Timing of product introduction in network economies under heterogeneous demand

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Abstract

This paper studies the introduction of a new technology in a spatial market with network externalities. Faced with an incumbent firm supplying a competing, incompatible technology, the sponsor of the new technology must decide on *when* to enter and *where* to locate in this economy. Firms' relative values, as a function of the entrant's choices, determine adoption patterns and the scope for private rents. The relationship between choices of timing and location is investigated, and the region of potential equilibria outcomes is derived. It is shown that private firms always take on too little R&D and introduce products that are too similar to the existing ones compared to the social optimum, because of their private business-stealing incentives. Private choices made under the regime of incompatibility are compared to those made when standards are fully compatibility. It is shown that full compatibility leads to perfect alignment between the private and the social planner's incentives. Numerical methods are used to support analysis.

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1 Introduction

For many technologies in network markets, communication with other users is an integral part of a product's value. The business of telecommunication is a good example. In a market where consumers have heterogeneous preferences, the location choice of a firm will influence not only the scope of potential markups that can be made but also the market shares now and in the future, since positive network externalities makes past sales an asset in present competition. The ability to choose horizontal product characteristics helps the new firm to target certain parts of the market. By lowering the degree of product differentiation of its product in comparison to the existing one, the entrant may improve its share of the market, but loose out on higher markups at the same time. Another choice facing the sponsor a new technology is to pick the level of research which increases the intrinsic value of the product, raising the general valuation of the technology. One may think of the entering firm as incorporating state-of-the-art innovations into the product. On the other hand, by devoting much time to R&D the entrant foregoes the opportunity to start the creation of a private installed base of users, leaves the incumbent to build up a network uncontested, and postpone realization of profits. Once introduced further upgrading is not possible, making changes in relative network sizes the only channel through which firms' mutual strengths change. This model analyses how the sponsor decides on *when* to enter and *where* to position its product in a spatial market, when battling a rival technology currently hosting a rival technology, and investigates the interaction between how vertical and horizontal attributes of technology are optimally chosen. The choices made by the newcomer determine the speed and direction of changes in market shares. This model allows for the introduction of technologies that only in time either preempts or become preempted by the competing standard. This conclusion is new to the literature on endogenous timing of introduction, as homogeneity of consumer preferences either leads to a complete over take of the market immediately upon arrival or no entry at all. Such radical changes in adoption patterns do not seem very descriptive for many standards wars. Another adverse consequence of the assumption of homogeneity in population's tastes is that new technologies enter only if there exists a scope of gaining a positive market share in order to sell at least one unit of technology, as shown in the articles of Katz and Shapiro (1992) and Regibeau and Rockett (1996). By identical buyers, however, positive sales equates with making every sale, and so all new introductions become dominant in their industries. I also find this observation at odds with reality. By including within-period heterogeneity in buyers' preferences, this model allows a new technology to capture an intermediate markets share upon introduction, and, as a function of characteristics chosen, shows the resulting transition in market shares positive or negative. The combination of within-period heterogeneity of demand and endogenous entry in network markets, at least to the best of my knowledge, not been much appreciated in the literature. The literature most closely related to the present analysis is Katz and Shapiro (1992) and Regibeau and Rockett (1996), both studying firms' endogenous choices of introduction dates. Katz and Shapiro (1992) focus on the endogenous choice of timing of product introduction of the second-generation technology when consumers hold perfect foresight on future network sizes and introduction times. Buyers in Regibeau and Rockett (1996) are myopic, but solve for the optimal entry dates of introduction for both generations of technology. Both models assume that buyers hold homogeneous preferences. Leão and Santos (2006) setup a two period model of competition between two sponsors of private networks. In a model without endogenous product introductions, they study how within-period heterogeneity and uncertain future valuations affects firms' market coverage and pricing decisions.

The model is outlined as follows. Following the setup, equilibrium pricing strategies are derived. Using these, the transition function describing intertemporal demands as a function of the entrant's choice of variables is derived. Five general cases are identified, and resulting prices and profits are found. The model is then solved where possible, and it is proved that equilibrium satisfies certain limitation with respect to the optimal level of product differentiation as a function of timing of introduction. The welfare implications are considered, and the case of full compatibility is solved and compared to the incompatibility case. Lastly, numerical methods are used to support findings.

2 Setup

The model describes competition between two private sponsors of the inherently incompatible networks, A and B. Firms are asymmetric in that firm A enters exogenously at time zero with an intrinsic quality level of α and starts the accumulation of a network from that point on. Firm B is a potential entrant with the ability to endogenously choose the quality level of the technology it is going to introduce to the economy, modeled as the amount of time it delays entry. Think of this "waiting period" as the R&D effort of firm B, such that the product it introduces always involves the state-of-the-art technology. Formally, the technological improvement follows a linear function of time, namely $\beta + \theta t$, where β is the *stand-alone value*[2] of firm B's technology, and θ is the addition to quality per unit of R&D time. Define the date of introduction for technology B as Δ. The quality chosen is a vertical attribute shared by all consumers. Once a technology has been introduced, no further improvements are possible. Incapability of upgrading technologies seems most appropriate whenever technology is "hard-coded" or when maintaining backwards compatibility to earlier adopters is expensive. Buyers adopting a given technology enter into the sponsor's network as part of the installed base, which is linearly proportional to the number of users that have adopted this network in the past. Let γ measures the importance of network effects, where $\gamma \geq 0$. When $\gamma = 0$ firms' relative strengths are unchanged across periods, since network sizes are without interest to users, reducing competition to playing the ordinary Hotelling horizontal differentiation model each period. In order to induce firm B to choose positive levels of R&D assume that $\theta \geq \gamma$ such that the technological value created during a period of research is no smaller than the value increase experienced by the

[2]Stand-alone value is the value of a technology without network benefits. Liebowitz and Margolis (1995) refer to it as the *autarky value*.

incumbent via its network accumulation. This restricts the applicability of the model to industries where innovation rates are sufficiently high. Let the unit line $x \in [0;1]$ represent the market. Firm B chooses its location in the market by picking b, $b \in [0;1]$, defining its position in the market via $x = 1 - b$. By assumption firm A is located at $x = 0$, and taking this fact into account, firm B uses its location to differentiate its product from the existing one. Competition is Bertrand. Both firms are able to perfectly discriminate between customers in the market, leading to discriminatory pricing of first degree. Hence, any excess valuation that some consumer has for a technology over the best alternative, can be soaked up through a tailored take-it-or-leave-it price. Firms produce at zero marginal cost. Alternatively, think of firms' qualities as net of any marginal costs incurred. No fixed costs of production. Time is discounted by the factor δ, assumed to satisfy the inequality $\delta > \frac{\gamma}{d(1-b)}$ in order for profits to be finite. Products are infinitely durable. A firm seizing to make positive sales remains in the market as an option to buyers.

