Strategic forward contracting in the wholesale electricity market[1]

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Abstract

This paper analyses a wholesale electricity market with supply function competition. Trade in the forward and spot markets is represented by a two-stage game, and its subgame perfect Nash equilibrium (SPNE) is characterized. It is verified that increased forward sales of a producer constitute a credible commitment to aggressive spot market bidding. The paper identifies market situations when this pro-competitive commitment is unilaterally profitable for the producer. It is also proven that a producer has incentives to sell in the forward market in order to reduce competitors' forward sales, which softens their spot market offers.

Keywords: supply function equilibrium, forward market, strategic contracting, strategic substitutes, oligopoly, wholesale electricity market

JEL codes: C72, D43, D44, G13, L13, L94

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1 INTRODUCTION

It is well known that forward sales mitigate the market power of the electricity producers and thus, a better understanding of the mechanisms behind forward contracting can improve the design of electricity markets. It has been shown both empirically (Wolak, 2000; Bushnell et al., 2008) and theoretically (Newbery, 1998b; Green, 1999a) that forward sales reduce producers' mark-ups. The reason is that if a fraction of the output has been sold in the forward market, a profit-maximising producer will gain less from mark-ups in the spot market. A producer whose output is larger than its forward sales will use its market-power to sell above its marginal cost in the spot market. On the other hand a producer whose output is less than its forward sales, will use its market-power to buy back power at a price below its marginal cost in the spot market. From a welfare perspective, the best contracting outcome is when output equals forward sales. In this case, profit-maximizing mark-ups are zero in the spot market.

It is evident that hedging and arbitrage opportunities play an important role in producers' contracting decisions. These mechanisms are well-studied for markets with perfect competition. But competition is often imperfect in electricity markets, so forward contracting is also driven by strategic interaction with competitors and potential entrants. A strategic producer will choose its forward sales such that competitors reduce their output for a given spot price. With such a soft response the producer can increase its total sales at competitors' expense. Thus contracting is stimulated when increased forward sales result in a less aggressive (soft) response from competitors.

Selling forward makes a strategic producer more interested in output and less interested in mark-ups in the spot market. Hence, it is a credible commitment to aggressive spot market bidding. Newbery (1998b) shows that producers can use this commitment to deter entry. This can be classified as a 'top dog' strategy in the taxonomy of business strategies outlined by Fudenberg and Tirole (1984), and Bulow et al. (1985).

In economics and game theory, the decisions of two or more players are called strategic complements if they mutually reinforce one another, and they are called strategic substitutes if they mutually offset one another (Bulow et al., 1985). In my context, strategies are substitutes when a commitment to high output results in reduced output from competitors for a given spot price. In this case producers have incentives to use forward sales as a 'top dog' strategy also against incumbent competitors. This was illustrated by Allaz and Vila (1993) in a two-stage Cournot model of a homogeneous duopoly product market. This set-up introduces a prisoners' dilemma for producers: given competitors' forward positions, each incumbent producer has incentives to sell forward to get a Stackelberg first-mover advantage. But when they all do so, all producers end up worse off. As a result, competition is tougher on the spot market and welfare is improved as compared to a situation without forward trading. Thus contracting is pro-competitive when strategies are substitutes, which has also been verified experimentally by Brandts et al. (2008). However, Mahenc and Salanié (2004) show that the reverse result can occur when strategies are complements, i.e. when a commitment to high output results in increased output from competitors for a given spot price. They analyse a market with differentiated goods and price competition, and show that a commitment to low mark-ups, due to forward sales, is met with a tough response, competitors also lower their mark-ups. To avoid the tough response, firms buy in the forward market (negative contracting) in order to soften competition in the spot market. This corresponds to the 'puppy dog' strategy in Fudenberg and Tirole (1984) and it increases mark-ups in the spot-market. Thus, forward trading is anti-competitive and it reduces welfare in the model by Mahenc and Salanié (2004).

In addition to the strategic commitment effect introduced by Allaz and Vila (1993), Green (1999a) identifies an additional effect, which I call strategic forward price manipulation. A producer can have incentives to increase its forward sales to lower the forward price and thereby the competitor's forward sales (along their committed forward supply curve) in order to soften competitors' spot market bidding. Again all producers have incentives to shift their forward supply curves outwards (given competitors' contracting strategies) as long as mark-ups are positive, so they all end up worse off and competition is improved. Similar to the strategic commitment effect, strategic forward price manipulation is moderated in that increased forward sales reduce the market price. Thus Green (1999a) shows that the level of strategic contracting resulting from strategic forward price manipulation is higher when competitors have more elastic forward supply, i.e. when competitors' forward sales can be reduced at the cost of a low price reduction.

Since forward contracting incentives are often diametrically different for price and quantity competition, the competition process should be considered in detail when analysing strategic contracting in a market. This paper analyses forward contracting in a wholesale electricity market without entry. Similar to previous strategic contracting models of the electricity market by Newbery (1998b) and Green (1999a), the supply function equilibrium model is used to represent competition in the spot market. The paper contributes to this literature by analysing an asymmetric duopoly market with general cost and demand functions. It is verified that contracting is a credible commitment to aggressive spot market bidding and that strategic forward price manipulation contributes to positive (pro-competitive) contracting.

It is also analysed under what circumstances the strategic commitment contributes to positive (pro-competitive) contracting. When both producers' outputs exceed their forward sales it is shown that contracting strategies are substitutes (pro-competitive) for a firm with convex marginal costs and a concave residual demand. Marginal costs are approximately convex in most markets. Residual demand can normally be made less convex and more concave by lowering the price cap; capacity payments can be introduced as a compensation to maintain investment incentives. Irrespective of the shape of the residual demand and marginal cost curves, strategies will be substitutes for at least one firm for prices near the price cap. When marginal costs are constant, strategies are neutrals for one firm and substitutes for the other. But positive contracting cannot be ensured for all circumstances. In the region where both firms are over-contracted, strategies are likely to be complements for one of the firms.

The comparative statics method used in the analysis results in a system of differential equations, which can be generalised to several firms. With numerical methods it would be possible to calculate subgame perfect Nash-equilibria of contracting decisions for more realistic examples. Such a method could also be applied to observed bid-curves to empirically estimate to what extent strategies are substitutes in real markets.

The structure of the paper is as follows. Section 2 describes the wholesale electricity market in detail. In Section 3 this description is used to motivate the assumptions of the model. Moreover, the mechanisms driving strategic contracting in electricity markets and how they can be modelled are explained in greater detail. The model is presented in Section 4. Section 5 first analyses strategic bidding in the last stage of the game. Thereafter it is analysed under what conditions the strategic commitment effect has a positive influence on forward sales. Next, a condition of optimal contracting is derived. It is evaluated for a simple case where the strategic commitment effect is isolated. Section 6 concludes the paper.

2 THE WHOLESALE ELECTRICITY MARKET

Electricity is produced by many different technologies that often have different marginal costs. The production cost of a plant is to a large extent determined by fuel costs and its production efficiency. These are well-known parameters that are common knowledge. The plants of a producer are used in merit order, starting with those that have the lowest marginal cost, such as nuclear power or hydro-power. Last in the merit-order are typically peaking power plants, such as open-cycle gas turbines burning natural gas. They are characterized by high marginal costs and low investment costs, which is a comparative advantage for plants used only under peak demand. Due to the merit-order, the marginal costs of a producer increase with output. There are some local deviations from this trend, due to the start-up costs of plants, which introduce local non-convexities in the production costs, but they are normally neglected in analyses of electricity markets. In many markets marginal costs are approximately convex, see Green and Newbery (1992) for the shape of the marginal cost curve in the electricity market of Britain. Even if electricity is produced by various technologies, it is still a very homogeneous good suitable for trade at commodity exchanges and auctions.

In wholesale electricity markets, producers sell electricity to retailers. In turn, retailers sell electricity to consumers in the retail market. Typically, there are few producers and many retailers in the wholesale market. In addition, electricity consumption is to a large extent exogenously determined, e.g. by the weather and it is very inelastic, especially in real time. Thus, the market power that can be exercised on the wholesale market by retailers and consumers is small as compared to the market power of power producers, which can be significant.

Due to ramp-rates in many production technologies, production plans are scheduled the day before delivery, and the day-ahead market is an important component in this planning process. It is a forward market, as electricity is not delivered until the day after. The day-ahead market is relatively liquid and often provides strike prices for financial contracts, so sometimes it is therefore referred to as the spot market. To avoid any confusion, I will therefore avoid this term in the context of electricity markets. The day-ahead market is organized as a double auction to which consumers and producers submit non-increasing bid curves and non-decreasing offer curves, respectively. The market clearing price is determined by the intersection of bids and offers. As in a uniform-price auction, all accepted bids from retailers pay the market clearing price and all accepted offers from producers are paid the same price.[3] Normally, there is a separate price and auction for each delivery period, which typically last 0.5 or 1 hours.

Electricity is special in that it is very expensive to store, so production must equal consumption at every moment. The system operator uses a real-time or balancing market to make necessary adjustments in production (and consumption to the extent that it is elastic). During the delivery period producers offer increments or decrements relative to their physical forward sales, which normally include sales in the day-ahead market. Thus even if the real-time market is seldom referred to as the spot market, it is a *de facto* spot market. Offer curves

[3] Most day-ahead and real-time markets are organized as uniform-price auctions, but the real-time market in Britain, which is organized as a pay-as-bid auction, is an exception.

to the balancing market are submitted before the beginning of the delivery period and are valid during the whole delivery period. Thus, the system operator's demand is uncertain when offers are submitted.

System imbalances arise because of unexpected changes in wind and temperature, unexpected production outages or unexpected transmission-line failures disconnecting consumers in cities. In very extreme cases, occurring of the order once every 10-100 years, multiple unexpected events can lead to the scheduled reserve capacity—expected available production capacity subtracted by expected demand—not being enough to meet the system imbalance. The loss-of-load probability (LOLP) is typically very small, but always positive. No matter how large the reserve margin is, a sufficiently large number of simultaneous unexpected events that decrease the production or reserve capacity, or increase the demand, will lead to a power shortage. LOLP during a particular delivery period can be estimated ex-ante from the scheduled reserve margin, the probability distribution of demand and from the probabilities of production failures in individual plants. Newbery (1998a) shows that the LOLP estimated ex-ante by the system operator in Britain decreases exponentially with the scheduled reserve margin. Using data provided in Newbery's paper and adjusting for the fact that the system operator consistently overestimated LOLP, I roughly estimate LOLP during half an hour to 0.1% when the scheduled reserve margin is 10% and that it roughly decreases by a factor of 100 for every additional 10% of reserve capacity.[4] However, these estimates are indeed very uncertain, and are only intended to give some feeling of the magnitude of LOLP and how it depends on the reserve capacity.

It should be noted that even if the demand distribution has long tails, these are normally very thin. As argued above the support of the demand's probability density is wide, but the standard deviation of the demand is normally small in electricity markets. Garcia and Kirschen (2006) show that the shock distribution is approximately normally distributed in Britain. Given the central limit theorem, this is not surprising as the demand shock to a large extent results from a large number of independent actions/accidents.

In the rare situations when electricity demand exceeds market supply, demand has to be rationed to avoid a system collapse, and the price is set equal to the price cap. One motivation for the use of price caps is that consumers who do not switch off their equipment when the electricity price becomes very high do not necessarily have a high marginal benefit of power.

[4] The values are estimated from Fig. 5 in Newbery (1998a) and from the fact that capacity payments in the old pool were proportional to LOLP. It is assumed that the system operators have overestimated the LOLP by a factor of 50. This very uncertain factor is based on the reasoning in the appendix in Newbery (1998a).

