Equilibrium Supply Security in a Multinational Electricity Market with Renewable Production

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Abstract

An increasing reliance on solar and wind power has raised concern about system ability to consistently satisfy electricity demand. This paper examines countries’ unilateral incentives to achieve supply security through capacity reserves and market integration in a multinational electricity market. Capacity reserves protect consumers against blackouts and extreme prices, but distort consumption and investment. Market integration alleviates supply constraints, but requires costly network reinforcement. Capacity reserves can be up- or downward distorted, but network investment is always insufficient in equilibrium. Capacity reserves are smaller when there are financial markets or when dispatched solely to resolve domestic supply constraints.

Key words: Capacity mechanism, decentralized policy making, multinational electricity market, network investment, security of supply.

JEL codes: D24; H23; L94; Q48

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1 Introduction

Support schemes to increase the production of energy from renewable sources now are common in many parts of the world as part of a policy to reduce greenhouse gas emissions and the dependence on energy imports. The subsidization of renewable electricity often has sparked investments predominantly in solar and wind power. The output fluctuations inherent to solar and wind power have subsequently raised concern about the ability to continuously satisfy demand in a system that relies on such intermittent electricity production.

In circumstances of a substantial shortfall of renewable output, the system operator may be forced to disconnect consumers from the grid in order to maintain system stability. Such rolling blackouts (curtailment) represent the most dramatic manifestation of supply shortage, but scarcity affects consumers negatively also in less extreme circumstances. Price insensitive short-run demand for electricity and capacity constraints in production and transmission imply that the market-clearing spot price of electricity can be very high in event the system is supply constrained even if not on the verge of collapse. The tolerance for blackouts and extreme prices is very limited in advanced economies. A key feature of a viable electricity system based upon renewable electricity production therefore is to maintain a security of supply, i.e. ensure that there is adequate generation capacity to satisfy demand at acceptable consumer prices.

There are two main ways how countries can achieve supply security. The first is to keep capacity reserves as backup in event of supply shortages in the spot market. Reserves often are procured by the use of capacity mechanisms such as auctions for generation capacity. Typical mechanisms address the problem of blackouts by requiring that available production capacity has a sufficient reserve margin to prevent the loss of load probability from exceeding some target level. They limit consumer price exposure by establishing trigger levels in the spot market above which capacity reserves are activated; see Neuhoef et al. (2016) for a characterization of common mechanisms.

See, for instance, the EU Renewables Directive (2009/28/EC) for a formulation of such objectives.

Germany is a leading example of a country that has started a transition to an electricity system based on renewables. Approximately one fourth of the country’s annual electricity production came from renewable sources in 2014. The corresponding figure was 6% at the turn of the millennium. Two-thirds of this increase can be attributed to solar and wind power. The data were retrieved from www.iea.org/statistics/ November 4, 2016.

The Union of the Electricity Industry in Europe (Eurelectric, 2006, p.15) defines security of electricity supply as “the ability of the electrical power system to provide electricity to end-users with a specified level of continuity and quality in a sustainable manner.” This definition appears to encompass curtailment alone, but in the subsequent discussion Eurelectric emphasizes that “energy prices can also have an influence on security of supply. For instance, if electricity prices were to rise enduringly to levels which were not affordable for a substantial portion of customers (households and industry), there would be an impact on security of supply.” Oren (2005) similarly views capacity reserves as an insurance both against curtailment and high prices.

The loss of load probability is the likelihood that available production capacity is insufficient to cover demand within a given period. For instance, ERCOT (Texas) and PJM (North-East USA) apply the same “one day in ten years” loss of load criterion for reserve margins. France and Great Britain use a very similar criterion.

Trigger prices often are explicit. For instance, NEM (Eastern and Southern Australia) and PJM define a specific price cap in the short-term market for situations of supply scarcity. Columbia and New England instead use capacity mechanisms based upon the more unusual reliability options. Producers are forced to issue call options for the contracted capacity reserve at some regulated strike price and to pay consumers the difference between the spot price and the strike price. By way of this construction, consumers de facto pay the minimum of the strike price and the spot price for their electricity (Cramton et al., 2013).

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The second solution is to increase network capacity and thereby improve the flow of electricity within the system. Better market integration reduces the likelihood of supply shortage and lowers market prices by allowing demand and supply fluctuations in different parts of the network to offset one another. Network expansion is regulated and undertaken by the network owner.

In a multinational electricity market, the price effects associated with capacity reserves and network investment propagate through to surrounding countries. Decisions at the national level concerning security of supply therefore run the risk of impairing the overall market performance insofar as local policy makers fail to fully account for the effects of their decisions. The concerns expressed by the European Commission (2015, p.10) in the recent framework strategy for an Energy Union about "divergent national market arrangements" and a necessity to ensure that "capacity mechanisms and support for renewable electricity are fully in line with existing rules and do not distort the internal energy market" bear testimony to this perception.

**Scope**  The purpose of this paper is to contribute to our understanding of the incentives for introducing capacity mechanisms in markets with intermittent renewable electricity generation. It emphasizes the implications of and consequences for market integration by couching the problem in a multinational electricity market setting. A main objective is to identify and account for foreign external effects and assess the overall welfare consequences of decentralized policy making associated with security of supply problems.

**Model description**  I consider a theoretical model of two symmetric and interconnected national electricity markets. Market integration is measured in terms of network reliability. The market is perfectly integrated and spot prices the same in both countries if the interconnection is fully operational. The two markets are separate and spot prices are set at the national level in the alternative scenario when the interconnection is down. Supply shortages sometimes arise because short-term demand is independent of the spot price of electricity (Joskow and Tirole, 2007), renewable production is stochastic, and thermal production capacity is constrained. There exists no market-clearing price in this case (Cramton and Stoft, 2006). Instead, the price is set at a price cap. The capacity reserve required to cover supply shortages in the spot market is larger if the price cap is smaller because then long-term demand for electricity is higher and spot market-based investment in thermal capacity is constrained. Conversely, a larger capacity reserve implies that a smaller price cap is sufficient to generate enough market-whenever demand in the spot market exceeds supply at a price equal to the maximal observed supply bid. The capacity reserve is supplied to clear the spot market at this price. This means that the spot price of electricity in Sweden cannot exceed the short-term marginal production cost of the most expensive unit in the market if the spot market is otherwise competitive.

7This is really a model of market integration between jurisdictions, where each jurisdiction unilaterally decides the size of its capacity reserve and network investment. In the present context, these jurisdictions are countries, but one could equally well assume them to be states, such as in the U.S.

8Reliability criteria, such as those mentioned in footnote 4, are strict. Accordingly, rolling blackouts are very infrequent events in most restructured electricity markets. A sector inquiry in the EU found one single instance of consumers being disconnected during the last five years. This happened during a heat wave in Poland in August 2015 (European Commission, 2016). For simplicity, the model assumes a target level of curtailment equal to zero.
based investment in thermal capacity to cover demand. A larger capacity reserve therefore is equivalent to a higher security of supply, all else equal.

Findings  The socially optimal capacity reserve balances the marginal benefit of an increased security of supply against excessive consumption and insufficient thermal investment resulting from a downward distortion in the long-run (expected) price of electricity. These effects spill over to the foreign country in an integrated market, but policy makers that maximize domestic surplus account for none of them. Still, decentralized policy making does not entail any welfare loss if market integration is perfect and capacity reserves are efficiently deployed. Symmetry then implies that decision makers effectively internalize all externalities abroad of changes in the domestic capacity reserve, and the social optimum can be implemented as a Nash equilibrium.

Equilibrium capacity reserves are distorted in the general case of partial (imperfect) market integration, but the magnitude and direction of the distortion depends on two opposing effects. On the one hand, the probability of a supply shortage is relatively small under market integration because of trade and imperfect correlation of renewable output. This portfolio effect of market integration calls for smaller capacity reserves in social optimum. On the other hand, an integrated market allows for a more efficient use of a given capacity reserve. This cost efficiency of market integration increases the socially optimal capacity reserve. The net foreign externality is negative (positive) if the portfolio effect dominates cost efficiency, in which case the equilibrium capacity reserve is too large (small) in a partially integrated market relative to the social optimum.

I endogenize market integration by allowing investment in network reliability, either at the central level to maximize total welfare, or at the national level. An increase in the capacity reserve decreases (increases) the marginal value of market integration if the foreign externality is negative (positive) and thereby reduces (increases) network investment. This strategic substitutability (complementarity) between capacity reserves and market integration causes downward distortions of network reliability because the capacity reserve is too large (small) from a social point of view under a negative (positive) foreign externality. Hence, investment in network reliability is unambiguously downward distorted. Decentralized network investment exacerbates this underinvestment problem further insofar as domestic policy makers ignore the positive effects abroad of improved market integration.

A main motive for capacity reserves is a concern over prices when the spot market is supply constrained. An obvious solution would seem to be that consumers worried about prices instead sign financial contracts to hedge their spot price risk. I show that the socially optimal capacity reserve is indeed close to zero if consumers can purchase call options in a competitive financial market that renders the equilibrium option price equal to the expected option payment. The market diversifies away all risk in this case. But consumers would still prefer the capacity mechanism because the capacity payments to producers are distributed across all consumers, even those who do not demand any hedge, whereas the financial contract is a private cost. A policy maker who attached more weight to specific consumer interests would have an incentive to introduce capacity reserves even if inefficient. Furthermore, it is doubtful whether sellers
can always diversify away all risk. For instance, they can be liquidity constrained retailers or producers. Capacity reserves arise in equilibrium and can be welfare improving even under financial contracting in case buyers and sellers strictly benefit from risk reduction.

I finally consider the effect of defining supply shortage at the national level instead of at the aggregate level, and requiring that capacity reserves be directed towards solving domestic capacity problems. The resulting dispatch of the capacity reserve then is inefficient, which makes market-based outcomes comparatively more attractive from an efficiency viewpoint. This reduces the socially optimal and equilibrium capacity reserve.

Related literature Notwithstanding the policy discussion surrounding electricity markets with renewable production, this paper is one of only a few to endogenize the security of electricity supply. An explanation for the lack of research can be that standard economic theory posits that specific measures are unnecessary to ensure the security of supply. A competitive "energy-only" market—where customers only pay for the amount of energy they consume and generators only are paid for the amount of energy they produce—is sufficient. Price hikes in times of scarcity will create just enough rent to render the socially optimal investments in thermal capacity privately profitable (Hogan, 2005; Oren, 2005 and Joskow, 2007).

The efficiency of an energy-only market arises under ideal market conditions where demand is price sensitive enough always to deliver some, possibly very high, price that clears the market. It is arguable whether current electricity markets fit this description, not least because many households are on contracts that do not incite them to respond to short-term price signals. Cramton and Stoft (2006) and Cramton et al. (2013) argue that appropriately designed capacity mechanisms are an efficient way of resolving associated supply constraints.

Joskow and Tirole (2007) show in their seminal contribution that price insensitive short-term demand alone is insufficient to vindicate capacity mechanisms on efficiency grounds. Instead, capacity obligations have the potential to improve efficiency if curtailment is inefficient or if price signals are distorted, for example as a result of market power or because of regulatory intervention. Joskow and Tirole (2007) explore in detail capacity obligations in relation to imperfect competition. Creti and Fabra (2007) and Schwenen (2014) illustrate in a similar vein how capacity reserves mitigate strategic withholding of production from the spot market.

There can be reasons for maintaining capacity reserves even in a competitive electricity market with efficient curtailment. Efficiency requires that the price cap is set at the consumer cost of involuntary rationing, the value of lost load (VOLL), so that consumers on average are indifferent between being rationed or not in scarcity situations (Stoft, 2002). The general applicability of such a policy can be disputed, not only because VOLL is difficult to estimate correctly, but also because it may be politically infeasible to permit the electricity price to

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9 An illustrative example is the California electricity market at the turn of the millennium. The price hedge consisted of a regulated retail price with retailers carrying the full spot price risk. All three investor-owned retailers subsequently ran into serious financial difficulties as spot prices soared to record levels in the summer of 2000, and one of them went bankrupt. See Wolak (2003) for diagnosis of the famous California electricity crisis. Producers carry the spot price risk under the system of reliability options, unless they themselves manage to hedge this risk. Neuhoff et al. (2016) discuss the distribution of risks associated with reliability options.
increase by a factor of 100 or more above its average level to achieve VOLL (Cramton et al., 2013). Furthermore, investors may question the credibility of VOLL pricing, in which case the desired investments will not come about (Joskow and Tirole, 2007). Neuhoff et al. (2016, p.258) argue that trigger prices must be set sufficiently low in capacity mechanisms else they would "clearly affect the social acceptance of the energy market design." The present paper incorporates the idea of politically acceptable prices by assuming that supply constraints has negative market external consequences for a subset of consumers. Policy makers account for these consumer effects in the choice of capacity reserves. The equilibrium capacity reserve (and price cap) balances the marginal benefit of protecting consumers against blackouts and high prices against the price distortions to long-run demand and thermal investment. Placing the problem in a multinational electricity market setting permits an analysis of the interaction between capacity reserves and market integration and to shed light on consequences of decentralized policy making.

Meyer and Gore (2015) simulate the cross-border effects of capacity mechanisms within a two-country numerical model. The purpose is to examine how different types of capacity mechanisms with exogenous properties affect investment distortions arising from differences in market power between countries. Both countries choose reliability options in equilibrium under the parameters of the model, and this equilibrium welfare dominates energy-only markets. The present paper employs a competitive model to endogenize the size of the capacity reserves and analyze the magnitude of the missing money problem in equilibrium. I derive exact conditions under which capacity reserves are upward- or downward distorted relative to the social optimum depending on two opposing forces: the portfolio effect and cost efficiency effects of market integration. I also extend the analysis in a number of new directions by endogenizing market integration and considering financial contracting and different allocation rules for capacity reserves.

