	-0.00041	-0.00034	-0.00035
	(0.0000)	(0.0000)	(0.0000)
Control variables:			
Voting rights restricted to parents		0.1800	0.1989
		(0.0000)	(0.0000)
Concurrent election		0.1131	0.1049
		(0.0367)	(0.0638)
Lagrange multiplier test (p-value)	60.13 (0.0000)	3.64 (0.0565)	5.21 (0.0224)
Estimated variance of u_i; $\hat{\sigma}_u^2$	0.0016	0.0005	0.0002
Estimated variance of ε_{ij}; $\hat{\sigma}_\varepsilon^2$	0.0245	0.0224	0.0257
R^2	0.48	0.57	0.50

Regression 2 differs from the first by its inclusion of binary variables that control for changes in voting rights (cf. the restriction of the right to vote to parents only) and the concurrence of ordinary

should have used ex ante pereceptions of the referendum outcomes or relevant instrumental variables in stead of ex post closeness data. Such variables are not available, and we have no choice but resort to actual splits of the registered votes.

elections. This modelling approach increases R^2 to 0.57, and leaves the effects of closeness and electoral size on voter turnout in both advisory and semi-binding referendums relatively unaltered. However, the effect of referendum design drops to 3.2 percentage points and ceases to be statistically significant. [21] Moreover, we find that confining voting right to a subset of the adult population with children attending the relevant school(s) leads to a marked increase in voter turnout equal to 18 percentage points. Referendums that occur simultaneously with ordinary elections display significantly higher voter turnout, albeit only nine of the referendums in the sample were conducted in connection with either local or national elections. The main finding, then, is that the referendum design effect appears not to be robust when controlling for alterations in voting rights and the concurrence of ordinary elections.

The last regression is similar to Regression 2, but has omitted the closeness variables. Previously we have argued that expected closness can be approximated by actual ex post closness if voters have rational expectations. In practice, it is not clearcut how – and whether – voters construct such expectations. Noticeably, opinion polls, to the best of our knowledge, in general have not been conducted prior to any of the referendums in our data set. However, when the electorate is 'small', it could be argued that public debate meetings (which sometimes have been conducted) or conversations amongst members of the local communities suffice to produce expectations as to the vote shares of nynorsk and bokmål in an upcoming referendum. Nevertheless, when (expected) closeness is disregarded as a possible determinant of voter turnout, the results from Regression 3 reveal essentially the same correlations between referendum design, electoral size and the two control variables as produced by the second regression analysis: First, substituting semi-binding referendums for advisory referendum rules increases voter turnout, but not in any statistically significant manner. Second, the negative effects on voter turnout of increases in electoral size prevail, and their magnitude remain approximately constant. Third, voter turnout goes up by an average of 19 percentage points when voting rights are confined to parents only. Lastly, the effect of concurrent elections on voter turnout prevails, but is now only weakly statistically significant.

The p-values listed in connection with the (Breusch-Pagan) Lagrange multiplier test under each regression in Table 3 measures the probability of the null hypothesis that no heterogeneity in the form

[21] One referee suggests that the estimated effect of changes in referendum type (advisory vs. semi-binding) is affected by multicollinearity when the econometric analysis controls for voting rights changes. However, the standard error of the referendum type coefficient is not markedly inflated when the control variables are included; the standard error is 0.039 in regression 2 compared to 0.037 in regression 1. Moreover, the statistics depicted in Table 2 clearly indicate that changes in voting rights affect voter turnout. Thus, we believe that regression 1 suffers from omitted variable bias, and that the specification used in regression 2 offers a comparatively more truthful treatment of the data.

of random (time) effects obtains. Such time effects are present in the first and third regressions, and weakly so in the case of the second regression. This is evidence in favour of adopting a random effects approach to the econometric analysis of our theoretical predictions.