It may be that the residential consumer is not at home or that he/she is not aware of the high price. Moreover, residential consumers may not face the real-time price but buy whatever they want at a contracted price, and distribution companies often do not observe who consumes what in real-time. Due to these imperfections, it will be welfare maximizing to ration demand at some very high reservation price, which is often set to an estimate of the value of lost load (VOLL) (Stoft, 2002).

Electricity prices are volatile, partly because electricity is not suitable for large-scale storage. To hedge their risks, market participants can buy and sell various derivative contracts, e.g. futures and forward contracts. These contracts commit the parties to buy and sell the contracted quantity in the real-time market at the agreed delivery price. Forward contracts are normally traded over the counter and futures contracts are traded at power exchanges. Differences between these contracts are in most circumstances sufficiently small to be ignored (Hull, 1997). In this paper, I use the term forward contract for both types of contracts. In most cases, the contracts are financial, so no physical transaction takes place. There is a procedural difference between the two contracts in that offers to the real-time market are net of physical sales. But unless there are imbalance penalties on delivery deviations from physical contracts the revenue stream of a firm is not influenced by whether the contract is physical or financial. The day-ahead market is different from most other forward markets in that it is a multi-unit auction.

It is reasonable to assume that the correlation between regional electricity prices and the global stock market is weak, i.e. the risk in the electricity market is mainly non-systematic. Ideally, speculators can completely eliminate non-systematic risks by holding a well-diversified portfolio (Hull, 1997). One would therefore expect no or only a small risk premium in the forward electricity price on top of the expected spot price. However, many empirical studies suggest that the risk premia can be significant, for example in Longstaff's and Wang's (2004) study of the PJM market (U.S.). Kristiansen (2007) has a similar conclusion for Nord Pool, the electricity market of the Nordic countries. It is possible that a premium in the forward price arises because the electricity market is immature; at least the Nordpool market shows such signs (Kristiansen, 2007). Green (1999b) motivates the risk premium by retailers having thinner margins and therefore being more risk-averse than producers. Anderson and Hu (2008c) show that strategic retailers preferring to buy in the forward market in order to reduce producer mark-ups in the real-time market results in equilibria where the forward price is higher than the expected real-time price.

Financial trading is anonymous in most power markets, and a firm's forward and futures positions are not revealed to competitors. But a producer can still get a rough estimate of changes in competitors' total forward positions by analysing changes in the turnover in the forward market and the forward price; see the model by Ferreira (2006) for example.

3 STRATEGIC CONTRACTING IN THE WHOLESALE ELECTRICITY MARKET

This section starts with a discussion of strategic bidding in real-time markets. Next, the section gives a detailed description of the mechanisms driving strategic contracting in electricity markets and how they can be modeled.

3.1 Strategic bidding in the real-time market

In the real-time market (the *de facto* spot market), producers choose supply functions (net of physical forward sales) before demand has been realized and production costs are common knowledge. Bolle (1992) and Green and Newbery (1992) observed that these characteristics of electricity auctions agree very well with the market assumptions behind the supply function equilibrium (SFE) under uncertain demand, which was introduced by Klemperer and Meyer (1989). If there are offers that are either accepted with certainty or rejected with certainty in equilibrium; these out-of-equilibrium offers can be used to support a continuum of equilibria (Klemperer and Meyer, 1989). However, the range of equilibria is constrained by capacity constraints (Green and Newbery, 1992; Baldick and Hogan, 2002; Genc and Reynolds, 2004; Anderson and Hu, 2008a). In particular, a unique equilibrium will be singled out if maximum demand is sufficiently high to make the capacity constraints of all (but possibly one) firms bind with a positive probability, which could be arbitrarily small (Holmberg, 2007; Holmberg, 2008). This unique equilibrium has been adopted by Aromi (2007), Newbery (2008) and Wilson (2008). It also corresponds to the unique (linear) SFE that arises when marginal costs and demand are linear, and demand shocks, capacities and prices are unbounded (Klemperer and Meyer, 1989). As argued in Section 2, it rarely happens that the lowest offers are rejected or that the highest offers in the auction are accepted. It requires multiple unexpected accidents to make this happen and such events coincide of the order once every 10 to 100 years. Hence, optimizing bids that are almost accepted or rejected with certainty only improve the expected payoff by a very slight amount. Still it is natural to assume that producers optimize these bids as well.

3.2 The literature on strategic contracting in wholesale electricity markets

In the forward market, strategic producers choose their contracting level in order to shift competitors' output curve (production as a function of the real-time price) inwards, so that the producer can increase its own output at competitors' expense. Together, strategic forward price manipulation and the strategic commitment determine the optimal level of contracting. I start with a description of the strategic commitment mechanism, where competitors adjust their bidding in the real-time market after observing the producer's contracting position. In the context of wholesale electricity markets, this Allaz and Vila effect has been analysed by means of two-stage models with supply function competition in the real-time market. Previous studies are restricted to linear demand and that producers have constant (Newbery, 1998b) or linear marginal costs (Green, 1999a; Chung et al., 2004). Rather than assuming a unique equilibrium, Newbery (1998b) considers three natural equilibrium selections, one where producers coordinate to obtain the most profitable SFE, one where producers always coordinate to obtain the same maximum price, and another where strategies are neutrals. The last choice implies that a competitor does not adjust its output curve after observing a change in the producer's forward sales. In Green (1999a), producers are assumed to choose a linear SFE, as it can be regarded as a focal point. As explained above this equilibrium also corresponds to the unique equilibrium that would result in a market with linear demand and linear marginal costs if no bid is accepted or rejected with certainty (shocks are unbounded).

The strategic commitment effect is often weaker when the real-time market has an SFE rather than a Cournot equilibrium. The reason is that unlike the Cournot equilibrium, the supply function equilibrium is ex post optimal to a range of additive demand shocks. Thus a linear shift of the demand curve (net of forward contracts) in the real-time market, due to increased forward sales, has in itself no influence on competitors' optimal offers. With an SFE in the real-time market, a strategic producer will use contracting sales to commit itself to less elastic net-offer curves in the real-time market, because this will increase competitors' mark-ups, so that their output curves shift inwards. But the slope of a producer's supply curve in the linear SFE is not influenced by its own forward sales (Green, 1999a). Accordingly competitors do not respond to increased forward sales of a producer, and strategies are neutrals for the linear SFE.

Strategic forward price manipulation occurs when a producer's forward sales change the forward price which, in turn, changes competitors' forward sales (along their committed forward supply curves) and later their offer curves in the real-time market. Green (1999a) shows that strategic forward price manipulation stimulates contracting (pro-competitive) for a linear SFE. The effect is stronger when most of the liquidity in the forward market is provided by producers rather than consumers. Committing to a forward supply curve mimics bidding in day-ahead electricity markets, but it could also be used as an approximate model of trade in other forward markets. Green (1999a) assumes that all bidders have the same information in the forward market and the residual forward demand of each producer is known with certainty in equilibrium. This implies that there will be a continuum of equilibria. In particular, the slopes of the supply functions in the forward market cannot be endogenously determined. This is related to the multiple-equilibrium problem when producers compete with linear supply functions under demand certainty in a one-shot game (Klemperer and Meyer, 1989).

3.3 The model in this paper

This paper assumes that the tails of the demand distribution are sufficiently long, so that there is a unique subgame equilibrium in the real-time market for given contracting levels as in Holmberg (2007, 2008). This equilibrium choice also makes the analysis consistent with Green (1999a) and it is also consistent with one of the SFE considered by Newbery (1998b).

The model is applied to an asymmetric duopoly market with general costs, and it is verified that increased forward sales constitute an (aggressive) commitment to reduced mark-ups in the real-time market. Hence, a producer that increases its forward sales will shift its total output curve outwards. This also implies that the competitor's response would be softer if its forward sales could be reduced. Increased forward sales reduce the forward price and the competitor's forward sales (along its committed forward supply curve), so that the competitor shifts its total output curve inwards. Hence, strategic forward price manipulation generally stimulates positive contracting.

Circumstances under which strategies are substitutes are non-trivial. It is useful to use the contracting point as a reference when describing these results. At its contracting point, a producer's its net-sales are zero (output equals forward sales) and its mark-up is zero. Above the contracting point, net-supply and mark-ups are positive. For real-time prices above both producers' contracting points, it is shown that strategies are substitutes for a producer with convex marginal costs that faces a concave residual demand. This result can be intuitively explained as follows. When the producer increases its forward sales, the marginal cost curve as a function of the net-supply (net of forward contracts) will shift inwards, and so will the net-supply curve. The inward shift of the net-supply curve is largest at points where the marginal cost curve is steep (as illustrated in Fig. 1) and at points where the mark-up is small. Hence, if marginal costs are convex and the residual demand is concave, selling in the forward market will shift the offer curve more inwards at high real-time prices compared to low prices. This will make the producer's net-supply curve less elastic for all real-time prices, and anticipating this from observed forward positions, competitors' optimal net-supply curves will become soft (less aggressive) as well.

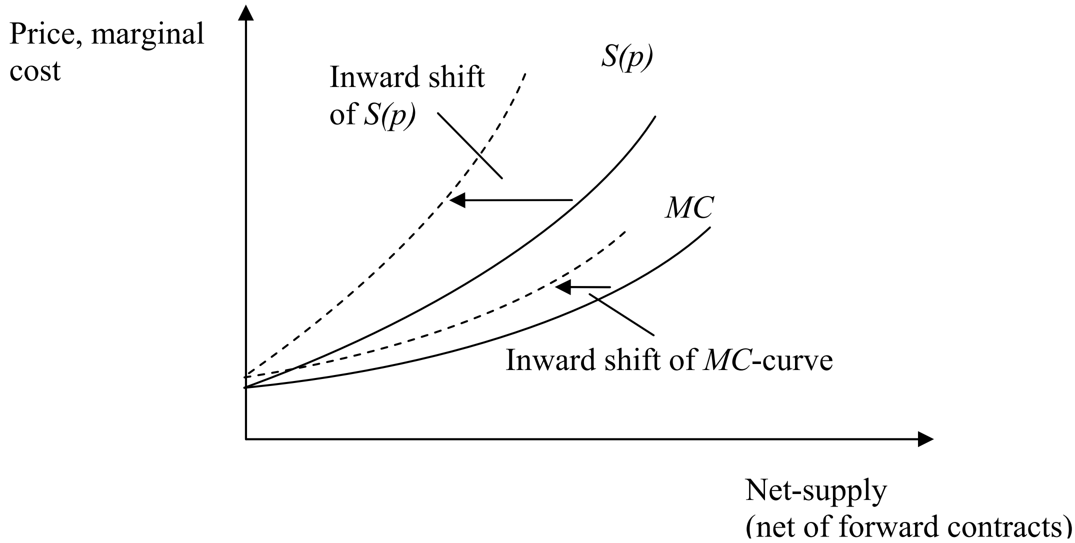

Figure 1. *The marginal cost curve as a function of net-supply (supply net of physical forward contracts) shifts inwards when forward sales increase, and there is a corresponding inward shift in the net-supply curve. If marginal costs are strictly convex, the inward shift tends to increase with the real-time price, and this makes the net-supply curve less elastic for all price levels.*

In the paper, I also derive a general contracting condition for a two-stage game, where the first stage represents the forward market and the second stage the real-time market. As in the model by Ferreira (2006), producers and retailers/consumers are assumed to submit their offer and bid curves simultaneously to the forward market, which is the case for day-ahead markets. Contracting levels and information that can be used to deduce them are not disclosed until the day-ahead market has closed. With risk-neutral agents, the forward price will equal the expected real-time price in the sequentially rational equilibrium. But as other agents' contracting strategies are not observable until the forward market has closed, no agent is given the opportunity to up-date its contracting strategy with respect to this information, so arbitrage opportunities may occur if one agent deviates from the equilibrium. This set-up differs from models by e.g. Allaz and Vila (1993), Newbery (1998b) and Green (1999a), where the forward demand is adjusted after the forward supply curves have been chosen by the producers to ensure that the forward price always equals the expected real-time price, also out of the equilibrium path. The Allaz and Vila assumption would for example apply in markets where producers first commit and announce their forward sales and buyers then decide how much the contracts are worth (given announced contracting levels influence on the real-time price). I show that the contracting condition can be simplified without this “out-of-equilibrium” assumption. Since sequentially rational real-time bidding leads to an envelope result, a marginal change in the real-time price does not influence the expected profit. Retailers, who represent consumers in the wholesale market, are assumed to be non-strategic and bid their marginal expected utility in the forward market.

