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VANITY AND CONGESTION: A STUDY OF RECIPROCAL EXTERNALITIES

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Vanity and congestion:
A study of reciprocal externalities

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Abstract

This paper models a private goods oligopoly market characterized by negative and reciprocal externalities. Although firms compete in prices and products are undifferentiated in equilibrium, the price-cost margin turns out to be positive. From a social perspective, the equilibrium price is higher than what is motivated by the negative externality. Hence, welfare can be improved by means of a price-ceiling. Finally, industries with high fixed costs would be expected to exhibit a high degree of concentration on the supply side and considerable price-cost margins.

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1. Introduction

The pleasure derived from consuming a good is sometimes affected by the consumption pattern of other people. Such consumption externalities may be of a one-way type, as when a living-room view is obstructed by neighboring houses, or reciprocal, as when driving a car reduces the street space available for other drivers, making driving less enjoyable. In this paper we study welfare aspects of negative and reciprocal externalities, of which congestion is a special case.

Negative externalities have long been a favorite topic of economic inquiry but the studies have normally abstracted from strategic behavior on the production side. For many applications this is a natural assumption to make, for instance, when studying optimal capacity and fee structures for publicly provided goods, like street space. [See e.g. Vickery (1969).] [2]

Reciprocal externalities are, however, likely to be important also on markets for private goods and services in that they affect the strategic interaction between firms. In the literature on clubs, Bertrand competition is shown not to ensure marginal cost pricing in the presence of congestion.[3] The reason is that increased demand result in more congestion which reduces consumers' willingness to pay for the good. Hence, price cuts tend to be unattractive. On the other hand, the socially efficient price is also higher than marginal cost to compensate for the negative externality. The question is whether prices are high enough or perhaps too high. Another example of reciprocal consumption externalities is given by markets for prestigious brand name goods where substantial output expansions may cause brand name debasement. For instance, if everyone wore Rolex watches, wearing one yourself would do little to enhance your prestige. [See e.g. Veblen (1899) and Hirsch (1976)]

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[2] Treatments of the more general problem of designing corrective taxes in the presence of externalities can be found in Diamond (1973) or Green and Sheshinski (1976). For the case of equal and reciprocal externalities Diamond shows that a uniform price, in excess of marginal cost by the value of the externality, permits the competitive equilibrium to be Pareto optimal.

[3] In Scotchmer (1985a), private clubs subject to congestion, like golf courses and sports clubs, are shown to choose membership fees above marginal cost in a Bertrand game. In contrast to our framework, consumer demand is assumed to be perfectly inelastic so the question of price efficiency cannot be addressed. This assumption is relaxed in Scotchmer (1985b) but instead firms choose a two-part tariff consisting of membership fees and user charges. In equilibrium, firms tend to set low charges in order to increase the consumer surplus captured by the membership fee. In this paper, it will become clear that competition in linear prices have quite different implications.

Historically, policy makers have been inclined to thouroghly regulate some congested markets. The transportation sector is perhaps the best example. In most countries practically all transportation services, (airlines, the trucking industry, railroads, taxis, etc), have been subject to extensive regulation, both in terms of price and entry. It is easy to see that congestion is a real issue on such markets. For instance, flights are less likely to be overbooked the smaller the number of other passengers and the availability of taxis decreases, i.e. waiting time increases, with per cab demand. Whether negative consumption externalities provide a rationale for regulatory intervention depends on the strength of the externalities relative the costs of regulation. Such costs would seem to depend on the context, availability of information etc, and optimal regulation is only used as a benchmark in the analysis.

The aim of this paper is to study price formation and economic efficiency on oligopolistic private goods markets characterized by reciprocal consumption externalities and price competition. The paper is organized as follows: In section 2, the basic model is presented and the price equilibrium is characterized. In section 3, welfare issues are examined while endogenous entry is discussed in section 4. Finally, in section 5, some concluding remarks are made.

2. The model

There are two types of goods. One is available in a number of different brands of identical intrinsic quality and the other is a composite good representing consumption of everything else. For the brand name good, consumer utility is assumed to be increasing in the amount consumed, but at a decreasing rate. Furthermore, brands can be differentiated in terms of exclusiveness (i.e. total sales) and utility is increasing in exclusiveness (decreasing in the volume of sales of a certain brand). The marginal utility from consuming the composite good is assumed to be approximately constant for reasonable ranges of income. The utility function of a consumer j purchasing brand name good i can be written;

$$U_{j,i}=U(y_{j,i},\ q_{j,i},\ Q_i) \qquad \textbf{(1)}$$

where $y_{j,i}$ and $q_{j,i}$ denote consumption of the composite good and of the brand name good respectively and Q_i represents total sales of brand i. By assumption, U_1, $U_2>0$, $U_3<0$, $U_{11}=0$ and $U_{22}<0$.