**Structure of the paper** Section 2 presents the model and explores the basic trade-off associated with capacity reserves in the two polar cases of national electricity markets and perfect market integration. The intermediary case of partial market integration and the consequences of decentralized policy making for equilibrium capacity reserves and network investment are analyzed in Sections 3 and 4. Section 5 introduces financial markets. Section 6 considers national allocation rules for capacity reserves. Finally, Section 7 concludes with some policy implications.

### 2 Capacity reserves in national or perfectly integrated markets

There are two countries, identical in terms of consumer preferences, income and production technologies. The benchmark model encompasses two polar degrees of market structure. The first case, indexed by $N$, is that of autarchy by which electricity markets are entirely national. Instead, there are transmission lines with sufficient capacity to equalize the electricity price across the two countries in the second case of perfect market integration, indexed by $I$. I consider the intermediary case of partial market integration in Section 3.

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10 See also European Commission (2016).
11 Joskow and Tirole (2007) discuss capacity reserves in relation to an exogenous price cap in the spot market.
2.1 The model

Demand There are two types of representative consumers: households and an electricity intensive industry. Households pay the expected (long-run) wholesale price of electricity $E[p]$. Their consumption $q_0$ therefore is independent of short-term price fluctuations and chosen to maximize quasi-linear utility $u(q_0) + q_0$ subject to the budget constraint $E[p]q_0 + q_0 + T \leq Y_0$, where $q_0$ is a numeraire good, $T$ is a fixed fee, and $Y_0$ represents income. Let $u(\cdot)$ be twice continuously differentiable, strictly increasing in the relevant domain and strictly concave, and assume that income $Y_0$ is large enough that the demand for both goods is strictly positive.

A representative energy intensive industry pays the short-run price $\tilde{p}$ and converts each MWh of electricity one-for-one into a good sold in the international market at price $\phi > 0$ net of other variable operating costs. Energy intensive industries depend on stable production conditions to run efficiently and therefore cannot respond to short-term price increases by reducing electricity consumption. I therefore assume that the industry has inelastic demand for $q_n \geq 0$ MWh electricity independently of $\tilde{p}$. In particular, the industry suffers an operating loss if $\tilde{p} > \phi$. Its surplus then equals $q_n(\phi - \tilde{p} - B(\tilde{p} - \phi))$. The term $B(\cdot)$ represents the shadow cost of the loss, which is continuously differentiable, increasing and convex for all $\tilde{p} > \phi$, with $B(\tilde{p} - \phi) = B'(0) = 0$ for all $\tilde{p} \leq \phi$. The asymmetry between profits and losses could stem for instance from liquidity constraints or from profit taxes that treat operating gains and losses asymmetrically, i.e. losses are not fully deductible. $B(\cdot)$ represents a negative externality that creates a demand for capacity reserves to reduce price risk. One would expect the industry also to hedge risk in the financial market or through long-term contracts. I consider financial contracting in Section 5. For now, it is sufficient to note that the analysis under financial contracting is qualitatively the same as below and in Sections 3 and 4 under the plausible assumption of risk aversion on both the buyer and the seller side, as in the seminal contribution by Bessembinder and Lemmon (2002). The assumption that only household demand is long-run price sensitive is for simplicity.

Supply Electricity is competitively supplied in the short and the long-run. Let $c(x)$ be the variable cost (fuel cost, variable O&M) of producing the $x$th MWh of thermal electricity in the country, a cost that is strictly increasing, convex and continuously differentiable. There is also a capital cost of installing thermal capacity that for simplicity is assumed to be constant and equal to $\delta > 0$ per MWe.

Renewable output $(r_1, r_2) \in [0, \bar{r}]^2$ in the two countries is intermittent (stochastic) and jointly distributed with cumulative distribution function $F(r_1, r_2)$ and density $f(r_1, r_2)$. Renewable production is symmetric, meaning $f(r_1, r_2) = f(r_2, r_1)$ in the entire domain. Let the marginal distribution be $F_N(r)$, with density $f_N(r) = \int_0^r f(r, \tilde{r})d\tilde{r}$. Denote by $F_I(r)$ the distribution of the average renewable output $r = \frac{r_1 + r_2}{2}$:

$$F_I(r) = \begin{cases} \int_0^{2r} F_N(2r - \tilde{r})f_N(\tilde{r})d\tilde{r} & \text{for } r \in [0, \bar{r}/2] \\ 1 - \int_{2r-\bar{r}}^{\bar{r}} (1 - F_N(2r - \tilde{r}))f_N(\tilde{r})d\tilde{r} & \text{for } r \in [\bar{r}/2, \bar{r}] \end{cases}$$

Renewable electricity production has zero marginal production cost. The capacity is politically
determined, so I treat it as exogenous throughout. Gains from electricity trade arise in a perfectly integrated market even if countries are ex ante symmetric insofar as renewable outputs \( r_1 \) and \( r_2 \) are imperfectly correlated.

**Short-run equilibrium** Assume that the market-based thermal capacity \( x \) (i.e. excluding any capacity reserve) is the same in both countries. The equilibrium price of electricity is implicitly defined by the market-clearing condition \( c^{-1}(\tilde{p}) + r = q_k + q_n = q \) if renewable output is large enough, where \( r \) indicates the renewable output in the representative country when electricity markets are national. If \( x < q \), then there is no market clearing price for low realizations of renewable output. I assume that the wholesale price is set at a price cap \( \tilde{p} \) if the market fails to clear. Hence,

\[
\tilde{p}(q - r) = \begin{cases} 
    c(q - r) & \forall r \geq q - x \\
    \tilde{p} & \forall r < q - x,
\end{cases}
\]  

(1)

identifies the short-term price of electricity.\(^{12}\) The price cap \( \tilde{p} \) is endogenous, but has no implications in the short-run besides redistributing income between consumers and electricity producers. Its importance will be apparent through its effects on long-run demand and investment in thermal capacity.

**Long-run equilibrium** The long-run household demand \( D_{M}(\tilde{p}) \) and the market-based investment level \( X_{M}(\tilde{p}) \) in thermal capacity depend on the market structure \( M = N, I \) because the relevant distribution of renewable output does so. The point at which the marginal utility of electricity consumption equals the expected price defines the equilibrium household demand:

\[
u'(D_{M}) = \int_{D_{M} + q_n - X_{M} - D_{M} + q_n - X_{M}} c(D_{M} + q_n - r)dF_{M}(r) + \tilde{p}F_{M}(D_{M} + q_n - X_{M}).
\]  

(2)

The corresponding market-based investment level in thermal capacity equates the expected scarcity rent of the marginal capacity with the marginal capital cost:

\[(\tilde{p} - c(X_{M}))F_{M}(D_{M} + q_n - X_{M}) = \delta.
\]  

(3)

Demand is decreasing and market-based thermal investment is increasing in the price cap \( \tilde{p} \); see Appendix A.1.

**Capacity reserves** The market-based supply of thermal capacity is insufficient to cover demand for low realizations of renewable output, i.e. whenever \( r < D_{M}(\tilde{p}) + q_n - X_{M}(\tilde{p}) \), for any finite price cap \( \tilde{p} \). To maintain system stability, the system operator can either activate capacity reserves, or, if that option has been exhausted, disconnect consumers. If system balance were to be attained entirely by curtailment, this would yield a disconnection (loss of load) probability equal to \( F_{M}(D_{M}(\tilde{p}) + q_n - X_{M}(\tilde{p})) > 0 \). I assume that it is politically unacceptable for sys-
tem operators to deliberately disconnect consumers. The remaining solution then is to procure enough capacity reserves that curtailment will not occur.

Under the assumption of national electricity markets, \( \bar{p}_N = \bar{P}_N(k) \) defined by

\[
D_N(\bar{P}_N) + q_n - X_N(\bar{P}_N) = k
\]

represents the smallest price cap that would generate precisely enough market-based investment to ensure that total thermal capacity equals total demand given the national capacity reserve \( k \). For any price cap above \( \bar{P}_N(k) \), there would be overinvestment and under-utilization of the capacity reserve. Conversely, there would not be enough capacity in the market to cover demand in all possible contingencies for a price cap below \( \bar{P}_N(k) \).

Denote by \( k = \frac{k_1 + k_2}{2} \) the average capacity reserve under perfect market integration, where \((k_1, k_2)\) are the capacity reserves in the two countries. The price cap \( \bar{p}_I = \bar{P}_I(k) \) defined by

\[
D_I(\bar{P}_I) + q_n - X_I(\bar{P}_I) = k
\]

is the smallest one required to generate enough market-based investment to ensure security of supply in the integrated market given the average capacity reserve \( k \).\(^{13,14}\) I assume that the activated capacity reserve is divided equally among the two countries under scarcity, i.e. whenever \( r = \frac{k_1 + k_2}{2} < k \). This allocation rule is ex post efficient here because it equates the marginal thermal costs across the two countries. The price cap is smaller when the capacity reserve is larger under both market structures \( M = N, I \):

\[
\bar{P}_M'(k) = \frac{1}{D_M'(P_M(k)) - X_M'(P_M(k))} < 0.
\]

For future reference, let

\[
\bar{k}_M = D_M(\bar{\phi}) + q_n - X_M(\bar{\phi}) > 0
\]

be the minimal capacity reserve necessary to fully protect the electricity intensive industry from losses under market structure \( M \).

Most wholesale electricity markets feature a bid cap above which the market participants cannot submit bids or offers. In some markets, this bid cap is set at VOLL.\(^{15}\) The price cap analyzed in this paper is the one implied by the target loss of load probability (which is zero) and the size of the capacity reserve, and can be substantially smaller than the bid cap. Hence,

\(^{13}\)In the present context, the price cap \( \bar{P}_M(k) \) is implicitly defined by the size of the capacity reserve. Alternatively, one can consider an explicit price cap \( \bar{p} \) and an implied capacity reserve \( k_M(\bar{p}) = D_M(\bar{p}) + q_n - X_M(\bar{p}) \). The two approaches are formally equivalent in a national electricity market, but may have different implications in an integrated market because of strategic interaction.

\(^{14}\)One could instead specify a target loss of load of load probability \( \theta \geq 0 \). Within this more general framework, \( D_M(\bar{P}_M) + q_n - X_M(\bar{P}_M) = k + F_M^{-1}(\theta) \) characterizes the price cap \( \bar{P}_M(k, \theta) \) that for a capacity reserve \( k \) yields precisely enough market-based investment in thermal capacity to generate a loss of load probability \( \theta \) under market structure \( M \). Actual \( \theta \)'s are very small. For instance, an annual loss of load probability of 0.1 days implies \( \theta < 0.0003 \). For simplicity, I let \( \theta = 0 \), such that \( \bar{P}_M(k) = P_M(k, 0) \).

\(^{15}\)Examples include ERCOT (Texas) and NEM (Eastern and Southern Australia).
situations may occur in which capacity reserves are activated at prices below VOLL and without there being any substantial risk of rolling blackouts.\footnote{Sweden is an illustrative case in point. It has not experienced even a single hour of curtailment since liberalization of its electricity market in 1996. Nor has the electricity price ever hit the bid cap of 2000 Euro/MWh during this period. Yet, the system operator has intervened on a number of occasions, most recently during the cold winter of 2009-10. This pattern is consistent with security of supply being defined also in terms of avoiding very high prices instead of only averting curtailment. Naturally, there have been several uncontrolled blackouts in Sweden, the most severe of which was the consequence of Hurricane Gudrun in 2005.}

For renewable output \( r \geq k \), there is enough thermal output offered at market terms to clear the market at the short-term marginal cost. If renewable output falls below the critical level \( r < k \), then it becomes necessary to invoke some of the capacity reserve to avoid supply shortage. In this case, the capacity reserve is bid into the market at the price cap. Hence, the short-term price of electricity can be characterized by

\[
p_M(r, k) = \begin{cases} c(x_M(k) + k - r) & \forall r \geq k \\ \bar{P}_M(k) & \forall r < k \end{cases}
\]

as a function of renewable output \( r \) and the capacity reserve \( k \), where \( x_M(k) = X_M(\bar{P}_M(k)) \) is the market-based thermal capacity, and \( r \) and \( k \) represent averages across the two countries if markets are perfectly integrated.

Henceforth, I make the simplifying assumption that

\[
c(x_M(k)) < \phi \forall k > 0.
\]

This assumption implies that the electricity intensive industry earns an operating profit under normal market conditions, i.e. as long as the market clears at the marginal thermal production cost. In other words, the industry runs into profitability problems only in situations of supply scarcity, i.e. when \( r < k \).

Because of the price cap, the income generated in the market is insufficient to cover the production cost of the capacity reserves. Additional capacity payments must therefore be put in place in order to ensure supply security whenever the (average) production of renewable electricity falls below \( k \). The capacity payments are assumed to be lump-sum and will not play any role in what follows. For the sake of completeness, I derive the least cost capacity payments in Appendix A.2.

**Household, industry and producer surplus** As the industry’s marginal utility of income is larger than that of the households, it is socially optimal that households finance the entire capacity payment in this model (which is also technically convenient and politically plausible). Letting \( q_M(k) = D_M(\bar{P}_M(k)) + q_n \) denote consumption in the representative country as a function of the (average) capacity reserve \( k \), the expected consumer surplus becomes

\[
CS_M(k) - T_M(k) = u(q_M(k) - q_n) + q_n \phi - \int_0^T p_M(r, k) dF_M(r) q_M(k) - q_n B(\bar{P}_M(k) - \phi) F_M(k) - T_M(k).
\]
The terms on the first row above are the gross utility of electricity consumption minus the expected payments. The first term on the second row is the expected shadow cost of the industry loss. The final term is the capacity payment $T_M(k)$. The optimal capacity reserve features a trade-off between insurance and efficiency, but is nonetheless different from a standard moral hazard problem: it is the electricity intensive industry that is exposed to price risk, but the households that pay the insurance cost in terms of the capacity payment.