Consumers have heterogeneous tastes for technologies as captured by their position x in the market, distributed with an uniform density of one. The Euclidian distance between the buyer and the firms represents how well the product's (horizontal) characteristic is aligned with the buyer's taste. The reduction in customers willingness-to-pay is quadratic in the distance traveled, with d measuring the magnitude of this reduction, $d > 0$. Buyers' preferences are otherwise identical. A simplifying assumption of $\alpha \geq d$ is made to ensure that the incumbent can profitably attract the buyer who is least inclined to buy its product when its value is at its lowest. Buyers have inelastic demands for one unit of technology and they are infinitely lived. Customers are small in the sense that they do not take the strategic effect of their own action onto others' actions into account, an assumption much in tune with there being a continuum of buyers. At each moment in time a new, uncommitted group of buyers arrives in the market. Due to the myopia of consumers future events does not enter into their decision making and adoption hinges entirely on a comparison of utilities currently offered. The concept of Markov Perfect Equilibrium is used to describe the pricing game played by firms. Formally a Markov Perfect Equilibrium is a profile of Markov strategies leading to Nash equilibrium in every proper subgame. Special for Markov strategies, or *state-space* strategies, is that the effect of the history of the game on current competition is completely summarized in the state variable, in this case being firms' values as a function of past play. Buyers have the option not to adopt a technology, in which case they disappear from the market forever. However, since firms can price discriminate perfectly no customers are left uncommitted in equilibrium. Having joined a network the user has no further actions to take, but persists as a permanent addition to the firm's installed base. Utility offered to the consumer at x at time t by firm A[3]: $U^A(x,t) = \alpha + \gamma\Delta - d(x)^2 - p^A(x,t) + \gamma \int_0^t \tilde{x}(s)ds$. Likewise, the utility offered to some consumer at x at time t by firm B: $U^B(x,t) = \beta + \theta\Delta - d(1-b-x)^2 - p^B(x,t) + \gamma \int_0^t (1-\tilde{x}(s))ds$. Note that in the above equations, the terms $\gamma \int_0^t \tilde{x}(s)ds$ and $\gamma \int_0^t (1-\tilde{x}(s))ds$ represents

[3]Note how time is "reset" in the equations relating to the duopoly, such that $t = 0$ is Δ periods after the incumbent entered the market.

the installed bases associated with networks A and B respectively accumulated during the first t periods of the duopoly. The term $\gamma\Delta$ in equation **??** is the value of the installed base accumulated by firm A during its monopoly period. The value of a firm that is no longer adopted by new users remains fixed.

3 Equilibrium pricing

The ability to price discriminate perfectly across time and space enable the higher value firm able to soak up the excess value created for each individual over the best alternative available to her. Since the value of a network is a function of time and the customer's position then so is the price. It can be proven that it is an equilibrium for A to charge a price equal to the value it creates for a given buyer during its monopoly period, since buyers' best alternative to adopting network A is not to adopt at all. The proof is available on request. Due to the myopia in consumers' expectations, adopters in the monopoly were not able to claim a reduction in price of firm A as the expected entry date of the second-generation technology draws nearer[4]. Duopoly competition is different as buyers benefit from the threat of reverting to their individually best alternative. A general proof of equilibrium pricing in the duopoly is provided here. It extends quite easily to suit each of the five cases of competition described in section 4.1.

Proposition 1. At all points in the market it is an equilibrium for the highest value network to set a price equal to the value it creates in excess of the competing firm. The lower value firm prices at cost.

Proof in Appendix A.

4 The transition function

The transition function describes in a dynamic way the intertemporal position of the pivotal agent. The position is useful in separating firms' market shares over time, as a function of the entrant's timing and location choices. Let $\tilde{x}(t)$ denote this function. The transition function is found by equating the duopoly utilities offered by firms, using the fact that both firms find it optimal to price at cost to the indifferent consumer. The derivation is reported in Appendix B.

$$\tilde{x}(t) = \left[\frac{\alpha - \beta - (\theta - \gamma)\Delta - d(1-b)b}{2d(1-b)}\right] e^{\frac{\gamma}{d(1-b)}t} + \frac{1}{2} \tag{4.1}$$

By construction, all consumers at $x < \tilde{x}(t)$ join A, and all consumers at $x > \tilde{x}(t)$

[4]Perfect foresight on consumers' behalf in Katz and Shapiro (1992) enable them to foresee the introduction of technology B and hence demand a reduction in firm A. The authors show that firm A's price declines steadily reaching marginal cost just at the moment firm B enters. As consumers in the present model are myopic they cannot make any predictions on future events, and the incumbent's monopoly pricing structure is of a different nature. However, since firms do make accurate predictions on future play, their behaviors are basically the same under both assumptions of buyers' expectations.

join B[5]. As consumers are distributed $[0;1]$, the demand facing firm B at time t equals $1-\tilde{x}(t)$. Increasing the quality of technology B via a longer R&D phase, leads to higher market shares once entered. The entrant's choice of location, b, influences market share $(1-\tilde{x}(t))$ in two ways. First, choosing a higher b increases the market share during the first period of duopoly competition[6] amplifying future market shares, as the entrant increases its appeal to consumers closer to the incumbent, moving the indifferent consumer towards a smaller x. Second, higher b increases the speed at which the market tips, since products become less differentiated, making buyers' valuations of the two products less different. As long as the market tips in favor of the entrant, both effects of increasing b work in the same direction. But when the market tips against the entrant the two effects are countervailing; one gives a higher demand in early periods, but the other speeds up tipping. The wedge between the effects of raising b is later shown to result in a discontinuity in the entrant's choice of variables for a continuous change in parameters. More heterogeneity in population tastes slows down the speed of tipping, as the network must be of higher value to counter the increased transportation costs. Also, higher degrees of heterogeneity make initial demands converge to the midpoint of firms' locations, since the closer firm becomes more attractive to the buyer. Only if B captures less than half the market are both these effects beneficial. Higher rates of technological improvement, θ, unilaterally make B's demand higher, since the quality for some R&D effort increases. The effect of changing γ is two-fold. On one hand higher γ leads to an increase in the value of the network size that firm A builds during the initial period, lowering the relative quality of firm B at the time of entry, *ceteris paribus*, but strong network effects can also be beneficial for the faster growing firm.

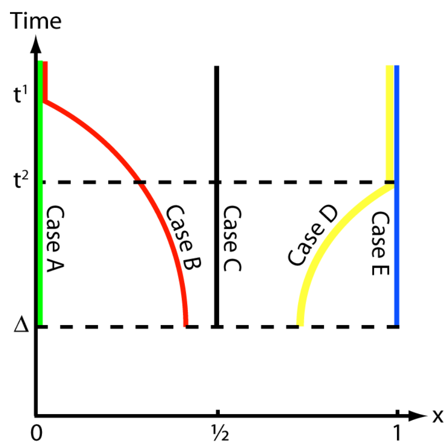
4.1 Market outcomes

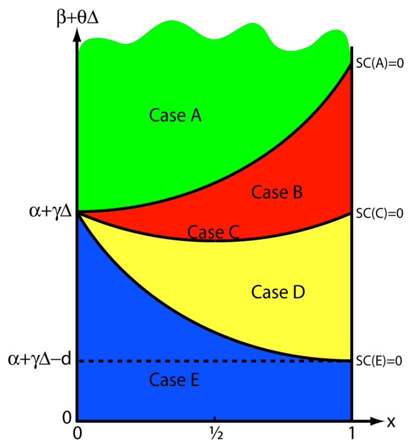
Intertemporal market shares can behave in different ways during the duopoly as a function of the combination of timing and location chosen by the entrant. The transition function identifies a set of formal conditions on these relationships, being useful for later analysis. Five general cases, denoted A, B, C, D, and E, are briefly described below. Case A has every buyer adopting the new technology immediately upon its introduction. This makes demands change abruptly when going from the monopoly to duopoly. This adoption pattern here is exactly what is observed in Katz and Shapiro (1992) and Regibeau and Rockett (1996), as homogeneous buyers all make the same choice during a period. Formally it must hold that $\tilde{x}(\Delta, b) \leq 0$ at $t=0$. Case B is defines a situation in which the new technology is not sufficiently valuable to capture all buyers immediately upon entry, but still attracts more than half the market. As network sizes is the only factor that changes relative valuations of products once introduced in the market, the market will tip in favor of firm B, which will come to dominate the entire market at time $t^1 = \ln\left(\frac{-d(1-b)}{[\alpha-\beta-(\theta-\gamma)\Delta-d(1-b)b]}\right)\frac{d(1-b)}{\gamma}$. At time zero

[5] The (zero mass) user at $x=\tilde{x}(t)$ is assumed to be assigned at random to either of the networks.