We assume that there is a continuum of identical consumers. The demand of an individual consumer patronizing firm i is derived by maximizing (1) with respect to $y_{j,i}$ and $q_{j,i}$ given that consumers correctly anticipate the equilibrium Q_i, and subject to the budget constraint $p_i q_{j,i} + y_{j,i} \leq I$, where the price of the composite good is normalized to unity. Furthermore, consumers do not perceive their own demand to influence the price-setting behavior of the firms. Nor do they take into account the effect of their own demand on exclusiveness.[4]

Let there be n firms each producing one brand name good, possibly differentiated by exclusiveness. Consumers, being utility maximizers, would never buy from a firm unless it is the best deal around. Thus, for given prices, market shares, m_i, will adjust so that customers are indifferent between buying from different firms in equilibrium.[5] Consequently, expressed in terms of indirect utility;

$$V(p_1, Q_1, I) = V(p_2, Q_2, I) = \ldots = V(p_n, Q_n, I) \tag{2}$$

which amounts to n-1 equations. The demand of a representative consumer patronizing firm i is derived using Roy's identity, yielding another n equations. Finally, the market shares add up to 1 so there are 2n equations altogether. The total number of consumers is normalized to one so the aggregate demand facing a firm equals individual demand times the market share,

$$Q_i = q_{j,i} m_i \tag{3}$$

which can be solved for explicitly using the 2n equations.

For the sake of tractability consumer preferences are assumed to have the simplest possible functional form consistent with the assumptions made. Consumer j's utility function is given by;

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[4]A notable exception to this, however, is Groucho Marx's famous remark about joining clubs.

[5]Consumers being indifferent between firms of course introduces the need for some invisible hand guiding demand so that indifference actually holds. If, for example prices, are equal and all "indifferent" consumers happen to patronize the same firm, they would not any longer be indifferent but rather realizing that they all made a mistake.

$$U_{j,i} = y_{j,i} + (1-\alpha q_{j,i})q_{j,i} - \beta Q_i q_{j,i} \tag{4}$$

The first term on the right-hand-side is consumer j's consumption of the composite good, $y_{j,i}$ while the second term gives the quadratic gross utility from consuming the differentiated good, $q_{j,i}$. The last term reflects individual j's disutility of the consumption of others', Q_i, which is assumed to increase in his own consumption of variety i. Hence marginal utility and individual demand depend on exclusiveness. The decrease in utility of additional consumption is parameterized by α while β measures the impact of the negative externality.[6] The individual demand and the indirect utility function are given by;

$$q_{j,i} = \frac{1-p_i-\beta Q_i}{2\alpha} \tag{5}$$

$$V(p_i, Q_i, I) = \frac{(1-p_i-\beta Q_i)^2}{4\alpha} + I \tag{6}$$

so in this case expression (2) implies;

$$p_1 + \beta Q_1 = p_2 + \beta Q_2 = \dots = p_n + \beta Q_n \tag{7}$$

Let **p** be the vector of prices charged by the firms. The marginal willingness to pay for one unit is at most one and thus $p_i \leq 1$, so **p** is a point in the price simplex $P=[0,1]^n$. The demand facing firm i can now be expressed as a function of **p**.

Lemma 1: The aggregate demand facing firm i is:

$$Q_i = \frac{2\alpha(\sum_{j\neq i} p_j-(n-1)p_i) + \beta(1-p_i)}{\beta(2\alpha n + \beta)}$$

Proof: In appendix.

