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COST REDUCING INNOVATIONS AND ENDOGENOUS SPILLOVERS

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Cost Reducing Innovations and Endogenous Spillovers

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ABSTRACT

R & D consortia, know-how trading and geographic agglomeration of firms are readily observable phenomena in many industries. The prevalence of such arrangements merits an inquiry into what their economic motivation might be. This paper analyzes the incentives for firms in an industry facing Cournot-competition in the product market to share cost-reducing information, and the determinants of these incentives. Not surprisingly, accumulated R & D capital strongly influences a firm's willingness to share its knowledge. Perhaps less obvious is the very strong tendency to cluster. In analyzing the impact on market structure both spillovers and investments in R & D are determined simultaneously. It turns out that there are two types of feasible equilibria. First, all firms may cluster together and invest little in R & D. Second, the alternative market structure features two coexisting behaviors. Some firms agglomerate whereas others seek isolation.

Moreover, when both R & D investments and the degree of interaction are decided on simultaneously in the first stage, before the Cournot game, firms will always cluster. Even though the incentives to cluster prove to be strong there are two feasible types of equilibria when firms are faced with a simultaneous choice of interaction level and R & D investments. First, all firms may cluster together and second some firms may opt for high investments and isolation while the others join the cluster and spend significantly less.

2. The Model

Consider a model with n firms, competing in an oligopolistic product market, whose costs depend on their technological know-how. The technological level is measured as accumulated R & D, which simply is the aggregate of knowledge gained by inhouse research and knowledge that has spilled over from competitors. To make the problem more tractable I assume Cournot-competition in the product market with linear demand, and the effect of cost-reducing R & D on marginal cost is assumed to take on a specific functional form.[1] Let the firms' costs be a function of previous investments in R & D plus spillovers, z_i. The unit cost of an individual firm is given by

$$c_i = e^{-r[z_i]}, \tag{1}$$

where c_i is decreasing in z_i and the parameter r determines the efficiency, or cost, of R & D investments.

In order to study endogenous determination of spillovers the Spence model needs to be modified to allow for individual choices of closeness to other firms in terms of the intensity of information exchange between firms. The parameter, θ, representing the spillover propensity in the industry as a whole is replaced with individual parameters for

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[1] In Spence's formulation costs are given by $c_i(t)=F(z_i(t))$, which is decreasing in z_i, and $z_i=M_i+\theta\sum_{j\neq i} M_j$.

each firm, $\theta_i \in [0,1]$, which measure both the likelihood of information inflow as well as information leakage. That is, spillovers from one firm to another are assumed to ge given by the product of the involved firms degree of openness times the accumulated R & D capital of the former firm. Furthermore, it is assumed that a firm can only choose its degree of openness towards the market and not vis-à-vis specific firms. Thus, this analysis is neither addressing unilateral information gathering decisions like the appropriate budget for industrial espionage nor bilateral agreements on information-sharing. The proportion of firm i's R & D that leaks out is θ_i and the fraction that in turn is absorbed by firm j is θ_j. The spillover from i to j is thus $\theta_i\theta_j$ times firm i's investments, M_i. The accumulated R & D capital of firm i is then

$$z_i = M_i + \theta_i \sum_{j \neq i} \theta_j M_j \, . \tag{2}$$

This can be thought of as it being difficult to unilaterally extract more information from competitors about their corporate secrets without giving them something in return. We could imagine firms deciding on the number of conferences on which to send their very perceptive but notoriously blabbing engineers.

After the investments and spillovers in the first period, firms compete in the product market. In an undifferentiated oligopolistic market with Cournot competition the market price is determined by the aggregate production of the firms. Assuming linear demand, the market price is given by the inverse demand function

$$P = \alpha - Q, \;\; Q = \sum_{i=1}^{n} q_i \, , \tag{3}$$

where Q is the aggregate quantity supplied by the n firms. The profit for an individual firm, i, is

$$\pi_i = P(Q)q_i - C_i(q_i), \tag{4}$$

The firms' costs are assumed to be linear in output and, thus, consist of a constant marginal cost component, c_i, and a fixed cost representing previous R & D investments, i.e., M_i,