[6] Provided that the combination of timing and location satisfies proposition 2.

the transition function satisfies $\tilde{x}(\Delta, b) \in \left(0; \frac{1}{2}\right)$. I find it intuitively appealing that new technologies capture only a fraction the market demand on arrival, and only with time grow strong enough to take command over the full market. As discussed later, Park (2004) finds that, in the standards battle of VHS and Betamax, it were in fact network effects that made the market tip in favor of the VHS and not quality differences during the period 1981-1988, supporting the workings of this model. Collectively cases A and B are denoted as being "revolutionary" as they changes adoption pattern of all buyers at some point in time. Case C has firms in a stalemate, as initial demand is split evenly, such that they both accumulate value at the exact same pace, and so market shares are constant at $\tilde{x}(t) = \frac{1}{2}$ for all t. Empirically not every new introduction in network market comes to dominate previous technologies. This seems to be a part of reality; The CD (introduced 1982) remains preferred widely to the MiniDisc (introduced 1992)[7] even though the MiniDisc were introduced 10 years later. However the timing literature have not accounted for this observation. Case D has the entrant introducing an inferior product such that $\tilde{x}(\Delta, b) \in \left(\frac{1}{2}; 1\right]$ initially. Network accumulation is small, and B is preempted at time $t^2 = \ln\left(\frac{d(1-b)}{\alpha-\beta-(\theta-\gamma)\Delta-d(1-b)b}\right)\frac{d(1-b)}{\gamma}$. Such introductions can be thought of as being "evolutionary" in the sense that technological improvement offered is only high enough to attract a small segment of the market, thus the product never becomes the market wide standard. Only if the rate of technological progress is insufficient to keep up with the network benefit that A creates during each of B's research periods is entry never profitable. Under homogeneity in buyers' preferences is case E the only alternative to case A. The left figure below illustrates examples of possible transitions paths in intertemporal demands for cases A-E. The figure on the right shows the regions of (Δ, b) combinations that will result in outcomes belonging to each of the cases.





[7] See the statistics in the "World Sales" publications from IFPI, The International Federation of the Phonographic Industry, on how sales are distributed on different formats; http://www.ifpi.org/content/section_resources/index.html

5 Solving the model

While A, C, and E are the only cases that allow for analytical solutions, case A is the only one interesting for economic analysis. Equilibrium choices are compared to those made under homogeneous demand. It is shown that for a given level of timing, there exist an upper- and lower boundary on the entrant's choice of location in equilibrium. Then some general regularities in optimal choices are discussed.

5.1 Analytical solutions

The profit function of firm B in case A as a function of its choice variables can be expressed as $\pi_A^B = \frac{1}{\delta}e^{-\delta\Delta}\left(\frac{\gamma}{\delta} - (\alpha - \beta - (\theta - \gamma)\Delta - d(1-b)b)\right)$. Using the Kuhn-Tucker method for constrained optimization, maximization this function with respect to Δ and b on the interior of case A gives[8]:

$$b_A^* = \frac{1}{2} \text{ and } \Delta_A^* = \frac{\alpha - \beta - \frac{d}{4}}{\theta - \gamma} + \frac{\theta - 2\gamma}{\delta(\theta - \gamma)} \tag{5.1}$$

Where $\theta > 2\gamma + \frac{d\delta}{2}$ must hold. Since case A is defined by combinations of (Δ, b) that make all buyers in the market adopt B immediately. Locating directly in the middle of demand served minimizes average transportation cost, which maximizes average valuation for network B that can be soaked up through prices. The equilibrium level of timing reflects the balance between the loss for remaining an outsider to the market for another period, $\beta - \alpha + \frac{d}{4} + (\theta - \gamma)\Delta$, in order to do more research, versus the present value of the net gain for actually doing an additional period of research, $\frac{\theta - 2\gamma}{\delta}$. This last term shows that increasing the delay in entry improves quality by θ, but at the same time fails to accumulate a network of its own (of size γ) and allows the competitor to grow in value (of size γ). If $(\alpha - \beta)$ increase such that the incumbent becomes relatively more valuable, firm B respond by raising Δ_A^*. The term $\frac{d}{4}$ is the *saved average travel expense* of firm B's buyers during a period over the travel expense they would otherwise incur had they traveled to firm A[9] at $x = 0$. One should interpret this as the entrant's gain from product differentiation. Observe how greater heterogeneity in population tastes lowers equilibrium timing; higher d increases gains from product differentiation leading to higher markups[10], making the research delay relatively more expensive. Higher θ always adds positively the present value of entrant's profits, but optimal timing increases iff $\frac{\gamma}{\delta} > \alpha - \beta - \frac{d}{4}$. If $\frac{\gamma}{\delta} > \alpha - \beta - \frac{d}{4}$ then higher γ has a positive impact on profits, but only has a positive impact on equilibrium timing iff $\frac{\theta}{\delta} < \alpha - \beta - \frac{d}{4}$. Lastly, an increase in the discount rate will always decrease the optimal

[8] Checking the Hessian for concavity of the objective function is not possible. However, the numerical methods introduced below shows that this is in fact a maximum.

[9] Average travel costs when A covers the market are $\int_0^1 d(x)^2 dx = \frac{d}{3}$. Average travel costs when B covers the market and locates at $b = \frac{1}{2}$ are $\int_0^1 d(\frac{1}{2} - x)^2 dx = \frac{d}{12}$.

[10] Note that higher d lowers the values of both firms, but as the average value of firm A decreases the most, due to the convexity of travel costs, the difference between these values increases.

timing of entry as long as $\theta > 2\gamma$ holds, which is guaranteed by for every combination on the interior of case A and equilibrium choices.

5.1.1 Comparison to the case of homogenous buyers

By forcing the entrant to pick $b = 1$ the above result can be compared to those that would prevail under homogeneous demand, as the option use product differentiation is eliminated. The structure of this market then resembles those seen in Katz and Shapiro (1992) and Regibeau and Rockett (1996), as the entering firm's choice reduce to a mere choice of timing of entry alone. The case A profit function using $b = 1$ leads to an optimal timing of entry as:

$$\Delta^*_{b=1} = \frac{\alpha - \beta}{\theta - \gamma} + \frac{\theta - 2\gamma}{\delta(\theta - \gamma)} \tag{5.2}$$

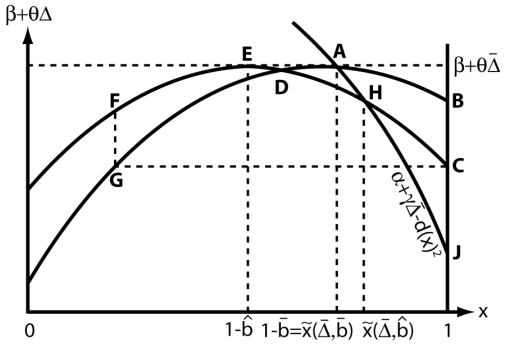
Clearly $\Delta^*_{b=1}$ is greater than Δ^*_A for all $d > 0$. Only if buyers incur no costs of compromising with their tastes, do the two solutions collapse to the same level of timing, since a firm's choice of location does not effect profits. One should therefore conclude that allowing firms to engage in product differentiation leads to faster timing of entry. Why is this so? If technological quality is the only way to created differences in buyers' valuations, it is natural that this incentive alone determines timing. However, if the newcomer is free to choose location, it will forego positive profits from product differentiation during the research phase. This gives stronger incentives for fast introduction. At least the entrant can always mimic behavior under homogeneous demand and be no worse by the additional freedom. However, if product differentiation is possible then the firm can target a segment of the market that has low valuation for the existing network, and so, especially if discounting makes a long delay very costly, fast entry without full dominance in the market can be a better strategy for a new firm.