The general contracting condition considers risk-aversion, arbitrage opportunities, the two strategic mechanisms and how sensitive the forward price is to a producer’s forward sales. The condition is evaluated for a simple case, where Allaz and Vila’s commitment effect is isolated by assuming that the long tails of the demand distribution are so thin and the variance in the demand distribution is so small that the demand outcome is almost certain (think of a normally or lognormally distributed demand density with a very small standard deviation), so that the influence of risk aversion becomes negligible. A similar assumption is used in Green (1999a), but he does not need the long tails assumption as the SFE is assumed to be a focal point. Almost certain demand implies that retailers can make risk-free arbitrage between the day-ahead and real-time price, so their demand in the forward market is perfectly elastic at the (almost sure) sequentially rational real-time price of the equilibrium. Consumers’/retailers’ contracting strategies are submitted before producers’ contracting strategies have been announced, so consumers’/retailers’ contracting strategies are not influenced by producers’ deviations from the equilibrium. Thus the forward price is fixed by retailers/consumers, which prevents strategic forward price manipulation.

In the models by Allaz and Vila (1993), Newbery (1998b) and Green (1999a), strategic contracting is moderated because consumers adjust the forward price when producers’ contract levels change. With perfectly elastic forward demand, the anti-competitive or pro-competitive effects are therefore stronger in this paper compared to the studies by Green (1999a) and Newbery (1998b).

4 MODEL

Producers’ game-theoretical trading strategies in the electricity market are modelled by a two-stage game. The first stage represents the forward market. Similarly to the day-ahead market, forward contracts are traded in a uniform-price auction to which sellers and buyers submit forward offer curves and forward demand curves, respectively. They are submitted simultaneously under symmetric information about the demand shock that is realized after the second stage. The forward market is cleared at the point where forward demand equals forward supply, and afterwards all agents' contracts are observable. The second stage represents the real-time market (the *de facto* spot market). It is another uniform-price auction to which sellers submit supply functions. I use the convention that these offers are net of the firm's forward sales, i.e. forward sales are physical. Imbalance penalties are neglected, so producers' revenue streams would have been the same with financial contracts. After offers to the real-time market have been submitted, an additive demand shock is realized. It is assumed that bids in the real-time market are sequentially rational. Thus, we calculate the subgame perfect Nash equilibrium of the two-stage game.

For simplicity, I restrict most of the analysis to two producers. The producers' forward offer curves are assumed to be non-decreasing and smooth in the forward price, p^f, and they are denoted by $\mathbf{x}(p_f)\equiv\{x_1(p_f), x_2(p_f)\}$. Let $x(p_f)\equiv x_1(p_f)+x_2(p_f)$ and $x_{-i}(p_f)\equiv\sum_{j\neq i}x_j(p_f)$.

Producer i's output (production) in stage 2 is denoted by q_i. Its net supply (output net of forward contracts) in the real-time market is non-decreasing in the real-time price, p, and it is denoted by $s_i(p)\equiv q_i(p)-x_i(p_f^*)$, where p_f^* is the realized forward price. All offers and bids must be between a price floor, $\underline{p}$, and a price cap, $\overline{p}$. Let $q(p)\equiv q_1(p)+q_2(p)$,

$q_{-i}(p)\equiv\sum_{j\neq 1}q_j(p)$, $s_{-i}(p)\equiv\sum_{j\neq i}s_j(p)$, and $s(p)\equiv s_1(p)+s_2(p)$. The cost function of firm i, $C_i(q_i)$, is common knowledge. It is increasing, convex and twice continuously differentiable up to the capacity constraint, k_i. The contracting points of the producers will be of importance for bidding in the real-time market. At its contracting point the output of a producer equals its forward sales. Hence, its net-supply in the real-time market is zero at this point. We let $p_0^{\min}=\min\left[C_1'\left(x_1\left(p_f^*\right)\right),C_2'\left(x_2\left(p_f^*\right)\right)\right]$ and $p_0^{\max}=\max\left[C_1'\left(x_1\left(p_f^*\right)\right),C_2'\left(x_2\left(p_f^*\right)\right)\right]$. Let m be the firm with the highest marginal cost at the contracting point, i.e. $p_0^{\max}=C_m'\left(x_m\left(p_f^*\right)\right)$, and let n be the firm with the lowest marginal cost at the contracting point, i.e. $p_0^{\min}=C_n'\left(x_n\left(p_f^*\right)\right)$.

Electricity demand is subject to an exogenous additive shock, ε, which is a random variable with a positive, continuously differentiable probability density everywhere on the support $\left[\underline{\varepsilon},\bar{\varepsilon}\right]$. The shock is realized after offers and bids have been submitted to the real-time market. A sufficiently wide support of the shock is needed to pin down a unique subgame equilibrium in the real-time market. But otherwise, the probability density of the shock will not influence bidding in the real-time market (Klemperer and Meyer, 1989). It will have more influence on the expected real-time price and bidding in the forward market. Conditional on the symmetric information available to consumers and producers in stage 1, the shock density and the probability distribution are denoted by $f(\varepsilon)$ and $F(\varepsilon)$, respectively. Consumers are represented by retailers in the wholesale market. They are all assumed to be small and non-strategic. Retailers' total purchases in the forward and spot market will sum up to the consumers' demand $\varepsilon + D(p)$. Given the equilibrium contracting of producers, retailers' forward demand is given by their marginal utility of increased forward purchases.

A producer's payoff for a given shock realization ε is given by:

$$\pi_i\left(\varepsilon, \mathbf{x}\left(p_f^*\right)\right) = p_f^* x_i\left(p_f^*\right) + p(\varepsilon) s_i\left(p(\varepsilon)\right) - C_i\left(x_i\left(p_f^*\right) + s_i\left(p(\varepsilon)\right)\right). \tag{1}$$

The risk-aversion of each producer is represented by a Bernoulli utility function $u_i(\pi_i)$ and in stage 1, the forward market, each producer acts in order to maximize its von-Neumann-Morgenstern expected utility,

$$U_i(\mathbf{x}) = \mathrm{E}\left(u_i\left[\pi_i(\varepsilon, \mathbf{x})\right]\right) = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} u_i\left[\pi_i(\varepsilon, \mathbf{x})\right] f(\varepsilon) d\varepsilon. \tag{2}$$

5 ANALYSIS

The subgame perfect Nash equilibrium of the two-stage game can be solved using backward induction. Sequentially rational real-time market bids are analysed in Section 5.1. Section 5.2 uses comparative statics to analyse the influence of forward sales on the equilibrium in the real-time market. Given the sequentially rational real-time market bids, the conditions for optimal contracting can be derived in Section 5.3. Section 5.4 isolates contracting incentives related to the strategic Allaz and Vila effect by assuming that the tails of the demand distribution are very thin.

5.1 The real-time market

In the last stage of the game, producers observe/infer competitors' forward contracts and then submit net-supply function offers to the uniform-price auction of the real-time market. It is a standard assumption in the SFE literature to consider ex-post optimal SFE (Klemperer and Meyer, 1989; Green and Newbery, 1992), which Anderson and Hu (2008a, 2008c) refer to as strong SFE. As a consequence of this assumption, there will be a one-to-one correspondence between the real-time price p and the demand shock ε. Thus, in the subgame equilibrium of the real-time market, an arbitrary firm i will, given the competitor's real-time market bid, $s_{-i}(p)$, choose its net-supply function such that its profit (and utility) is independently maximized for each price and the corresponding shock outcome. For a given shock outcome, the utility of the pay-off in (1) can be written as

$$u_i\left[\pi_i\left(\varepsilon, \mathbf{x}\left(p_f^*\right)\right)\right] = u_i\left[p_f^* x_i\left(p_f^*\right) + p\underbrace{\left[\varepsilon + D(p) - x\left(p_f^*\right) - s_{-i}(p)\right]}_{s_i} - C\underbrace{\left(\varepsilon + D(p) - x_{-i}\left(p_f^*\right) - s_{-i}(p)\right)}_{s_i + x_i}\right]. \tag{3}$$

By differentiating this expression with respect to p, we get

$$\frac{\partial u_i\left[\pi_i\left(\varepsilon, \mathbf{x}\left(p_f^*\right)\right)\right]}{\partial p} = u_i'(\cdot)\left\{\underbrace{\left[\varepsilon + D(p) - x\left(p_f^*\right) - s_{-i}(p)\right]}_{s_i} + \left[p - C'\underbrace{\left(\varepsilon + D(p) - x_{-i}\left(p_f^*\right) - s_{-i}(p)\right)}_{s_i + x_i}\right]\left[D'(p) - s_{-i}'(p)\right]\right\}. \tag{4}$$

Thus, as in Anderson and Xu (2005), sequentially rational real-time market bids (net of forward contracts) satisfy the first-order condition derived by Klemperer and Meyer (1989) for any level of forward contracting[5]

$$s_i + \left[\frac{\partial D}{\partial p} - \frac{\partial s_{-i}}{\partial p}\right]\left[p - C_i'(q_i)\right] \equiv 0. \tag{5}$$

The corresponding conditions are derived by Newbery (1998b), Green (1999a), and Chung et al. (2004) for cases with constant and linear marginal costs. The equation in (5) can be rewritten as follows

$$\frac{p - C_i'(q_i)}{p} = \frac{-s_i / p}{\frac{\partial D}{\partial p} - \frac{\partial s_{-i}}{\partial p}}.$$

An intuitive interpretation of the KM condition is that each producer acts as a monopolist with respect to its residual demand curve for each shock outcome and the optimal price of a

[5] Note that the notation used in this paper is different from that in Anderson and Xu (2005). In their paper, output is denoted by $S_i(p)$ and forward sales by Q_i.

producer is given by the inverse elasticity rule (Tirole, 1988) for each shock outcome. As the profit is maximized independently for each shock outcome, this implies that the optimal supply curves in the real-time market do not depend on risk aversion or on the probability distribution of the shock outcomes.