An increase in the capacity reserve reduces the expected price of electricity. The first term below is the direct benefit of redistributing income from the power producers to consumers (the quantity effect is of second-order importance):

$$CS_M'(k) = -q_M(k) \int_0^\tau \frac{\partial p_M(r,k)}{\partial k} dF_M(r) + SS_M(k, \phi).$$

The second term is the marginal expected security of supply (see Appendix A.8 for the details):

$$SS_M(k, \phi) = -q_n B' (\hat{P}_M(k) - \phi) F_M(k) - q_n B (\hat{P}_M(k) - \phi) f_M(k) q_M(k).$$ (7)

One the one hand, an increase in the capacity reserve reduces the maximal price, which tends to increase the security of supply. On the other hand, a larger capacity reserve crowds out market-based investment in thermal capacity and thereby increases the probability that the market cannot clear, which tends to reduce the security of supply. Because of crowding out, a higher capacity reserve need not necessarily be associated with a higher expected security of supply.\(^{17}\)

The corresponding expected profit of the electricity producers equals

$$\Pi_M(k) + T_M(k) = \int_0^\tau [p_M(r,k)q_M(k) - \int_0^{q_M(k)} c(\tilde{r}) d\tilde{r}] dF_M(r) - \delta q_M(k) + T_M(k).$$

The marginal effect

$$\Pi_M'(k) = q_M(k) \int_0^\tau \frac{\partial p_M(r,k)}{\partial k} dF_M(r) - \psi_M(k) q_M'(k)$$
on generation profit of increasing the capacity reserve is negative, excluding the effect on the capacity payment. Besides redistributing income to the consumers, the price reduction also drives a wedge between the marginal long-run cost of thermal capacity and the marginal willingness to pay for electricity. The second term is the marginal inefficiency associated with this price distortion:

$$\psi_M(k) = \delta + \int_0^\tau (c(q_M(k) - r) - p_M(r,k)) dF_M(r)$$

$$= \int_0^k [c(q_M(k) - r) - c(q_M(k) - k)] dF_M(r) > 0.$$ (8)

Instead of underinvesting relative to the competitive equilibrium, as would be the case under imperfect competition, the power industry is actually overinvesting (in terms of the sum of market-based investment and the capacity reserve). Overinvestment relative to the competitive

\(^{17}\)However, I show in Appendix A.8 that the price effect dominates crowding-out, at least for sufficiently small capacity reserves.
equilibrium creates a markdown, \( \psi_M(k) \). This price distortion is, moreover, increasing in the size of the capacity reserve

\[
\psi'_M(k) = \int_0^k \left[ c'(q_M(k) - r)q'_M(k) + c'(q_M(k) - k)(1 - q'_M(k)) \right] dF_M(r) > 0
\]

because \( q'_M(k) \in (0, 1) \); see equation (25) in Appendix A.1.

### 2.2 The socially optimal capacity reserve

Aggregate welfare in the representative country is the sum of consumer and producer surplus as a function of the domestic capacity reserve \( k \) when electricity markets are national. The capacity payment merely represents a lump-sum transfer between households and electricity producers and therefore has no bearing on aggregate welfare in this model (hence, it is not important for the welfare analysis that the capacity market is fully competitive as long as the capacity reserve is dispatched in an efficient manner). Symmetry, full price equalization, efficient dispatch of the capacity reserve and lump-sum capacity payments imply that the welfare is the same in both countries under perfect market integration and a function of the average capacity reserve \( k \). Hence, the welfare in the representative country can be written as

\[
W_M(k) = CS_M(k) + \Pi_M(k)
\]

for both market structures \( M = N, I \).

I assume throughout that the problem of optimizing the capacity reserve is well-behaved under both market structures:\(^{18}\)

\[
\begin{align*}
W''_N(k) &< 0 \text{ for all } k \in (0, 2 \max\{\tilde{k}_N; \tilde{k}_I\}], \\
W''_I(k) &< 0 \text{ for all } k \in (0, \max\{\tilde{k}_N; \tilde{k}_I\}], \\
\lim_{k \to 0} W'_M(k) &> 0, \quad M = N, I.
\end{align*}
\]

Solving the first-order condition yields the following result (the proof is in Appendix A.3):

**Proposition 1** Assume that electricity markets are either national or perfectly integrated. The socially optimal capacity reserve \( k^*_{M} \in (0, \tilde{k}_M) \) under market structure \( M = N, I \) entails a trade-off between the marginal benefit of increased security of supply against the marginal cost of distorting consumption and investment:

\[
SS_M(k^*_{M}, \phi) = \psi_M(k^*_{M})q'_M(k^*_{M}).
\]

The social optimum can be implemented as a pay-off dominant Nash equilibrium under both market structures if countries set capacity reserves non-cooperatively to maximize domestic welfare.

The assumption that capacity reserves are set by policy makers in each country in a decentral-
ized and non-cooperative manner does not necessarily represent any large source of inefficiency. Each country de facto internalizes the welfare effect abroad in their choice of capacity reserve in case of symmetry, perfect market integration and if capacity reserves are allocated in an ex post efficient manner.

**Comparative statics** The trade-off facing policy makers is qualitatively the same independently of whether electricity markets are national of perfectly integrated. However, the magnitudes of the marginal effects differ between the two market structures. On the one hand, a fully integrated electricity market allows for a more efficient use of a given total capacity reserve \( k_1 + k_2 \) because reserves can be activated in such a manner as to increase efficiency by equalizing marginal thermal production costs across countries. This *cost efficiency of market integration* can be represented as the ratio of the expected cost distortion under market integration over the expected cost distortion when markets are national,

\[
\frac{\psi_1(k)}{\psi_N(k)},
\]

and tends to increase the socially optimal capacity reserve under full market integration relative the case when electricity markets are national.

On the other hand, the probability of a shortage of renewable electricity is relatively smaller under market integration because of trade and the imperfect correlation of renewable output. This *portfolio effect of market integration* can be represented as the adjusted probability that the capacity reserve is invoked under market integration relative to the adjusted probability that it is invoked in the national market,

\[
\frac{F_I(k) B'(P_I(k) - \phi) \mathbb{P}_I(P_I(k)) + B(P_I(k) - \phi) f_I(k)}{F_N(k) B'(P_N(k) - \phi) \mathbb{P}_N(P_N(k)) + B(P_N(k) - \phi) f_N(k)},
\]

and tends to reduce the socially optimal capacity reserve under full market integration relative the case when electricity markets are national.\(^{19}\) The effect of market integration on the socially optimal capacity reserve depends on the relative magnitudes of those two effects (the proof is in Appendix A.4):

**Proposition 2** The socially optimal capacity reserve is larger under perfect market integration compared to the case when electricity markets are national \( k_{fb}^I > k_{fb}^N \) if cost efficiency dominates the portfolio effect of market integration:

\[
\frac{\psi_I(k)}{\psi_N(k)} \leq \frac{F_I(k) B'(P_I(k) - \phi) \mathbb{P}_I(P_I(k)) + B(P_I(k) - \phi) f_I(k)}{F_N(k) B'(P_N(k) - \phi) \mathbb{P}_N(P_N(k)) + B(P_N(k) - \phi) f_N(k)}, \quad k \in \{k_{fb}^I, k_{fb}^N\}.
\]

The opposite result holds if the inequality is reversed so that the portfolio effect dominates.\(^{19}\)

\(^{19}\text{If, for instance } c(x) = cx, \text{ and } (r_1, r_2) \text{ are stochastically independent with distribution } F_N(r) = \frac{r}{2}, \text{ then } \frac{\psi_I(k)}{\psi_N(k)} = \frac{2k}{2k} = 1, \text{ and } \frac{F_I(k)}{F_N(k)} = \frac{2k}{2k} = 1 \text{ are both below unity for } k \leq k_M \leq \frac{2}{3}.\)
3 Capacity reserves in partially integrated markets

The analysis has so far relied on assumptions that markets either are entirely national or perfectly integrated. This section allows markets to be partially integrated in the sense that there is trade between them, but trade flows are sometimes restricted.

3.1 Model extension

The analysis of electricity markets under transmission constraints is notoriously difficult, especially under the assumption of strategic interaction among players. One reason is that optimal behavior is discontinuous at trading volumes around which the constraint is just binding; see Holmberg and Philpott (2012) and references therein. To maintain tractability of the model while still capturing the flavour of network constraints, I assume that the transmission network has enough installed capacity to handle all trade flows, but the network breaks down with probability $1 - \sigma \in [0, 1]$. If this happens, markets are completely separated and thus become entirely national. Instead, the market is fully integrated if the transmission network operates at full capacity. Under this simplified structure, $\sigma$ is a measure of market integration. While obviously a technical simplification, there is a grain of truth to this way of modeling networks because transmission capacity sometimes is reduced for scheduled or unscheduled maintenance reasons.

I also make a small reinterpretation of the time frame of the model. The analysis in Section 2 was cast in terms of the long-term problem of ensuring enough thermal investment to cover demand while simultaneously avoiding price spikes. Many countries in the EU actually are in a situation of overcapacity (European Commission, 2016). Instead, renewable production has driven down prices so far that the expected market revenue is insufficient to cover the fixed costs of keeping thermal capacity available for the spot market. Assume now that $\delta$ is the fixed cost of keeping a unit of thermal capacity available and $c(\cdot)$ its variable production cost. Consider the intermediary problem of keeping enough thermal capacity online to ensure supply security.

The timing of the game is as follows. The policy makers in the two countries procure capacity reserves $(k_1, k_2)$ in the first stage. Network reliability is realized, subsequent to which the markets are either perfectly integrated or national. Consumers decide how much electricity to purchase and power producers how much thermal capacity to make available to the short-term market depending on the market structure $M = N, I$. Finally, renewable output is realized in the two countries. The real-time wholesale market clears all prices if renewable output and/or transmission capacity is sufficient to handle the residual flow of electricity between markets. Otherwise capacity reserves are activated in one or both markets.\textsuperscript{20,21}

\textsuperscript{20}An alternative timing would be to assume that consumers and power producers make their choices prior to the revelation of market structure. Demand and thermal supply in each country would then depend on the full range of price caps $(\bar{p}_{N1}, \bar{p}_{N2}, \bar{p}_I)$. The trade-off facing policy makers would remain qualitatively intact, but the analysis of decentralized policy making would be obscured by an intractability of second-order conditions.

\textsuperscript{21}One could also maintain a long-term framework and assume that network owners with probability $\sigma$ make an incremental investment to remove bottlenecks. I endogenize $\sigma$ in Section 4.
The expected welfare in country \( i \) simply becomes the weighted average

\[
W(k_i, k_j) = \sigma W_I(k) + (1 - \sigma) W_N(k_i)
\]

under this structure, where \( k = \frac{k_i + k_j}{2} \) represents the average capacity reserve. The corresponding expected welfare equals \( W(k, k) \) in the representative country under symmetric capacity reserves, \( k_1 = k_2 = k \).

### 3.2 Equilibrium capacity reserves

Consider the social optimum as a benchmark. The first-best optimal capacity reserve \( k^{fb}(\sigma) \) is symmetric and trades-off the marginal effect in the integrated market against the marginal effect when markets are national:

\[
\sigma W'_I(k^{fb}) + (1 - \sigma) W'_N(k^{fb}) = 0.
\]

Now let policy makers in each country set their capacity reserves non-cooperatively to maximize the domestic welfare \( W(k_i, k_j) \). The first-order condition becomes

\[
\frac{\partial W(k_i, k^*)}{\partial k_i}|_{k_i = k^*} = \frac{1}{2} \sigma W'_I(k^*) + (1 - \sigma) W'_N(k^*) = 0
\]

in symmetric equilibrium, \( k^*_1 = k^*_2 = k^*(\sigma) \). Whereas an electricity market with zero or full integration generates the efficient outcome in the present model, the market with partial integration does not. By comparing equilibrium condition (16) with the optimality condition (15), it is quite obvious that the decentralized (non-cooperative) equilibrium generally will be inefficient because the policy maker in the home country does not take into account the marginal effect \( \sigma W'_I(k)/2 \) abroad of expanding the capacity reserve at home. What is less clear, is whether decentralized policy making leads to upward or downward distortions of the capacity reserve under partial market integration.

To evaluate the effects of decentralized policy making, consider the symmetric capacity reserve \( k_1 = k_2 = \kappa(t, \sigma) \) implicitly defined by the solution to

\[
\frac{1 + t}{2} \sigma W'_I(\kappa) + (1 - \sigma) W'_N(\kappa) = 0.
\]

The parameter \( t \) measures the degree to which policy makers internalize the externality abroad of changes in the domestic capacity reserve. Policy makers internalize the full effect if \( t = 1 \), in which case the first-best solution obtains: \( \kappa(1, \sigma) = k^{fb}(\sigma) \). The non-cooperative solution obtains in the opposite case when policy makers do not internalize any of the effects abroad: \( \kappa(0, \sigma) = k^*(\sigma) \).

The difference between the socially optimal capacity reserve and the non-cooperative solution equals

\[
k^{fb} - k^* = \int_0^1 \frac{\partial \kappa(t, \sigma)}{\partial t} dt = \int_0^1 \frac{\sigma W'_I(\kappa(t, \sigma))}{-[(1 + t)\sigma W''_I(\kappa(t, \sigma)) + 2(1 - \sigma) W''_N(\kappa(t, \sigma))]} dt.
\]
The denominator of (18) is strictly positive by assumption (9). Hence, decentralized policy making leads to downward (upward) distortions in the equilibrium capacity reserve if the foreign externality is positive (negative), which is very intuitive. The sign of the externality in turn depends on the relative strengths of the marginal effects of market integration:

**Lemma 1** The foreign externality is positive [negative] if cost efficiency is stronger [weaker] than the portfolio effect of market integration ($\sigma W'_{I}(\kappa(t, \sigma)) > [\leq] 0$ for all $t \in [0, 1]$ and $\sigma \in (0, 1]$ if inequality (13) is satisfied [violated]).

