Advanced Microeconomic Theory EC104

Problem Set 2

(* is easy, ** is difficult, *** is more difficult)

1. * In the following two games find the strategy profiles that survive iterated elimination of dominated strategies. (In part b do not forget mixed strategies).

a.

	x_2	y_2	z_2
x_1	4,3	5,1	6,2
y_1	2,1	8,4	3,6
z_1	3,0	9,6	2,8

b.

	x_2	y_2
x_1	2,0	-1,0
y_1	0,0	0,0
z_1	-1,0	2,0

2. * For each of the following two player games, find all equilibria. As usual, player 1 chooses the row and player 2 chooses the column. In part e, player 1 chooses between rows, player 2 between columns and player 3 between the 'boxes'.

a.

	x_2	y_2
x_1	2,1	1,2
y_1	1,5	2,1

b.

	x_2	y_2
x_1	3,7	6,6
y_1	2,2	7,3

c.

	x_2	y_2
x_1	7,3	6,6
y_1	2,2	3,7

d.

	x_2	y_2	z_2
x_1	4,2	5,1	0,3
y_1	1,3	0,1	2,2

e.

x_3			y_3		
	x_2	y_2		x_2	y_2
x_1	5,2,3	6,1,2	x_1	1,2,2	6,1,1
y_1	4,5,1	8,6,4	y_1	9,0,0	3,2,5

3. * Consider the following normal form game:

1/2	L	R
T	a,b	c,d
B	e,f	g,h

3a. Determine the conditions for (B,R) to be a Nash Equilibrium of the game.

3b. Assume $a = h = 6, b = g = 1, c = d = 0, e = f = x$. For what values of x does the game has dominant strategy equilibria?

3c. Assume $a = h = 2, b = g = 1$, $c = d = e = f = 0$. Assume player 1 plays the mixed strategy $(r, 1 - r)$ and player 2 the mixed strategy $(q, 1 - q)$, where $r \in [0, 1]$ and $q \in [0, 1]$. Derive the best response mapping for both players and find all the Nash Equilibria of the game. Represent both best response mappings and equilibria in an appropriate diagram.

4. * Suppose you play in a football team, and you are about to take a penalty kick. You have to decide whether to kick to the left or right corner of the goal. Your opponent team's goalkeeper, in turn, has to decide whether to dive left or right. To put some numbers to this, assume that if the goalkeeper dives left (right) when you kick left (right), then the goalkeeper blocks the kick with probability one. On the other hand, if you kick left (right) and the goalkeeper dives right (left), then you will definitely score a goal with probability one.

4a. Model this story as a normal form game (use a matrix in which the payoffs for the penalty kicker and the goalkeeper are the probabilities of scoring a goal and blocking a kick, respectively, for any combination of strategies).

4b. Find all the Nash Equilibria of the game (in pure and/or mixed strategies).

4c. Find all the Nash Equilibria of the game when the penalty kicker has 2/3 chance of scoring if he kicks left and the goalkeeper dives left, and only 1/3 chance if he kicks right and the goalkeeper dives right.

5. **(Mas-Colell, Whinston and Green)

Consumers are uniformly distributed along a boardwalk that is 1 mile long. Ice-cream prices are regulated, so consumers go to the nearest vendor because they dislike walking (assume that at the regulated prices all consumers will purchase an ice cream even if they have to walk a full mile). If more than one vendor is at the same location, they split the business evenly.

5a. Consider a game in which two ice-cream vendors pick their locations simultaneously. Model this situation as a strategic game. In particular, give the exact values of the payoff functions of the two vendors as a function of their relative locations.

5b. Show that there exists a unique pure strategy Nash equilibrium and that it involves both vendors locating at the midpoint of the boardwalk.

5c. Show that with three vendors, no pure strategy Nash equilibrium exists.

6. *** Consider the Cournot duopoly model in which two firms 1 and 2, simultaneously choose the quantities they will sell on the market, q_1 and q_2. The price each receives for each unit given these quantities is: $P(q_1, q_2) = a - b(q_1 + q_2)$. Their costs are c per unit sold.

