"Game Theory"
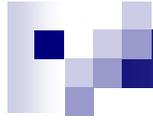
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Lecture notes 1

- What is game theory?

- Strategies

Course requirements

- One final exam: 100 percent of the total grade

Aim of the course

After completing the course, the student is expected to be able to:

- Account for the central parts of modern game theory and to explain how this method of analysis can be applied to the strategic interaction among economic agents.

- Explain the origins and functions of different economic and political economy structures with the help of game theory.

- Solve practical problems involving strategic interaction for economic agents and to provide an intuitive explanation for the methods used and the results.

What is game theory?

- Analysis of strategic interaction.
- Compare to e.g. consumer theory – no strategic uncertainty...
- GT modeling aims at "laying bare" the fundamentals of some situation involving strategic interaction.
- Remember! A model is not reality, but helps us to structure our thoughts and focus our attention on important mechanisms that shape reality.

What is game theory?

- A set of analytic tools to study situations involving strategic interactions.

- What is **strategic interaction**? One that involves figuring out the best course of action for me. It depends on what one or more other people do. I have to guess what other people or other person will do. If there is only one person in the game, all you're doing is deciding on the odds. When another person is involved, my actions depend on what I think you're going to do and your actions depend on what you think I'm going to do.

What is game theory?

- Game theory creates a model for these situations and predicts how these interactions are going to turn out.

- Strategic interactions involve small numbers of actors, countries, or firms.

Example: US campaign for president

- McCain's announcement of Vice President.

- There is discussion about when he should announce. Whenever he announces, that story will dominate that day. So McCain has at least one 24 hour news cycle in his pocket. McCain can take attention away from Obama through this announcement. Since the Republican Convention comes after Democratic Convention, McCain can see who Obama names and take that name into consideration. Based on who Obama chooses, McCain can choose an optimal VP candidate: that's his strategic interaction.

- What McCain is going to do depends on what Obama did.

Example: Iraq

- One of the major issues in campaign is Iraq. Do you name a fixed deadline for withdraw of troops? If you state a deadline, you affect the negotiations going on within Iraq. How hard factions within Iraq bargain depends on whether they think American troops are going to stay.

- One argument for setting deadline is that they were compromise more if they think troops will leave. The counterargument is that resurgent will hold off until after troops leave, then use their strength to further their ends.
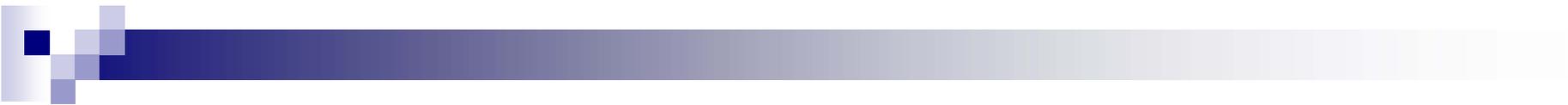
Example: Russia and Georgia

- Georgia takes action to try to reinvent Georgian control. The Russians push back very hard. You could argue that if the Georgian President had known how Russia would react, he wouldn't have made that decision. Why did he misjudge? Maybe he didn't think Russia would push back as hard or that NATO and the US would back him more. He is trying to guess what other actors will do in reaction to his action.

Example: Buyer, Seller negotiations (haggling)

- Strategies: low ball, show that you will walk away. The goal is the get the lowest possible price. If you are convinced that seller will stick to a price and the price worth it to you, you will stick to that price. But is the seller bluffing about his named price? This is strategic interaction.

Example: Ebay

- Auctions are a form of strategic interactions. Different institutional or strategic situations change incentives.
- There are different types of auctions.
- English Auction: prices start low, you bid up, winner pays the price they bid, no one else pays anything.
- Dutch Auction: start at a high price, go lower, until someone is willing to pay stated price.
- Second Price Auction: bid up the price, winner pays second highest price.
- All Pay Auction: the highest bid wins, but everyone pays the price they bid.
- How do these variations affect how everyone bids?
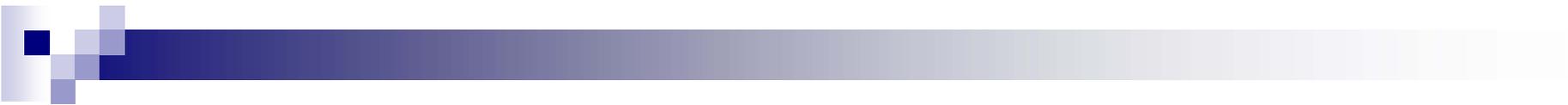
How to work with GT models

- Step 1: Identify an interesting problem that contains strategic interaction.

- Step 2: Try to capture the core "set up" of the problem in the model while using as few ingredients as possible - this is truly difficult.

- Step 3: Analyse the model. Then;
  - ☐ The analysis gives interesting results, or
  - ☐ The model tells you something is wrong... Back to step 2, and so on...

History of GT

- Breakthrough: John von Neumann and Oscar Morgenstern's book "Games and Economic Behavior" (1944).

- John Nash (nobel prize 1994) – wrote his famous paper in 1950, introducing the concept of Nash equilibrium – although he did not call it this way…
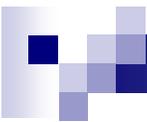
Benchmark case: agents are rational

- An agent chooses the best action according to his/her preferences that is available to her.
- Rational preferences are complete and transitive.
- Preferences are described by "payoff functions" (often utility functions).

Ordinal preferences:

- The payoff function u represents an agent's preferences if, for any action a or b in the the action set A,

$$u(a) > u(b) \text{ iff } a \succ b$$

Are people always rational?