**Proof.** Assume that $(t, \sigma) \in [0, 1] \times [0, 1)$. Strict quasi-concavity of $W_I(k)$ and $W_N(k)$ imply $\frac{1}{2} \sigma \omega W'_I(k) + (1 - \sigma) W'_N(k) > (\leq) 0$ for all $k < \min\{k^f_I; k^b_I\}$ ($k > \max\{k^f_N; k^b_N\}$). Hence, $\kappa(t, \sigma) \in [\min\{k^f_N; k^b_N\}; \max\{k^f_I; k^b_I\}]$. If inequality (13) is satisfied [violated], then $\kappa(t, \sigma) \in [k^f_I; k^b_I]$ $[\kappa(t, \sigma) \in [k^f_N, k^b_N]]$ by Proposition 2. Strict quasi-concavity of $W_I(k)$ then implies $W'_I(\kappa(t, \sigma)) > (\leq) 0$ if inequality (13) is satisfied [violated].

A marginal increase in the domestic capacity reserve increases the security of supply even abroad in an integrated market, but the lower price cap exacerbates the distortions to consumption and investments abroad. The marginal distortion owing to an increase in the capacity reserve is small (large) in magnitude compared to the supply security effect if the cost efficiency of market integration is strong (weak). The foreign externality is positive (negative) if cost efficiency dominates (is weaker) than the portfolio effect of market integration ($\sigma W'_{I}(\kappa(t, \sigma)) > [\leq] 0$ for all $t \in [0, 1]$ and $\sigma \in (0, 1]$ if inequality (13) is satisfied [violated]).

**Proposition 3** Assume that the electricity markets are partially integrated, $\sigma \in (0, 1)$, and that the countries choose capacity reserves non-cooperatively to maximize domestic welfare. The capacity reserve $k^*(\sigma) \in [\min\{k^f_N; k^b_N\}; \max\{k^f_I; k^b_I\}]$ in the unique symmetric equilibrium solves

$$\frac{1}{2} \sigma \omega_k I(k^*, \phi) + (1 - \sigma) \omega_N I(k^*, \phi) = \frac{1}{2} \sigma \psi_I(k^*) q_I^*(k^*) + (1 - \sigma) \psi_N(k^*) q_N^*(k^*).$$ (19)

The equilibrium capacity reserve is downward [upward] distorted if cost efficiency dominates [is dominated by] the portfolio effect of market integration ($k^*_I(\sigma) < [>] k^b_I(\sigma)$ if $\sigma \in (0, 1)$ and inequality (13) is satisfied [violated]).

### 4 Network investment to increase market integration

The price spikes associated with losses in renewable output can be mitigated either by means of capacity reserves or by market integration. Capacity reserves achieve this by imposing an implicit cap on the price of electricity that incites sufficient thermal capacity to cover consumption. Under market integration, output reductions in one country can be alleviated by increased production in other markets, thereby increasing productive efficiency and limiting price increases.

Because of the price caps, the market provides insufficient incentives to invest in thermal capacity and therefore has to be complemented by a mechanism that generates additional capacity payments. But the market also provides insufficient incentives for improving network reliability. Network owners typically earn their income from buying electricity at a low price in one area
and selling it at a higher price in another when network constraints prevent all areas in the market from clearing at a single price. Unfortunately, the market generates no such congestion rent here. Either the transmission network is fully operational, in which case the market is integrated and there are no price differences, or the network is completely down, in which case there is no trade between the countries. The lack of profitability is particularly visible in the present context, but applies more broadly to the problem of investing in network reliability. To account for this "missing money" problem in network reliability, I assume that the transmission networks are regulated. I consider both the case when regulation of network investment is centralized and when network investment is decentralized to the individual countries along with the choice of capacity reserves.

4.1 Centralized network investment

Under centralized network regulation, total reliability $\sigma_I$ is chosen to maximize the expected total welfare

$$2\sigma_I W_I(k) + (1 - \sigma_I)(W_N(k_1) + W_N(k_2)) - 2C(\sqrt{\sigma_I})$$

across the two countries, taking the capacity reserves $(k_1, k_2)$ as given and subject to the twice continuously differentiable, increasing and strictly convex cost function $C(\cdot)$, where assumptions that $\frac{C''(y)}{C'(y)} > 1$ for all $y > 0$, $\lim_{y \to 0} C'(y)/y < W_I(k_{N}^{fb}) - W_N(k_{N}^{fb})$ and $C'(1)/2 > W_I(k_1^{fb}) - W_N(k_1^{fb})$ ensure existence of an interior solution. Each country chooses its capacity reserve to maximize the domestic welfare, taking network reliability $\sigma_I$ and the capacity reserve in the other country as given.

The optimal degree $R_I(k)$ of network reliability under centralized regulation is a trade-off between the marginal value of market integration and the marginal cost of increasing network reliability

$$W_I(k) - W_N(k) = \frac{C'(\sqrt{R_I})}{2\sqrt{R_I}}$$

as a function of the symmetric capacity reserve $k_1 = k_2 = k$.

The equilibrium degree of market integration $\sigma_I(t)$ is implicitly characterized by the solution to $\sigma_I = R_I(\kappa(t, \sigma_I))$ as a function of the degree $t$ to which policy makers internalize the foreign externality of capacity reserves. The first-best degree of market integration satisfies $\sigma_I^{fb} = \sigma_I(1)$, whereas the equilibrium degree of market integration equals $\sigma_I^* = \sigma_I(0)$. Hence,

$$\sigma_I^{fb} - \sigma_I^* = \int_0^1 \sigma_I'(t) dt = \int_0^1 \frac{R_I'(\kappa(t, \sigma_I(t)))}{1 - R_I'(\kappa(t, \sigma_I(t)))} \frac{\partial \kappa(t, \sigma_I(t))}{\partial \sigma} dt$$

measures the effect on market integration of decentralizing the choice of capacity reserves under centralized network regulation. The denominator of the fraction is positive in stable equilibrium (Dixit, 1986). By

$$R_I'(\kappa) = \frac{(1 + t)\sigma_I + 2(1 - \sigma_I)}{1 - \sigma_I} \frac{2\sigma_I^3 W_I'(\kappa)}{C''(\sqrt{\sigma_I})\sqrt{\sigma_I} - C''(\sqrt{\sigma_I})},$$

17
an increase in the capacity reserve tends to increase the marginal value of market integration and drive up network investment if the foreign externality is positive. Capacity reserves and market integration are strategic complements in this case. Instead, capacity reserves and market integration are strategic substitutes if the foreign externality is negative. Whether equilibrium capacity reserves are above or below the social optimum under decentralized policy making also depends on the magnitudes of the two effects of market integration, see (18). Multiplying the two effects yields

$$R'_I(\kappa) \frac{\partial k}{\partial t} = \frac{2\sigma_I^2(1+t)\sigma_I+2(1-\sigma_I)}{C''(\sqrt{\sigma_I})\sqrt{\sigma_I} - C''(\sqrt{\sigma_I}) - [(1+t)\sigma_IW''_{II}(\kappa(t,\sigma)) + 2(1-\sigma_I)W''_{NN}(\kappa)]} > 0,$$

and the following result becomes immediately obvious:

**Proposition 4** Market integration is unambiguously downward distorted if network investment is centralized and the countries choose capacity reserves non-cooperatively ($\sigma_I^* < \sigma^{fb}$ in stable equilibrium).

A decentralized choice of capacity reserves at the individual country level has an unambiguous effect on market integration, despite the ambiguous effect on capacity reserves. Capacity reserves are downward distorted if the cost efficiency of market integration is comparatively strong, which in turn leads to a downward distortion of network investment by strategic complementarity. Instead, capacity reserves are upward distorted if the portfolio effect of market integration is comparatively strong, which again leads to a downward distortion of network investment, this time by strategic substitutability.

### 4.2 Decentralized network investment

Assume now that the two countries invest in domestic network reliability ($y_1, y_2$) in a non-cooperative manner. The total network reliability becomes $y_1y_2$ under the assumption that network reliability is stochastically independent across the two countries. The welfare in country $i$ then equals

$$W(k_i, k_j, y_i, y_j) = y_iy_jW_I(k) + (1 - y_iy_j)W_N(k_i) - C(y_i)$$

as a function of the capacity reserves ($k_i, k_j$) and network reliability ($y_i, y_j$).

Country $i$’s welfare function is not necessarily quasi-concave in the domestic policy variables ($k_i, y_i$) although it is quasi-concave in each of the two arguments $k_i$ and $y_i$. To circumvent any existence problems caused by non-concavity, I assume that $k_i$ and $y_i$ are decentralized to different policy makers in country $i$ and chosen independently of one another. Any Nash equilibrium under a coordinated choice of ($k_i, y_i$) is contained in the set of Nash equilibria under a non-cooperative choice of $k_i$ and $y_i$.

The total network reliability $R_N(k) = y^2_N(k)$ under decentralized network investment is
characterized by the solution to

\[ W_I(k) - W_N(k) = \frac{C'(\sqrt{R_N})}{\sqrt{R_N}} \]

in interior symmetric equilibrium for a symmetric capacity reserve \( k_1 = k_2 = k \). The equilibrium degree of market integration \( \sigma_N(t) \) under decentralized network investment is implicitly characterized by the solution to \( \sigma_N = R_N(\kappa(t, \sigma_N)) \) as a function of the degree \( t \) to which policy makers internalize the foreign externality of capacity reserves.

By following the same procedure as in the case of centralized network investment, it is easy to verify that market integration is smaller when domestic policy makers fail to internalize the external effects of capacity reserves compared to the case when all such effects are internalized: \( \sigma_N^* < \sigma_N(1) \). The next question is whether decentralized network investment further accentuates those distortions, i.e. whether \( \sigma_N^* < \sigma_I^* \). To analyze this question, define \( R(k, \tau) \) by

\[ W_I(k) - W_N(k) = \frac{C'(\sqrt{R})}{(1 + \tau)\sqrt{R}} \]

and \( \sigma(t, \tau) \) by \( \sigma = R(\kappa(t, \sigma), \tau) \). By construction, \( \sigma_I^* = \sigma(0, 1) \) and \( \sigma_N^* = \sigma(0, 0) \), so that the difference in network reliability between the two regimes becomes:

\[ \sigma_I^* - \sigma_N^* = \int_0^1 \frac{\partial \sigma(0, \tau)}{\partial \tau} d\tau = \int_0^1 \frac{\frac{\partial R(\kappa(t, \tau), \sigma)}{\partial \tau}}{1 - \frac{\partial R(\kappa(t, \sigma), \tau)}{\partial \sigma}} \frac{\partial \kappa(t, \sigma)}{\partial \sigma} d\tau. \]

The denominator is positive in stable equilibrium, so that the effect on market integration is determined by the direct effect:

\[ \frac{\partial R(\kappa, \tau)}{\partial \tau} = \frac{1}{1 + \tau} \frac{2\sigma C'(\sigma^2)}{C'(\sqrt{\sigma})\sqrt{\sigma} - C'(\sqrt{\sigma})} > 0, \]

and it follows that:

**Proposition 5** Market integration is further downward distorted if both network investment and capacity reserves are decided non-cooperatively by the two countries compared to the case when network investment is centralized (\( \sigma_N^* < \sigma_I^* < \sigma_I^{fb} \) in stable equilibrium).

Domestic investment in network reliability has positive effects abroad because of improved market integration. A country concerned entirely with the maximization of domestic surplus neglects these positive external effects, which causes the total network reliability to be smaller under decentralized than centralized network investment. Hence, the welfare distortions associated with decentralized decision making are additive in this model.

---

\(^{22}\)Observe that \( y_1 = y_2 = 0 \) constitutes a Nash equilibrium under decentralized network investment because network reliability is zero independently of \( y_i \) if \( y_i = 0 \). I consider the more interesting case of positive market integration.
5 Financial markets

Capacity reserves are beneficial because they protect consumers against blackouts and financial losses by reducing price spikes. An alternative way to hedge price risk would be through a financial market. This section investigates how financial markets interact with the socially optimal capacity reserves and those that would arise in equilibrium. In particular, would the distortions associated with decentralized policy making prevail or vanish in a competitive and well-functioning financial market?

5.1 Model extension

Let the industry in country $i$ purchase $q_n$ call options for one MWh each with strike price $s$. Assume that the financial market is perfectly competitive and that realized gains and losses are treated symmetrically in the financial market; the seller is risk neutral and can clear any losses one for one against other profits. The equilibrium option price in country $i$ then simply equals the expected option payment:

$$v(k_i, k_j, s) = \sigma \int_0^\bar{r} \max\{p_I(r, k) - s; 0\} dF_I(r) + (1 - \sigma) \int_0^\bar{r} \max\{p_N(r, k_i) - s; 0\} dF_N(r)$$

under partial market integration ($\sigma \in [0, 1]$ and exogenous).

Financial contracting leaves the profit of the power producers unaffected. The expected welfare in country $i$ thus becomes

$$W(k_i, k_j, s) = \sigma (CS_I(k, s) + \Pi_I(k)) + (1 - \sigma)(CS_N(k_i, s) + \Pi_N(k_i)) - q_n v(k_i, k_j, s),$$

where

$$CS_M(k, s) = u(q_M(k) - q_n) - \int_0^\bar{r} p_M(r, k) dF_M(r) q_M(k)$$

$$+ q_n \int_0^\bar{r} [\phi - \min\{p_M(r, k); s\} - B(\min\{p_M(r, k); s\} - \phi)] dF_M(r)$$

represents the consumer surplus under market structure $M$ gross of the option cost $q_n v(k_i, k_j, s)$.23 The corresponding expected welfare in the representative country becomes $W(k, s) = W(k, k, s)$ under symmetric capacity reserves, $k_1 = k_2 = k$.