5.2 Refining the feasible region

While analytical solutions cannot be derived for cases B and D, another approach to shrink the feasible region of the entrant's choices is pursued. The profitability of a firm's choice of timing and location can be boiled down to two effects; size of current and future market shares and size of current and future markups. However, there exist three boundaries that can be shown to dominate different subsets of potential equilibria of timing and location combinations. These involves both smaller current markups *and* a smaller initial market share, which in turn leads to future market shares and markups being smaller than they otherwise could have been under the alternative combination on the boundary. Such choices cannot be part of equilibrium. The analysis rests upon the fact that only research time is costly for the firm, the choice of location is free, and then comparing markups and market shares resulting from different choices. This allows for the consideration of how, for some fixed research delay, profits are connected to the choice of location. Let "average markup" denote the sum of valuations a firm creates in excess of its competitor during a period.

Proposition 2. The level of research time Δ, is positively related to the maximal, feasible level of the location variable b for all $\Delta \in \left[\frac{\alpha-\beta-d}{\theta-\gamma}; \frac{\alpha-\beta}{\theta-\gamma}\right]$.

The figure illustrates competition as it unfolds during the first period of duopoly given two different choice pairs $(\bar{\Delta}, \hat{b})$ and $(\bar{\Delta}, \bar{b})$ where $\hat{b} > \bar{b}$ and $(\bar{\Delta}, \bar{b})$ satisfies the property $\tilde{x}(\bar{\Delta}, \bar{b}) = 1 - \bar{b}$ during the initial period of competition. Values created across the market under these two combinations are compared. The incumbent's value function is represented by AHJ.



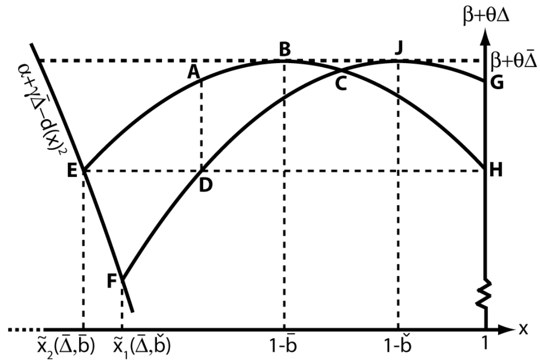
Combination $(\bar{\Delta}, \bar{b})$ has firm B attaining its maximum value at point A in the figure. Likewise $(\bar{\Delta}, \hat{b})$ attains its maximum value at point E, which has the same vertical height as point A as the entry delay is the exact same. Two effects: 1) Since firms' value functions are downward sloping for all $x \in [0; 1]$ then $\hat{b} > \bar{b}$ has $\tilde{x}(\bar{\Delta}, \hat{b}) < \tilde{x}(\bar{\Delta}, \bar{b})$. 2) The average markup earned under $(\bar{\Delta}, \bar{b})$, area ABCJHA, is at least as big as the one earned under $(\bar{\Delta}, \hat{b})$, area HCJH, since the difference, area ABCHA, is positive for all $\hat{b} > \bar{b}$ since $d > 0$. The equation characterizing the combinations of Δ and b defining the boundary of the feasible area is

$$\bar{b}_{\max}(\bar{\Delta}) = 1 - \left(\frac{\alpha - \beta - (\theta - \gamma)\bar{\Delta}}{d}\right)^{\frac{1}{2}} \tag{5.3}$$

See derivation in Appendix C. Given some $\bar{\Delta} \in \left[\frac{\alpha-\beta-d}{\theta-\gamma}; \frac{\alpha-\beta}{\theta-\gamma}\right]$ all $b \in [\bar{b}; 1]$ are *excluded* from the region containing the feasible outcome by proposition 2. Equation 5.3 shows a negative relationship between research time and the minimal, feasible level of product differentiation, coinciding with firm A's value function. The interpretation of this result is that an entrant with a relatively poor product should not aim for a strategy of little product differentiation; if so the value B would offer to buyers close to A would be too small to make them join B, while those buyers not much in favor of A would incur high travel costs for reaching B. Instead it serves the entrant better to focus on minimizing travel costs for the segment of the market that has tastes very different from A's product. Another region of combinations can be excluded from the set of optimal solutions to the entrant's problem.

Proposition 3. There is a positive relationship between research time and minimum, feasible level of b.

Compare differences in markups made under combinations $(\bar{\Delta}, \bar{b})$ to $(\bar{\Delta}, \check{b})$, where $\bar{b} = \frac{|H-E|}{2}$ and $\check{b} < \bar{b}$. It is shown that the former combination dominates the latter for all such locations.



Firstly, choosing $(\bar{\Delta}, \bar{b})$ results a difference in average markup equal to the area ABCDFEA, while $(\bar{\Delta}, \check{b})$ results in an excess markup difference of area JGHCJ. Since transportation costs are symmetric around firms' locations, midway between points B and J lies point C, through which a vertical axis mirrors area JGHCJ to ABCDA. However, area ADFEA represents the additional markup earned by choosing $\bar{b}$ over $\check{b}$. This area is positive for all $\check{b} < \bar{b}$ by construction. Next consider how the choice of b affects current demand. Since the value function of firm A is downward sloping for all $x \in [0;1]$, choosing $\check{b}$ smaller than $\bar{b}$ makes value functions intersect at a higher x, $1 - \tilde{x}(\bar{\Delta}, \bar{b}) > 1 - \tilde{x}(\bar{\Delta}, \check{b})$. Since future demands in equilibrium is a deterministic function of initial demands, then choosing $\check{b} < \bar{b}$ not only leads to a smaller average markup today, but also to smaller market shares, and such combinations can be ruled out of the class of potential optimizers. The boundary can be expressed as

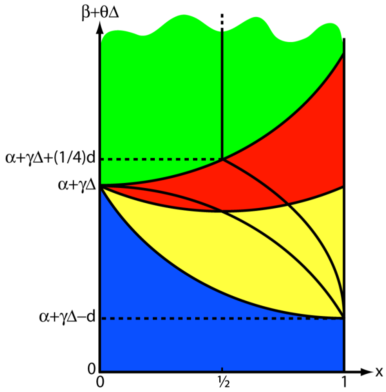
$$\bar{b}_{\min}(\bar{\Delta}) = \frac{2}{3} - \left(\frac{1}{9} + \frac{\alpha - \beta - (\theta - \gamma)\bar{\Delta}}{3d}\right)^{\frac{1}{2}} \tag{5.4}$$

Given some $\bar{\Delta}$ all $b \in [0; \bar{b}]$ are excluded from the set of potential optimizing combinations. For this reason one should not expect to observe new product introductions of technologies that are of high quality and very different from the existing technology. High quality products ensures the sponsor a high market share which, given the desire to affect tipping and earn high markups, is best exploited by locating on the half of initial demand closest to the incumbent, in order to gain high market shares in future periods, and to appeal broadly current and future adopters, properly discounted.

Lastly, rewrite the condition that separates cases A and B

$$\bar{b}_{top}(\bar{\Delta}) = 1 - \left(-\frac{(\alpha - \beta - (\theta - \gamma)\bar{\Delta})}{d}\right)^{\frac{1}{2}} \tag{5.5}$$

The equilibrium solution derived in equation 5.1 shows those combinations on the interior of case A that dominates all other pairs. Hence $\bar{b}_{top}(\bar{\Delta})$ shows the maximal b given Δ, where $\Delta \in \left(\frac{\alpha-\beta}{\theta-\gamma}; \frac{\alpha-\beta+\frac{d}{4}}{\theta-\gamma}\right]$, that is not being ruled out by the solution to case A. The potential equilibria ruled out all leads to immediate coverage of the full market, but which smaller average markups than what could otherwise have been realized. Together the set of potential equilibria that cannot be dominated form the feasible region, coupled with the results from equation 5.1, is illustrated in the figure below.



This result shows that there exist both an upper- and a lower bound on characteristics chosen in equilibrium for new innovations as a function of research time. When the entrant takes over the entire market on entry these boundaries coincide. The region can serve as a way to predict the general outcome of future competition, like the currently raging HD DVD versus Blu-ray standards war.