Discussion

One of our key empirical findings is that referendum design – to wit, semi-binding vs. advisory referendum rules – does not exert any statistically significant effect on voter turnout. Thus, the evidence seems to refute our theoretical prediction that voter turnout increases when the referendum rules change from advisory to semi-binding. The recalcitrant evidence reported above forces us to reconsider our theoretical approach. Consider our assumption that the winning alternative is implemented with exogenous probability $k \in (0.5,1)$ under an advisory referendum. Does this assumption stand up to scrutiny? It is fairly obvious that, from the perspective of the local authorities, the implementation probability k is in fact *endogenous* and in part determined by the information that any given advisory referendum generates. To fix ideas, consider two extreme cases. If voter turnout is low and the split of registered votes is approximately perfect, the referendum has produced a 'fuzzy' advice. In this case, the implementation probability k may well be low; in particular if the majority alternative prescribes a costly policy change. However, if turnout is high and the outcome of the referendum is near unanimous, the advice given by voters to the local authority is undoubtedly clearer and should carry greater political weight. Hence, we would be inclined to assign high value to the implementation probability k if this outcome occurs. In other words, it seems reasonable to argue that both voter turnout and the distribution of votes in practice influence the probability of the winning alternative being implemented – *ex post*.

Nevertheless, prior to the referendum and from the voters' point of view, neither the precise value of the implementation probability k nor its determinants may be readily ascertainable. Before the referendum takes place, voters only know that it is not certain that the local authority chooses to implement the majority alternative. In this study we have not endeavoured to analyse how the implementation probability varies with the referendum outcome. Consequently, we model the probability as exogenous, constant and identical for all referendums.

The local authorities overruled the majority alternative in 12 of the 73 semi-binding referendums that turned out to be *de facto* advisory and in 9 of the 108 *de jure* advisory referendums. All in all, approximately 12% of the *de jure* and *de facto* advisory referendums produced advice that was disregarded by the local authorities. Although the implementation probability k in practice does not

equal 1, our econometric analysis suggests that k is not sufficiently low to generate a statistically discernible effect on voter turnout. Thus, if *voters* perceive the implementation probability as being close to unity, both advisory and semi-binding referendums are approximately binding from the voters' point of view. In that case, rational choice theory predicts voter turnout to remain largely unaffected by changes in referendum type. Rather than falsifying our theoretical prediction flat out, the recalcitrant evidence may therefore be interpreted to imply that voters act *as if* the different referendum types were equivalent.[22]

The referendum design effect disappears once we control for variations in voting rights. Restricting voting rights to parents obviously decreases the electoral size, on average from 671 to 213 voters. Thus, it might be argued that this effect is 'picked up' by the variable that measures the number of eligible voters. Nonetheless, regressions 2 and 3 both identify a separate effect of limiting voting rights to parents, even after controlling for electoral size effects. This empirical finding suggests to us that the variation in voting rights also affects the distribution of benefits that voters assign to the electoral alternatives; bokmål and nynorsk. Restricting voting rights may exclude people that do not care (as much) about the outcome of language referendums. If parents are more concerned about which language their children are to be taught than the public at large, rational choice theory would predict an increase in voter turnout following a restriction in voting rights.

The empirical investigation has identified changes in the voting rights of parents to have a substantial quantitative impact on turnout. This finding points to the fundamental democratic question of who should be allowed to vote. If "substantial" voter turnout is required to secure the legitimacy of referendum outcomes, restricting suffrage to the members of the population expected to be most affected by, interested in and concerned with the issue at hand would be optimal. However, the purpose of referendums is precisely to reveal how policy alternatives are valued across the population, an argument which tends to favour the implementation of universal suffrage.

Our finding that concurrent elections positively affect voter turnout in local referendums is consistent with evidence reported by Geys (2006). Reasonably, the cost of participating in a simultaneous election is lower than the cost of voting in a series of elections. Therefore, an added election on the

[22] A related issue, pointed out to us by one of the referees, is whether voters have sufficient information regarding the different referendum types. Anecdotal evidence suggests to us that at least Noregs Mållag, when campaigning for nynorsk victories in semi-binding referendums during the 1970s, calculated in advance the number of nynorsk votes needed to secure the (binding) implemention of nynorsk. In recent years, the official announcements of upcoming referendums have always stated that the referendum is advisory only. However, we are in no position to make the unqualified argument that voters in general always have been fully aware of the distinction between, and the implications of, different referendum types.

ballot effectively lowers the average cost per election, which – according to rational choice theory – should increase voter turnout.

A related methodological issue is whether a rational choice approach to voter turnout is appropriate at all. We used the smallness of electoral size and the fact that many of the referendums were close to argue that individual voter influence would most likely be high in many of the referendums in our dataset. Plugging the data on voter turnout, electoral size and vote distribution into ϕ, we can estimate the expected equilibrium pivotal probability in each referendum to gauge whether rational choice could reasonably explain voter turnout.