5.2 Equilibrium capacity reserves vs. the social optimum

Assume that the capacity reserves are symmetric and so small that the option is in the money when renewable resources are scarce under both market structures, i.e. $\bar{P}_I(k) > s$ and $\bar{P}_N(k) > s$.

23It would be appropriate to denote the shadow cost $B(\min\{p_M(r, k); s\} + v(k_i, k_j, s) - \phi)$ under financial contracting because the electricity intensive industry turns an operating profit if and only if $\phi \geq \min\{p_M(r, k); s\} + v(k_i, k_j, s)$. However, the options are purchased prior to the resolution of any uncertainty and therefore represents a sunk cost at the production stage. To avoid uninteresting complications, I assume that only the variable part of the profit represents a shadow cost to the firm.
The welfare effect of an increase in the capacity reserve equals:

\[
\frac{\partial W(k,s)}{\partial k} = -q_n B(s - \phi)[\sigma f_1(k)q'_1(k) + (1 - \sigma)f_N(k)q'_N(k)]
\]

\[ -\sigma \psi_1(k)q'_1(k) - (1 - \sigma)\psi_N(k)q'_N(k). \tag{20} \]

The sum of the two terms on the second row is the marginal expected distortion of consumption and investment in a partially integrated market. The term on the first row is the marginal insurance effect. It is zero if the strike price is below the industry’s break-even price so that the financial market already offers complete insurance \((B(s - \phi) = 0 \text{ for all } s \leq \phi)\). The marginal insurance effect is strictly negative when the firm is exposed to price risk \((s > \phi)\). Recall that the welfare benefit of an increase in the capacity reserve works through the reduction in the maximal price, \(P^*_M(k) < 0\), when there are no financial contracts; see Proposition 1. This security of supply benefit vanishes under option contracting because then it is the strike price \(s\) that marks the maximal price for the electricity intensive industry. The only remaining effect of the capacity reserve is to crowd out market-based investment in thermal capacity, which increases the likelihood that the price cannot clear in the market. Crowding-out represents the first term in (20) above. Hence, (the proof is in Appendix A.6):

**Proposition 6** Assume that consumers can hedge risk by purchasing call options in a competitive financial market that renders the equilibrium option price equal to the expected option payment. The socially optimal capacity reserve \(k^{fb}(\sigma, s)\) is zero for any degree of market integration \(\sigma \in [0,1]\) and any option strike price \(s < \infty\). The social optimum can be implemented as a pay-off dominant Nash equilibrium if countries set capacity reserves non-cooperatively to maximize domestic welfare.

Financial markets completely remove the need for capacity reserves because they distort prices and investments without providing any hedging benefits beyond what can be achieved through financial contracting alone. The efficiency of energy-only markets does not hinge upon financial markets being able to hedge all consumers’ price risk \((s \leq \phi)\). All that matters is that the price risk is bounded \((s < \infty)\). The expected shadow cost of losses is driven to zero as capacity reserves become small because the probability \(F_M(k)\) of supply scarcity vanishes.

There are no inefficiencies associated with decentralized policy making, not even under incomplete market integration. No country has anything to gain by unilaterally introducing a capacity market in an energy-only market with financial contracting because there are no domestic hedging benefits to be achieved, only distortions.

Proposition 6 points to at least two reasons why countries would introduce capacity markets in a market with financial contracting. Domestic policy makers could have other objectives than to maximize the sum of domestic consumer and producer surplus. If, for example, the expected profit of the energy intensive industry weighs more heavily than the other groups in the economy, a motive for introducing a capacity mechanism would be to push down the expected option payment and thereby reduce the cost to the industry of financial contracting.
An efficiency argument in favour of capacity markets arises in an imperfect financial market unable to hedge all risk. There could for instance be volume risk, which I have ignored by assuming constant demand \( q_n \). But there could also be remaining price risk. Assume that the sellers of financial contracts cannot diversify away all risk. To facilitate comparison with the analysis in Section 3, assume that \( B(\cdot) \) now denotes the shadow cost of losses faced by the sellers of the option contracts, whereas \( \tilde{B}(\cdot) \) represents the industry’s shadow cost.\(^\text{24}\) In a competitive financial market, the option price equals the expected option payment plus the risk correction:

\[
v(k, k, s) = \sigma \int_0^r B(\max\{p_I(r, k) - s; 0\}) dF_I(r) + (1 - \sigma) \int_0^r B(\max\{p_N(r, k) - s; 0\}) dF_N(r).
\]

The option price will be very high in an energy-only market if \( B(\cdot) \) is large for large option payments, even if the financial market is competitive and despite the option payment being bounded in expectation.\(^\text{25}\) Capacity reserves again improve performance in the financial market by limiting market participants’ exposure to price spikes. The welfare effect of a small increase in the capacity reserve equals

\[
\frac{\partial W(k, s)}{\partial k} = \sigma S_I(k, s) + (1 - \sigma) S_N(k, s)
- \sigma [\psi_I(k) + q_n \tilde{B}(s - \phi) f_I(k)] q'_I(k) - (1 - \sigma) [\psi_N(k) + q_n \tilde{B}(s - \phi) f_N(k)] q'_N(k).
\]

This trade-off is qualitatively similar to the one that arises with consumer risk aversion, but no financial markets. A minor difference is that the reference price now equals the strike price \( s \) instead of the industry break-even price \( \phi \). If \( s = \phi \), then the solution is exactly the same as in Proposition 3. Hence, it is only under strong assumptions about the financial market in terms of competitiveness and the diversifiability of risk that the need for capacity reserves vanishes.

### 6 National allocation rules for capacity reserves

I have so far assumed that all available capacity reserves are used in an efficient manner under market integration, independently of where the system is constrained the most. In this section, I instead assume that countries are responsible for handling their own supply problems separately. This change is of no consequence in a situation with national markets, because then there would be no flow of electricity between the countries anyway. For illustration, consider therefore the opposite polar case of perfect market integration.

In a perfectly integrated market, total consumption \( q \) and market-based investment \( x < q \) are identical in the two countries independently of the how supply constraints are handled because all consumers and producers face identical prices. There is enough thermal capacity to clear the market if and only if \( r \geq q - x \). In the opposite case of a supply constrained market, I define

\(^{24}\)Now there is risk aversion both on the seller and buyer side. A sufficient condition for gains from trade in the financial markets given \( s > \phi \) is \( B(\tilde{p} - \phi) - B(\tilde{p} - s) > B(\tilde{p} - s) \) for all \( \tilde{p} > s \).

\(^{25}\)It is easy to verify that \( \lim_{k \to 0} v(k, k, s) \leq \sigma \lim_{k \to 0} u'(q_I(k) - q_n) + (1 - \sigma) \lim_{k \to 0} u'(q_N(k) - q_n) < \infty. \)
the national supply constraint in country $i$ as

$$\begin{align*}
\max \{ q - x - r_i; 0 \} & \text{ if } r < q - x \text{ and } r_j < q - x \\
2(q - x) - r_1 - r_2 & \text{ if } r < q - x \text{ and } r_j \geq q - x.
\end{align*}$$

Country $i$ faces a national supply constraint only if the domestic market-based supply is insufficient to cover the domestic demand: $x + r_i < q$. If this situation occurs also in country $j$, then the domestic excess demand defines the national supply constraint in both countries. If instead country $j$ has excess supply, $x + r_j \geq q$, then the national supply constraint in country $i$ is the difference between the domestic excess demand and net imports.

The price cap $P_I(k)$ of Section 2 was defined to generate precisely enough market-based thermal investment $x_I(k)$ to cover residual demand $q_I(k) - k$ in the worst case scenario without renewable production anywhere and if the two countries have the same capacity reserve, $k_1 = k_2 = k$. If the two countries have chosen different capacity reserves, $k_1 \neq k_2$, then $P_I(k)$ is still necessary and sufficient to ensure the security of supply in both markets if now $k = \min\{k_1; k_2\}$.

The symmetry of demand and market-based thermal investment implies that total thermal output only depends on $k = \min\{k_1; k_2\}$ even if $k_1 \neq k_2$. In this case, there is excess thermal capacity $k_i - \min\{k_1; k_2\}$ in one country. Importantly, the thermal production

$$\begin{align*}
q_I(k) - r_i & \text{ if } r_i < k \text{ and } r_j < k \\
2q_I(k) - x_I(k) - r_i - r_j & \text{ if } r < k \text{ and } r_j \geq k \\
x_I(k) & \text{ if } r < k \text{ and } r_i \geq k
\end{align*}$$

in country $i$ displays more variability under a national supply constraint than under an aggregate supply constraint where thermal production equals $q_I(k) - r$. This variability implies an inefficiency because of the convexity of the thermal production cost. The welfare in the representative country can then be written as

$$W_{I\text{nat}}(k) = CS_I(k) + \Pi_I(k) - \Omega_I(k)$$

for symmetric capacity reserves $k_1 = k_2 = k$, where $\Omega_I(k)$ represents the production inefficiency associated with the national supply constraint, and $\Omega'_I(k) = \omega_I(k_I^{rb})q'_I(k_I^{rb}) > 0$ is the corresponding marginal production inefficiency; see equations (26) and (27) in Appendix A.7 for a characterization and a proof of the following:

**Proposition 7** Assume that electricity markets are perfectly integrated, but supply constraints are defined at the national level. Any constrained socially optimal capacity reserve satisfies $k_I^{rb} < k_I^{fb}$ and is characterized by:

$$SS_I(k_I^{rb}, \phi) = [\psi_I(k_I^{rb}) + \omega_I(k_I^{rb})]q'_I(k_I^{rb}).$$

(21)

The constrained social optimum can be implemented as a pay-off dominant Nash equilibrium if countries set capacity reserves non-cooperatively to maximize domestic welfare.
National allocation rules imply that the socially optimal capacity reserve \( k_{sb}^{*} \) falls below the level \( k_{fb}^{*} \) that would arise under an efficient dispatch of capacity reserves because the marginal distortion associated with a capacity reserve is larger in the former case. However, there are no particular distortions associated with decentralized policy making in the perfectly integrated market. Symmetry across countries and the fact that the price cap \( \bar{P}_t(k) \) is determined by the minimal capacity reserve \( k = \min\{k_1; k_2\} \) imply that each country internalizes all welfare effects by the unilaterally optimal choice of capacity reserve.

7 Policy discussion

This paper has studied countries’ unilateral incentives for increasing security of supply by means of capacity reserves and network investment in a two-country model of interconnected electricity markets with fluctuating renewable production. Capacity reserves offer consumers protection against price spikes and running blackouts in situations of renewable production shortfalls, but also distort long-run investment and consumption decisions in the market. Network reinforcements reduce national supply constraints, but are costly.

A first finding is that a non-cooperative choice of capacity reserves not necessarily is inefficient. National policy makers effectively internalize the foreign externalities if countries are symmetric, perfectly integrated, and capacity reserves are deployed in an efficient matter. Hence, necessary conditions for inefficient policy making are country asymmetries and/or imperfectly integrated markets. This paper emphasizes distortions associated with market integration.

Equilibrium capacity reserves can be too large or too small in an imperfectly integrated market depending on the relative magnitude of two cross-border externalities. On the one hand, a larger foreign capacity reserve benefits the home country by improving supply security in the entire market. Free-riding on foreign capacity reserves tends to generate capacity reserves that are too small. On the other hand, a larger domestic capacity reserve exacerbates consumption and investment distortions abroad. Such international spill-overs cause excessive capacity reserves. Because of these ambiguous effects, it is impossible to make general recommendations about whether countries should be encouraged to increase domestic capacity reserves or discouraged from doing so. The net effect depends quantitatively on the strength of a portfolio relative to a cost efficiency effect of market integration.

Network underinvestment is a pervasive problem. First of all, congestion rent is an inappropriate measure of the social value of network reinforcements to increase system reliability. For instance, congestion rents are always zero in the present model independently of network reliability. Hence, the optimal level of network investment cannot be decided on the basis of market signals alone. Centralizing the choice of network investment improves matters because of the positive foreign externalities associated with improved market integration. However, even a regulation that causes network owners to invest in order to maximize total welfare is insufficient if countries choose capacity reserves non-cooperatively. In light of this finding, the current EU guidelines for cross-border interconnections subject to which (European Union, 2013, p.44) 

"[t]he costs for the development, construction, operation and maintenance of projects of com-
mon interest should in general be fully borne by the users of the infrastructure" are likely to be suboptimal from a social welfare perspective. One way to reduce the inefficiency of domestically chosen capacity reserves is to establish a regulation that induces network investors to attach a stronger weight to the marginal value of increased market integration relative to the cost of improving the network and thus to overinvest all else equal. This suggests that users should either pay in excess of the full network costs, or network investment should be subsidized at central EU level to offset the distortions associated with capacity reserves.

A major benefit of capacity reserves is to shelter consumers against short-term price spikes in the market. This benefit is reduced if consumers also can hedge price risk in a financial market. Financial contracting thus reduces the need for capacity mechanisms. Put differently, a larger share of the thermal investment necessary to ensure security of supply can be left to the market if consumers have the possibility to insure themselves against the price spikes necessary to accomplish this investment. In fact, the optimal capacity reserve is close to zero in the limit when the financial market is efficient and able to absorb all price risk.  

A fundamental property of an efficient market design therefore is the development of an efficient financial market (European Commission, 2016). However, this market is more likely to develop if capacity reserves are in place to protect market participants against extreme prices. Consequently, capacity and financial markets are not necessarily substitutes for one another.

The socially optimal and equilibrium capacity reserves are smaller when reserves are deployed solely to resolve domestic supply constraints, because the marginal thermal production cost associated with capacity reserves then is higher than necessary. A national perspective on supply constraints therefore transforms into larger than necessary price spikes to ensure the security of supply in an integrated electricity market with large shares of renewable production. Instead, a multinational approach to capacity mechanisms increases efficiency and the security of supply, for instance a system in which domestic capacity reserves can be invoked so as to relieve supply security problems abroad.  