$$C_i = c_i q_i + M_i, \tag{5}$$

Each firm makes Cournot-conjectures about the behavior of the other firms and thus choose q_i so as to maximize (4) taking other firms' strategies as given. The optimal production quantity for firm i is then given by

$$q_i = \alpha - c_i - Q, \tag{6}$$

thus, summing q_i over i and rearranging

$$Q = \frac{n\alpha - C}{1+n}, \tag{7}$$

where $C=\sum_{i=1}^{n} c_i$. Hence, in equilibrium the profit for the individual firm is given by

$$\pi_i = \left[\frac{\alpha + C}{1+n} - c_i\right]^2 - M_i \,. \tag{8}$$

In the analysis above, firms choose θ and M in the first stage so as to maximize the profit in the Cournot game played in the subsequent period, i.e. expression (8). Throughout the paper it is assumed that in the absence of spillovers it will be profitable for the firm to undertake cost-reducing investments. The motivation is that it could seem rather bizarre to study spillovers in a context where there is nothing to spill over. The equilibrium concept is that of perfect Nash equilibria.

3. Equilibrium analysis

In this section it is shown that when firms have roughly similar stocks of R & D they gain by sharing this knowledge since a general reduction in costs benefits all firms in the subsequent Cournot-game, which is a positive-sum game in cost reductions.

Furthermore it is shown that there may exist equilibria where some firms choose isolation in conjunction with high R & D expenditures while other firms cluster together and spend significantly less. First, the payoffs corresponding to isolation and interaction are shown to be strictly decreasing and strictly increasing in the proportion of firm choosing isolation. Second, if there is such an equilibrium it can be shown that it is stable in a sense described below. The accumulated investment vector, M, is initially assumed to be exogenously given. Firm i chooses its level of interaction $\theta_i \in [0,1]$ as a best response to the choices of other firms, θ_{-i} and M. In fact, it turns out that there are no feasible equilibria involving intermediate values of θ.

Proposition 1: All firms will choose either $\theta=0$ or $\theta=1$.
Proof: In Appendix.

This raises the question of which firms will select which level of θ? Intuitively one would expect firms with high R & D spendings to be less willing to share their knowledge than firms with modest investments in R & D. Letting B denote the set of firms choosing $\theta = 1$ we have that

Corollary 1: $M_i > \sum_{j \in B} M_j$ *is a necessary condition for $\theta_i = 0$ being optimal for firm i.*

Proof: In Appendix.

This confirms the intuition that firms with a history of low investments in R & D are more likely to locate in a cluster but also indicates that there is a very strong general tendency for firms to cluster in the sense that large differences in accumulated R & D investments are necessary to make it attractive for any firm to seek isolation. It is also worth noting that in a Cournot-market firms with low costs, which in this model reflects high R & D spendings, have a large market share. This, implies that small firms would be expected to be clustered whereas really big firms might consider keeping to themselves.

The more general problem of simultaneous determination of θ and M is facilitated by proposition 1, ensuring that all firms will choose θ to equal either zero or one regardless of the vector M. Circumstances like market demand, cost of investment and market concentration may induce many firms to cluster together and choose low investment level but there will no equilibria featuring a wide spectra of investment levels; in fact, there will be at most two investment levels.

Lemma 1: Firms with the same degree of openness, θ, will also choose the same level of investment, M.
Proof: In Appendix

This implies that there can be no equilibrium involving only one firm, or no firm at all, selecting $\theta=1$. The reason is that if all firms settle for $\theta=0$ and thus also choose the same investmentlevel then any firm would be better off sharing its knowledge one of the other firms. Although this is a Nash equilibrium, it does not satisfy perfection. The slightest probability that any of the other firms would select $\theta=1$ makes $\theta=1$ a dominant strategy for the firm. Thus, in equilibrium either all firms share their information or there is an asymmetric equilibrium where firms adopt different strategies in terms of openness and investments.

In an non-atomistic population of identical firms an asymmetric equilibrium arises when the payoffs corresponding to the strategies are equalized for some proportion of the firms selecting each strategy. The number of firms in this model is however a discrete variable (to assume a continuum would have strong undesirable ramifications on the Cournot game) and the payoffs associated with different strategies may never be exactly equalized. Even though the payoffs accruing to the players subscribing to one of the strategies may be higher than that of the players choosing the alternative strategy when populations are finite this does not in any way preclude asymmetric equilibria. It is sufficient that no player would gain by altering his or her strategy choice.