With sufficiently inelastic demand in the duopoly market, no capacity constraints bind below the price cap and Klemperer and Meyer's (1989) first-order condition is satisfied for both firms in the interval $\left(p_0^{\max}, \overline{p}\right)$. Thus, from (5) we get

$$\begin{cases} s_1 + \left[\dfrac{\partial D}{\partial p} - \dfrac{\partial s_2}{\partial p}\right]\left[p - C_1'(s_1 + x_1)\right] = 0 \\ s_2 + \left[\dfrac{\partial D}{\partial p} - \dfrac{\partial s_1}{\partial p}\right]\left[p - C_2'(s_2 + x_2)\right] = 0 \end{cases} \quad \forall p \in \left(p_0^{\max}, \overline{p}\right). \tag{6}$$

It is straightforward to rewrite the system of differential equations on the standard form:

$$\begin{cases} \dfrac{\partial s_2}{\partial p} = \dfrac{\partial D}{\partial p} + \dfrac{s_1}{p - C_1'(s_1 + x_1)} \\ \dfrac{\partial s_1}{\partial p} = \dfrac{\partial D}{\partial p} + \dfrac{s_2}{p - C_2'(s_2 + x_2)} \end{cases} \quad \forall p \in \left(p_0^{\max}, \overline{p}\right). \tag{7}$$

Anderson and Hu (2008c) show that in an ex-post optimal (strong) SFE, the supply function $s_i(p)$ satisfies one of the following properties at each price:[6]

(a) if $p<C'_i(x_i(p_f^*))$ then $s_i(p)<0$ and $p<C'_i(q_i)$,

(b) $s_i(C'_i(x_i(p_f^*))) = 0$, (8)

(c) if $p>C'_i(x_i(p_f^*))$ then $s_i(p)>0$ and $p >C'_i(q_i)$.

Under the long-tails assumption all market prices between the price floor and cap occur with a positive probability. Hence, a necessary property of solutions satisfying (6) is that firms with a non-binding capacity constraint and non-binding price cap must face a smooth residual demand. If demand is sufficiently inelastic, the supply functions of both firms will have a positive slope bounded away from zero. This implies that no capacity constraint can bind below the price cap. Hence, the capacity of one firm, typically the smaller firm, starts to bind exactly when the real-time price reaches the price cap (Holmberg, 2007; Holmberg, 2009). Without loss of generality I refer to this as firm 2. The other firm, which I refer to as firm 1, offers its remaining net-supply capacity with a perfectly elastic segment along the

[6] Note that this is stated somewhat differently in Anderson and Hu (2008c), as s_i represents gross-supply in their paper.

binding price cap (Holmberg, 2007; Holmberg, 2009). The length of this segment is chosen such that there is a non-decreasing solution for the interval $\left(p_0^{\max}, \overline{p}\right)$ (Holmberg, 2007; Holmberg, 2009). Thus, the end-conditions of the system of first-order equations are:

$$\begin{cases} s_1\left(\overline{p}-\right) \le k_1 - x_1\left(p_f^*\right) \\ s_2\left(\overline{p}\right) = k_2 - x_2\left(p_f^*\right). \end{cases} \tag{9}$$

Moreover, it follows from (8) that the solution of firm m, the one with the highest marginal cost at its contracting point, must satisfy

$$s_m\left(p_0^{\max}\right) = 0. \tag{10}$$

Aromi (2007) considers a duopoly market with perfectly inelastic demand, a price cap and a positive loss of load probability. He verifies that there exists a unique monotonic solution to the system of differential equations in (7) when producers are uncontracted, i.e. both firms have non-negative net-supply. With contracts it follows from (8) that both firms have positive net-supply for prices $p \ge p_0^{\max}$. Thus in this price-range, Aromi's (2007) argument can be applied also to firms with given contract levels, because as a function of net-supply, contracting simply shifts the marginal cost curve and capacity constraints. The equilibrium in the real-time market is illustrated in Fig. 2.

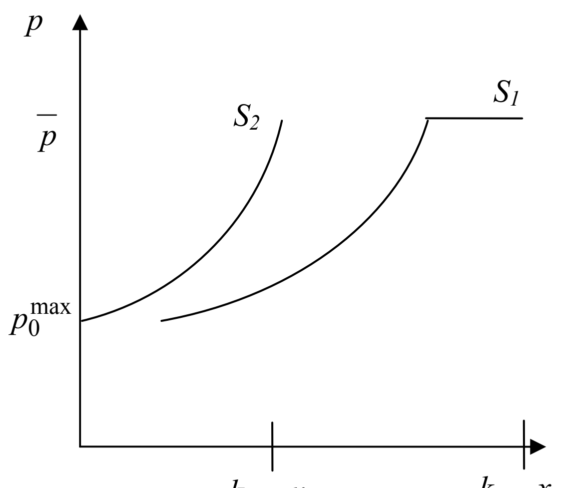

Figure 2. *The capacity constrained SFE in a duopoly market. In this case m=2, i.e. firm 2, has the highest marginal cost at its contracting point.*

5.2 The influence of strategic contracting on capacity constrained SFE

The strategic commitment effect is determined by the sensitivity of the SFE in the real-time market to marginal changes in forward sales. I use the assumption of sufficiently long tails in the demand distribution to pin down a unique subgame equilibrium of the real-time market for the price range $\left(p_0^{\max}, \overline{p}\right)$, and I will perform a comparative statics analysis of the unique equilibrium. Analogous results can be derived for the range $\left(\underline{p}, p_0^{\min}\right)$. Unfortunately a comparative statics analysis cannot be performed for the range $\left(p_0^{\min}, p_0^{\max}\right)$, which would be relevant for asymmetric equilibria such that producers have different marginal costs at zero net-supply.

The equations in (7) are valid for any subgame, i.e. for any level of forward contracts. Thus, we can perform comparative statics by differentiating these expressions with respect to the contracting level, x_i, of one producer, while we keep the contracting level of the other producer, j, fixed. After changing the order of differentiation for terms that are also differentiated with respect to the price, we get a new system of differential equations that can be used to determine the marginal shifts $\delta_{ki}(p)=\frac{\partial s_k(p)}{\partial x_i}$ in the net-supply curves. Before stating this result, it is useful to introduce the following definitions:

$$T_i(p)=\frac{s_i(p)C_i''(q_i(p))}{p-C_i'(q_i(p))} \tag{11}$$

and

$$\delta_{ii}^*(p)=\frac{-T_i(p)}{1+T_i(p)}. \tag{12}$$

Proposition 1. Let $i, j \in 1,2$ and $j \neq i$. Assume that firm i increases its forward sales by an infinitesimally small amount, dx_i. Then, the resulting marginal shifts, $\delta_{ki}(p)=\frac{\partial s_k(p)}{\partial x_i}$, in the sequentially rational net-supply curves satisfy the following system of differential equations:

$$\begin{cases} \dfrac{\partial \delta_{ji}(p)}{\partial p}=\dfrac{\delta_{ii}(p)\left[p-C_i'\left(s_i(p)+x_i\left(p_f^*\right)\right)\right]+s_i(p)C_i''\left(s_i(p)+x_i\left(p_f^*\right)\right)\left(1+\delta_{ii}(p)\right)}{\left[p-C_i'\left(s_i(p)+x_i\left(p_f^*\right)\right)\right]^2} \\ \dfrac{\partial \delta_{ii}(p)}{\partial p}=\dfrac{\delta_{ji}(p)\left[p-C_j'\left(s_j(p)+x_j\left(p_f^*\right)\right)\right]+s_j(p)C_j''\left(s_j(p)+x_j\left(p_f^*\right)\right)\delta_{ji}(p)}{\left[p-C_j'\left(s_j(p)+x_j\left(p_f^*\right)\right)\right]^2} \end{cases}$$

$\forall p \in \left(p_0^{\max}, \overline{p}\right)$ and the following boundary conditions

$$\delta_{21}\left(\overline{p}\right)=0,\ \delta_{22}\left(\overline{p}\right)=-1,\ \delta_{mn}\left(p_0^{\max}\right)=0, \text{ and } -1<\delta_{mm}\left(p_0^{\max}\right)=\delta_{mm}^*\left(p_0^{\max}\right)\leq 0.$$

Proof: See the appendix.

Using (11) we can also write the first differential equation as follows:

$$\frac{\partial \delta_{ji}(p)}{\partial p} = \frac{\delta_{ii}(p) + T_i(p)(1+\delta_{ii}(p))}{p - C_i'\left(s_i(p) + x_i\left(p_f^*\right)\right)}.$$

By means of (12) it is now straightforward to show that

$$\frac{\partial \delta_{ji}(p)}{\partial p} \begin{cases} > 0 & \text{if } \delta_{ii}(p) > \delta_{ii}^*(p) \\ = 0 & \text{if } \delta_{ii}(p) = \delta_{ii}^*(p) \\ < 0 & \text{if } \delta_{ii}(p) < \delta_{ii}^*(p). \end{cases}$$

Morerover, it follows from Proposition 1 that $\frac{\partial \delta_{ii}(p)}{\partial p}$ has the same sign as $\delta_{ji}(p)$. From the signs of the derivatives $\frac{\partial \delta_{ii}(p)}{\partial p}$ and $\frac{\partial \delta_{ji}(p)}{\partial p}$ we can draw the following phase portrait, which indicates the trajectories of solutions to the differential equation in Proposition 1 for different states *(δ_{ii},δ_{ji})*.

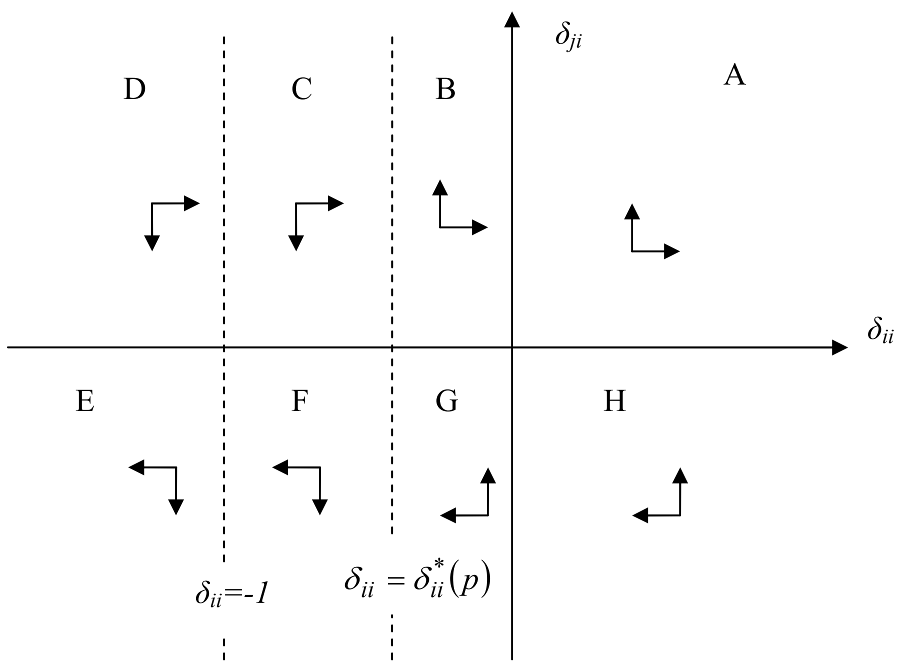

Figure 3. *The phase portrait of the system of differential equations in* Proposition 1 for *$p > p_0^{max}$.*

It is clear from the phase portrait that the point $\delta_{ii}(p)=\delta_{ii}^{*}(p)$ and δ_{ji}=0 is a saddle point. We also note that the solution must start in region *D* or *H* to end up in one of these regions, but this would violate the boundary conditions at $p_0^{\max}$ in Proposition 1. Moreover, we realize that if the solution ever enters region *A* or *E* then the solution has to stay inside these regions, which would violate the boundary conditions at $\overline{p}$ in Proposition 1. Thus we can rule out solutions where δ_{ii}<-1 or δ_{ii}>0 and conclude

Proposition 2. If firm *i* increases its forward sales by an infinitesimally small amount dx_i, then its sequentially rational net-supply curve, $s_i(p)$, shifts (weakly) inwards by an amount (weakly) less than dx_i, i.e. $-1 \le \delta_{ii}(p)=\frac{\partial s_i(p)}{\partial x_i} \le 0 \;\; \forall p \in \left(p_0^{\max}, \overline{p}\right)$.