Table 4. Estimated expected pivotal probabilities

Year	County	Municipality	School district	Pivotal prob.
1978	Nordland	Hattfjelldal	Grubben	0.5
1975	Møre og Romsdal	Rindal	Skogen	0.189
1980	Nord-Trøndelag	Verdal	Garnes	0.179
1983	Telemark	Drangedal	Henseid	0.119
1972	Nordland	Leifjord	Tverlandet	0.116
1980	Sør-Trøndelag	Rennebu	Nerskogen	0.108
1973	Aust-Agder	Gjerstad	Gjerstad	0.103
1984	Møre og Romsdal	Gjemnes	Heggem	0.102
1979	Nord-Trøndelag	Steinkjer	Følling	0.086
1972	Oppland	Ringebu	Strand	0.076

Table 4 reports the highest pivotal probabilities in the sample, for simplicity assuming that the referendums were believed to be binding. The pivotal probabilities are high in some instances. In *Grubben* school district in the municipality of *Hattfjelldal,* all thirty members of the electorate participated in the 1978 referendum. Half of them voted in favour of nynorsk, the rest in favour of bokmål. If every elector expected everyone else to vote and knew that the electorate was split perfectly in half, each voter would expect to be decisive and the referendum to be a draw. This implies an expected pivotal probability of 0.5, which is as high as it can get. The probability of being pivotal was above one in one thousand in 88 of the referendums. Given the importance of the question at hand, the choice of language to be used in schools, we would not be surprised to find the benefit of choosing policy to surpass the net cost of voting by a factor of one thousand.

However, we do not wish to argue that the rational choice "pivotal" voting model necessarily is the most appropriate one. Other models, such as Shachar and Nalebuff's (1999) "follow-the-leader" model and the "group-utilitarian" model studied by Coate and Conlin (2004), generate similar predictions. However, all of these belong to the same class of models in which the benefit of voting depends crucially on perceived influence on the election outcome. In the pivotal model voters consider perceived individual influence. In the other two, behaviour is determined by group influence. Such "influence" models can be distinguished from "expressive" models in which voting is an act of civic duty or an expression of the electors' views and not an effort to affect the outcome of the election.[23] It is far from obvious why referendum design and the distribution of voter preferences as measured by the degree of closeness and electoral size should matter for participation in expressive models. Hence, one might interpret our empirical results as providing general support of influence models in small elections. Nonetheless, Coate, Conlin and Moro (2004) offer a critical view of the appropriateness of the pivotal voter model. They analyse data from Texas liquor referendums and find a model of voter turnout in which electors vote if they feel strongly enough about the issue at hand to outperform the pivotal voter model in terms of predicting out-of-sample closeness of referendums.

Nor do we claim that voters use calculus *only* when deciding whether to participate. Consider, for example, the case of *Vera* school district in the municipality of *Verdal*. In 1977, the whole electorate (all six of them) unanimously decided to vote down the proposal to implement *nynorsk*. Here, the most favoured policy would have been implemented irrespective of the absence of one or more voters. Owing to the small size of the electorate, it is not unlikely that all voters knew this in advance and thus, the expected probability of being pivotal was likely to be zero for each individual and for every group of voters. Pure calculus of voting cannot explain voter participation in this case.[24]

5. Conclusion

We have analysed voter turnout using field data from local Norwegian language referendums. Voter turnout is negatively correlated with the size of the electorate and increasing in the intensity of electoral competition as measured by the closeness of the referendum. Also, the participation rate is significantly higher when suffrage is constrained to parents with children in school age and in the few instances when the referendum has occurred at the same date as a local or general election. The

[23] See Dhillon and Peralta (2002) for a recent survey.

[24] There could have been a pre-emptive motive for voting in this case. A binding referendum guarantees the language of choice to be the official one for five years to come, *i.e.* for as long as the children of the eligible voters remain at school.

evidence does not corroborate our prediction that semi-binding referendums display a higher voter turnout than advisory referendums.

In our sample, restricted voting rights tend to exclude the very young voters (below the age of 25) from participating in language referendums. Franklin (2004) provides evidence in support of the hypothesis that the act of voting is very much a habit acquired at young age: Young voters are much more sensitive to the general factors affecting voter turnout than older voters. Following Franklin (2004), one would expect the changes in local referendum voting rights to have left a foot mark on turnout in subsequent elections by affecting the voting propensity of those generations of electors excluded from (or allowed to vote in) a referendum at a young age. Tracing the dynamic effects of changes in voting rights lies outside the scope of this paper, but poses an interesting venue for future research. Moreover, the magnitude of any dynamic effect has normative implications. To the extent that early participation in a referendum is a strong indicator of future voting behaviour and a high turnout is desirable to secure the democratic legitimacy of elections, local direct democracy has the potential for being a seedbed of an inclusive democracy.