In markets with relatively slow technological progress and/or high network effects, one should expect a new introduction to be very different from the existing one. An extreme example is the standards-war in keyboard layouts between the QWERTY and Dvorak systems. Arguably, technological progress in the keyboard layout industry is rather slow, and knowing this Dvorak designed his keyboard to be highly differentiated from QWERTY; only keys A and M are placed at identical spots[11]. Obviously the technological improvement offered by Dvorak[12] has been insufficient to overcome the network benefits of QWERTY, and get the bandwagon rolling in its favor. See Shy (2001, p. 43) for an illustration of both layouts and a short case study.

Entering the market with a revolutionizing technology, which is a technology that will come to dominate the market, a very high degree of product differentiation is no longer required. Rather location can be used as a mean of business-stealing. The VCR battle of fought by the VHS- and Betamax standards seems to be applicable, where the choice between VCRs seems to me as less subject to personal tastes than the choice of keyboard, especially if allowing for prior training. Betamax, the incumbent, had a lead of a year and a half before Matsushita introduced the VHS in 1977. Once introduced, the VHS offered tapes that could store up to 4 hours of video, while the Betamax managed only 2 hours at the time. Arguably recording time is an important characteristic of the usefulness/quality of the product, if for an example one is going to record the Super Bowl. This sparked stronger sales for the VHS during the late 1970s, and got a bandwagon rolling. Park (2004) reports data showing that the installed base of VHS were twice the size of that of Betamax around 1981, increasing to the VHS

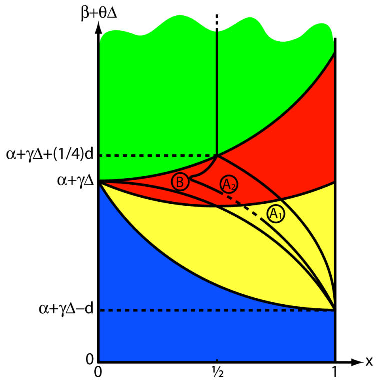
[11]During the first decades of the QWERTY standard's lifetime, several minor changes were made to the individual key's placements.

[12]David (1985) argues that Dvorak is in fact superior to QWERTY in terms of typing speed.

network being 7 times greater in 1988. Park concludes that it were in fact network effects, and not differences in intrinsic qualities or prices, that made the U.S. market for VCRs tip during the years 1981-1988 when differences in technological qualities were about the same level.

5.3 Regularities on the equilibrium path

Having derived the feasible set of combinations (Δ, b), this section analyses in more detail the tensions that go into optimizing behavior of the entrant as a function of the general attractiveness of R&D, being the result of the parameters representing the market. Two general regularities are shown to exist[13]. A) For all positive levels of network effects there exists a pivotal market condition around which the entrant's optimal choice set exhibits a discontinuity. B) The introduction of revolutionary technologies exhibits a backward-bended nature of location choices as the technological quality level increase. The figure below illustrates the two properties.



Ad A) Around the pivotal market parameterization two different equilibria yield the same level of present value profits, giving rise to a discontinuity on the equilibrium path. Let A_1 and A_2 in the figure above marks such two equilibria, and all combinations in between leads to present value profits that are inferior to those at the boundary points. The discontinuity is rooted in the fact that network effects has different effects on profits depending on whether the technology is revolutionary or evolutionary; as discussed in relation to the transition function, higher b increases both basis demand *and* speed of tipping. Introducing a revolutionary product both these effects benefit the entrant, and to capitalize a high b is therefore chosen. This in turn increases levels of current and future demands, making the optimal research level higher, since the value created by doing R&D finds greater return. For evolutionary products, increased speed of tipping leads to faster preemption, and optimal b is thus lower as only one of the two effects on demand for changing b is advantageous to the entrant. This leads to a smaller market share, which reduces the research effort since small demand means small return to R&D. For these reasons does the set of optimal combinations exhibits a discontinuity even though one considers an epsilon change in the parameterization of the market.

[13]A discrete version of the model shows these points in a formal way. Available on request.

The result is that the newcomer must choose between two general strategies of fast entry and high product differentiation or slow entry and a lesser degree of product differentiation. One should therefore expect to observe either long lived technologies that offer a great quality improvement over its predecessor along with a broad appeal to the market, or short-lived technologies that are inferior to the existing technology appealing to a highly specialized segment of the market. Only when network sizes are unimportant to buyers, ($\gamma = 0$), or when network are completely open, does the entrant not use b its location to control intertemporal network sizes, eliminating this wedge. It is the assumption of heterogeneous demand that allows for appreciation of this point.

Ad B) The backward-bended nature of equilibrium choices belonging is the result how location is related to the intertemporal stream of profits. The point "B" in the figure above marks this. For cases where return to R&D is small, while still enough to make the entrant develop a revolutionizing product, it is not worthwhile to introduce a product of very high quality and so a relatively high degree of differentiation is optimal to avoid cannibalization of markups. However, as R&D becomes less expensive, if for an example the discount rate decreases or technological improvement rate increases, firm B is willing to enter with a higher quality product. To ease the burden of preempting firm A, a smaller level of product differentiation can be chosen to both cut into current and future network sizes of firm A and to increase the speed of tipping, without too heavy losses on early markups. Lastly, imagining a situation where it is feasible for B to enter with a quality that is close to actually capturing all demand immediately. Since location is chosen to maximize the present value of profits there is the tradeoff between choosing a high b in order to reap early profits through quickly gaining a high market share, versus the incentive for later gains from a location that maximizes average valuation once firm A is preempted (or relatively close to it). The higher initial quality is, the faster is the incumbent preempted, and the higher is the optimal level of product differentiation. In the limit where firm A captures just an infinitesimal part of demand at the moment of entry, the entrant locates at approximately $b = \frac{1}{2}$, because this is the optimal location in competition during the next, as well as all future, instant(s) as it then covers the entire market. This result fits nicely with our result from case A, stating that firm B always locates right in the middle of demand. Taken together, the optimal combinations of the entrant have a "backward-bended" nature as intrinsic quality increase.

6 Welfare

This section investigates the first-best choices as made by a social planner, who aims at maximizing total welfare of the economy as a whole, and compares it to the private incentives. The objective of the social planner is to choose the combination of Δ and b that maximizes the discounted sum of value created by the higher value firm at each point in the market, at each point in time[14]. The analytical solution when equilibrium belongs to case A equals the value of firm A for all $t \in [0; \Delta)$ and the value created by firm B for all $t \geq \Delta$. This can be expressed as $W_A = \frac{1}{\delta} e^{-\delta\Delta} \left(\beta - \alpha + (\theta - \gamma)\Delta + d(1-b)b\right) + \frac{\gamma}{\delta^2} + \frac{\alpha - \frac{d}{3}}{\delta}$. Constrained optimization yields

$$b_A^{SP} = \frac{1}{2} \text{ and } \Delta_A^{SP} = \frac{\alpha - \beta - \frac{d}{4}}{\theta - \gamma} + \frac{1}{\delta} \tag{6.1}$$

The location choice is the one that minimizes the loss to transportation. The timing choice is a balancing of the welfare realized by entering now, $\beta - \alpha + \frac{d}{4} + (\theta - \gamma)\Delta$, versus the present value of another period of R&D, $\frac{\theta - \gamma}{\delta}$. The latter term reflects that $\frac{\theta}{\delta}$ is the present value increase for an unit of R&D, mitigated by $\frac{\gamma}{\delta}$ being the loss from having to fight a stronger incumbent in all future periods. Comparing the welfare maximizing choices in equation 6.1 to B's private incentives in case A, equation 5.1, it is seen that the location choices are identical, but optimal public delay in timing is greater than the private one, $\Delta_A^{SP} > \Delta_A^*$. The reason for this discrepancy is the fact that while firm B internalizes the full effect of minimizing average transportation cost by locating in the middle of the market, it does not internalize the negative effect of its choice of timing on incumbent's profits. It can be shown that case A-entry is always welfare improving. Introduction of a product of very poor quality may reduce welfare; even though some buyers find it worthwhile to adopt the new technology, splitting of the market is associated with a loss of potential network benefits, and if the technological improvement is not strong enough to outweigh this loss, overall welfare is reduced. The stronger network effects are, the stronger is the externalities between buyers actions, and the greater is the scope such loss. Conversely, only if network effects are sufficiently low, will the introduction of evolutionary technologies not reduce welfare.