Let the proportion of firms that choose isolation, $\theta=0$, be denoted by p, a rational number between 0 and 1. Using Lemma 1 the payoffs associated with choosing $\theta=0$ and $M_{\theta=0}$ or $\theta=1$ and $M_{\theta=1}$, where the investment levels $M_{\theta=0}$ and $M_{\theta=1}$ are chosen optimally given the choices of the other players, are given by

$$\pi_{\theta=0} = \frac{1}{(1+n)^2}[\alpha + n(pc_{\theta=0}+(1-p)c_{\theta=1}) - (1+n)c_{\theta=0}]^2 - M_{\theta=0}$$
$$\pi_{\theta=1} = \frac{1}{(1+n)^2}[\alpha + n(pc_{\theta=0}+(1-p)c_{\theta=1}) - (1+n)c_{\theta=1}]^2 - M_{\theta=1} \quad (10)$$

where, as was shown earlier, $M_{\theta=0} > \Sigma M_{\theta=1}$ implying that $c_{\theta=0} < c_{\theta=1}$ and $f_{\theta=0} > f_{\theta=1}$. The equilibrium values of M, and thus of c, depend on p. If all firms but one prefer seclusion then that firm is indifferent between $\theta=0$ and $\theta=1$, that is, $\pi_{\theta=0}((n-1)/n) = \pi_{\theta=1}((n-1)/n) = \pi_{\theta=0}(1)$.[2] Furthermore, the payoffs turn out to be strictly monotonic in p up to the point where p equals (n-2)/n.

Lemma 2: $\pi_{\theta=0}\left(\frac{x}{n}\right) > \pi_{\theta=0}\left(\frac{x+1}{n}\right)$ *and* $\pi_{\theta=1}\left(\frac{x-1}{n}\right) < \pi_{\theta=1}\left(\frac{x}{n}\right)$ $x=1,2,\ldots,n-2$.

Proof: In Appendix.

An increase in the proportion of firms pursuing "independent" R & D results in a decrease in these firms' profit level and an increase in the payoffs of the firms sharing their knowledge. Thus, incumbents in a cluster are better off the smaller the proportion of firms in the industry inhabiting the cluster.

Undoubtedly, investments are more efficiently used in a cluster and it seems likely that, despite the suboptimally low investment level, the profit of a firm in a market where all firms interact should be greater than that of a firm i an environment where all firms have chosen isolation. Not surprisingly it can be shown that

Lemma 3: $\pi_{\theta=1}(p=0) > \pi_{\theta=0}(p=1)$.

Proof: In Appendix.

This also serves to rule out the possibility of equilibria where all firms choose $\theta=0$ through a weak dominance argument. (If all firms but one, say firm i, set $\theta=0$ then firm i is indifferent with respect to different θs. However if some firm happens to deviate firm i would have been better off setting $\theta=1$ since $\pi_{\theta=1}$ is increasing in p.) Lemma 2 and 3 provide the necessary foundation for establishing the conditions for existence of an asymmetric equilibrium.

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[2]For obvious reasons $\pi_{\theta=0}(0)$ and $\pi_{\theta=1}(1)$ are not defined.

conjecture that the propensity to cluster increases with industry demand and market concentration, the fewer firms there are in the industry the more attractive is cooperation.)

5. Conclusions

This paper has been concerned with the endogenous determination of R & D spillovers in an oligopolistic market where R & D lowers production costs. The main results of this paper are that the incentive to cluster is quite strong in the sense that large differences in accumulated R & D and thereby in marginal cost, are necessary to make it attractive for any firm to eschew interaction. In a Cournot oligopoly differences in marginal cost correspond to differences in market share. Thus, it is only the really big firms that may consider it worthwhile to conduct R & D in seclusion.

When both R & D investments and spillovers are determined simultaneously two types of outcomes are feasible. First, all firms may find it optimal to agglomerate and invest moderately in R & D. Second, if it would be profitable for any of the firms in the cluster to deviate and to undertake a more ambitious R & D program then this gives rise to an asymmetric equilibrium where most firms remain clustered while some firms pursue their R & D ventures in isolation.