Our study contains a novel theoretical analysis of referendum design, and in theory legally (semi-) binding referendums generate higher turnout and produces better information about the preferences of the electorate. Conversely, we could argue that advisory referendums are superior to binding ones in as much such elections allow the political authorities room for considering additional sources of information upon which to base a final decision. Therefore, binding referendums are optimal from society's point of view only to the extent that an increased decisiveness in the referendum leads to a significant increase in voter turnout.[25] Noticeably, our empirical findings suggest that voter turnout may, in practice, to a large extent be invariant to changes in referendum design. As the decisiveness of the referendum in practice is endogenous, whereas we treat it as exogenous, further empirical work, perhaps controlled experimental studies, is needed to gauge whether voter turnout is truly invariant with respect to referendum design.[26]

[25] This argument presupposes that a correct decision actually exists, and the electorate holds the key to the decision. The first point requires that costs and benefits can be aggregated across individuals, see Ledyard and Palfrey (1994 and 2002) for a normative analysis of referendums. If inter-personal utility comparisons are impossible, all outcomes are (Pareto)optimal as some are likely to benefit and others likely to lose from any decision. The second point requires that the preferences of the electorate are representative of the preferences of the entire population, which raises the question of optimal suffrage.

[26] Laboratory experiments on voter turnout have exclusively considered binding referendums, *cf.* Schram and Sonnemans (2005) for a survey and Levine and Palfrey (2006) for a recent study.

Regarding the generalisability of our findings, Scarrow (2001) finds that a large number of European countries have either introduced or expanded the opportunities for holding local referendums during the last three decades. We thus anticipate our empirical estimates to be challenged by future analyses of voter turnout in local referendums in other European countries.

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Appendix A

This appendix derives the large sample properties of the expected probability of being pivotal under a semi-binding referendum. Let $N = M+m+1$ be the size of the electorate, with M and $m+1$ denoting the size of the majority and minority group, respectively. Assume that the majority is a constant fraction $\omega > 0.5$ of the electorate, *i.e.* $M = \omega N$ and $m+1 = (1-\omega)N$. Note that majority and minority refers to the size of the electorates, not actual turnout. Let θ be the threshold above which the referendum is binding. We assume that $N\theta \leq m$. The other generic case, $N\theta > m+1$, is simpler because pivotalness then arises if and only if the referendum is de facto advisory. The pivotal probability in that case is equal to that where the referendum is advisory by rule. We derive the approximate pivotal probability for a member of the minority. Write q_m [q_M] the unconditional probability that a random minority [majority] voter participates in the referendum, and assume all voters to make their decisions simultaneously and independently. The expected probability that minority alternative wins as perceived by a minority voter who has decided to participate is given by

$$p'_m = \sum_{Y=0}^{\theta N-1}\sum_{X=0}^{\theta N-2} h(q_m, m, X)h(q_M, M, Y)R(X+1, Y)$$

$$+\sum_{Y=0}^{\theta N-1}\sum_{X=\theta N-1}^{m} h(q_m, m, X)h(q_M, M, Y)P(X+1, Y)$$

$$+\sum_{Y=\theta N}^{M}\sum_{X=0}^{m} h(q_m, m, X)h(q_M, M, Y)P(X+1, Y),$$

where X [Y] is the (stochastic) number of minority [majority] voters,

$$h(q, M, Z) = \frac{M!}{Z!(M-Z)!} q^Z (1-q)^{M-Z},$$

and $P(\cdot,\cdot)$ [$R(\cdot,\cdot)$] the probability of the minority alternative winning the election as a function of voter turnout in a binding [advisory] referendum. The first term in the probability above refers to the case in which voter turnout falls below the legal threshold for both alternatives and the referendum is de facto advisory. The two subsequent terms refer to the case in which at least one of the alternatives gains enough votes to render the referendum de facto binding. If one minority voter drops out, the probability of the minority still winning the referendum falls to

$$\underline{p}_m = \sum_{Y=0}^{\theta N-1}\sum_{X=0}^{\theta N-1} h(q_m,m,X)h(q_M,M,Y)R(X,Y)$$