Firms maximize profits with respect to price taking into account the strategic

[6]For positive externalities, $\beta<0$, the equal utility condition, expression (7), will not hold (or be unstable) and there is a tendency towards natural monopolies. The strategic implications of positive externalities are discussed in the literature on networks. See for example Katz and Shapiro (1985) and (1986).

interdependence between price choices. Consequently, the appropriate equilibrium concept is the Nash equilibrium. Marginal production costs, c_i, are assumed to be constant and strictly less than one. The profit function of firm i is

$$\pi_i = (p_i - c_i)Q_i \tag{8}$$

Having characterized consumer behavior and firm behavior, the next step is to characterize the market equilibrium. Substituting the demand of firm i into its profit function and maximizing with respect to p_i, taking the other firms' prices as given, yields the best response function, φ_i, of firm i.[7]

$$\varphi_i(p_{-i}) = \frac{1}{2}\left[c_i + \frac{2\alpha\sum_{j\neq i} p_j + \beta}{2\alpha(n-1) + \beta}\right] \tag{9}$$

The reaction functions are linear and upward sloping in the competitors' prices implying strategic complementarity. Furthermore, cost differences affect the intercepts, but not the slopes, i.e. the responsiveness to other firms' actions. In figure 1, which illustrates the duopoly case, $c_1 > c_2$ making $p_1 > p_2$ in equilibrium.

Figure 1 about here.

If ß is large relative to α, the influence of the other firms' prices is very limited and the optimal price will be close to $(c_i + 1)/2$, i.e the price that would be chosen by a profit maximizing monopolist. This, however, does not mean that extreme congestion is likely to be desirable from the firms' perspective. On the contrary, if ß approaches infinity, consumers' valuation of the good is reduced to such extent that firm demand goes to zero.

For a duopoly market, the existence of a unique and symmetric price equilibrium is intuitively clear and it can easily be established also in the n-firm case.

Proposition 1: There exists a unique equilibrium.

Proof: First, the price simplex, P, is a non-empty, compact and convex set. Furthermore,

[7]Assuming Cournot competition does not change the analysis much. Equilibrium prices would then be somewhat higher but qualitatively, all results go through.

the vector valued best response function, $\Phi(\mathbf{p})$, is linear and thus u.h.c. and convex. Finally, it can easily be shown that $\Phi(\mathbf{p}) \subset P$ and thus Kakutani's theorem guarantees a fixpoint. Uniqueness then follows directly since $\Phi(\mathbf{p})$ is a contraction. □

Corollary 1: If $c_i = c$ for all i, then the equilibrium will be symmetric with $p_1=p_2=\ldots=p^*$.

$$p^* = \frac{2\alpha(n-1)c+\beta(c+1)}{2\alpha(n-1)+2\beta}$$

Proof: Identical costs yield symmetric reaction functions ensuring a symmetric equilibrium. Solving (9) for $p_i=p_j$ yields p^*. □

Proposition 2: The equilibrium price, p, is increasing in ß and for ß = 0, p*= c. When ß approaches infinity, p* approaches the monopolistic price, (c+1)/2.*

Proof: Follows from differentiating p*. □

Hence, equilibrium prices are above marginal cost despite that firms compete in prices and products are undifferentiated in equilibrium, costs being equal. The undercutting strategy becomes unattractive since output expansions affect quality negatively. Technically speaking, in a standard Bertrand game, firm demand and profits are discontinuous at the lowest price charged by the competitors. This discontinuity is smoothed out by reciprocal externalities allowing a price differential to be compensated for by differences in quality. Hence, it is not possible to capture the entire market by undercutting the rival slightly. If ß is small, the situation is nevertheless very similar to the standard Bertrand game with prices close to marginal cost and basically no profits. This suggests that there may be incentives for firms to deliberately try to influence the impact of congestion on consumer utility. [8]

Proposition 3: The equilibrium price, p, is decreasing in n and it approaches c as n approaches infinity.*

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[8]If, for example, transportation firms could commit to lower their capacity, it could be interpreted as an increase in ß, possibly leading to higher profits.These issues are discussed more thoroughly in Häckner and Nyberg (1991).

Proof: Follows from differentiating p*. □

Not surprisingly, an increase in the number of firms induces a more competitive market structure leading to lower prices.

3. Social welfare implications

Consumers do not take into account the negative externality they inflict on their fellow consumers when buying an exclusive product making it less exclusive and hence less desirable for others. Thus, the equilibrium consumption of exclusive items, given a certain price, can be expected to be too high from the consumers' point of view. Indeed, this can easily be demonstrated to be the case. As shown above, the externality affects the strategic interaction between producers generating an equilibrium price that is above marginal cost. The question is, however, whether this price is sufficiently high to compensate for the externality or whether it is actually too high from a social point of view.