An interesting extention of the model could be to allow for the possibility of limiting entry to a cluster and consequently also for the formation of several cluster. These, would be differentiated with respect to level of investment since, by the same logic as in the paper, clusters at similar technology-levels would have an incentive to merge.

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APPENDIX (Tentative - not thoroughly checked)

Proof of Proposition 1: First it is shown that the first order condition for an interior equilibrium cannot be simultanously satisfied for two or more firms. Second, points such that the first order condition is satisfied for a single firm are shown to be local minimia. Hence, firms choose either $\theta=0$ or $\theta=1$.

Let A be the set of firms choosing interior solutions, B the set of firms opting for $\theta_j=1$ and C the firms setting $\theta_j=0$. The first-order condition for an interior solution for a firm i is given by

$$\frac{\partial \pi_i}{\partial \theta_i} = -\frac{2r}{(1+n)^2}(\alpha+C_{-i}-nc_i)M_i c_i \sum_{j\neq i}\theta_j\left(c_{ji}-n\frac{M_j}{M_i}\right) = \Phi_i \sum_{j\neq i}\theta_j\left(c_{ji}-n\frac{M_j}{M_i}\right) = 0 \tag{A1}$$

where $c_{ji} = c_j/c_i$. Thus, the FOC requires the last factor to equal zero. Dividing out Φ_i and rearranging yields,

$$M_i\Big(n\theta_i+\sum_{j\neq i}\theta_j c_{j,i}\Big) - n\sum_{j=1}^{n}\theta_j M_j = 0 \tag{A2}$$

where the last term is equal for all firms in A. Using the FOC of two firms in A, say firms 1 and 2, noting that $c_{j,i} = c_{j,k}\, c_{k,i}$ we arrive at;

$$\left(1-\frac{M_2}{M_1}c_{1,2}\right)\sum_{j\neq 1,2}\theta_j c_{j,1} + \theta_1\left(n-\frac{M_2}{M_1}c_{1,2}\right) + \theta_2\left(c_{2,1}-n\frac{M_2}{M_1}\right) = 0 \tag{A3}$$

Suppose the last term of (A3) is greater than or equal to zero then, recalling that $c_{2,1} = 1/c_{1,2}$, the two preceding terms must clearly be greater than zero which contradicts the equality. Thus, the last term must be negative. This holds in all pairwise comparisons between firms in A. Hence, there cannot exist an interior solution including more than one firm.

Now, if all firms but firm i settle for either $\theta=0$ or $\theta=1$ (A1) can be written as

$$\Phi_i \sum_{j\in B}\left(c_{j,i}-n\frac{M_j}{M_i}\right) = \Phi_i\left(\sum_{j\in B} e^{r[M_i-\sum_{k\in B, k\neq i} M_k]} - n\frac{\sum_{j\in Bi} M_j}{M_i}\right) \tag{A4}$$

To satisfy (A1) M_i must be greater than $\sum_{j\in B} M_j$ which however violates the second-order condition since,

$$\frac{\partial^2 \pi_i}{\partial \theta_i^2} = \Phi M_i\left[\frac{\sum_{j\neq i}\theta_j M_j}{M_i}\sum_{j\neq i}\theta_j c_{j,i} - \sum_{j\neq i}\theta_j^2 c_{j,i}\right] = \Phi M_i \sum_{j\in B} c_{j,i}\left[\frac{\sum_{j\in B} M_j}{M_i} - 1\right] > 0 \tag{A5}$$

Thus in all equilibria firms will either choose $\theta=0$ or $\theta=1$ regardless of the initial distribution of M's. □

Proof of Corollary 1: Recalling equation (A4) and noting that the number of firms in B is less than of equal to n it is clear that if $M_i \leq \sum_{j\neq i} M_j$ then $\frac{\partial \pi_i}{\partial \theta_i} > 0$ for all θ_i which implies that $\theta_i=1$. □