$$+\sum_{Y=0}^{\theta N-1}\sum_{X=\theta N}^{m} h(q_m,m,X)h(q_M,M,Y)P(X,Y)$$

$$+\sum_{Y=\theta N}^{M}\sum_{X=0}^{m} h(q_m,m,X)h(q_M,M,Y)P(X,Y),$$

By majority rule, $P(X,Y) = 1$ for all $X > Y$, $P(Y,Y) = 0.5$ (both alternatives win with equal probability in case of a tie) and $P(X,Y) = 0$ otherwise. By assumption, $R(X,Y) = k$ for all $X > Y$, $R(Y,Y) = 0.5$ and $R(X,Y) = 1\text{-}k$ otherwise. $k > 0.5$ is the exogenous probability of the winner being implemented under the advisory referendum. Subtract $\underline{p}_m$ from p'_m to get the pivotal probability:

$$p_m = 0.5\sum_{Y=0}^{m} h(q_M,M,Y)(h(q_m,m,Y-1)+h(q_m,m,Y))$$

$$-(1-k)\sum_{Y=0}^{\theta N-1} h(q_M,M,Y)(h(q_m,m,Y-1)+h(q_m,m,Y))$$

$$+(1-k)\sum_{Y=0}^{\theta N-1} h(q_M,M,Y)h(q_m,m,\theta N-1).$$

The expression can be explained as follows. Assume that exactly Y members of the majority have decided to vote. A minority voter is decisive in two circumstances in a binding referendum. In the first case, participation leads to a tie, wheras abstention would imply a sure loss, *i.e.* $X+1 = Y$. In the second, participation guarantees a win, whereas abstention would lead to a tie, *i.e.* $X = Y$. In both cases the probability of winning the election increases by one half. Aggregating over possible turnout levels gives the first term in p_m. However, not all referendums are binding. If participation drops below the critical threshold, the referendum is merely advisory. In that case even the losing alternative might be implemented. This loss in influence is captured by the second term. Finally, there is a possibility that the participant swings the referendum from advisory to binding. This happens in the event that exactly $N\theta\text{-}1$ of her fellow electors and $N\theta\text{-}1$ or less of the opposers participate. In that case the probability of the favoured alternative being implemented increases from k to unity. This effect is captured by the third term in p_m.

Turning next to the approximation, the first thing to note is that the probability of any single event is close to zero when N is large. Hence, the final term in p_m vanishes in the limit. Second, $h(q,M,Z) \approx h(q,M,Z+1)$ and $h(q,M,Z) \approx F(Z+0.5|a,b^2)- F(Z\text{-}0.5|\ a,b^2)$ with $F(\cdot|a,b^2)$ the cumulative of the normal distribution $f(\cdot|a,b^2)$ with expectation $a = Mq$ and variance $b^2 = Mq(1\text{-}q)$ (see *e.g.* DeGroot, 1989). Hence,

$$p_m \approx \sum_{Y=0}^{m} \int_{-0.5}^{0.5} f(w+Y \mid Mq_M, Mq_M(1-q_M))dw \int_{-0.5}^{0.5} f(w+Y \mid mq_m, mq_m(1-q_m))dw$$

$$-2(1-k)\sum_{Y=0}^{\theta N-1} \int_{-0.5}^{0.5} f(w+Y \mid Mq_M, Mq_M(1-q_M))dw \int_{-0.5}^{0.5} f(w+Y \mid mq_m, mq_m(1-q_m))dw$$

for N sufficiently large. Define four new variables $\mu_m = N(1-\omega)q_m$, $\sigma^2_m = N(1-\omega)q_m(1-q_m)$, $\mu_M = N\omega q_M$ and $\sigma^2_M = N\omega q_M(1-q_M)$. These variables represent the expected turnout and the variance in turnout of the two groups, respectively. For N large, we have

$$\int_{-0.5}^{0.5} f(w+Y \mid \mu_m, \sigma_m^2)dw \approx f(Y \mid \mu_m, \sigma_m^2)$$

A similar approximation holds for M. Hence,

$$p_m \approx \sum_{Y=0}^{m} \frac{1}{2\pi\sigma_M\sigma_m} e^{-\frac{1}{2}\left(\frac{Y-\mu_M}{\sigma_M}\right)^2 - \frac{1}{2}\left(\frac{Y-\mu_m}{\sigma_m}\right)^2} - 2(1-k)\sum_{Y=0}^{N\theta} \frac{1}{2\pi\sigma_M\sigma_m} e^{-\frac{1}{2}\left(\frac{Y-\mu_M}{\sigma_M}\right)^2 - \frac{1}{2}\left(\frac{Y-\mu_m}{\sigma_m}\right)^2}$$