- No, people sometimes make mistakes.

- Also, sometimes our theory falls short of predicting human behaviour.

- Does this mean we should
  a) Abandon our theory?
  b) Build on our theory by means of eg experiments and the use of psychological findings?

Game Theory

The study of multi-person decision problems (<u>few</u> agents).
The study of <u>rational behaviour</u> in situations involving <u>interdependency</u>.
A set of tools to formally describe situations of <u>strategic interaction</u>.

We can use Game Theory to model:

- Trading processes (auctions, bargaining)
- Strategic voting in committees (Bank of England)
- Competition in oligopolistic markets
- Competition/collusion among countries in choosing tariffs/trade policies/environmental standards
- Interaction between monetary authority and unions

What is a Game?

A set of rules specifying:

- Players
- Alternative choices/actions players choose from
- Order of play
- Outcomes and payoffs

Four basic classes of games:

- Non-cooperative vs. cooperative games
- Strategic (or normal form) games vs. extensive form games
- Games with complete vs. incomplete information

- Static vs. dynamic games

Game Theory provides **solution concepts** (notions of equilibrium):

- Dominance
- Nash Equilibrium
- Subgame-perfect Nash Equilibrium
- Bayesian Nash Equilibrium
- Perfect Bayesian Nash Equilibrium

Normal Form Representation of Games

- Players: $I=\{1,2,...,N\}$
- (pure) strategies: $\forall i \in I,\ s_i \in S_i$ (strategy space)
- Players choose their strategies simultaneously (or without knowledge of the others' strategies)
- Payoff functions (vNM utility functions)

$$u_i : S_1 \times S_2 \times...\times S_N \to \Re$$

In complete information games these are *common knowledge* among players.

***Def*.: *Strategy profile*:** $s=(s_1,..., s_N) \in S_1 \times S_2 \times...\times S_N$

Or alternatively…. $s=(s_i, s_{-i}) \in (S_i , S_{-i})$

…every s induces an **outcome** in the game ($u_i(s)$ for each i).

What constitutes a game on strategic (normal) form?

- A set of *players*,

- for each player, a set of *actions (sometimes referred to a strategies),*

- for each player, *preferences* over the set of action profiles (usually in the form of a *payoff function*),

- decisions are made simultaneously "once and for all".

Strategies

- A Strategic Form Game has the following:

- A set of alternatives (or actions) for each player.

- Ranking for each player for all of the possible ways the game could be played.
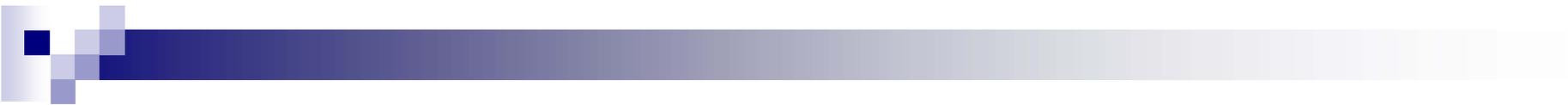
Strategies

- Game with three players

- Set of actions for each player.

- If I pair all of Player I's actions with Player II's and III's, pair all of Player II's actions with Player I's and Players III's, pair all of Player III's actions with Player I's and Player II's

- I have all the possible ways the game can be played.

- So if I give you a set of alternatives and rankings, then you have a game in strategic form.

Example: R&D Competition between Two Firms A and B

- Decisions: Invest, Not Invest
- Four possible ways the game can play out: (A invest, B not invest) (A invest, B invest) (A not invest, B not invest) (A not invest, B invest)
- The game is a competition between two firms to develop new technology.

Example: R&D Competition between Two Firms A and B

		FIRM B	
		Invest	Not Invest
FIRM A	Invest	2,2	4,1
	Not Invest	1,4	3,3

- This is the game in strategic form. Payoffs rank from 1 (the worst) to 4 (the best). Row players are the first number and Column players are the second number.
- Both Players' preferences rank as following:
- 4 = (I invest, you don't)
- 3 = (I don't, you don't)
- 2 = (I invest, you invest)
- 1 = (I don't, you invest)
- This is a game called the **Prisoner's Dilemma**.

Example: the original PD game

- Players: two thieves
- Actions: *Quiet* or *Fink*
- Preferences: See matrix

Thief 2

	Quiet	*Fink*
Thief 1 *Quiet*	-1, -1	-3, 0
Fink	0, -3	-2,-2

Example: Russia and EU (Game of Chicken)

Russia and EU are competing to see who will stand firm longer. If one compromises, the other one wins. If both stand firm, there is conflict.

- Two actions: Stand firm, back down

Example: Russia and EU (Game of Chicken)

		RUSSIA	
		Stand firm	Back down
EUROPEAN UNION	Stand firm	1,1	4,2
	Back down	2,4	3,3

Both Players' preferences rank as follows:
4 = (I stand firm, you back down)
3 = (I back down, you back down)
2 = (I stand firm, you stand firm)
1 = (I back down, you stand firm)

Other classic games...

Battle of the sexes

Partner 2

		Hockey	*Bach*
Partner 1	*Hockey*	2, 1	0, 0
	Bach	0, 0	1, 2

Matching Pennies

Player 2

		Head	*Tail*
Player 1	*Head*	1, -1	-1, 1
	Tail	-1, 1	1, -1

Stag hunt

Hunter 2

		Stag	*Hare*
Hunter 1	*Stag*	3, 3	0, 2
	Hare	2, 0	1, 1

Security dilemma

State 2

		Disarm	*Arm*
State 1	*Disarm*	3, 3	0, 2
	Arm	2, 0	1, 1