Proof of Lemma 1: The FOC for an optimal investment level for firm i, M_i, is given by

$$\frac{\partial \pi_i}{\partial M_i} = \frac{2r}{(1+n)^2}\left(\alpha+\sum_{j=1}^{n} c_j-(1+n)c_i\right)\left(nc_i-\theta_i\sum_{j\neq i}\theta_j c_j\right) - 1 = 0 \tag{A6}$$

which is a quadratic expression in c_i yielding two potential solutions. Since the coefficient of the quadratic term is negative only the lower value of c_i is compatible with a stable equilibrium featuring positive investments. The FOC for a firm choosing either $\theta=1$ or $\theta=0$ can be rearranged so that

$$\begin{aligned} &c_i^2 - \left(\frac{\alpha+\sum_{j=1}^{n} c_j}{1+n} + \frac{\sum_{j\in B} c_j}{n}\right)c_i + \left(\frac{1+n}{2nr} + \frac{\alpha+\sum_{j=1}^{n} c_j}{1+n}\frac{\sum_{j\in B} c_j}{n}\right) = 0 \\ &c_i^2 - \left(\frac{\alpha+\sum_{j=1}^{n} c_j}{1+n}\right)c_i + \frac{1+n}{2nr} = 0 \end{aligned} \tag{A7}$$

where, for every solution candidate c = $(c_1, c_2,\dots ,c_n)$, the sums in the brackets are the same for all i. In any equilibrium all firms with the same θ face the same FOC and will consequently choose the same c. Thus, for any vector M and corresponding vector c there can only be one $c_{\theta=1}$ and one $c_{\theta=0}$ that satisfy (A7) and are compatible with an equilibrium. □

Proof of Lemma 2: Define the functions, $f_{\theta=0}$ and $f_{\theta=1}$, as the continuous, in p, counterparts of $\pi_{\theta=0}$ and $\pi_{\theta=1}$ in expression (10) and let $s_{\theta=0}$ and $s_{\theta=1}$ be the solutions to a continuous version of (A7). Thus, in admissible point the function values coincide, i.e. $f_{\theta=0}(x/n) = \pi_{\theta=0}(x/n)$ and $f_{\theta=0}((x+1)/n) = \pi_{\theta=0}((x+1)/n)$ for $x=1,2,\dots ,n-2$.

If $f_{\theta=0}$ can be shown to be strictly decreasing in p for all $p\in[1/n, (n-2)n]$ then it follows that $f_{\theta=0}(x/n) > f_{\theta=0}((x+1)/n)$ and strict monotonicity of $\pi_{\theta=0}$ is ensured. The same argument is used for $f_{\theta=1}$.

Differentiating the f functions with respect to p yields

$$\begin{aligned} \frac{df_{\theta=0}}{dp} &= \frac{\partial f_{\theta=0}}{\partial p} + \frac{\partial f_{\theta=0}}{\partial s_{\theta=0}}\frac{\partial s_{\theta=0}}{\partial p} + \frac{\partial f_{\theta=0}}{\partial s_{\theta=1}}\frac{\partial s_{\theta=1}}{\partial p} \\ \frac{df_{\theta=1}}{dp} &= \frac{\partial f_{\theta=1}}{\partial p} + \frac{\partial f_{\theta=1}}{\partial s_{\theta=0}}\frac{\partial s_{\theta=0}}{\partial p} + \frac{\partial f_{\theta=1}}{\partial s_{\theta=1}}\frac{\partial s_{\theta=1}}{\partial p} \end{aligned} \tag{A8}$$

where the partial derivatives of f are easily derived. Differentiating the "continuous (A7)" with respect to $s_{\theta=0}$, $s_{\theta=1}$ and p, and using the implicit function theorem and rearranging (quite extensively in the $\theta=1$ case) we obtain (will be slightly altered from here and downward)

$$\begin{aligned} \frac{ds_{\theta=0}}{dp} &= \frac{1}{\left[2s_{\theta=0}-\frac{\alpha+n(ps_{\theta=0}+(1-p)s_{\theta=1})}{1+n}+\frac{np}{1+n}s_{\theta=0}\right]}\frac{n}{1+n}s_{\theta=0}\left[s_{\theta=0}-s_{\theta=1}+(1-p)\frac{ds_{\theta=1}}{dp}\right] \\ \frac{ds_{\theta=1}}{dp} &= \frac{n}{1+n}\left[s_{\theta=0}-s_{\theta=1}+p\frac{ds_{\theta=0}}{dp}\right] \end{aligned} \tag{A9}$$

