

Microeconomic Theory EC3.64

Problem Set 2

1. In the following two games find the strategy profiles that survive iterated elimination of dominated strategies. (In part b do not forget mixed strategies).

		x_2	y_2	z_2
a.	x_1	4,3	5,1	6,2
	y_1	2,1	8,4	3,6
	z_1	3,0	9,6	2,8

		x_2	y_2
b.	x_1	2,0	-1,0
	y_1	0,0	0,0
	z_1	-1,0	2,0

2. For each of the following two player games, find all equilibria. As usual, player 1 chooses the row and player 2 chooses the column. In part e, player 1 chooses between rows, player 2 between columns and player 3 between the 'boxes'.

		x_2	y_2
a.	x_1	2,1	1,2
	y_1	1,5	2,1

		x_2	y_2
b.	x_1	3,7	6,6
	y_1	2,2	7,3

		x_2	y_2
c.	x_1	7,3	6,6
	y_1	2,2	3,7

		x_2	y_2	z_2
d.	x_1	4,2	5,1	0,3
	y_1	1,3	0,1	2,2

	x_3		y_3	
		x_2	y_2	
e.	x_1	5,2,3	6,1,2	
	y_1	4,5,1	8,6,4	

3. Consider the following normal form game:

1/2	L	R
T	a,b	c,d
B	e,f	g,h

- a) determine the conditions for (B,R) to be a Nash Equilibrium of the game.
 - c) Assume $a = h = 6, b = g = 1, c = d = 0, e = f = x$. For what values of x does the game has dominant strategy equilibria?
 - d) Assume $a = h = 2, b = g = 1, c = d = e = f = 0$. Assume player 1 plays the mixed strategy $(r, 1 - r)$ and player 2 the mixed strategy $(q, 1 - q)$, where $r \in [0, 1]$ and $q \in [0, 1]$. Derive the best response mapping for both players and find all the Nash Equilibria of the game. Represent both best response mappings and equilibria in an appropriate diagram.
4. Suppose you play in a football team, and you are about to take a penalty kick. You have to decide whether to kick to the left or right corner of the goal. Your opponent team's goalkeeper, in turn, has to decide whether to dive left or right. To put some numbers to this, assume that if the goalkeeper dives left (right) when you kick left (right), then the goalkeeper blocks the kick with probability one. On the other hand, if you kick left (right) and the goalkeeper dives right (left), then you will definitely score a goal with probability one.
- a) Model this story as a normal form game (use a matrix in which the pay-offs for the penalty kicker and the goalkeeper are the probabilities of scoring a goal and blocking a kick, respectively, for any combination of strategies).
 - b) Find all the Nash Equilibria of the game (in pure and/or mixed strategies).
 - c) Find all the Nash Equilibria of the game when the penalty kicker has 2/3 chance of scoring if he kicks left and the goalkeeper dives left, and only 1/3 chance if he kicks right and the goalkeeper dives right.

5. Each of n farmers can costlessly produce as much wheat as he chooses. Suppose that the k th farmer produces W_k , so that the total amount of what produced is $W = W_1 + W_2 + \dots + W_n$. The price p at which wheat sells is then determined by the demand equation $p = e^{-W}$.
- Show that the strategy of producing one unit of wheat strongly dominates all of a profit-maximizing farmer's other strategies. Check that the use of this strategy yields a profit of e^{-n} for a farmer.
 - Explain why the best of all agreements that treat each farmer equally requires each to produce only $\frac{1}{n}$ units of wheat. Check that a farmer's profit would then be $\frac{1}{en}$. Why would such an agreement need to be binding (that is, signed as a legally binding contract) for it to be honored by a profit-maximizing farmers?
 - Confirm that xe^{-x} is largest when $x = 1$. Deduce that all the farmers would make a larger profit if they all honored the agreement rather than each producing one unit and so flooding the market.
 - You would have realized what the exercise went through was version of the "tragedy of the commons". Why is such an n -player game a generalization of the Prisoners' Dilemma?
6. Consider the following case of a *differentiated good Cournot model*. Firm i produces type i widgets at a constant unit cost of c_i , $i = 1, 2$. If q_1 and q_2 are the quantities of the two varieties produced, the respective prices for the two goods are determined by the demand equations $p_1 = M - 2q_1 - q_2$ and $p_2 = M - q_1 - 2q_2$. Adapt Cournot's duopoly model to this new situation, and find:
- The firms' reaction functions
 - The quantities produced in equilibrium and prices at which the goods are sold and the equilibrium profits.
7. Two firms set prices in a market whose demand curve is given by $Q = D(p)$, where $D(p)$ is a downward-sloping function and p is the lower of the two prices. The lower priced firm meets all of the demand; if the two firms post the same price, then they each get half the market. Assume costs of production are zero and that prices can only be quoted in dollar units (0, 1, 2...).
- Show that if the rival firm charges a price above the monopoly price p_m , then the best response is to charge the monopoly price.
 - Show further that if the rival firm charges a price $p(> 1)$ at or below the monopoly price, then the best response is to charge a price below p .
 - Conclude from the preceding arguments that the unique Nash Equilibrium price must be for each firm to price at 1 dollar.
 - What would be the Nash Equilibrium if there were 3 firms in this market? More generally, if there were N firms? Explain.