Two general "business-stealing" effects distort the private incentives from the socially optimal ones. These are referred to as the *weakened-rival effect* and the *location-boosted demand effect*. The weakened-rival effect[15] is the private incentive of the entrant to halt the network accumulation of the incumbent, by entering faster than it otherwise would have done, in order to limit the value it has to fight in the future. However, since this tactic is purely a private incentive of firm B, the social planner always has a greater incentive to do R&D whenever network effects exist. Only if $\gamma = 0$ does private and public incentives coincide.

The location-boosted demand effect, which to the best of my knowledge has not been captured in a model of endogenous timing yet, refers to the incentive to choose a product that is socially too closely related to the existing product *for a given level*

[14] The social planner's problem can be recast in terms of the incumbent firm's choices; if firm A was the sponsor of both technologies, though still remaining incompatible, the firm would internalize the effect of the new technology's characteristics on both profit streams. The outcome would therefore be perfectly aligned with the social planner's choices.

[15] See Katz and Shapiro (1986) p.835.

of research time. Choosing a low degree of product differentiation makes the new product appeal much to the group of buyers that otherwise have a strong valuation for the established network. Offering these consumers a closer substitute, firm B is able to increase its own relative market shares now and in the future, while at the same time weakening the network accumulation of the incumbent. This gives B a stronger relative position in future competition. From a social perspective this incentive is wasteful, as it is purely a private gain for firm B to promote its own interests, which exerts a negative externality on firm A's demands in the process. When users have no interest in network sizes this inefficiency becomes eliminated.

Comparing welfare results under case A obtained above to what would result from a market of homogeneous demand, as usually represented by forcing the outcome $b = 1$, it can be shown the option to minimize the social loss to transportation by locating somewhere else in the market, makes the social planner prefer relatively quicker entry. The simple reason is the possibility of reducing the wasteful loss to transportation, makes the option to enter the market more attractive in comparison to the value of doing another period of R&D, which is unaffected by such considerations in case A. For case A it was also concluded that heterogeneous demands makes the entrant enter faster than it otherwise would have under the already skewed case of homogeneous demand, nevertheless faster entry under heterogeneous demands is socially more desirable for the very same reason. In fact using the analytical solution to case A, it can be shown that the size of the inefficiencies under the two demand regimes are identical. This is due to the fact that the discrepancy between private and social incentives lies not in the benefits foregone for not entering, which is fully internalized by the entrant in case A, but in differences in their assessments of the value of further R&D, which does not involve travel costs incurred in case A. For cases involving changes to intertemporal market shares the relationship will likely be less clear cut, and one can conjecture that the size of the inefficiency relatively bigger under heterogeneous demand in such cases.

7 Compatibility

Maintained through out the paper has been the assumption of strict incompatibility between networks. This section relaxes this assumption to observe how compatibility influences the optimal choices of firm B. Under a regime of full compatibility between technologies, users are able to communicate across networks; when a buyer joins one network, he or she increases the values of both networks identically. In a sense this makes firms' private installed bases public, and for this reason individual firms cannot use previous sales strategically i.e. to charge positive markups. Since demand is a function of relative values the market will never tip because compatibility inhibits shifts in firms' relative network sizes, even if one firm is adopted by all consumers[16]. Therefore duopoly prices are constant in equilibrium, since prices

[16] Comparing values of networks A and B taking into account that no networks are private, the counterpart of the transition function under full compatibility becomes $\frac{\alpha-\beta-\theta\Delta+d(1-b)^2}{2d(1-b)} = \tilde{x}_{FC}$

are set equal to the difference in firms' values. Solving the case A profit function $\pi^B_{A_{FC}} = \frac{1}{\delta}e^{-\delta\Delta}\left(\beta + \theta\Delta - \alpha + d(1-b)b\right)$ with respect for timing and location yields

$$b^*_{A_{FC}} = \frac{1}{2} \text{ and } \Delta^*_{A_{FC}} = \frac{\alpha - \beta - \frac{d}{4}}{\theta} + \frac{1}{\delta}$$

The parameter restriction $\theta \geq \frac{\delta d}{2}$ must be satisfied. Not surprisingly, the choice of location under perfect compatibility is the same as under incompatibility of standards, because the firm maximizes the average markup by minimizing average transportations costs across the market. However, the speed of entry can be shown to be slower under full compatibility since the entrant's markup is not reduced through foregone network benefits and fast entry is not needed to mitigate such a negative effect. For case B the profit function equals $\pi^B_{B_{FC}} = \frac{1}{\delta}e^{-\delta\Delta}\left\{\frac{(\alpha-\beta-\theta\Delta+d(1-b)^2)^2}{4d(1-b)} - (\alpha - \beta - \theta\Delta - d(1-b)b)\right\}$. Maximize $\pi^B_{B_{FC}}$ with respect to Δ and b subject to the separating conditions defining case B_{FC} gives

$$b^*_{B_{FC}} = \frac{1}{2} - \frac{1}{2}\left(1 - \frac{2\theta}{\delta d}\right)^{\frac{1}{2}}$$

$$\Delta^*_{B_{FC}} = \frac{2}{\delta} + \frac{\alpha - \beta - d\left(\frac{1}{2} + \frac{1}{2}\left(1 - \frac{2\theta}{\delta d}\right)^{\frac{1}{2}}\right)\left(\frac{3}{2} - \frac{1}{2}\left(1 - \frac{2\theta}{\delta d}\right)^{\frac{1}{2}}\right)}{\theta}$$

These two solutions satisfies the two separating conditions defining case B_{FC} provided that $\theta < \frac{\delta d}{2}$. These results show that there exists a negative relationship between research time and the degree of product differentiation. The high quality offered, the greater becomes the resulting market share, and the higher b is chosen to maximize average markups by locating in the middle of demand served. Higher rates of technological improvement, θ, makes longer R&D phases more attractive, and with higher levels of timing comes higher b's maximize average markup. By the envelope theorem, the total effect of a change in parameters equals to direct effect only. Partial differentiation reveals that $d \uparrow$ *always* leads to faster entry and lower b. Intuitively, a higher degree of heterogeneity in demand makes traveling more expensive for buyers, making the closer firm relative more attractive, and since the equilibrium markup of the winning firm has been shown to equal the excess value over the second best offer, markups always increase as a result. In response the entrant moves the date of introduction forward and with it a correspondingly lower b to remain at a position that minimizes average transportation costs. Even though the discount rate δ does not enter the first order condition for b, higher δ influences the entrant's choice of location indirectly as higher discounting makes waiting more costly, leading to faster entry associated with smaller b. If α increases, or β decreases, markups earned by firm B decreases correspondingly, leading to a longer period of R&D. Changes to γ influences both firms identically, making this effect vanish from equilibrium choices. Note that equilibrium choices are continuous functions of parameters showing that there is no discrete shift as seen under incompatible standards (property A), due to the fact that market shares are time invariant.