The proven inequality $-1 \le \delta_{ii}(p)=\frac{\partial s_i(p)}{\partial x_i}$ implies that firm *i* will shift its output curve

$q_i(p)=s_i(p)+x_i$ (weakly) outwards when it increases its forward sales, i.e. $\frac{\partial q_i(p)}{\partial x_i} \ge 0$.

Thus, as expected, increased forward sales constitute a credible commitment to increased output. How competitor *j* responds to the aggressive commitment of producer *i* depends on whether it will face a more or less elastic residual demand. In the following we will show that the response is very much related to the shape of $T_i(p)$, which determines the behaviour of $\delta_{ii}^{*}(p)$. We first consider the case when $T_i(p)$ is non-decreasing. We can show that *j* will shift its net-supply curve inwards, i.e. the response is soft, under the following circumstances:

Proposition 3. Consider a market equilibrium where $T_i(p)$ is non-decreasing $\forall p \in \left(p_0^{\max}, \overline{p}\right)$. Now, if firm *i* increases its forward sales by an infinitesimally small amount dx_i, then firm *j* shifts its net-supply curve (weakly) inwards, i.e. strategies are substitutes and

$\delta_{ji}(p)=\frac{\partial s_j(p)}{\partial x_i} \le 0 \; \forall p \in \left(p_0^{\max}, \overline{p}\right)$.

Proof: We know from Proposition 2 that the solution cannot enter the regions: *A, D, E* and *H* in Fig. 3. A non-decreasing $T_i(p)$ implies that $\frac{d\delta_{ii}^{*}(p)}{dp}=\frac{-T_i'(p)}{\left(1+T_i(p)\right)^2} \le 0$, so the solution will be trapped in region *B* if the state ever enters this region. But this would violate the boundary conditions $\delta_{21}(\overline{p})=0$ (if i=1) and $\delta_{22}(\overline{p})=-1$ (if i=2). Hence, the solution can never enter region *B*. This implies that the solution can never enter region *C* unless it starts there. But starting in this region is not possible, because it would violate the boundary conditions $\delta_{mn}\left(p_0^{\max}\right)=0$ (if i=n) and $\delta_{mm}\left(p_0^{\max}\right)=\delta_{mm}^*\left(p_0^{\max}\right)$ (if i=m). □

Using (5) and (11) we can write

$$T_i(p)=\left(s'_j(p)-D'(p)\right)C_i''(q_i(p)), \tag{13}$$

and accordingly strategies are substitutes in the range $\left(p_0^{\max},\overline{p}\right)$ when marginal costs are convex and residual demand, $D(p)-s_j(p)$, is concave. As argued in Section 2, it is reasonable to assume that marginal costs are convex in many electricity markets. In equilibrium, a net-supply function, $s_i(p)$, is normally concave when the price cap selects a solution to (7) among the upper trajectories, but $s_i(p)$ is normally convex for solutions among the lower trajectories (Klemperer and Meyer, 1989; Green and Newbery, 1992).[7] With little elasticity in the demand curve, the latter would result in concave residual demand. Thus it seems that reducing price caps (and possibly introducing capacity payments as compensation to producers) would stimulate strategic contracting in the right direction.

The following result proves that strategies are generally substitutes for firm 2 when prices are sufficiently close to the price cap. Intuitively, we can explain this by the following argument. When firm 2's capacity constraints binds, which it does at the price cap, firm 2 must shift its net-supply curve inwards by the same amount as it increases its forward sales. Inward shifts of the net-supply curve cannot be larger (because of Proposition 1) at lower prices. Thus the elasticity of firm 2's supply curve cannot increase at the price cap, and consequently firm 1's response to firm 2's increased forward sales will be to shift its supply near the price cap (weakly) inwards.

Proposition 4. Strategies are substitutes for firm i=2*,* i.e. $\delta_{12}(p)=\dfrac{\partial s_1(p)}{\partial x_2}\leq 0$, in some range $\left(p^*,\overline{p}\right)$.

[7] Note that I am referring to convexity/concavity of the function $s_i(p)$, i.e. price on the horizontal axis and supply on the vertical axis.

Proof: We know from Proposition 2 that the solution cannot enter the regions: *A, D, E* and *H* in Fig. 3. It follows from the phase portrait that we cannot meet the boundary condition $\delta_{22}(\bar{p}) = -1$ in region C, as it would imply that $\delta_{22}(p) < -1$ for prices slightly below $\bar{p}$. Hence, it follows from the phase portrait that the solution must be in region *F* for prices slightly below the price cap. □

We now show that strategies are complements for all $p \in \left(p_0^{\max}, \bar{p}\right)$ if i=1 and

$T_1(p) = \dfrac{s_1(p) C_1''(q_1(p))}{p - C_1'(q_1(p))}$ is non-increasing in this interval.

Proposition 5. If $T_1(p)$ is non-increasing $\forall p \in \left(p_0^{\max}, \bar{p}\right)$, then strategies are complements for firm i=1, i.e. $\delta_{21}(p) = \dfrac{\partial s_2(p)}{\partial x_1} \geq 0 \quad \forall p \in \left(p_0^{\max}, \bar{p}\right)$.

Proof: We know from Proposition 2 that the solution cannot enter the regions: *A, D, E* and *H* in Fig. 3. A non-increasing $T_1(p)$ implies that $\dfrac{d\delta_{11}^*(p)}{dp} = \dfrac{-T_1'(p)}{(1 + T_1(p))^2} \geq 0$, so the state will be trapped in region *F* if the solution ever enters this region. But this would violate the boundary condition $\delta_{21}(\bar{p}) = 0$. Hence, the state can never enter region *F*. The state can never enter region *G* unless it starts there. But starting in this region is not possible, because it would violate the boundary conditions $\delta_{mn}\left(p_0^{\max}\right) = 0$ (if i=n) and $\delta_{mm}\left(p_0^{\max}\right) = \delta_{mm}^*\left(p_0^{\max}\right)$ (if i=m). □

We get the following by combining Propositions 3 and 5.

Corollary 1. If $T_1(p) = \dfrac{s_1(p) C_1''(q_1(p))}{p - C_1'(q_1(p))}$ is constant $\forall p \in \left(p_0^{\max}, \bar{p}\right)$, then strategies are neutral for firm i=1, i.e. $\delta_{21}(p) = 0 \ \ \forall p \in \left(p_0^{\max}, \bar{p}\right)$.

This verifies Green's (1999a) result that strategies are neutrals if marginal costs, demand and supply functions are linear. The linear SFE considered by Green is a unique SFE for unbounded shocks and infinite capacity constraints (Klemperer and Meyer, 1989). Thus, the linear SFE is related to the capacity-constrained SFE. But with a price cap and a price floor at infinite prices, the boundary conditions at these points are no longer effective, and the result in Corollary 1 applies to both firms.

With constant marginal costs, we get $T_i(p) = \frac{s_i(p) C_i''(q_i(p))}{p - C_i'(q_i(p))} = 0$ and $\delta_{ii}^*(p) = 0$. This removes region B and G in the phase portrait in Fig. 3.

Proposition 6. Consider a market where producer i has constant marginal costs. If firm i has a horizontal segment at the price cap (i=1), then the sequentially rational response of firm j=2 is neutral, i.e. $\delta_{21}(p) = \frac{\partial s_2(p)}{\partial x_1} = 0$. If firm i does not have a horizontal segment at the price cap (i=2), then strategies are substitutes, i.e. $\delta_{12}(p) = \frac{\partial s_1(p)}{\partial x_2} \le 0 \quad \forall p \in \left(p_0^{\max}, \bar{p}\right)$.

Proof: The result that $\delta_{ii}^*(p) = 0$ removes both region B and G in the phase portrait in Fig. 3. The state cannot start in region C, because this would violate the initial conditions $\delta_{mn}\left(p_0^{\max}\right) = 0$ or $\delta_{mm}\left(p_0^{\max}\right) = \delta_{mm}^*\left(p_0^{\max}\right) = 0$. This leaves us with two remaining options, either the solution is stuck in region F or stuck in the saddle point $\delta_{ii}(p) = \delta_{ji}(p) = 0$. If i=1, the former would violate the end-condition $\delta_{21}\left(\bar{p}\right) = 0$. If i=2, the other case would violate the end-condition $\delta_{22}\left(\bar{p}\right) = -1$. □

Note that the result $\delta_{21}(p) = \frac{\partial s_2(p)}{\partial x_1} = 0$ corresponds to one of the equilibria selected in Newbery's (1998b) analysis of strategic contracting in markets with constant marginal costs.

It is straightforward to make an analogous analysis for prices $p<p_0^{min}$. Production needs to be non-negative; a firm cannot buy back more power than it has sold in the forward market. In the unique equilibrium, this non-negative production constraint of one firm, typically the smaller firm, starts to bind exactly when the real-time price reaches the price floor (Holmberg, 2007). Without loss of generality I refer to this as firm β, which does not necessarily equal 2. The other firm, which I refer to as firm α, buys back its remaining forward sales with a perfectly elastic segment along the binding price floor (Holmberg, 2007). Again the argument by Aromi (2007) can be used to verify that there exists a unique monotonic solution to the

system of differential equations in (7) and the boundary conditions analogous to (9) and (10) for prices in the range $p \leq p_0^{\min}$.

The system of differential equations in Proposition 1 is still valid, but (8) implies that mark-ups and net-supply are negative below the contracting points. Hence, the signs of the derivatives $\frac{\partial \delta_{ii}(p)}{\partial p}$ and $\frac{\partial \delta_{ji}(p)}{\partial p}$ change and accordingly also the direction of each arrow in the phase portrait. It is straightforward to verify that the relevant boundary conditions for this region are: $\delta_{\beta\alpha}(\underline{p}) = 0$, $\delta_{\beta\beta}(\underline{p}) = -1$, $\delta_{nm}(p_0^{\min}) = 0$, and $-1 < \delta_{nn}(p_0^{\min}) = \delta_{nn}^*(p_0^{\min}) \leq 0$.