Define three new variables,

$$\mu = \frac{\mu_m\sigma_M^2 + \mu_M\sigma_m^2}{(\sigma_m^2 + \sigma_M^2)},\ \sigma^2 = \frac{\sigma_m^2\sigma_M^2}{(\sigma_m^2 + \sigma_M^2)} \text{ and } \kappa = \frac{(\mu_M - \mu_m)^2}{\sigma_m^2 + \sigma_m^2}.$$

Now,

$$\frac{1}{2}\left(\frac{Y-\mu_M}{\sigma_M}\right)^2 + \frac{1}{2}\left(\frac{Y-\mu_m}{\sigma_m}\right)^2 = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2 + \frac{1}{2}\kappa \text{ and } \sigma_M\sigma_m = \sigma\sqrt{(\sigma_M^2 + \sigma_m^2)}$$

imply

$$p_m \approx \frac{e^{-\frac{1}{2}\kappa}}{\sqrt{2\pi(\sigma_M^2 + \sigma_m^2)}} \left(\sum_{X=0}^{m} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} - 2(1-k)\sum_{X=0}^{N\theta} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \right)$$

Rewrite and use the definition of ϕ in the main text to obtain

$$\frac{p_m}{\phi} \approx \frac{\sum_{X=0}^{m} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} - 2(1-k)\sum_{X=0}^{N\theta} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2}}{1 - 2(1-k)F(N\theta \mid \mu, \sigma^2)}.$$

Finally,

$$\sum_{X=0}^{m} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \approx \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} dX = 1$$

and

$$\sum_{X=0}^{N\theta} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} \approx \int_{-\infty}^{N\theta} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2} dX = F(N\theta \mid \mu, \sigma^2)$$

for m large imply

$$\lim_{m\to\infty}\frac{p_m}{\phi}=1$$

for $\theta < 1-\omega$. In the other case, that is, when the minority has support below the threshold level, F =1, the pivotal probability is the same as in an advisory referendum. Following the same steps as above it is easy to verify that even $p_M/\phi\to 1$ as $m\to\infty$.

Appendix B: Proof of Proposition 1

Let $\phi_B(q)$, $\phi_A(q)$ and $\phi_S(q)$ be the pivotal probabilities in the binding, advisory and semi-binding referendums, respectively, and z_B, z_A and z_S the symmetric equilibrium expected voter turnout. $\lim_{q\to 1}\phi_B(q)=0$ and $G^{-1}(1)=\gamma_h>0$, $\lim_{q\to 0}\phi_B(q)>1$ and $G^{-1}(0)=\gamma_l<1$ and continuity of ϕ and G imply the existence of z_B. The existence proof for z_A is analogous. $G^{-1}(q)$ strictly increasing in q and ϕ_B and ϕ_A both strictly decreasing in q under assumption I imply uniqueness of z_B and z_A. Implicit differentiation of the equilibrium condition with respect to k for $F = 1$ yields $z_B > z_A$. $z_S \in [z_A,z_B)$: $\phi_B(q) > \phi_S(q) \geq \phi_A(q)$ is obvious. Suppose that $z_S \geq z_B$. In this case, $\phi_S(z_S)-G^{-1}(z_S) < \phi_B(z_S)-G^{-1}(z_S) \leq \phi_B(z_B)-G^{-1}(z_B) = 0$ where the second inequality follows from monotonicity of $\phi_B(\cdot)$ under assumption I, and the equality is true by definition. Thus $z_S \geq z_B$ cannot be an equilibrium. Conversely, $z_S < z_A$ cannot be an equilibrium either, as $\phi_S(z_S)-G^{-1}(z_S) \geq \phi_A(z_S)-G^{-1}(z_S)>\phi_A(z_A)-G^{-1}(z_A) = 0$. $\phi_S(z_A)-G^{-1}(z_A) \geq\phi_A(z_A)-G^{-1}(z_A) = 0$, $\phi_S(z_B)-G^{-1}(z_B) < \phi_B(z_B)-G^{-1}(z_B) = 0$ and continuity of ϕ_S and G imply the existence of $z_S \in [z_A,z_B)$. *QED*