Since time does not influence market shares in duopoly the entrant looses the incentive it had under incompatibility for choosing a low degree of product differentiation to shift network accumulation in its favor. For this reason the optimal location for a given level of R&D under compatibility is always the one that maximizes markups across the share of the market covered in each of every future period of competition. In fact, this location is the one that maximizes value created for the economy as a whole under compatibility, and one can conclude that private- and public location incentives are perfectly aligned. Timing wise, openness of networks completely eliminates the preemptive incentive of the entrant of fast entry in order to limit the network accumulation of the incumbent, since compatibility of networks does not provide the existing network with a strategic advantage of its past sales. In other words, the weakened-rival effect does not come into play when installed bases are made public. Comparing the level of timing in this model of heterogeneous demand and full compatibility to equilibrium timing had demand been homogeneous, shows that the entrant *always* enters faster under heterogeneous demand, because the possibility to locate freely in the market allows the firm to make a higher level of average markup, making the balance between product improvement and the loss to discounting occur sooner. Indeed, this conclusion is also drawn by the articles of Regibeau and Rockett (1996) and Katz and Shapiro (1992), while not capturing the relationship with product differentiation.

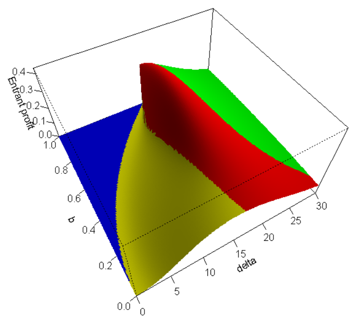
In the special case where buyers put no value on network sizes, $\gamma = 0$, the solutions derived in this section are identical to those that would prevail under a regime of incompatibility. In this case the equilibrium derived here describes the combinations along the boundary derived in proposition 4, equation 5.4, and on the interior of case A given by equation 5.1.

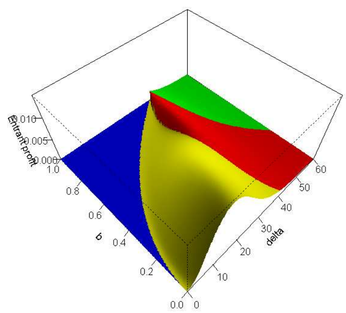
8 Numerical methods

This section investigates the mechanisms of producer B's profit maximizing behavior using numerical methods. Numerical integration is used to calculate the entrant's present value of profits for a given set of parameters, $S_j \equiv \{\theta, \gamma, \alpha, \beta, d, \delta\}$, describing an economy across a two-dimensional grid of the endogenous choice variables. The table below reports optimal choices of the entrant, the social planner, and those made under full compatibility for an economy characterized by the parameter set $S = \{variable, 0.25, 4, 0, 4, 0.2\}$ as a function of the technological improvement rate.

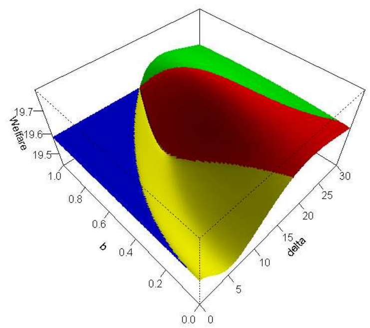
	Incompatibility		Welfare		Compatibility	
θ	Δ	b	Δ	b	Δ	b
0.9	7.8	0.50	9.6	0.50	8.4	0.50
0.8	8.5	0.54	10.5	0.50	8.8	0.50
0.7	10.0	0.57	11.8	0.50	9.3	0.50
0.6	11.8	0.60	13.7	0.52	10.0	0.50
0.5	15.4	0.61	17.6	0.54	11.0	0.50
0.4	23.3	0.58	26.5	0.55	12.5	0.50
0.3872	24.8	0.56	28.5	0.55	12.0	0.45
0.3872	16.2	0.17	28.5	0.55	12.0	0.45
0.35	15.5	0.10	37.0	0.52	11.3	0.35
0.3	15.1	0.04	68.0	0.45	10.8	0.25
0.2	--	--	--	--	10.4	0.14
0.1	--	--	--	--	10.2	0.06

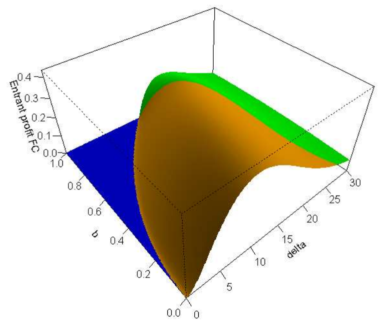
The table shows that under incompatibility of standards the entrant always chooses a level of timing that is lower than the welfare maximizing one due to the weakened-rival effect. The entrant's incentive to choose a lower degree of product differentiation than socially optimal is less clear from this kind of representation, as this analysis draws its conclusions on the optimal b's holding research level fixed. Consider two economies described by parameterizations $S_1 = \{0.5, 0.25, 4, 0, 4, 0.2\}$ (left) and $S_2 = \{0.35, 0.25, 4, 0, 4, 0.2\}$ (right). The figure shows the entrant's present value of profits as a function of potential variable combinations.





In the figure to the left profits are maximized at $(\Delta, b) = (15.4, 0.61)$ belonging to the red case B. In the figure to the right $(\Delta, b) = (15.5, 0.1)$. The relatively higher rate of technological progress under S_1, compared to S_2, induces the entrant to undergo a longer period of research before entering the market, as the value created per period is higher. Entering with a high quality product makes the entrant capture a greater market share and so move closer to the incumbent to not only secure a high average markup, but also ensure help the tipping of the market along. The lower quality under S_2 is not enough to capture the majority of buyers, so b is chosen to cater to those buyers that have low valuations for the existing technology. The left figure below illustrates how the welfare level of the economy parameterized by S_1 is related to B's choices.





One sees that the choice of the social planner chooses the pair $(\Delta^{SP}, b^{SP}) = (17.6, 0.54)$. Observe that welfare is actually reduced if firm B enters the market with a product of poor quality, as the buyers adopting the product are clearly better off, but all current and future buyers of technology A valuations' are reduced since they do not get the benefit of the being compatible to those adopting B. The figure on the right shows the entrant's profits under full compatibility of networks for a market with parameterization $S_3 \equiv \{\theta, \alpha, \beta, d, \delta\} = \{0.3, 4, 0, 4, 0.2\}$ leading to the maximum point $(\Delta, b) = (10.8, 0.25)$ belonging to case B_{FC}.

9 Conclusions

This model has analyzed how the sponsor of a new technology optimally chooses product quality and level of product differentiation when competing against an incumbent firm in a network market. The addition of heterogeneity in demand eliminates the unrealistic feature of the homogeneous demand models, predicting that all buyers following the introduction of a new technology join the new standard, without any transitional period in market shares. Additionally, this model allows for the introduction of "evolutionary" technologies, encompassing the empirical fact that not all new introductions become full blown successes, an observation that could not be reconciled in the existing literature. Upper and lower boundaries on feasible levels of product differentiation as a function of research time were shown to exist. When initial demands are relatively low, timing is negatively related to product differentiation; higher quality calls for a better match between the product offered and average taste users served. However, as the initial market share is close to one, the entrant knows that the established firm is soon preempted, making less differentiation more attractive in order to be able to earn higher markups once it does take over the market. The private choice of timing were shown to be too small compared to the socially optimal level, the weakened-rival effect, and, given the level of timing chosen by the entrant, involves too little product differentiation, the location-boosted demand effect. Both effects are consequences of the entrant not internalizing the consequences of its action on the established firm's value. Full compatibility between standards were shown to eliminate the inefficient incentives of the private firm since the existing network gains no strategic advantage in competition for the period the entrant spends doing R&D.

10 Appendix A - Equilibrium pricing

Proof 1. Define $\tilde{x}(t)$ as the location of a some buyer who values products A and B equally at time t, before prices are taken into account. It is an undercutting proof equilibrium for both firms to price at cost at this point. Let both firms initially set a price above marginal cost. Without loss of generality, let firm B slightly undercut the price of firm A by setting the price $p^B = p^A - \varepsilon$, where ε in some infinitely small, positive value. Anticipating this undercut, firm A finds itself better off by undercutting firm B by charging a price slightly below p^B. Equilibrium is reached only when both firms reach marginal cost pricing.