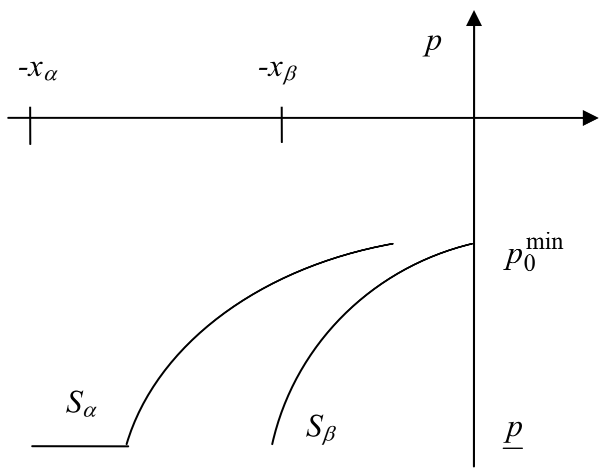

Figure 4. *The capacity constrained SFE in a duopoly market* for $p<p_0^{min}$. *In this case* $n=\beta$, *i.e. firm* β *has the lowest marginal cost at its contracting point.*

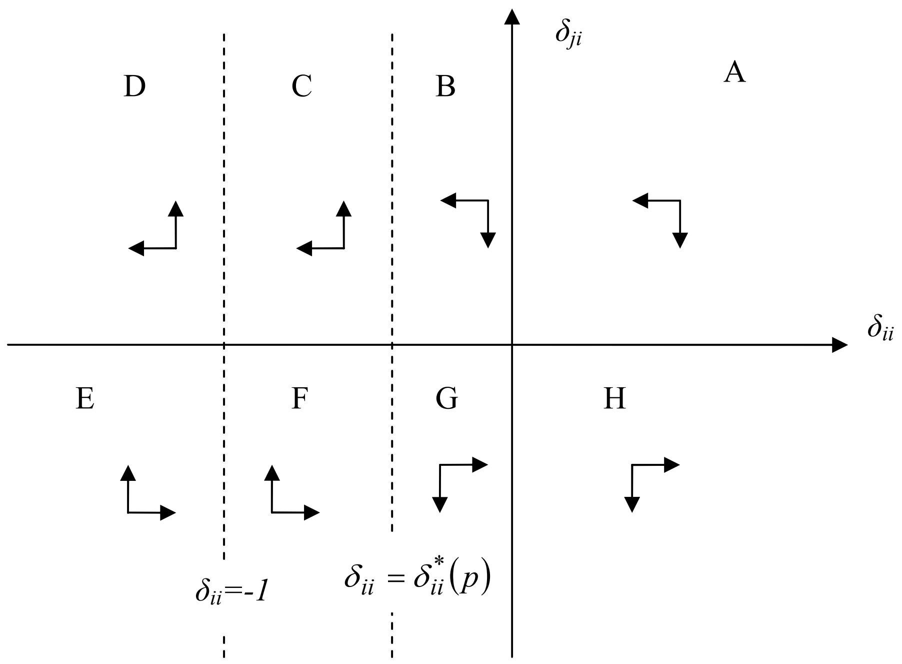

Figure 5. *The phase portrait of the system of differential equations in* Proposition 1 for $p<p_0^{min}$.

Proposition 7. Consider an SFE of a duopoly market. Assume that firm *i* increases its forward sales by an infinitesimally small amount dx_i. Then

i) The producer's sequentially rational net-supply curve, $s_i(p)$, shifts (weakly) inwards by an amount (weakly) less than dx_i, i.e. $-1 \le \delta_{ii}(p) = \dfrac{\partial s_i(p)}{\partial x_i} \le 0$ $\forall p \in \left(\underline{p}, p_0^{\min}\right)$, and the total output curve shifts (weakly) outwards.

ii) If $T_i(p)$ is non-increasing $\forall p \in \left(\underline{p}, p_0^{\min}\right)$, then the sequentially rational response of firm *j* is to shift its net-supply curve (weakly) inwards, i.e. $\delta_{ji}(p) = \dfrac{\partial s_j(p)}{\partial x_i} \le 0$ $\forall p \in \left(\underline{p}, p_0^{\min}\right)$.

iii) If firm *i* does not have a horizontal segment at the price floor (*i=β),* then the sequentially rational response of firm *j=α* is to shift its net-supply curve (weakly) inwards, i.e. $\delta_{\alpha\beta}(p) = \dfrac{\partial s_\alpha(p)}{\partial x_\beta} \le 0$ in some range $\left(\underline{p}, p^*\right)$.

iv) If firm *i* has a horizontal segment at the price floor ($i=\alpha$) and $T_\alpha(p)$ is non-decreasing $\forall p \in \left(\underline{p}, p_0^{\min}\right)$, then the sequentially rational response of firm $j=\beta$ is to shift its net-supply curve weakly outwards, i.e. $\delta_{\beta\alpha}(p) = \dfrac{\partial s_\beta(p)}{\partial x_\alpha} \geq 0$ $\forall p \in \left(\underline{p}, p_0^{\min}\right)$.

v) If firm *i* has a horizontal segment at the price floor ($i=\alpha$) and $T_\alpha(p)$ is constant $\forall p \in \left(\underline{p}, p_0^{\min}\right)$, then the sequentially rational response of firm $j=\beta$ is neutral. Its supply curve is unchanged, i.e. $\delta_{\beta\alpha}(p) = 0 \quad \forall p \in \left(\underline{p}, p_0^{\min}\right)$.

vi) Consider a market where producer *i* has constant marginal costs. If firm *i* has a horizontal segment at the price floor ($i=\alpha$), then the sequentially rational response of firm $j=\beta$ is neutral, i.e. $\delta_{\beta\alpha}(p) = \dfrac{\partial s_\beta(p)}{\partial x_\alpha} = 0$. If firm *i* does not have a horizontal segment at the price floor *($i=\beta$),* then the sequentially rational response of firm $j=\alpha$ is to shift its net-supply curve (weakly) inwards, i.e. $\delta_{\alpha\beta}(p) = \dfrac{\partial s_\alpha(p)}{\partial x_\beta} \leq 0 \quad \forall p \in \left(\underline{p}, p_0^{\min}\right)$.

Proof: Follows from the phase portrait in Fig. 5 and arguments analogous to proofs of Propositions 1 to 6 and Corollary 1.

We realize from the second claim in this proposition and (13) that strategies are substitutes in the range $\forall p \in \left(\underline{p}, p_0^{\min}\right)$ when marginal costs are concave and residual demand is convex. But in practice one would expect marginal costs to be convex, and as the distance between the marginal cost and price floor is normally relatively short, the shape of the residual demand is uncertain in this range. There is a considerable risk that strategies will be complements (anti-competitive) for firm α.

We have seen from the phase portraits in Fig 3 and Fig. 5 that in the regions where both producers are either over- or under-contracted, the solution of the system of differential equations in Proposition 1 has a saddle point. But if one would draw a phase portrait for the range where $p \in \left(p_0^{\min}, p_0^{\max}\right)$ the solution would change character to a spiral, and the arguments used to derive conditions for pro- and anti-competitive contracting would not work. This concurs with the analysis of Anderson and Hu (2008c), who claim that a unique equilibrium cannot be found for this range and provides numerical examples of this. The reason is that producers can have perfectly elastic segments at competitor's contracting points and the length of these segments cannot be uniquely determined (see Fig. 6). This implies that a comparative statics analysis cannot be performed for this price range.

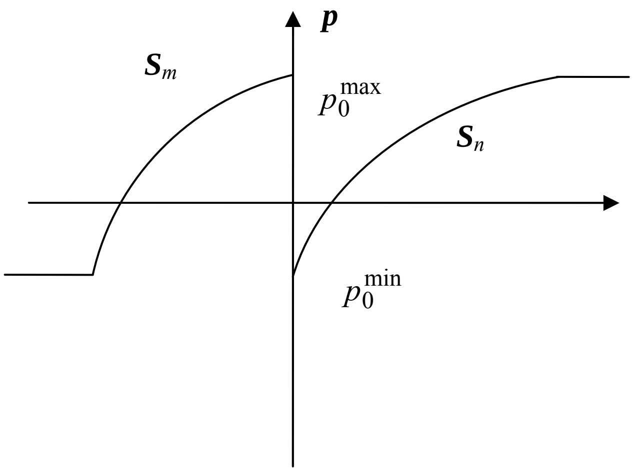

Figure 6. *The capacity constrained SFE in a duopoly market between the contracting points cannot be uniquely determined.*

Increased sales in the forward market cannot increase the forward price. As a competitor's forward supply is non-decreasing, this implies that the competitor's forward sales cannot increase if producer i increases its sales, i.e. $\frac{\partial x_j}{\partial p_f}\frac{dp_f}{dx_i} \le 0$. As a consequence of Proposition 2 and the first point in Proposition 7, we have $\frac{\partial q_j}{\partial x_j} = 1 + \frac{\partial s_j}{\partial x_j} \ge 0$. Hence, we can draw the following conclusion:

Corollary 2. In a duopoly market it is always the case that $\frac{\partial q_j(p,\mathbf{x})}{\partial x_j}\frac{\partial x_j}{\partial p_f}\frac{dp_f}{dx_i} \le 0$

$\forall p \in \left(\underline{p}, p_0^{\min}\right)$ and $\forall p \in \left(p_0^{\max}, \overline{p}\right)$.

This proves that reducing competitors' forward positions by selling in the forward market has a softening effect on competitors' output, i.e. strategic forward price manipulation gives incentives to increased forward sales.

5.3 The forward market

Proceeding with the backward induction method, I will now calculate the optimal contracting level given sequentially rational bidding strategies in the real-time market. This analysis applies to any number of competitors and I let q_{-i} denote the total output of competitors to firm i. The total profit of firm i (including forward sales) for a given shock realisation ε is

$$\begin{aligned}\pi_i\left(\varepsilon,\mathbf{x}\left(p_f^*\right)\right) &= \underbrace{\left[s_i\left(p\left(\varepsilon,\mathbf{x}\left(p_f^*\right)\right),\mathbf{x}\left(p_f^*\right)\right)+x_i\left(p_f^*\right)\right]}_{q_i}p\left(\varepsilon,\mathbf{x}\left(p_f^*\right)\right) \\ &+ x_i\left(p_f^*\right)\left[p_f^* - p\left(\varepsilon,\mathbf{x}\left(p_f^*\right)\right)\right] - C_i\underbrace{\left[s_i\left(p\left(\varepsilon,\mathbf{x}\left(p_f^*\right)\right),\mathbf{x}\left(p_f^*\right)\right)+x_i\left(p_f^*\right)\right]}_{q_i}.\end{aligned} \tag{14}$$

The first term is the revenue in the real-time market. The second term is the arbitrage profit and the third term is the total cost. Strategic forward contracting is driven by shifts in the competitor's output curve. To analyse these shifts, firm i's output is expressed in demand and the competitor's output. Thus (14) can be written:

$$\begin{aligned}\pi_i &= \overbrace{\left[D\left(p(\varepsilon,\mathbf{x}),\varepsilon\right)-q_{-i}\left(p(\varepsilon,\mathbf{x}),\mathbf{x}\right)\right]}^{q_i}p(\varepsilon,\mathbf{x})+x_i\left[p_f^*-p(\varepsilon,\mathbf{x})\right] \\ &- C_i\underbrace{\left[D\left(p(\varepsilon,\mathbf{x}),\varepsilon\right)-q_{-i}\left(p(\varepsilon,\mathbf{x}),\mathbf{x}\right)\right]}_{q_i}.\end{aligned} \tag{15}$$

Assume that $q_{-i}\left(p\left(\varepsilon,\mathbf{x}\left(p_f^*\right)\right),\mathbf{x}\left(p_f^*\right)\right)$ is the competitor's sequentially rational (subgame equilibrium) response and let us calculate the equilibrium contracting level of firm i. The marginal change in firm i's profit after a marginal change in x_i is:

$$\begin{aligned}\frac{d\pi_i}{dx_i} &= \left(\frac{\partial D}{\partial p}\frac{dp}{dx_i}-\frac{\partial q_{-i}}{\partial p}\frac{dp}{dx_i}-\frac{\partial q_{-i}}{\partial x_i}-\sum_{j\neq i}\frac{\partial q_{-i}}{\partial x_j}\frac{dx_j}{dx_i}\right)\left[p-C_i'(q_i)\right] \\ &+ q_i\frac{dp}{dx_i}+p_f^*-p(\varepsilon,\mathbf{x})+x_i\left(\frac{dp_f^*}{dx_i}-\frac{dp}{dx_i}\right).\end{aligned} \tag{16}$$

Note that the total derivative $\frac{dp}{dx_i}$ not only includes the direct effect $\frac{\partial p}{\partial x_i}$ but also indirect changes in the real-time price via changes in competitors' forward contracting levels due to changes in the forward price. We see that the profit of firm i for a given shock realisation ε can change through four mechanisms. 1) Changes in output, both because of shifts in competitors' output curve and a changed real-time price. The additional profit is then given by the increased output times the mark-up, $p-C_i'(q_i)$. 2) Changes in the real-time price also change the mark-up for total sales, q_i. 3) The arbitrage profit on any additional contract is given by $p_f^* - p(\varepsilon, \mathbf{x})$. 4) The arbitrage profit from the firm's contracts changes if

$\frac{dp_f^*}{dx_i} \neq \frac{dp}{dx_i}$.

