Microeconomic Theory: Problem set 4
Bayesian Games

Yves Zenou

Exercise 1: The Bayes' rule

One evening, individual A was knocked down by a taxi driver who run away. The informations gathered by the police show that, among the taxis that were out that night, 75% were green and 25% were blue (in this city, taxis can only be blue or green). However, an eyewitness has been found and he asserts that it was a blue taxi. Several experiments have been undertaken and they show that under identical circumstances to that of the accident, an eyewitness sees the correct color in 80% of the cases.

(1a) Without the information about the eyewitness, which company should the lawyers of individual A prosecute?

(1b) With the information about the eyewitness but without the experiments, which company should the lawyers of individual A prosecute?

(1c) With the informations about the eyewitness and the experiments, lawyers will update their beliefs by using the Bayes' rule.. In this case, which company should the lawyers of individual A prosecute?

(1d) Assume now that, among the taxis that were out that night, 85% were green and 15% were blue. With the informations about the eyewitness and the experiments, lawyers will update their beliefs by using the Bayes' rule. In this case, which company should the lawyers of individual A prosecute?

Exercise 2: A fight with imperfect information about strengths

Two people are involved in a dispute. Person 1 does not know whether person 2 is strong or weak; he assigns probability α to person 2's being strong. Person 2 is fully informed. Each person can either fight or yield. Each person's preferences are represented by the expected utility of a Bernoulli payoff function that assigns the payoff of 0 if he yields (regardless of the other person's action) and a payoff of 1 if he fights and his opponent yields. If both persons

fight then their payoffs are $(-1, 1)$ if person 2 is strong and $(1, -1)$ is person 2 is weak.

(2a) Formulate this situation as a Bayesian game

(2b) Find its Nash equilibria if $\alpha < 1/2$ and if $\alpha > 1/2$.

Exercise 3: Certification by a monopolist

Consider a seller of a good of quality v, with $v \in [a, b]$. The seller knows v, which is drawn from a uniform distribution on $[a, b]$. The seller has no production cost. The consumers are heterogenous and characterized by their willingness to pay for the good; this willingness is denoted by θ. The parameter θ is uniformly distributed on $[0, 1]$. The utility function of a consumer of type θ who buys a good of quality v is given by

$$U(\theta, v) = \theta\, v - p$$

where p is the price by for the acquisition of the good. If the consumer does not buy the good, his utility is equal to 0. Let us denote k the cost of certification. Certification consists in revealing in a credible way the value of v.

(3a) Calculate the demand function under *perfect information* for a good of quality v sold at a price p.

(3b) Calculate the profit function under *perfect information* of a seller of a good of quality v.

(3c) Find the Nash equilibrium of the Bayesian game (*imperfect information* on the quality v of the good) with certification by assuming that if type v is certified, then any type $v' > v$ is also certified and if type v is not certified, then any type $v' < v$ is also not certified