For all $x > \tilde{x}(t)$ is firm B the higher value firm, and can always make a profitable undercut to any price above cost posted by firm A. As firm A has no better strategy than to offer its product for sale at cost, a weakly dominating strategy for the losing firm, firm A sets $p^A = 0$ for all $x > \tilde{x}(t)$. Show that it is optimal for firm B to choose a price that equals the excess of buyers' valuations at all such x. To see this, suppose that firm B were to pick a higher price, for i.e. $p^B = V^B - V^A + \varepsilon$. Consumers would now be faced with the choice of receiving utility $U^A = V^A - p^A = V^A$ from joining network A, but only receiving $U^B = V^B - p^B = V^A - \varepsilon$ from adopting network B. Hence, producer B does not win this buyer. If firm B charges a lower price than the proposed equilibrium price, i.e. $p^B = V^B - V^A - \varepsilon$, buyers receive utility V^A from network A and utility $V^A + \varepsilon$ from B. Now firm B still wins the buyer, but earns a smaller markup. This establishes that $p^B = V^B - V^A$ is the equilibrium price for all $x > \tilde{x}(t)$. Basically, this is a standard game of Bertrand competition where prices are lowered until the last firm but one cannot set prices any lower. Symmetry implies that equilibrium prices are $p^A = V^A - V^B$ and $p^B = 0$ for all $x < \tilde{x}(t)$. These strategies hold for any t. This completes the proof.

11 Appendix B - The transitions function

Equate utilities $U^A(x,t)$ to $U^B(x,t)$ to find the location of the indifferent buyer as a function of time

$\tilde{x}(t) = \frac{\alpha-\beta-(\theta-\gamma)\Delta+d(1-b)^2-\gamma t+2\gamma\int_0^t \tilde{x}(s)ds}{2d(1-b)}$

Define $D \equiv \alpha - \beta - (\theta - \gamma)\Delta + d(1-b)^2$ and $C \equiv 2d(1-b)$ to get

$$\tilde{x}(t) = \frac{D}{C} - \frac{\gamma}{C}t + \frac{2\gamma\int_0^t \tilde{x}(s)ds}{C} \tag{11.1}$$

Differentiate $\tilde{x}(t)$ wrt. t using Leibniz formula for differentiation of integrals with respect to a parameter appearing in the limits of integration

$\dot{x}(t) = -\frac{\gamma}{C} + \frac{2\gamma}{C}\tilde{x}(t)$, where $\frac{\partial\tilde{x}(t)}{\partial t} = \dot{x}(t)$

The general solution to this differential equation is

$$\tilde{x}(t) = We^{\frac{2\gamma}{C}t} + \frac{1}{2} \quad (11.2)$$

Where W is an unknown constant. The particular solution is obtained using the initial condition that firm B has accumulated no network immediately upon entry, and that firm A at this point has a network size of $\gamma\Delta$. Hence, at $t = 0$, equation 11.1 becomes

$\tilde{x}(0) = \frac{D}{C} - \frac{\gamma}{C}0 + \frac{2\gamma\int_0^0 \tilde{x}(s)ds}{C}$

where $\int_0^0 \tilde{x}(s)ds = 0$ by construction of the integral. Then $\tilde{x}(0) = \frac{D}{C}$. Equating this to equation 11.2 evaluated at $t = 0$ gives $W = [\frac{D}{C} - \frac{1}{2}]$. Inserting this W into equation 11.2 gives $\tilde{x}(t) = [\frac{D}{C} - \frac{1}{2}]e^{\frac{2\gamma}{C}t} + \frac{1}{2}$. Plugging back D and C and rewrite to get the "transition function"

$$\tilde{x}(t) = \left[\frac{\alpha - \beta - (\theta - \gamma)\Delta - d(1-b)b}{2d(1-b)}\right] e^{\frac{\gamma}{d(1-b)}t} + \frac{1}{2}$$

12 Appendix C - Refining the feasible region

Proof 2. First note that all Δ satisfying the condition $\beta + \theta\Delta < \alpha + \gamma\Delta - d$ belongs to case E and there exists no combination of Δ and b that can elevate firm B from a profit level of zero. For Δ satisfying $\alpha + \gamma\Delta - d \leq \beta + \theta\Delta \leq \alpha + \gamma\Delta$ the transition function guarantees the existence of a $b \in [0; 1]$ that results in at least an infinitesimal part of market demand, since $\tilde{x}(b) \in [0; 1]$ in this region. Consider the pair $(\bar{\Delta}, \bar{b})$ satisfying the property $\bar{b} = 1 - \tilde{x}(\bar{\Delta}, \bar{b})$ at time zero. This construction ensures that the firms' value functions intersect at the apex of value function B. Since transportation costs are quadratic, value functions decrease monotonically around the apex, and uniqueness of combination $(\bar{\Delta}, \bar{b})$ follows[17]. Take some $\hat{b} > \bar{b}$. It will now be shown that combination $(\bar{\Delta}, \bar{b})$ dominates all combinations $(\bar{\Delta}, \hat{b})$ by giving both higher markups and higher market shares for the same level of research time. It can be shown that buyers located at $x \geq (1 - \bar{b})$ has higher valuation for technology B when located at $\bar{b}$ over $\hat{b}$, and since maximal demand given $\bar{\Delta}$ equals $(1 - \bar{b})$ by construction, then it has been shown that $(\bar{\Delta}, \bar{b})$ dominates all $(\bar{\Delta}, \hat{b})$. Derive the equation characterizing the combinations of Δ and b defining the boundary of the feasible area by setting $\tilde{x}(\bar{\Delta}, \bar{b}, t = 0) = 1 - \bar{b}$

$$\bar{b}_{\max}(\Delta) = 1 - \left(\frac{\alpha - \beta - (\theta - \gamma)\bar{\Delta}}{d}\right)^{\frac{1}{2}}$$

Derivation of boundary in proposition 3. Let firm B pick the combination $(\bar{\Delta}, \bar{b})$ satisfying $\bar{b} = \frac{1 - \tilde{x}(\bar{\Delta}, \bar{b})}{2}$, where $\bar{b}$ is the location midway between $x = 1$ and $x = \tilde{x}(\bar{\Delta}, \bar{b})$ at $t = 0$. Given some $\bar{\Delta}$ satisfying $\alpha + \gamma\Delta - d \leq \beta + \theta\Delta \leq \alpha + \gamma\Delta + \frac{d}{4}$,

[17] If $b = 1$ and $\alpha + \gamma\Delta = \beta + \theta\Delta$ then both firms locate at the exact same point on the market with identical values. In this case there would be infinitely many intersections between firms' value functions. In this case, however, B makes zero markups, and it can do strictly better by choosing any other b, so this situation is disregarded.

calculate the value of technology B during the initial period of duopoly at $x = 1$, and equate it to the value function of firm A, to find the x that satisfies the equality. By construction, the consumer at this x is indifferent between networks A and B, namely $\tilde{x}(\bar{\Delta}, \bar{b}) = \left[\frac{\alpha-\beta-(\theta-\gamma)\bar{\Delta}+d(\bar{b})^2}{d}\right]^{\frac{1}{2}}$. Us the identity $\bar{b} \equiv \frac{1-\tilde{x}(\bar{\Delta},\bar{b})}{2}$ leading to the second degree polynomial $0 = 3\bar{b}^2 - 4\bar{b} - \frac{\alpha-\beta-(\theta-\gamma)\bar{\Delta}-d}{d}$. Solving for b the (only relevant) solution is

$$\bar{b}_{\min}(\Delta) = \frac{2}{3} - \left(\frac{1}{9} + \frac{\alpha - \beta - (\theta - \gamma)\Delta}{3d}\right)^{\frac{1}{2}}$$

13 References

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