References


Cramton, Peter and Steven Stoft (2006): The convergence of market designs for adequate generating capacity. Report for the California Electricity Oversight Board.


26 See Galetovic et al. (2015) for a quantitative analysis of energy-only versus markets with capacity reserves and the role of financial markets in bridging the gap between the two.

27 Neuhofer et al. (2016) argue in favor of cross-border coordination of procurement and activation of capacity reserves. Newbery (2016) discusses the importance of market integration for the efficiency of capacity mechanisms.


Appendix

A.1 The continuous price extension

Let the short-term price be defined by

\[
\tilde{p}(q - r) = \begin{cases} 
  c(q - r) & \forall r \geq q - x \\
  \tilde{p}(q - r) & \forall r \in ((q - x)(1 - \varepsilon), q - x) \\
  \bar{p} & \forall r \leq (q - x)(1 - \varepsilon).
\end{cases}
\]

The only difference between this price and (1) in the main text is the inclusion of the twice continuously differentiable and increasing function \(\tilde{p}(\cdot)\) in a small interval \(r \in ((q - x)(1 - \varepsilon), q - x)\). In particular, \(\tilde{p}(x) = c(x)\) and \(\tilde{p}(q\varepsilon + x(1 - \varepsilon)) = \bar{p}\). The purpose is to avoid uninteresting and complicating discontinuities around the point of full capacity utilization, \(r = q - x\). All results in the main text are limiting results for \(\varepsilon \to 0\).

The optimality conditions

\[
\begin{align*}
  u'(D_M) &= \int_{(D_M + q_n - X_M)(1 - \varepsilon)}^{r} \tilde{p}(D_M + q_n - r) dF_M(r) + \tilde{p}F_M((D_M + q_n - X_M)(1 - \varepsilon)), \\
  F_{X_M} &= \int_{(D_M + q_n - X_M)(1 - \varepsilon)}^{r} (\tilde{p}(D_M + q_n - r) - \bar{p}) dF_M(r) + (\tilde{p} - c(X_M))F_M(D_M + q_n - X_M) = \delta
\end{align*}
\]

jointly define the equilibrium household demand \(D_M(\tilde{p})\) and market-based investment \(X_M(\tilde{p})\).

Straightforward differentiation of the two conditions yields:

\[
\begin{align*}
  D'_M(\tilde{p}) &= \frac{c(X_M)(D_M + q_n - X_M)(1 - \varepsilon)}{c'(X_M)F_M((D_M + q_n - X_M)(1 - \varepsilon))} < 0, \\
  X'_M(\tilde{p}) &= \frac{c'(D_M) - \int_{(D_M + q_n - X_M)(1 - \varepsilon)}^{r} \tilde{p}(D_M + q_n - r) dF_M(r)}{c(X_M)F_M((D_M + q_n - X_M)(1 - \varepsilon))}D'_M(\tilde{p}) > 0.
\end{align*}
\]

Combine the two market-clearing conditions to get

\[
\begin{align*}
  u'(D_M) &= \int_{(D_M + q_n - X_M)}^{r} c(D_M + q_n - r) dF_M(r) + c(X_M)F_M(D_M + q_n - X_M) + \delta.
\end{align*}
\]

Hence, the demand in the energy-only market, \(\lim_{\tilde{p} \to -\infty} D_M(\tilde{p}) = D^\infty_M\), is the solution to

\[
\begin{align*}
  u'(D^\infty_M) &= \int_{0}^{r} c(D^\infty_M + q_n - r) dF_M(r) + \delta,
\end{align*}
\]

whereas the market-based investment level satisfies \(X^\infty_M = \lim_{\tilde{p} \to -\infty} X_M(\tilde{p}) = D^\infty_M + q_n < \infty\).

By the definitions of \(P_M(k)\) and \(x_M(k)\) in the main text, I can then solve for the short-term price as a function of \(k\):

\[
p_M(r, k) = \begin{cases} 
  c(x_M(k) + k - r) & \forall r \geq k \\
  \tilde{p}(x_M(k) + k - r) & \forall r \in (k(1 - \varepsilon), k) \\
  \bar{p}_M(k) & \forall r \leq k(1 - \varepsilon).
\end{cases}
\]
Straightforward differentiation of \( q_M(k) = D_M(\tilde{P}_M(k)) + q_n \) yields

\[
q'_M(k) = \frac{c'(x_M(k))F_M(k)}{c'(x_M(k))F_M(k) + \int_k^1 c'(q_M(k) - r)dF_M(r) - u''(q_M(k) - q_n)} \in (0, 1).
\]

(A.2) Capacity payments

The activated capacity reserve is sold in the wholesale market at the administered price. Hence,

\[
T_N(k) = \int_0^{k_1} \int_0^{k_2} c(x_N(k) + z)dz - p_N(r, k)(k - r)dF_N(r) + \delta k
\]

represents the minimal capacity payment necessary to procure the desired capacity reserve \( k \) and ensure supply security at the price cap \( \tilde{P}_N(k) \) when electricity markets are national.

The minimal capacity payment necessary to implement a capacity reserve of \( k \) in both countries under perfect market integration is given by

\[
T_I(k) = \int_{\min\{2k; \tau\}}^{\max\{2k - \tau, 0\}} \int_0^{2k - r_2} c(x_I(k) + z)dz - p_I(r, k)(k - r)dF(r_1, r_2)
\]

\[
+ \int_0^{\max\{2k - \tau, 0\}} \int_0^{k_2} c(x_I(k) + z)dz - p_I(r, k)(k - r)dF(r_1, r_2) + \delta k.
\]

The renewable output in country 2 is large enough to clear the market independently of renewable output in country 1 if \( r_2 \geq \min\{2k; \tau\} \). At the other extreme, the capacity reserve in country 1 is always activated independently of domestic renewable production if \( r_2 < \max\{2k - \tau, 0\} \). This possibility is captured by the first term on the second row above. In the intermediate case, \( \max\{2k - \tau, 0\} \leq r_2 < \min\{2k; \tau\} \), the capacity reserve in country 1 is activated if and only if the domestic renewable output is too small: \( r_1 < 2k - r_2 \). This case represents the term on the first row above.

(A.3) Proof of Proposition 1

Existence and uniqueness Continuity of \( W_M(k) \) in \( k \) and compactness of the domain, \( k \in [0, \tilde{r}] \) imply that a social optimum \( k_M^{fb} \) exists. Any socially optimal capacity reserve \( k_M^{fb} \) is positive by the assumption that \( W'_M(k) > 0 \) for all \( k \) sufficiently close to zero. It is also the case that \( k_M^{fb} \leq \tilde{k}_M \) because \( \tilde{P}_M(k) < \phi \) for all \( k > \tilde{k}_M \) and any capacity reserve above \( \tilde{k}_M \) therefore would serve only to distort consumption and investment further without providing any additional insurance benefits. In fact, \( k_M^{fb} < \tilde{k}_M \) because

\[
\lim_{k \uparrow \tilde{k}_M} W'_M(k) = -\psi_M(\tilde{k}_M)q'_M(\tilde{k}_M) < 0,
\]

see (31). Strict concavity of \( W_M(k) \) in the domain \( (0, \tilde{k}_M) \) implies that the first-order condition \( W'_M(k_M^{fb}) = 0 \) uniquely characterizes the socially optimal capacity reserve, which is approximately equal to (10) for \( \varepsilon \) close to zero.
Implementation. This is trivial when electricity markets are national because then there is no strategic interaction between the policy makers in the two countries, and assuming that national policy makers choose the capacity reserve to maximize the domestic welfare. In the case of perfect market integration, expected welfare in country $i$ equals $W_i\left(\frac{k_i + k_j^b}{2}\right) \leq W_i(k_j^b)$ for all $k_i \neq k_j^b$, where the inequality follows from global optimality of $k_j^b$. Hence, choosing a capacity reserve of $k_i = k_j^b$ is a best-reply for country $i$ to the choice of capacity reserve $k_j = k_j^b$ in country $j \neq i$. There could be multiple Nash equilibria, but the one in which both countries choose $k_j^b$ is pay-off dominant because national welfare in both countries is proportional to aggregate welfare, which is maximized at $k_j^b$.

A.4 Proof of Proposition 2

Let $\rho_M(k, \phi)$ implicitly defined by $p_M(\rho_M, k) = \phi$ for $k < \tilde{k}_M$ and by $\rho_M(k, \phi) = 0$ for $k > \tilde{k}_M$ denote the threshold level of renewable output below which the electricity price rises above $\phi$. By the definition of $\tilde{k}_M$ in (4) and assumption (6), it follows that $\rho_M(k, \phi) \in (k(1-\varepsilon), k)$ if $k < \tilde{k}_M$. Observe that

$$
\frac{W'_I(k)}{q'_I(k)q_n} = \frac{\psi_I(k)}{\psi_N(k)q'_N(k)q_n} + \frac{\psi_I(k)H(k)}{\psi_N(k)q'_N(k)q_n} + \frac{\psi_I(k)}{\psi_N(k)}B(\hat{P}_N(k) - \phi)F_N(k - f_N(k(1-\varepsilon))) + \frac{\psi_I(k)}{\psi_N(k)}B(\hat{P}_N(k) - \phi)F_N(k - f_N(k(1-\varepsilon)))
$$

for $k < \min\{\tilde{k}_N; \tilde{k}_I\}$, where

$$
H(k) = \frac{1}{\psi_N(k)}[B'(\hat{P}_N(k) - \phi)F_N(k) - F_N(k) - F_N(k(1-\varepsilon))] + \frac{\psi_I(k)}{\psi_N(k)}B(\hat{P}_N(k) - \phi)F_N(k - f_N(k(1-\varepsilon)))
$$

$H(k_N^b) > 0$ and $H(k_I^b) > 0$ if inequality (13) is satisfied, whereas the terms on the last three rows of the above expression are negligible for $\varepsilon$ sufficiently close to zero. Assume first that $\tilde{k}_I \leq \tilde{k}_N$. As $W'_I(k_N^b) = 0$, it follows from the above expression that $W'_N(k_I^b) < 0$. Strict quasi-concavity of $W_N$ then implies $k_N^b < k_I^b$. Assume next that $\tilde{k}_N \leq \tilde{k}_I$. As $W'_N(k_N^b) = 0$, it follows that $W'_I(k_N^b) > 0$. Strict quasi-concavity of $W_I$ then implies $k_N^b > k_I^b$. All inequalities are reversed if inequality (13) is reversed.

A.5 Proof of Proposition 3

Uniqueness Let $Z(k, t, \sigma) = \frac{1+t}{2}\sigma W'_I(k) + (1-\sigma)W'_N(k)$. We already know from Lemma 1 that any solution $Z(k, t, \sigma) = 0$ must satisfy $\kappa(t, \sigma) \in [\min\{k_N^b; k_I^b\}, \max\{k_N^b; k_I^b\}]$ and that there exists at least one such solution $\kappa(t, \sigma)$ for every $(t, \sigma) \in [0, 1]^2$. Strict concavity of $W_I(k)$ and
$W_N(k)$ in the domain $[\min\{k_N^{fb};k_I^{fb}\}, \max\{k_N^{fb};k_I^{fb}\}]$ imply that $\kappa(t, \sigma)$ is unique. In particular, there exists a unique symmetric equilibrium candidate $k^*(\sigma) = \kappa(0, \sigma)$ which is, moreover, contained in $[\min\{k_N^{fb};k_I^{fb}\}, \max\{k_N^{fb};k_I^{fb}\}]$.

**Existence** Assume that $k_j = k^*(\sigma)$, and consider country $i$'s incentive to deviate from $k^*(\sigma)$. It can never be optimal for $i$ to deviate to $k_i > 2\max\{\bar{k}_N(\phi);\bar{k}_I(\phi)\} - k^*(\sigma)$ because then $k_i + k^*(\sigma) > \bar{k}_I$ and $k_i > \bar{k}_N$ so that $\bar{P}_I(k_i + k^*(\sigma)) < \phi$ and $\bar{P}_N(k_i) < \phi$. In this case, country $i$ only distorts investment and consumption at home without offering any additional insurance benefits to the domestic industry. Next,

$$
\frac{\partial^2 W(k_i, k^*)}{\partial k_i^2} = \frac{1}{4}\sigma W''_{1}\left(\frac{k_i + k^*(\sigma)}{2}\right) + (1 - \sigma)W''_{1}(k_i) < 0
$$

for all $k_i \in (0, 2\max\{\bar{k}_N;\bar{k}_I\} - k^*(\sigma)]$ by assumption (9), and $k^*(\sigma) \in (0, 2\max\{\bar{k}_N;\bar{k}_I\} - k^*(\sigma)]$ imply that $k_i = k^*(\sigma)$ is country $i$'s unique best-reply to $k_j = k^*(\sigma)$.

**Characterization** The first-order condition $Z(\kappa, 0, \sigma) = 0$ uniquely characterizes the symmetric equilibrium $k^*(\sigma) = \kappa(0, \sigma)$, which is approximately equal to (19) for $\varepsilon$ close to zero. The comparative statics follow directly from (18) and Lemma 1.