Using the assumption that real-time market bids are sequentially rational we will now simplify the expression in (16). However, we first note that $s_i = q_i - x_i\left(p_f^*\right)$ and that $\frac{\partial s_{-i}}{\partial p} = \frac{\partial q_{-i}}{\partial p}$, because $\frac{\partial x_{-i}}{\partial p} = 0$. Thus (16) can be written in the following form:

$$\frac{d\pi_i}{dx_i} = \frac{dp}{dx_i}\left\{\left(\frac{\partial D}{\partial p} - \frac{\partial s_{-i}}{\partial p}\right)\left[p - C_i'(q_i)\right] + s_i\right\}$$
$$-\left(\frac{\partial q_{-i}}{\partial x_i} + \sum_{j \neq i} \frac{\partial q_{-i}}{\partial x_j}\frac{dx_j}{dx_i}\right)\left\{p - C_i'(q_i)\right\} + p_f^* - p(\varepsilon, \mathbf{x}) + x_i \frac{dp_f^*}{dx_i}.$$

Sequentially rational real-time market bids satisfy the KM condition in (5) also when firms deviate from their equilibrium forward contracting levels. Thus, the first term is zero and, accordingly,

$$\frac{d\pi_i}{dx_i} = -\left(\frac{\partial q_{-i}}{\partial x_i} + \sum_{j \neq i} \frac{\partial q_{-i}}{\partial x_j}\frac{dx_j}{dx_i}\right)\left[p - C_i'(q_i)\right] + p_f^* - p(\varepsilon, \mathbf{x}) + x_i \frac{dp_f^*}{dx_i}. \tag{17}$$

Note that all $\frac{dp}{dx_i}$ derivatives have been cancelled out. Thus, in a subgame perfect Nash equilibrium, a marginal change in the real-time price in itself has no impact on the profit. This result and a corresponding result in the analysis of business strategies in sequential markets (Tirole, 1988) can be explained by the envelope theorem (Mas-Colell et al., 1995). With sequentially rational real-time market bids, the real-time price of the subgame equilibrium maximizes a firm's profit for each shock outcome. In a profit maximum, a marginal change in the real-time price has no first-order effect on the profit. From (17), we realize that, in equilibrium, there are only three transmission mechanisms left through which firm *i*'s profit is influenced by a marginal change in x_i: 1) the shift in the competitor's output curve, i.e. the extent to which the response is soft, 2) the arbitrage profit on the additional contract and 3) changes in the forward price.

In equilibrium, firm *i* maximises its expected utility and its forward contracting level is given by the condition

$$\frac{dE[u_i(\pi_i(\varepsilon,\mathbf{x}))]}{dx_i} = \int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \frac{d\pi_i(\varepsilon,\mathbf{x})}{dx_i} u_i{}'(\pi_i(\varepsilon,\mathbf{x})) f(\varepsilon) d\varepsilon = 0. \tag{18}$$

Hence, it follows from (17) that firm i's optimal forward position is implicitly determined by:

$$\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left\{ -\left(\frac{\partial q_{-i}}{\partial x_i} + \sum_{j \neq i} \frac{\partial q_{-i}}{\partial x_j} \frac{dx_j}{dx_i} \right) [p - C_i'(q_i)] + p_f^* - p(\varepsilon,\mathbf{x}) + x_i \frac{dp_f^*}{dx_i} \right\} u_i{}'(\pi_i(\varepsilon,\mathbf{x})) f(\varepsilon) d\varepsilon \tag{19}$$
$$= 0.$$

5.4 Isolating the strategic commitment effect

The subgame perfect Nash equilibrium of the game depends on many parameters, such as uncertainty in the market and risk aversion of the agents. Here, we will isolate the contracting incentives related to the Allaz and Vila effect by making a simplifying assumption on the demand distribution.

Unexpected events in the electricity market do occur, because of unexpected changes in wind and temperature, unexpected production outages or unexpected transmission-line failures etc. A combination of such events can result in large shocks, which are even rarer, motivating the long thin tails of the demand distribution that have been assumed. Assuming that producers can calculate their optimal supply curves at no cost, this will motivate them to choose optimal supply curves for all possible scenarios (even if they are extremely rare). But unexpected events do not normally occur, and in most cases electricity demand and production can be accurately predicted, so the standard deviation of the demand shock is very small. Although there is some uncertainty in real electricity markets, we can simplify our calculations by making the standard deviation smaller and smaller, and the thickness of the tails thinner and thinner. As for lognormal and normal distributions, the support of the shock density is assumed to be unchanged in this process, so the long tail assumption still holds. In the limit, where we make the approximation that the standard deviation approaches zero, the shock outcome is almost certain and the SPNE of the game will be independent of the agents' risk aversion.

In this limit, non-strategic, price-taking retailers will, given producers' equilibrium contracting levels, simply bid their marginal revenue of increased contracting in the forward market, which is to buy everything they can below the almost certain real-time price of the SPNE and sell as much as they can above the almost certain real-time price of the SPNE. Thus, in the limit, retailers' forward demand will be perfectly elastic at the almost certain real-time price of the equilibrium. Offers and bids to the forward market are submitted simultaneously, as in a day-ahead market, and no information about other agents contracting strategies is disclosed until the day-ahead market has closed. Hence, retailers and other players do not have the opportunity to change their contracting strategy if someone chooses a contracting level different from his/her equilibrium strategy. Also they do not have any reason to believe that anyone would deviate from the SPNE, as by definition no player has such incentives. The consequence is that, in the limit, retailers' forward demand will be perfectly elastic at the almost certain real-time price of the SPNE irrespective of producers' deviations. For marginal deviations from contracting levels of the SPNE, this will remove arbitrage opportunities and the influence of forward sales on the forward price. Moreover, the shock outcome is almost certainly known by the producers, and their contracting condition in (19) can be simplified to:

$$\frac{d\pi_i(\varepsilon,\mathbf{x})}{dx_i} = -\frac{\partial q_{-i}}{\partial x_i}\left[p - C_i'(q_i)\right] = 0. \tag{20}$$

In general, a contracting solution x_i^0 satisfying this first-order condition is a global profit maximum if

$$\frac{d\pi_i(\varepsilon,\mathbf{x})}{dx_i} > 0\ \forall x_i < x_i^0 \quad \text{and} \quad \frac{d\pi_i(\varepsilon,\mathbf{x})}{dx_i} < 0\ \forall x_i > x_i^0 .$$

Let $q_i(\mathbf{x})$ be the almost certain dispatch of producer i, given the set of contracting levels $\mathbf{x}\left(p_f^*\right)$. We know from (8) that $p - C_i'(q_i(\mathbf{x})) > 0$ if $q_i(\mathbf{x}) > x_i$ (under-contracting) and $p - C_i'(q_i(\mathbf{x})) < 0$ if $q_i(\mathbf{x}) < x_i$ (over-contracting). Thus, we realize that we have the following potential equilibria:

i) Full contracting for strategic substitutes: $p = C_i'(q_i(\mathbf{x}))$, $q_i(\mathbf{x}) = x_i^0$, and $\frac{\partial q_{-i}}{\partial x_i} \le 0\ \forall x_i$.

ii) Significant under-contracting for tough responses to increased forward sales: $p = \overline{p} > C_i'(q_i(\mathbf{x}))$, $q_i(\mathbf{x}) > x_i^0$, $\frac{\partial q_{-i}}{\partial x_i} \ge 0\ \forall x_i < q_i(\mathbf{x})$, and $\frac{\partial q_{-i}}{\partial x_i} \le 0\ \forall x_i > q_i(\mathbf{x})$.

iii) Locally strategic neutrals with under-contracting: $p > C_i'(q_i(\mathbf{x}))$, $q_i(\mathbf{x}) > x_i^0$,

$\frac{\partial q_{-i}}{\partial x_i} \le 0 \; \forall x_i < x_i^0$, $\frac{\partial q_{-i}}{\partial x_i} = 0$ if $x_i = x_i^0$, $\frac{\partial q_{-i}}{\partial x_i} \ge 0$ if $x_i^0 < x_i < q_i(\mathbf{x})$, and

$\frac{\partial q_{-i}}{\partial x_i} \le 0$ if $x_i > q_i(\mathbf{x})$.

iv) Locally strategic neutrals with over-contracting: $p < C_i'(q_i(\mathbf{x}))$ and $q_i(\mathbf{x}) < x_i^0$,

$\frac{\partial q_{-i}}{\partial x_i} \le 0$ if $x_i < q_i(\mathbf{x})$, $\frac{\partial q_{-i}}{\partial x_i} \ge 0$ if $q_i(\mathbf{x}) < x_i < x_i^0$, $\frac{\partial q_{-i}}{\partial x_i} = 0$ if $x_i = x_i^0$, and

$\frac{\partial q_{-i}}{\partial x_i} \le 0$ if $x_i > x_i^0$.

From a welfare perspective, the competitive outcome with full contracting and zero mark-ups is preferable. It occurs when strategies are substitutes, i.e. competitors' response to increased forward sales is to shift output inwards. This encourages forward sales of the producer as long as mark-ups are positive, but the contracting incentive stops as soon as the producer is fully contracted and mark-ups are zero. If strategies are complements, the strategic producer can shift competitors' output curves inwards by reducing forward sales. This will result in high mark-ups and large revenues for producers. But the outcome is not beneficial to welfare, as it implies underproduction if demand is elastic. There are also potential contracting equilibria where strategies are locally neutral at the contracting point. To ensure a global profit maximum in this case, profits should be strategic complements for contracting levels between $q_i(\mathbf{x})$ and x_i^0, and strategic substitutes for all other contracting levels.

5.4.1 Example with strategic contracting

In a duopoly real-time market where firms have identical constant marginal costs, perfectly inelastic demand and given forward positions, there is a simple unique supply function equilibrium in which both producers bid symmetric linear supply functions for all prices above the price floor and below the price cap (Holmberg, 2007). The net-supply and net-demand capacity will bind for at least one firm (not necessarily the same) at both these prices (Holmberg, 2007). Any excess net-supply of the other firm will be offered with a perfectly elastic segment along the binding price cap and any excess net-demand will be offered with a perfectly elastic segment along the binding price floor (Holmberg, 2007). The outlined equilibrium of the real-time market is shown in Figure 7. To isolate contracting incentives resulting from the strategic commitment, we will continue to assume that the shock outcome is almost certain (even if the tails of the demand distribution are long) so that retailers' forward demand becomes perfectly elastic.