To facilitate comparative static comparisons we examine a symmetric price equilibrium, where individuals choose the same q. Since consumers are identical social welfare can be measured by the utility of the representative individual minus the per capita cost of production.

$$W = y + (1-\alpha q)q - \frac{\beta q^2}{n} - cq \qquad (10)$$

Differentiating W with respect to q gives the socially optimal individual consumption;

$$q^{**} = \frac{n(1-c)}{2\alpha n+2\beta} \qquad (11)$$

Hence, the more severe the externality, the lower is the socially optimal consumption level. Moreover, this can be seen to be higher than the equilibrium quantity, derived by inserting the equilibrium price (Corollary 1) into aggregate demand. It thereby follows that the price maximizing social welfare, p^{**}, is lower than the equilibrium price.

Proposition 4 The socially optimal consumption level can be always be obtained by means *of a price-ceiling, the ceiling being:*

$$p^{**} = \frac{2\alpha cn+\beta(c+1)}{2\alpha n+2\beta}$$

Proof: Solving for the price that makes individual demand equal to q^{**} yields p^{**}. The difference between the equilibrium price, p^*, and p^{**}, is strictly positive for all $\beta>0$. □

Note that p^{**} approaches marginal cost as ß approaches zero. This is true for p^* as well so for an arbitrarily small ß, p^* will be arbitrarily close to p^{**} yielding an arbitrarily small welfare loss. It is not surprising that a negative consumption externality raises optimal prices above marginal cost. The important social welfare conclusion is that the anti-competitive feature of the market, also caused by the externality, will be too strong, thus motivating a price ceiling.[9]

Another interesting conclusion concerns empirical estimates of consumer surplus in the presence of negative externalities. Comparing (11) with the actual demand function of lemma 1, it is clear that the area below the demand function will be larger than the true consumer surplus. Consequently, any conventional method to estimate consumer surplus will yield biased results.

4. Entry

Until now, the number of firms has been exogenous. In absence of fixed costs or other entry barriers a free entry equilibrium would be characterized by an infinite number of firms each producing an infinitely small amount. Prices would be driven down to marginal cost, despite the externality, completely eroding firm profits. However, on many markets entry may involve incurring substantial initial costs. For example, in the transportation sector, large fixed investments in capacity, as well as in marketing, are generally needed when entering

[9]Of course, policy implications of this kind makes most sense in cases of physical externalities such as those of competing transportation systems. It seems difficult to argue convincingly for regulating the prices of Cartier and Rolex watches.

the market.[10] We therefore introduce a fixed cost, K, keeping the assumption of equal marginal costs across firms.

Proposition 5: Firm profits increase in market concentration and decrease in industry cost level.

Proof: See appendix.

Hence, the larger the fixed cost and the larger the marginal cost, the smaller the number firms that could enter profitably.

Proposition 6: Firm profits are quasiconcave in β and increases (decreases) in β for low (high) values of β.

Proof: Follows from simple differentiation of the profit function.

Thus, given a certain K, the equilibrium number of firms will be largest for some intermediate value of ß. The explanation is that for low values of ß, the market will be fairly competitive implying low profits and no opportunity for a large number of firms to cover their fixed costs. On the other hand, if ß is large, aggregate demand will be very low since the marginal utility from consuming the good will be reduced to a great extent. Hence, only a small number of firms would be able to enter profitably.

We may conclude that if fixed costs are not negligible it is reasonable to expect a small number of incumbent firms charging prices above marginal costs without being threatened by entry.

5. Conclusions

The introduction of consumption externalities into a standard Bertrand oligopoly model has

[10] In markets for exclusive brand name goods, marketing expenses are often very large when new products are introduced. If the simplifying assumption is made that marketing has only an informational value, and does not influence preferences directly, marketing may readily be thought of as a sunk cost.

several important implications. First, as would be expected, it induces over-consumption from the consumers' point of view, at any given price. Second, it changes the incentives of firms dampening competition. Firms may charge prices well above marginal cost despite Bertrand competition and despite goods being homogenous in equilibrium. In fact, if the externality is substantial, equilibrium prices may be close to the monopoly level. The anti-competitive effect dominates the over-consumption effect which translates into a market price that is too high from a social point of view. Thus, welfare can be improved by means of a price-ceiling, which should be noted is commonly practiced in markets for transportation services. Furthermore, we may note that any standard estimate of consumer surplus based on observed demand functions will be positively biased in the presence of negative externalities.

These conclusions are of course based on a specific parametrization of the utility function. However, in most cases linear demand functions and linear "crowding" costs are probably good approximations of real conditions.