### A.6 Proof of Proposition 6

**The social optimum** The expected welfare in country $i$ can equivalently be written as

$$
W(k_i, k_j, s) = W(k_i, k_j) - q_n\sigma \int_0^{\rho_1(k, \phi)} B(\min\{p_I(r, k); s\} - \phi)dF_I(r)
$$

$$
+ q_n(1 - \sigma) \int_0^{\rho_N(k_i, \phi)} B(\min\{p_N(r, k_i); s\} - \phi)dF_N(r),
$$

where

$$
\hat{W}(k_i, k_j) = \sigma[u(q_I(k) - q_n) + q_n\phi - \int_0^\tau p_I(r, k)dF_I(r)q_I(k) + \Pi_I(k)]
$$

$$
+ (1 - \sigma)[u(q_N(k_i) - q_n) + q_n\phi - \int_0^\tau p_N(r, k_i)dF_N(r)q_N(k_i) + \Pi_N(k_i)]
$$

denotes the expected welfare gross of the expected shadow cost of losses. At symmetric capacity reserves $k_1 = k_2 = k$:

$$
\hat{W}(k, k) \geq W(k, k, s) \geq \hat{W}(k, k) - q_nB(s - \phi)[\sigma F_I(\rho_I(k, \phi)) + (1 - \sigma)F_N(\rho_N(k, \phi))]
$$

where the first inequality follows from $B(\min\{p_M(r, k); s\} - \phi) \geq 0$ for all $r$, and the second from $B(s - \phi) \geq B(\min\{p_M(r, k); s\} - \phi)$ for all $r$. $\rho_M(k, \phi) \leq k$ yields

$$
W(0, 0, s) = \hat{W}(0, 0) = \sigma[u(D_N^{\infty}) + q_n\phi - \int_0^\tau \int_0^{D_N^{\infty} + q_n - r} c(\tilde{r})d\tilde{r}F_I(r) - \delta(D_N^{\infty} + q_n)]
$$

$$
+ (1 - \sigma)[u(D_N^{\infty}) + q_n\phi - \int_0^\tau \int_0^{D_N^{\infty} + q_n - r} c(\tilde{r})d\tilde{r}F_N(r) - \delta(D_N^{\infty} + q_n)]
$$
Assume that differentiation of Characterization

For completeness, I replicate the marginal welfare expression (20) for and the fact that zero capacity reserves is the first-best social optimum.

is non-negative for all \((k, j)\) because the shadow cost is non-negative and

$$\frac{\partial \bar{W}(k, j)}{\partial k_i} = -\frac{\sigma}{2} \psi(k) q_i'(k) - (1 - \sigma) \psi_N(k) q_N'(k) < 0, \quad \frac{\partial \bar{W}(k, j)}{\partial k_j} = -\frac{\sigma}{2} \psi(k) q_i'(k) \leq 0$$

imply \(\bar{W}(0, 0) \geq \bar{W}(k, j)\) for all \((k, j)\). \(W(0, 0, s) \geq W(k, j, s)\) for all \((k, j)\) implies that \(k_1^o(\sigma, s) = k_2^o(\sigma, s) = 0\) is the social optimum.

**Implementation** \(W(0, 0, s) \geq W(k, j, 0)\) for all \(k > 0\) implies that \(k_1^o(\sigma, s) = k_2^o(\sigma, s) = 0\) can be sustained as a Nash equilibrium when the two countries choose capacity reserves non-cooperatively to maximize domestic welfare. The equilibrium is pay-off dominant by symmetry and the fact that zero capacity reserves is the first-best social optimum.

**Characterization** For completeness, I replicate the marginal welfare expression (20) for \(s > \phi\).

Assume that \(k_1 = k_2 = k\) is sufficiently small that \(\bar{P}_I(k) > s\) and \(\bar{P}_N(k) > s\). Straightforward differentiation of \(W(k, s)\) yields

$$\frac{\partial W(k, s)}{\partial k} = -\sigma q_i'(k) [q_o \int_{\rho_I(k, s)}^{\rho_M(k, s)} B'(\bar{p}(q_i(k) - r) - \phi) \bar{p}'(q_I(k) - r) dF_I(r) + \psi_I(k)] - (1 - \sigma) q_N'(k) [q_o \int_{\rho_N(k, s)}^{\rho_M(k, s)} B'(\bar{p}(q_N(k) - r) - \phi) \bar{p}'(q_N(k) - r) dF_N(r) + \psi_N(k)],$$

which is strictly negative. An integration by parts yields

$$\int_{\rho_M(k, s)}^{\rho_M(k, s)} B'(\bar{p}(q_M(k) - r) - \phi) \bar{p}'(q_M(k) - r) dF_M(r)$$

$$= \int_{\rho_M(k, s)}^{\rho_M(k, s)} B(\bar{p}(q_M(k) - r) - \phi) f_M'(r) dr + B(s - \phi) f_M(\rho_M(k, s))$$

$$\approx B(s - \phi) f_M(k)$$

for \(\varepsilon\) close to zero. The approximation holds because \(k(1 - \varepsilon) < \rho_M(k, s) < \rho_M(k, \phi) < k\) for \(\bar{P}_M(k) > s > \phi\) implies \(\rho_M(k, \phi) \rightarrow \rho_M(k, s) \rightarrow k\) as \(\varepsilon \rightarrow 0\). Substituting \(B(s - \phi) f_M(\rho_M(k, s))\) into \(\partial W(k, s)/\partial k\) above produces (20).

**A.7 Proof of Proposition 7**

I first characterize the difference between the thermal production cost under a national versus
and the first row of
\[
\frac{\partial \Omega_{k}^2 (k, r_i, r_j)}{\partial k} \frac{1}{q_i(k)} = \frac{1}{2} c(2q_i(k) - x_i(k) - 2r) + \frac{1}{2} c(x_i(k)) - c(q_i(k) - r) + \frac{1}{2} \{c(x_i(k) + 2(k - r)) - c(x_i(k))\}(1 - \frac{K'_i(\tilde{p}_i)}{D'_i(\tilde{p}_i)})
\]
are both non-negative by weak convexity of $c(z)$ and (28). The expression on the second row of
\[
\frac{\partial \Omega_{k}^1}{\partial k} \frac{1}{q_i(k)}
\]
is strictly positive for all $r < k$ because $c'(z) > 0$ and
\[
1 - \frac{K'_i(\tilde{p}_i)}{D'_i(\tilde{p}_i)} = 1 + \int_{q_i(k)}^{\tilde{p}_i} c'(z) dF(z) = \frac{c'(q_i(k)-r) dF(r) - u''(q_i(k)-q_o)}{c'(x_i(k))F(x_i(k))} > 0.
\]

The marginal cost distortion converges to zero as $k$ becomes small:\n}\lim_{k \to 0} \Omega_{k}^1 = 0.\]
To see this, note that (35) implies $\lim_{k \to 0} q_i'(k) = 0$ and $\lim_{k \to 0}(1 - \frac{K'_i(\tilde{p}_i)}{D'_i(\tilde{p}_i)}) q_i'(k) = 1$. Hence,
symmetry and the assumption that

\[ \begin{align*}
A.8 \text{ Regularity assumptions} & \\
\text{Social optimum} & \text{A constrained social optimum } k^b_I \text{ exists by continuity of } W_{nat}(k) \text{ in } k \text{ and compactness of the domain: } k \in [0, \bar{r}]. \text{ Any socially optimal capacity reserve satisfies } k^b_I < \bar{k}_I \text{ because a capacity reserve above } \bar{k}_I \text{ would distort consumption and investment without providing any additional insurance benefits. I next show that } k^b_I > 0. W_{nat}(k) = W_I(k) - \Omega_I(k) \text{ implies} \\
W'_{nat}(k) & = -q_i \int_0^{\bar{r}} B'(p_I(r, k) - \phi) \frac{\partial p_I(r, k)}{\partial k} dF_I(r) - (\psi_I(k) + \omega_I(k)) q_i'(k) \\
\text{(30)} & \text{by the definition (29) of } \omega_I(k). \text{ } W'_I(k) > 0 \text{ for all } k > 0, \text{ but sufficiently close to zero by assumption (8), and because } \lim_{k \to 0} \Omega'_I(k) = 0, \text{ it follows that } W'_I(k) > 0 \text{ for all } k > 0, \text{ but sufficiently close to zero. Hence, a symmetric capacity reserve is a social optimum only if } W'_I(k^b_I) = 0, \text{ which is approximately equal to (21) for } \varepsilon \text{ close to zero.} \\
\text{Comparative statics} & \text{Strict quasi-concavity of } W_I(k), \text{ and } W'_I(k^b_I) = W'_I(k^b_I) + \Omega'_I(k^b_I) = \Omega'_I(k^b_I) > 0 \text{ imply } k^b_I > k^b_I. \\
\text{Implementation} & \text{The expected welfare in country } i \text{ equals} \\
W_{nat}(k_i, k^b_I) & = W_{nat}(\min\{k_i; k^b_I\}) - \delta(k_i - \min\{k_i; k^b_I\}) \\
\text{for } k_j = k^b_I. \text{ Hence,} \\
W_{nat}(k^b_I, k^b_I) - W_{nat}(k_i, k^b_I) & = W_{nat}(k^b_I) - W_{nat}(k_i) \geq 0 \\
\text{for all } k_i < k^b_I \text{ because } k^b_I \text{ maximizes } W_{nat}(k). \text{ Furthermore,} \\
W_{nat}(k^b_I, k^b_I) - W_{nat}(k_i, k^b_I) & = \delta(k_i - k^b_I) > 0 \\
\text{for } k_i > k^b_I. \text{ Hence, } k_i = k^b_I \text{ is a best-reply to } k_j = k^b_I. \text{ The equilibrium is pay-off dominant by symmetry and the assumption that } k^b_I \text{ is constrained socially optimal under a national supply constraint.}
\end{align*} \]

A.8 Regularity assumptions

This appendix derives sufficient conditions for assumption (9) to hold. Let \( A_M(k) = \int_{k(1-\varepsilon)}^{\rho_M(k, \phi)} B(p_M(r, k) - \phi)dF_M(r) \) be the expected shadow cost of operating losses. It is characterized by

\[ A_M(k) = \int_{k(1-\varepsilon)}^{\rho_M(k, \phi)} B(\hat{p}(q_M(k) - r) - \phi)dF_M(r) + B(M(k) - \phi)F_M(k(1-\varepsilon)) \]

for \( k < \bar{k}_M \) and by \( A_M(k) = 0 \) for \( k \geq \bar{k}_M \). The shadow cost is continuous because \( \rho_M(\bar{k}_M, \phi) = \bar{k}_M(1-\varepsilon) \) and \( B(0) = 0 \) imply \( \lim_{k \to \bar{k}_M} A_M(k) = 0 \). Under the assumption that \( k < \bar{k}_M \),

\[ A'_M(k) = q'_M(k) \int_{k(1-\varepsilon)}^{\rho_M(k, \phi)} B'(\hat{p}(q_M(k) - r) - \phi)\hat{p}'(q_M(k) - r)dF_M(r) + B'(\hat{p}_M(k) - \phi)F_M(k(1-\varepsilon)) \hat{p}_M(k) \]
Using the following integration by parts
\[
\int_{k(1-\varepsilon)}^{\rho_M(k,\phi)} B'(\hat{p}(q_M(k) - r) - \phi)\hat{p}'(q_M(k) - r)dF_M(r)
\]
\[
= \int_{k(1-\varepsilon)}^{\rho_M(k,\phi)} B(\hat{p}(q_M(k) - r) - \phi)f'_M(r) + B(\hat{P}_M(k) - \phi)f_M(k(1-\varepsilon))
\]
I obtain
\[
A'_M(k) = B'(\hat{P}_M(k) - \phi)F_M(k(1-\varepsilon))\hat{P}'_M(k) + q'_M(k)B(\hat{P}_M(k) - \phi)f_M(k(1-\varepsilon))
\] 
Observe that \(A'_M(k) = 0\) by the additional assumption that \(B'(0) = 0\). Hence, \(A_M(k)\) is continuously differentiable in \(k\). Furthermore, the term on the second row converges to zero as \(\varepsilon \to 0\) because \(\rho_M(k,\phi) \in (k(1-\varepsilon),k)\). Hence, the marginal benefit of hedging price spikes, 

\[-q_nA'_M(k),\] 

can be written approximately as (7) for \(\varepsilon\) close to zero.