6a. Argue that successive elimination of strictly dominated strategies yields a unique prediction in this game, which is the unique Nash equilibrium of this game.

6b. Would this be true if there were three firms instead of two?

7. (Osborne, Exercise 114.4)*** Swimming with sharks

You and a friend are spending two days at the beach; you both enjoy swimming. Each of you believes that with probability π the water is infested with sharks. If sharks are present, a swimmer will surely be attacked. Each of you has preferences represented by the *expected value* of a payoff function that assigns $-c$ to being attacked by a shark, 0 to sitting on the beach, and 1 to a day's worth of undisturbed swimming (where $c > 0$!).

If a swimmer is attacked by sharks on the first day, then you both deduce that a swimmer will surely be attacked the next day, and hence do not go swimming the next day.

If at least one of you swims on the first day and is not attacked, then you both know that the water is shark-free.

If neither of you swims on the first day, each of you retains the belief that the probability of the water's being infested is π, and hence on the second day swim only if $-\pi c + 1 - \pi > 0$ and sits on the beach if $-\pi c + 1 - \pi < 0$, thus receiving an expected payoff of $\max\{-\pi c + 1 - \pi, 0\}$.

7a. Model this situation as a strategic game in which you and your friend each decides whether to go swimming on your first day at the beach. If, for example, you go swimming on the first day, you (and your friend, if she goes swimming) are attacked with probability π, in which case you stay out of the water on the second day; you (and your friend, if she goes swimming) swim undisturbed with probability $1 - \pi$, in which case you swim on the second day. Thus your expected payoff if you swim on the first day is

$$\pi(-c + 0) + (1 - \pi)(1 + 1) = -\pi c + 2(1 - \pi),$$

independent of your friend's action.

7b. Find the mixed strategy Nash equilibria of the game (depending on c and π).

7c. Does the existence of a friend make it more or less likely that you decide to go swimming on the first day? (Penguins diving into water where seals may lurk are sometimes said to face the same dilemma, though Court (1996) argues that the evidence suggests that they do not.)

8. ***(Mas-Colell, Whinston and Green)

There are n firms in an industry. Each can try to convince Congress to give the industry a subsidy. Let h_i denote the number of hours of effort put in by firm i, and let

$$c_i(h_i) = w_i\,(h_i)^2$$

be the cost of this effort to firm i (w_i is a positive constant).

When the effort levels of the firms are $(h_1, h_2, ..., h_n)$, the value of the subsidy that gets approved for each firm i is:

$$\alpha \sum_{i=1}^{i=n} h_i + \beta \prod_{i=1}^{i=n} h_i = \alpha\,(h_1 + ... + h_n) + \beta\,(h_1 \times ... \times h_n)$$

where α and β are constants. This means that the utility of each firm $i = 1, ..., n$ is given by:

$$U_i(h_1, ...h_n) = \alpha \sum_{j=1}^{j=n} h_j + \beta \prod_{j=1}^{j=n} h_j - w_i\,(h_i)^2$$

8a. Consider a game in which the firms decide simultaneously and independently how many hours they will each devote to this effort. Model this situation as a strategic game.

8b. Show that each firm has a strictly dominant strategy if and only if $\beta = 0$. What is firm i's strictly dominant strategy when this is so?

9. *** Consider the Bertrand duopoly model in which two firms 1 and 2, simultaneously choose the prices they will sell on the market, p_1 and p_2. Assume that the strategy space S_i of firm i is the interval $[0, 1]$, i.e. $p_1 \in [0, 1]$ and $p_2 \in [0, 1]$. The linear market demand is given by:

$$D = 1 - 2p_i + p_j$$

Assume that there are zero marginal costs so that the profit of firm $i = 1, 2$ is equal to:

$$\pi_i = (1 - 2p_i + p_j)\,p_i$$

This game is called a *supermodular game*. It is important for the answer to 9b to observe that we have imposed the strategy space S_i to be a bounded interval so that prices p_i and p_j cannot be strictly less than zero and strictly greater than one.

9a. Determine the unique Nash equilibrium of this game.

9b. Argue that successive elimination of strictly dominated strategies yields a unique prediction in this game, which is exactly the unique Nash equilibrium of this game.