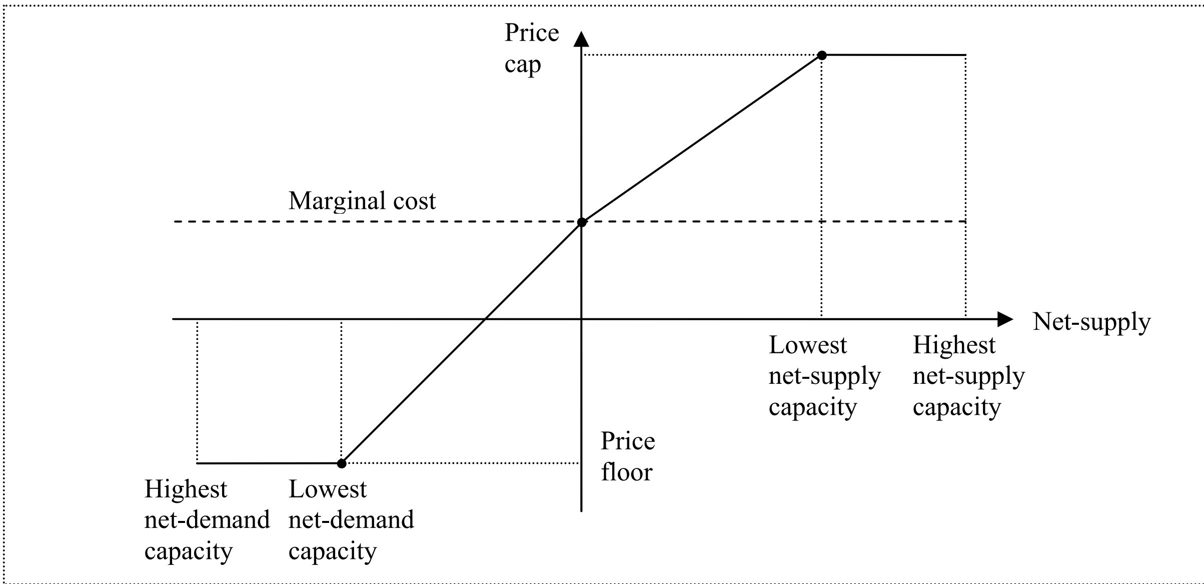

Figure 7. *The capacity constrained SFE of a duopoly market with identical constant marginal costs. Both firms have identical offers between the price floor and cap, but differences occur at the price cap and floor.*

As the equilibrium is symmetric for $p \in \left(\underline{p}, \overline{p}\right)$, we have from Proposition 1 that $\delta_{21}\left(\overline{p}\right) = 0$, $\delta_{11}\left(\overline{p}\right) = 0$, and $\delta_{12}\left(\overline{p}\right) = \delta_{22}\left(\overline{p}\right) = -1$. For identical constant marginal costs, *c*, the system of differential equations in Proposition 1 can be simplified to:

$$\begin{cases} \dfrac{\partial \delta_{ji}(p)}{\partial p} = \dfrac{\delta_{ii}(p)}{p - c} \\ \dfrac{\partial \delta_{ii}(p)}{\partial p} = \dfrac{\delta_{ji}(p)}{p - c}. \end{cases}$$

As both the system of differential equations and the boundary conditions are symmetric, we get a symmetric solution for $p \in \left(c, \overline{p}\right)$:

$$\delta_{ii}(p) = \delta_{ji}(p) = \begin{cases} 0 & \text{if } i = 1 \\ \dfrac{c - p}{\overline{p} - c} & \text{if } i = 2. \end{cases} \tag{21}$$

This result can be explained by the fact that it is producer 2, who has the lowest net-supply capacity, that determines the shape of the symmetric part of the equilibrium. Note also that the result concurs with Proposition 6. Strategies are neutrals for the firm with the higher net-capacity. But with enough forward sales, this firm will become the firm with the lowest net-supply capacity in the market. We have an analogous result for $p \in (\underline{p}, c)$, which is also confirmed by claim vi) in Proposition 7, and, accordingly, strategies are (weakly) substitutes for both firms in the whole price interval. We now have from (20) and (21) that the only possible subgame perfect equilibrium is when $p = C_i'(q_i(\mathbf{x}))$ and the almost certain output $q_i(\mathbf{x})$ equals forward sales, x_i^0. We also see from (21) that total output increases with the contracting level for prices near the contracting point, i.e. $1 + \frac{\partial s_i}{\partial x_i} + \frac{\partial s_j}{\partial x_i} > 0$ if *p-c* is sufficiently close to zero. This ensures that the real-time price crosses the marginal cost once when producer *i* increases its forward sales. Hence, $p - C_i'(q_i(\mathbf{x})) > 0$ if $x_i < x_i^0$ and $p - C_i'(q_i(\mathbf{x})) < 0$ if $x_i > x_i^0$. Together with (20) and the fact that strategies are substitutes, this implies that $\frac{d\pi_i(\varepsilon, \mathbf{x})}{dx_i} > 0 \; \forall x_i < x_i^0$ and $\frac{d\pi_i(\varepsilon, \mathbf{x})}{dx_i} < 0 \; \forall x_i > x_i^0$, which is sufficient for a subgame perfect Nash equilibrium. This is true for both producers, so we will have a subgame perfect equilibrium where both producers are fully contracted and the forward and the real-time price equal the marginal cost. Hence, all production is sold in the forward market.

6 CONCLUSIONS

This paper theoretically analyses strategic contracting in a wholesale electricity market by means of a two-stage game in which producers compete with supply function strategies. Two strategic effects are considered. First, competitors will adjust their strategic bids in the real-time market after observing changes in a producer's contracting position. This is the strategic commitment illustrated by Allaz and Vila (1993). Second, if a producer increases its forward sales, the forward price will decrease and competitors' forward sales will change (along their committed forward supply curve). In turn, this will change competitors' strategic bidding in the real-time market. I call this strategic forward price manipulation.

Sequentially rational offer strategies in the real-time market are calculated by the supply function equilibrium (SFE) with uncertain demand. A unique SFE is determined by the assumption that the tails of the shock distribution are sufficiently long to ensure that the capacity constraint of all firms but possibly the largest bind with a positive probability. The probability can be arbitrarily small. The long-tails assumption is realistic for electricity markets, because the loss of load probability (LOLP) is always positive, even if it is very small when demand is expected to be low. This equilibrium choice is also consistent with the analysis in Green (1999a) and Newbery (1998b).

Applying this assumption to a duopoly market, it is shown that a producer shifts its sequentially rational total output curve (weakly) outwards if it increases its forward sales. Thus positive forward sales is an aggressive commitment to low mark-ups in the real-time market. This also implies that the competitor's response is softer (less-aggressive) if its forward sales can be reduced. Thus strategic forward price manipulation stimulates positive contracting, which is pro-competitive. The strategic commitment is also pro-competitive when strategies are substitutes, i.e. an aggressive commitment is met by a soft response from competitors. It is shown that strategies are substitutes in the region where both firms are under-contracted if a producer has convex marginal costs and faces concave residual demand. A market is more likely to satisfy this pro-competitive condition if the price cap is low. Capacity payments can be introduced as a compensation if the price cap is lowered to maintain investment incentives. The pro-competitive condition is always satisfied for one firm near the price cap and the price floor. With constant marginal costs, strategies are substitutes for one firm and neutrals for the other. But it is likely that strategies will be complements for at least one firm if both firms are over-contracted, and this results in negative (anti-competitive) contracting. Results are ambiguous in the region where one firm is over-contracted and the other under-contracted. Both pro-competitive and anti-competitive effects of the strategic commitment would be amplified if forward positions were disclosed before the real-time market is opened.

Mimicking the design of day-ahead markets, producers and retailers (representing electricity consumers) are assumed to commit to their bid curves simultaneously in the forward market. I isolate contracting incentives for the strategic commitment by evaluating the contracting condition for the simplified case when the tails of the shock density are assumed to be very thin and the standard deviation of the demand shock is assumed to be very small. If strategies are substitutes, we get a subgame perfect Nash equilibrium with the competitive market outcome; full-contracting and zero mark-ups. This is a stronger result as compared to Allaz and Vila (1993). The reason is that producers' contracting positions are disclosed after the forward market has closed in my model, whereas Allaz and Vila (1993) allow consumers to adjust their contracting strategy when changes in producers' forward sales occur, and this moderates strategic contracting sales in their model. Similarly, the assumption in my paper amplifies strategic forward buying when strategies are complements in comparison to Mahenc and Salanié (2004).

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Appendix

Proof of Proposition 1

By differentiating the system of differential equations in (7) we get:

$$\begin{cases} \dfrac{\partial^2 s_j}{\partial p \partial x_i} = \dfrac{\dfrac{\partial s_i}{\partial x_i}\left[p - C_i'(s_i + x_i)\right] + s_i C_i''(s_i + x_i)\left(1 + \dfrac{\partial s_i}{\partial x_i}\right)}{\left[p - C_i'(s_i + x_i)\right]^2} \\ \dfrac{\partial^2 s_i}{\partial p \partial x_i} = \dfrac{\dfrac{\partial s_j}{\partial x_i}\left[p - C_j'(s_j + x_j)\right] + s_j C_j''(s_j + x_j)\dfrac{\partial s_j}{\partial x_i}}{\left[p - C_j'(s_j + x_j)\right]^2} \end{cases} \forall p \in \left(p_0^{\max}, \overline{p}\right). \tag{22}$$

Provided the derivatives exist we have $\dfrac{\partial^2 s_k}{\partial p \partial x_i} = \dfrac{\partial^2 s_k}{\partial x_i \partial p}$ according to Young’s theorem. We can therefore change the order of differentiation and somewhat simplify the expression by introducing a variable $\delta_{ki}(p) = \dfrac{\partial s_k(p)}{\partial x_i}$. The boundary conditions in (9) and (10) are also valid for all contracting levels. Thus, it is straightforward to show that

$\delta_{21}(\bar{p}) = \frac{\partial s_2(\bar{p})}{\partial x_1} = 0$, $\delta_{22}(\bar{p}) = \frac{\partial s_2(\bar{p})}{\partial x_2} = -1$, and $\delta_{mn}(p_0^{\max}) = \frac{\partial s_m(p_0^{\max})}{\partial x_n} = 0$, where n is the firm with the lowest marginal cost at the contracting point. To calculate $\delta_{mm}(p_0^{\max})$, we write (10) as $s_m(C_i'(x_m), \mathbf{x}) = 0$. Differentiating both sides with respect to x_m yields:

$\frac{\partial s_m(C_i'(x_m), \mathbf{x})}{\partial x_m} + C_m''(x_m) s_m'(p_0^{\max}) = 0$ and accordingly

$$\delta_{mm}(p_0^{\max}) = \frac{\partial s_m(C_i'(x_m), \mathbf{x})}{\partial x_m} = -C_m''(x_m) s_m'(p_0^{\max}). \tag{23}$$

As producer m's mark-up is zero at the net-supply zero, its mark-up needs to be increasing in output in order not to violate Anderson and Hu's (2008c) results in (8). Hence,

$s_m'(p_0^{\max}) < \frac{1}{C_m''(x_m)}$ and accordingly,

$$0 \geq \delta_{mm}(p_0^{\max}) = \frac{\partial s_m(C_i'(x_m), \mathbf{x})}{\partial x_m} = -C_m''(x_m) s_m'(p_0^{\max}) > -1.$$

By means of l'Hôpital's rule, it can be shown that

$$T_i(p_0^{\max}) = \frac{s_m(p_0^{\max}) C_m''(s_m(p_0^{\max}) + x_m)}{p_0^{\max} - C_m'(s_m(p_0^{\max}) + x_m)} = \frac{0}{0} = \frac{s_m'(p_0^{\max}) C_m''(x_m)}{1 - s_m'(p_0^{\max}) C_m''(x_m)} \tag{24}$$

Thus

$$\delta_{mm}^*(p_0^{\max}) = \frac{-T_i(p_0^{\max})}{1 + T_i(p_0^{\max})} = \frac{-s_m'(p_0^{\max}) C_m''(x_m)}{1 - s_m'(p_0^{\max}) C_m''(x_m) + s_m'(p_0^{\max}) C_m''(x_m)}$$
$$= -s_m'(p_0^{\max}) C_m''(x_m) = \delta_{mm}(p_0^{\max}).$$

□