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Appendix

Proof of Lemma 1: In equilibrium, equation (7) must hold. Using (3) and (5) we then have;

$$p_1 + \frac{\beta m_1(1-p_1)}{2\alpha+\beta m_1} = p_2 + \frac{\beta m_2(1-p_2)}{2\alpha+\beta m_2} = \ldots = p_n + \frac{\beta m_n(1-p_n)}{2\alpha+\beta m_n}$$

which implies that,

$$\frac{2\alpha p_1+\beta m_1}{2\alpha+\beta m_1} = \frac{2\alpha p_2+\beta m_2}{2\alpha+\beta m_2} = \ldots = \frac{2\alpha p_n+\beta m_n}{2\alpha+\beta m_n} = k$$

Thus, the number of customers buying from i can be written on the form;

$$m_i = \frac{2\alpha(k-p_i)}{\beta(1-k)}$$

which summing over all i yields an expression for k. Substituting for k results in,

$$m_i = \frac{\beta(1-p_i)-2\alpha(np_i-\sum_{j=1}^{n} p_j)}{\beta(n-\sum_{j=1}^{n} p_j)}$$

Recalling equations (3) and (5) and substituting for m_i yields the desired result. □

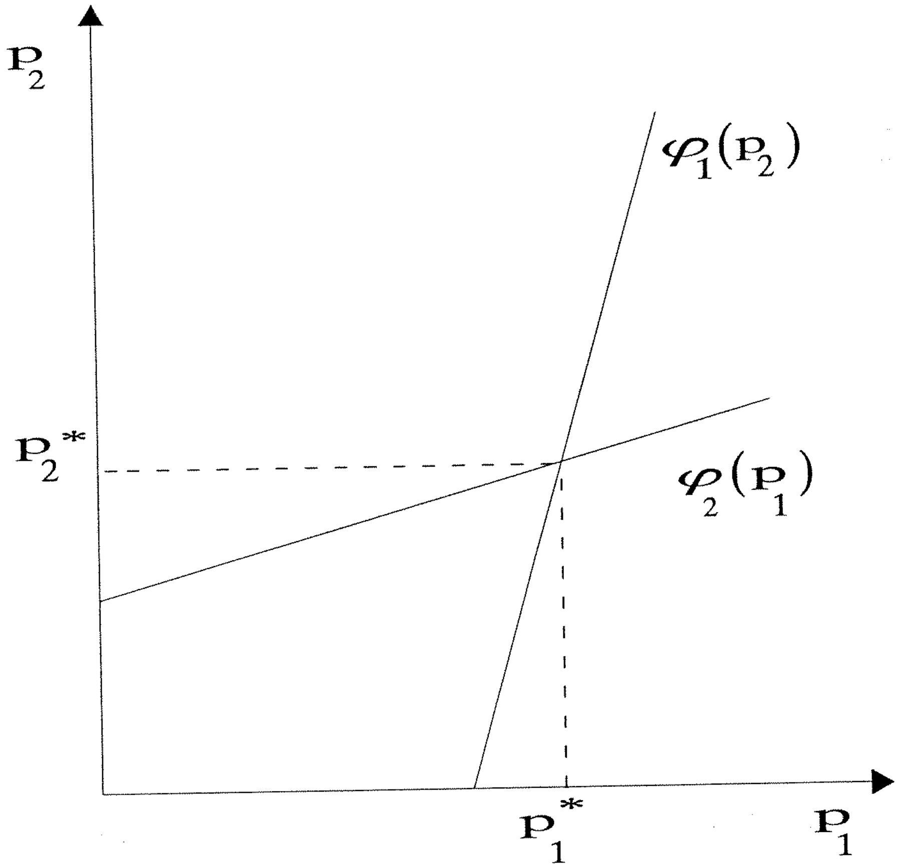
Proof of Proposition 5: Differentiating the profit function yields

$$\frac{\partial\pi}{\partial n} = \frac{-\alpha\beta(1-c)^2[2\alpha^2(n(2n-3)+1)+4\alpha\beta(n-1)+\beta^2]}{2(2\alpha n+\beta)^2(\alpha(n-1)+\beta)^3)} < 0$$

and

$$\frac{\partial\pi}{\partial c} = \frac{-\beta(1-c)(2\alpha(n-1)+\beta)}{2(2\alpha n+\beta)(\alpha(n-1)+\beta)^2} < 0$$

which establishes the proposition. □



p_2
$\varphi_1(p_2)$
p_2^*
$\varphi_2(p_1)$
p_1^*
p_1