which may be inserted into (A8) to yield

$$\frac{df_{\theta=0}}{dp} = \frac{2}{(1+n)^2}[\alpha+n(ps_{\theta=0}+(1-p)s_{\theta=1})-(1+n)s_{\theta=0}]\left[n(s_{\theta=0}-s_{\theta=1})\frac{1+n}{1+np}-\frac{\partial s_{\theta=0}}{\partial p}\right] - \frac{\partial M_{\theta=0}}{\partial s_{\theta=0}}\frac{\partial s_{\theta=0}}{\partial p}$$
$$\frac{df_{\theta=1}}{dp} = -\frac{\partial M_{\theta=1}}{\partial s_{\theta=1}}\frac{\partial s_{\theta=1}}{\partial p} \quad \text{(A10)}$$

This can be rewritten as

$$\frac{df_{\theta=0}}{dp} = \frac{2}{(1+n)^2}[\alpha+n(ps_{\theta=0}+(1-p)s_{\theta=1})-(1+n)s_{\theta=0}]\left[n(s_{\theta=0}-s_{\theta=1})\frac{1+n}{1+np}\right] < 0$$
$$\frac{df_{\theta=1}}{dp} = -\frac{1}{rs_{\theta=1}}\left[\frac{n}{1+np}(s_{\theta=0}-s_{\theta=1}) + \frac{np}{1+np}\frac{\partial s_{\theta=0}}{\partial p}\right] > 0 \quad \text{(A11)}$$

where the sign of the bottom expression is obtained by insertion of the derivative of $s_{\theta=0}$ with respect to p. □

Proof of Lemma 3: Lemma 1 states that firms choosing the same θ will also settle for the same M and thus they will have the same c. The optimal c when all firms cluster can then be obtained from the upper equation in (A7) which can be rewritten as;

$$c^2 - \alpha c + \frac{(1+n)^2}{2r} = 0 \quad \text{(A12)}$$

In the same way the lower expression in (A7) can be written,

$$c^2 - \alpha c + \frac{1}{n}\frac{(1+n)^2}{2r} = 0 \quad \text{(A13)}$$

when all firms choose not to interact. Since only the smaller roots can support equilibria it can be seen that firms in the isolated case will have lower costs than firms in the integrated case despite the sharing that goes on in the latter case i.e. $M_{\theta=0} > nM_{\theta=1}$. Let $M_{\theta=0} = \gamma n M_{\theta=1}$ where $\gamma > 1$. The corresponding profits is given by

$$\pi_{\theta=1} = \left[\frac{\alpha-e^{-M_{\theta=1}}}{1+n}\right]^2 - M_{\theta=1}$$
$$\pi_{\theta=0} = \left[\frac{\alpha-e^{-\gamma n M_{\theta=1}}}{1+n}\right]^2 - \gamma n M_{\theta=1} \quad \text{(A14)}$$

where it is easily seen that the former is greater than the latter when $\gamma=1$. Maximizing the latter equation with respect to γ yields that $\gamma=1$ is optimal. Hence, $\pi_{\theta=1}(p=0) > \pi_{\theta=0}(p=1)$. □

Proof of Proposition 2: Lemma 2 establishes that $\pi_{\theta=1}$ is strictly increasing and $\pi_{\theta=0}$ is strictly decreasing in p. Lemma 3 demonstrates that $\pi_{\theta=1}(p=0) > \pi_{\theta=0}(p=1)$ ensuring that the "lines" must intersect unless setting $\theta=1$ strongly dominates $\theta=0$ for all p, like in figure 2b.

Thus, there is always an $p^* = x/n$ s.t. $\pi_{\theta=1}(x/n) > \pi_{\theta=0}((x+1)/n)$ and $\pi_{\theta=0}(x/n) > \pi_{\theta=1}((x-1)/n)$ or s.t. $\pi_{\theta=1}(x/n) = \pi_{\theta=0}((x+1)/n)$. In the former case p^* is unique whereas in the latter case both proportions are equilibria. □