The next task is to evaluate \(\lim_{k \to 0} W'_M(k) = -q_n \lim_{k \to 0} A'_M(k) - \lim_{k \to 0} \psi_M(k)q'_M(k)\). To this end, I make the following assumptions beyond those specified in the main body of the text: \(f_M(\cdot)\) is bounded and twice continuously differentiable, with \(f'_M(\cdot)\) and \(f''_M(\cdot)\) bounded for \(M = N, I\). Furthermore,
\[
f'_M(k) \geq 0 \forall k \geq 0, \lim_{k \to 0} \frac{F_M(k)}{f_M(k(1-\varepsilon))} = 0, \lim_{\hat{p} \to \infty} B(\hat{p} - \phi) = \infty, \lim_{\hat{p} \to \infty} \frac{B'(\hat{p} - \phi)}{B(\hat{p} - \phi)} > 0.
\]

Rewrite \(-A'_M(k)\) as:
\[
-A'_M(k) = \frac{q'_M(k)}{F_M(k)}\frac{F_M(k)}{\hat{P}_M(k) - \phi)\frac{F_M(k)}{f_M(k(1-\varepsilon))} - f_M(\rho_M(k,\phi))\frac{F_M(k)}{f_M(k(1-\varepsilon))}} - f_M(\rho_M(k,\phi))\frac{F_M(k)}{f_M(k(1-\varepsilon))} + q'_M(k)\hat{P}'_M(k) [B(\hat{P}_M(k) - \phi) - B(\hat{p}(q_M(k) - r) - \phi)f'_M(r)dr.
\]
The term on the third row of (33) is non-negative for all \(k\) sufficiently close to zero by the assumption that \(f_M(r) \geq 0\) for all \(r\) sufficiently close to zero. Consider next the terms inside the square brackets on the second row of (33). The second term is negative, but vanishes in the limit as \(k \to 0\) by the assumption that \(f_M(r)\) is bounded and \(\lim_{k \to 0} \frac{F_M(k)}{f_M(k(1-\varepsilon))} = 0\). To evaluate the first term inside the square brackets, observe that
\[
\frac{F_M(k(1-\varepsilon))}{-D'_M(F_M(k))} = \int_k^\infty c'(q_M(k) - r)dF_M(r) + \int_k^\infty \hat{p}(q_M(k) - r)f'_M(r)dr - u''(q_M(k) - q_n)
\]
\[+ \hat{P}_M(k)f_M(k(1-\varepsilon)) - c(x_M(k))f_M(k)
\] 
after an integration by parts. Multiplying this expression by \(F_M(k)/f_M(k(1-\varepsilon))\) and substi-
tuting in the optimality condition (23) for market-based investment yields
\[
\frac{F_M(k(1 - \varepsilon))}{-D_M'(P_M(k))} \frac{F_M(k)}{f_M(k(1 - \varepsilon))} = \left[ \int_k^r q_M(k) - r) dF_M(r) - u''(q_M(k) - q_n) \right] \frac{F_M(k)}{f_M(k(1 - \varepsilon))} + \int_k^r \frac{\hat{p}(q_M(k) - r) - c(x_M(k)) f_M(r) dr}{f_M(k(1 - \varepsilon))} + \int_k^r \left( \hat{P}_M(k) - \hat{p}(q_M(k) - r) \right) dF_M(r) + \delta
\]

after simplification. The term on the first row is positive, the term on the second row is non-negative for \( k \) sufficiently close to zero by the assumption that \( f_M'(r) \geq 0 \) for all \( r \) sufficiently close to zero. The first term on the third row is also positive. It then follows that
\[
\frac{F_M(k(1 - \varepsilon))}{-D_M'(P_M(k))} \frac{F_M(k)}{f_M(k(1 - \varepsilon))} > \delta
\]

for all \( k \) sufficiently close to zero. By the additional assumption that \( \lim_{\hat{p} \to \infty} B'(\hat{\hat{\rho}} - \phi) > 0 \), it follows that the term inside the square brackets on the second row of (33) is strictly positive and bounded away from zero for all \( k \) sufficiently close to zero. Finally, evaluate the terms on the first row of (33). From (25), it follows directly that
\[
\lim_{k \to 0} \frac{q'_M(k)}{F_M(k)} = \frac{c(K_M^\infty)}{\int_0^\infty c'(D_M^\infty + q_n - r) dF_M(r) - u''(D_M^\infty)} > 0
\]

and bounded from above. By way of the optimality condition (23) for market-based investment, it follows that \( \hat{P}_M(k) > \frac{\delta}{F_M(k)} + c(x_M(k)) \). Monotonicity of \( B \) then implies
\[
\lim_{k \to 0} B(\hat{P}_M(k) - \phi) F_M(k) \geq \lim_{k \to 0} B\left( \frac{\delta}{F_M(k)} + c(x_M(k)) - \phi \right) F_M(k)
\]
\[
= \delta \lim_{k \to 0} B'\left( \frac{\delta}{F_M(k)} + c(x_M(k)) - \phi \right) > 0,
\]

where I have used L'Hôpital’s rule to get the second result. Hence,
\[
\lim_{k \to 0} B(\hat{P}_M(k) - \phi) f_M(k(1 - \varepsilon)) = \lim_{k \to 0} B(\hat{P}_M(k) - \phi) F_M(k) \frac{f_M(k(1 - \varepsilon))}{F_M(k)} = \infty.
\]

To conclude, \( \lim_{k \to 0} W_M'(k) = -q_n \lim_{k \to 0} A'_M(k) = \infty \) under the additional assumptions (32).

The final task is to evaluate \( W_M''(k) \). In doing so, I will make the following assumptions additional to (32):
\[
\begin{align*}
u''(q_n) & \geq 0, \\
36''(k) & \geq 0 \forall k \geq 0, \lim_{k \to 0} \frac{F_M(k(1 - \varepsilon))}{f_M(k(1 - \varepsilon))} < \infty, \\
\lim_{\hat{\rho} \to \infty} B'(\hat{\rho} - \phi) & = \infty, \lim_{\hat{\rho} \to \infty} \frac{B''(\hat{\rho} - \phi)}{B'(\hat{\rho} - \phi)} > 0 \text{ and sufficiently large.}
\end{align*}
\]

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Straightforward differentiation yields

$$A_M''(k) = B'(\tilde{P}_M(k) - \phi) \frac{F_M(k(1 - \varepsilon))}{D_M'(\tilde{P}_M(k))} + B'(\tilde{P}_M(k) - \phi) \frac{q_M''(k)F_M(k) + q_M''(k)}{D_M'(\tilde{P}_M(k))}$$

$$+ q_M'(k)B'(\tilde{P}_M(k) - \phi) \frac{dF_M(k(1 - \varepsilon))}{dk} + f_M(k(1 - \varepsilon))P_M'(k)$$

$$+ B(\tilde{P}_M(k) - \phi)[q_M''(k)f_M(k(1 - \varepsilon)) + (q_M'(k))^2f_M'(k(1 - \varepsilon))]$$

$$+ \int_{k(1-\varepsilon)}^{p_{M(k)}(k,\phi)} B(\tilde{p}(q_M(k) - r) - \phi)[q_M''(k)f_M'(r) + (q_M'(k))^2f_M''(r)]dr.$$

after using an integration by parts similar to the above and collecting terms.

Next, substitute

$$\frac{1}{q_M''(k)} \left[ \frac{d}{dk} \frac{F_M(k(1 - \varepsilon))}{D_M'(\tilde{P}_M(k))} + f_M(k(1 - \varepsilon))P_M'(k) \right]$$

$$= u''(q_M(k) - q_0) + c'(x_M(k))f_M(k) + \int_{k(1-\varepsilon)}^{k} [\tilde{P}_M(k) - \tilde{p}(q_M(k) - r)]f_M''(r)dr$$

$$- \int_{k(1-\varepsilon)}^{k} c''(q_M(k) - r) dF_M(r) - [\tilde{P}_M(k) - c(x_M(k))]f_M'(k)$$

into $A_M''(k)$ above to get

$$A_M''(k) = B'(\tilde{P}_M(k) - \phi)F_M(k(1 - \varepsilon))(\tilde{P}_M'(k))^2 \left\{ \frac{B''(\tilde{P}_M(k) - \phi)}{B'(\tilde{P}_M(k) - \phi)} + \frac{q_M''(k)}{q_M'(k)P_M'(k)} \right\}$$

$$- \left( \frac{D_M'(\tilde{P}_M(k)))^2}{F_M(k(1 - \varepsilon))} (\tilde{P}_M(k) - c(x_M(k))) \right) \left[ \int_{k(1-\varepsilon)}^{k} c''(q_M(k) - r) dF_M(r) - f_M'(k) \right]$$

$$+ (q_M'(k))^2B'(\tilde{P}_M(k) - \phi)u''(q_M(k) - q_0)$$

$$+ c'(x_M(k))f_M(k) + \int_{k(1-\varepsilon)}^{k} (\tilde{P}_M(k) - \tilde{p}(q_M(k) - r))f_M''(r)dr$$

$$+ B(\tilde{P}_M(k) - \phi)[q_M''(k)f_M(k(1 - \varepsilon)) + (q_M'(k))^2f_M'(k(1 - \varepsilon))]$$

$$+ \int_{k(1-\varepsilon)}^{p_{M(k)}(k,\phi)} B(\tilde{p}(q_M(k) - r) - \phi)[q_M''(k)f_M'(r) + (q_M'(k))^2f_M''(r)]dr.$$

Consider first the properties of $q_M''(k)$. By differentiating (25) and rearranging terms:

$$\frac{q_M''(k)F_M(k)}{q_M'(k)f_M(k)} = 1 - \frac{q_M'(k)F_M(k)}{F_M(k)f_M(k)} (1 - q_M'(k))c''(x_M(k)) \int_{k(1-\varepsilon)}^{k} c'(q_M(k) - r) dF_M(r) - \frac{u''(q_M(k) - q_0)}{c'(x_M(k))}$$

The right-hand side of this expression converges to 1 by the assumptions that $f_M'(k) \geq 0 \forall k$ sufficiently close to zero and $\lim_{k \to 0} \frac{F_M(k)}{f_M(k(1 - \varepsilon))} = 0$. Hence, $q_M''(k) > 0$ for all $k$ sufficiently close to zero. This result plus the assumptions that $u'' \geq 0$, $f_M'(k) \geq 0$ and $f_M''(k) \geq 0$ for all $k$ sufficiently close to zero imply that the final four rows of $A_M''(k)$ above all are positive for $k$ sufficiently close to zero. The next expression to evaluate is the one in curly brackets in $A_M''(k)$.\[36\]
By expanding:

\[
\frac{q''_M(k)}{q'_M(k)P'_M(k)} = \frac{q''_M(k) F_M(k) D'_M(\bar{P}_M(k)) f_M(k(1-\varepsilon)) F_M(k) F_M(k(1-\varepsilon)) f_M(k)}{q'_M(k) F_M(k(1-\varepsilon)) F_M(k) q'_M(k) f_M(k(1-\varepsilon)) F_M(k)}. 
\]

Term \( i \) converges to 1, term \( ii \) satisfies

\[
\lim_{k \to 0} D'_M(\bar{P}_M(k)) \frac{f_M(k(1-\varepsilon))}{F_M(k(1-\varepsilon))} \in [-\frac{1}{\delta}, 0) \quad (37)
\]

by (34), term \( iii \) is bounded from above by (35), and term \( iv \) is bounded from above by assumption (36). Hence, \( \lim_{k \to 0} \frac{q''_M(k)}{q'_M(k)P'_M(k)} \) is bounded from below and dominated by \( \frac{B''(\bar{p}_M(k)-\phi)}{B'(\bar{p}_M(k)-\phi)} \) for \( k \) sufficiently close to zero if \( \lim_{\bar{p}_M \to -\infty} \frac{B''(\bar{p}_M-\phi)}{B'(\bar{p}_M-\phi)} \) is sufficiently large. By expanding the next expression, I obtain

\[
\lim_{k \to 0} \frac{(D'_M(\bar{P}_M(k)))^2}{F_M(k(1-\varepsilon))} (\bar{P}_M(k) - c(x_M(k))) \\
\leq \lim_{k \to 0} \frac{(\bar{P}_M(k) - c(x_M(k)))F_M(k(1-\varepsilon))}{\delta^2} \lim_{k \to 0} \frac{F_M(k)}{f_M(k(1-\varepsilon))}
\]

by (37). Using again the optimality condition (23) for market-based investment,

\[
(\bar{P}_M(k) - c(x_M(k)))F_M(k(1-\varepsilon)) \\
= \delta - \int_{k(1-\varepsilon)}^k [\bar{p}(q_M(k) - r) - c(x_M(k))]dF_M(r) < \delta,
\]

Hence,

\[
\lim_{k \to 0} \frac{(D'_M(\bar{P}_M(k)))^2}{F_M(k(1-\varepsilon))} (\bar{P}_M(k) - c(x_M(k))) = 0.
\]

It follows that the term in curly brackets of \( A''_M(k) \) is strictly positive and bounded away from zero for all \( k \) sufficiently close to zero. By using the optimality condition (23) for market-based investment one final time, I obtain

\[
- \frac{F_M(k)}{q'_M(k) f_M(k(1-\varepsilon))} \bar{P}'_M(k) \\
= \delta + \frac{F_M(k)}{f_M(k(1-\varepsilon))} [\int_k^\infty c'(q_M - r)dF_M(r) - u''(q_M - q_n)]dF_M(r) \\
+ \int_{k(1-\varepsilon)}^k \frac{F_M(k)}{f_M(k(1-\varepsilon))} (\bar{p}(q_M - r) - c(x_M)) f'_M(r) + (\bar{P}_M(k) - \bar{p}(q_M - r)) f_M(r) dr
\]

so that

\[
\lim_{k \to 0} \left( \frac{F_M(k(1-\varepsilon))}{f_M(k(1-\varepsilon))} \bar{P}'_M(k) \right)^2 \geq \frac{(\delta c'(K_M))^2}{\int_0^\infty c'(D_M^\infty + q_n - r)dF_M(r) - u''(D_M^\infty)^2} > 0.
\]

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Hence,

\[
\lim_{k \to 0} B'(\tilde{P}_M(k) - \phi)F_M(\tilde{P}_M(k)(1 - \varepsilon))(\tilde{P}_M'(k))^2 \\
\geq \lim_{k \to 0} B'(\tilde{P}_M(k) - \phi)F_M(k)\frac{(\delta e'(K_{\tilde{M}}))}{\left[\int_k^p e'(D_{\tilde{M}}^N + q_n - r)dF_M(r) - u''(D_{\tilde{M}}^N)^2\right]}^2
\]

The proof that \(\lim_{k \to 0} B'(\tilde{P}_M(k) - \phi)F_M(k) > 0\) and bounded away from zero is identical to the proof that \(\lim_{k \to 0} B(\tilde{P}_M(k) - \phi)F_M(k) > 0\) and bounded from zero and therefore omitted.

To summarize these findings, assumptions (36) and (37) jointly imply that \(\lim_{k \to 0} A_M''(k) \to \infty\) and \(q_M''(k) > 0\) for all \(k\) sufficiently close to zero. It then follows that

\[
W_M''(k) = -q_n A_M''(k) - \psi_M'(k)q_M'(k) - \psi_M'(k)q_M''(k) < 0
\]

for all \(k\) sufficiently close to zero. By continuity, therefore, \(W_N(k)\) is strictly concave in the domain \((0, 2 \max\{\tilde{k}_N; \tilde{k}_I\})\), and \(W_I(k)\) is strictly concave in the domain \((0, \max\{\tilde{k}_N; \tilde{k}_I\})\) unless \(\tilde{k}_N\) and \(\tilde{k}_I\) are large, in which case the second-derivatives of \(W_N(k)\) and \(W_I(k)\) are indeterminate for large enough values of \(k\).